

Design of a New Synthetic Acceptance Sampling Plan

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Abstract: In this paper, we propose a new synthetic sampling plan assuming that the quality characteristic follows the normal distribution with known and unknown standard deviation. The proposed plan is given and the operating characteristic (OC) function is derived to measure the performance of the proposed sampling plan for some fixed parameters. The parameters of the proposed sampling plan are determined using non-linear optimization solution. A real example is added to explain the use of the proposed plan by industry.

Keywords: sampling plans; inspection; quality control; synthetic; producer's risk; consumer's risk

1. Introduction

Every industry in the world is keen to attract customers through the high quality of its products [1]. The assurance of high-quality products can be achieved by implementing some inspection criteria for accepting or rejecting at every level of production from raw material to final finished product [2]. The inspection is done through some specific rules known as a sampling plan. The sampling plans are well-defined rules for the inspection of the raw material to the finished product. During the inspection of a product, the cost of inspection is directly related to the sample size. The inspection per unit of sample increases as the sample size increases. The inspection cost of a sampling plan is minimized through a well-defined sampling plan. The sampling plan parameters are selected through non-linear optimization methods by minimizing the sample size [3]. The producer's risk and consumer's risk are associated with every sampling plan. The chance of rejecting a good lot is called the producer's risk and the chance of accepting a bad lot is called the consumer's risk. A well-defined sampling plan is also helpful in minimizing the two risks. Thus, inspection that is done through a well-defined sampling plan guarantees a minimum cost and a correct decision about the submitted product. A variety of acceptance sampling plans have been used in industry for the inspection of products—for example, applications of a sampling plan in a hospital pharmacy [4], in pulp manufacturing [5], and in food control [6].

Two types of sampling plans have been widely used in industry: attribute sampling and variable sampling plans. The attribute sampling plan is usually used when data are obtained through the counting process and a variable sampling plan is used with measurement data. Both sampling plans have been widely used in industry for the inspection of the finished product (for example, see [7–10]). According to Li et al. [11], in some practical situations it may be necessary to apply both sampling schemes simultaneously; that is called mixed sampling for the inspection of the finished product and can save money and time during inspection. Several authors focused on the design of a mixed sampling plan. Suresh and Devaarul [12] proposed a mixed sampling plan with chain sampling as an attribute plan. Aslam et al. [13] proposed a mixed repetitive sampling plan using a process capability index. Butt et al. [14] proposed the plan for the process yield. Balamurali et al. [15] designed a mixed variable plan using the process capability index. Balamurali [3] proposed a chain sampling plan for the

process capability index. Ramya and Devaarul [16] proposed the variable sampling plan for the Lomax distribution. More information about mixed sampling can be found in Suresh and Devaarul [12], Aslam et al. [17], and Aslam et al. [18].

Sometimes, it may be difficult to make a final decision about a submitted lot of product using information about the main quality of interest. In this situation, other variables called auxiliary variables are investigated that are correlated to the main variable of study. For example, in the steel industry, the tensile strength, the variable of interest, is difficult to measure while the hardness, the auxiliary variable, which is correlated to tensile strength, is easy to measure—see Aslam et al. [19]. Similarly, the single-strand break factor, the variable of interest, is correlated to the weight of textile fibers, the auxiliary variable; see Abdul Abdul Haq and Khoo [20]. As mentioned by Riaz [21], the study of the variable of interest with the auxiliary information may improve the precision of the decision. Aslam et al. [19] and Aslam et al. [22] proposed acceptance sampling plans using auxiliary information. Recently, Haq and Khoo [20] proposed a new synthetic control chart that uses statistical information on both the main variable and the auxiliary variable. They compared the performance of a new statistic with a classical statistic.

The sampling plan based on the synthetic statistic is the integration of the traditional single sampling plan and the plan based on conforming run length (CRL). According to Haq and Khoo [20], “In 100% inspection, the CRL is defined as the total number of inspected units between the two consecutive nonconforming units—including the ending nonconforming unit.” By exploring the literature, we found that, to the best of our knowledge, there has been no work done on designing a sampling plan using a synthetic statistic. In this paper, we present a sampling plan for the inspection of the finished product. We expect that the proposed sampling plan based on the synthetic statistic will perform better than the traditional single sampling plan. The complete structure of the proposed plan is given for normal distribution with known and unknown standard deviation. A real example is used for illustration purposes.

2. Design of the Proposed Sampling Plan

Suppose that $Y_1, Y_2, \dots, Y_n$ is an independent random sample of size n from lot quality characteristic Y that follows the normal distribution with mean $\hat{\mu}_Y = \sum_{i=1}^n Y_i/n$ and variance σ_Y^2 . Let U denote the upper specification limit (USL). The proposed sampling plan is stated as follows:

2.1. Plan When the Standard Deviation Is Known

Step 1: Select a random sample of size n from a lot at the inspection point and calculate $\hat{\mu}_Y$.

Step 2: The lot will be accepted and considered conforming if $v = (U - \hat{\mu}_Y)/\sigma_Y$ is larger than an acceptance number c_1 ; if smaller than an acceptance number c_2 , the lot will be rejected and declared as non-conforming. If $c_2 \leq v \leq c_1$, go to the next step.

Step 3: Count the number of inspected units between the current and the last non-conforming units. The number is taken as CRL, say d_1 .

Step 4: Accept the lot if $CRL \geq d$, where d denotes the non-conforming items; otherwise reject the lot.

The quality of interest is assumed to follow the normal distribution with known or unknown population standard deviation. Haq and Khoo [20] stated that CRL is a random variable. The distribution function of CRL, $F_p(CRL)$, is given by

$$F_p(CRL) = 1 - (1 - p)^{CRL}, \quad CRL = 1, 2, \dots \quad (1)$$

The CRL is normally distributed as

$$CRL \sim N\left(\frac{1}{p}, \frac{1-p}{p^2}\right), \quad (2)$$

where p is the proportion of non-conforming units.

The operating characteristic (OC) function of the proposed sampling plan is given by

$$P_a(p) = P\{v > c_1\} + P\{c_2 < v < c_1\}P\{CRL \geq d\}. \quad (3)$$

For abbreviation, $A_1 = P\{v > c_1\}$, $A_2 = P\{c_2 < v < c_1\}$ and $A_3 = \{CRL \geq d\}$.

So, the OC function in Equation (3) can be written as

$$P_a(p) = A_1 + A_2A_3. \quad (4)$$

Now, the OC function will be driven assuming the quality of interest Y follows the normal distribution with known population standard deviation.

A lot of product will be accepted if $P\{v > c_1\}$.

Let

$$A_1 = P\{(U - \hat{\mu}_Y)/\sigma_Y > c_1\}. \quad (5)$$

Now, according to Duncan [23],

$$c_1\sigma_Y + \hat{\mu}_Y \sim N(\mu_Y + c_1\sigma_Y, \sigma_Y^2/n). \quad (6)$$

Thus,

$$A_1 = P\left\{\frac{c_1\sigma_Y + \hat{\mu}_Y - (\mu_Y + c_1\sigma_Y)}{\sigma_Y/\sqrt{n}} < \frac{U - (\mu_Y + c_1\sigma_Y)}{\sigma_Y/\sqrt{n}}\right\}. \quad (7)$$

Let $z = \frac{\hat{\mu}_Y - \mu_Y}{\sigma_Y}$, where z is a standard normal random variable:

$$A_1 = P\left\{z < \frac{U - \mu_Y - c_1\sigma_Y}{\sigma_Y/\sqrt{n}}\right\}. \quad (8)$$

Let $z_p = (U - \mu_Y)/\sigma_Y/\sqrt{n}$, where z_p is the p -th percentile under the standard normal distribution.

$$A_1 = P\{z < (z_p - c_1)\sqrt{n}\} = \Phi((z_p - c_1)\sqrt{n}) \quad (9)$$

Similarly,

$$A_2 = P\{c_2 < v < c_1\} = P(v > c_2) - P(v > c_1) = \Phi((z_p - c_2)\sqrt{n}) - \Phi((z_p - c_1)\sqrt{n}). \quad (10)$$

Now, we derive this probability for A_3 as follows:

$$A_3 = P\{CRL \geq d\} = 1 - P\{CRL < d\} = 1 - 1 + (1 - p)^{CRL} = (1 - p)^{CRL}. \quad (11)$$

Finally, the OC function can be rewritten as

$$P_a(p) = \Phi((z_p - c_1)\sqrt{n}) + \{\Phi((z_p - c_2)\sqrt{n}) - \Phi((z_p - c_1)\sqrt{n})\} \times (1 - p)^{CRL}. \quad (12)$$

Let α be the producer's risk and β be the consumer's risk. The producer desires that the lot acceptance probability be larger than the confidence level, say $1 - \alpha$, at an acceptable quality level (AQL), while the consumer would like the lot acceptance probability for the bad lot to be smaller than β at limiting quality level (LQL). Let p_1 and p_2 denote AQL and LQL, respectively. The plan parameters of the proposed plan will be determined using non-linear optimization:

$$\text{Minimize } n \quad (13a)$$

Subject to

$$P_a(p_1) = \Phi((z_{p_1} - c_1)\sqrt{n}) + \{\Phi((z_{p_1} - c_2)\sqrt{n}) - \Phi((z_{p_1} - c_1)\sqrt{n})\} \times (1 - p_1)^{CRL} \geq 1 - \alpha \quad (13b)$$

$$P_a(p_2) = \Phi((z_{p_2} - c_1)\sqrt{n}) + \{\Phi((z_{p_2} - c_2)\sqrt{n}) - \Phi((z_{p_2} - c_1)\sqrt{n})\} \times (1 - p_2)^{CRL} \leq \beta \quad (13c)$$

The plan parameters were determined through the grid search method using R. There may exist multiple combinations of plan parameters that satisfy the constraints given in Equations (13). The plan parameters that have smaller values of sample size n were selected for the proposed plan.

The plan parameters of the proposed sampling plan when the population standard deviation is known are reported in Tables 1–6. Tables 1–3 are shown when $\alpha = 0.10$; $\beta = 0.10$ and $d = 3, 5$ and 10 . Tables 4–6 apply when $\alpha = 0.05$; $\beta = 0.05$ and $d = 3, 5$ and 10 .

Table 1. The plan parameters when $\alpha = 0.10$; $\beta = 0.10$ and $d_1 = 3$.

p_1	p_2	n	c_1	c_2
0.001	0.002	149	3.4794	2.9843
	0.003	59	3.4656	2.9172
	0.004	39	3.6467	2.8750
	0.006	22	3.9711	2.7909
	0.008	16	3.9849	2.7665
	0.010	14	3.9729	2.7304
0.0025	0.005	126	3.1160	2.6908
	0.010	29	3.3875	2.5655
	0.015	17	3.5652	2.4867
	0.02	13	3.7411	2.4192
	0.025	11	3.5434	2.3765
0.005	0.010	108	3.1973	2.4497
	0.015	42	3.8170	2.3767
	0.02	25	3.1853	2.3147
	0.03	15	3.4527	2.2232
	0.04	11	3.8465	2.1485
	0.05	9	3.7092	2.0973
0.01	0.02	91	3.9526	2.1913
	0.03	34	3.3476	2.1028
	0.04	22	3.4104	2.0251
	0.05	16	3.4434	1.9832
	0.10	7	3.5877	1.7948
0.03	0.06	66	3.7805	1.7163
	0.09	25	3.5193	1.6044
	0.12	15	3.9399	1.5323
	0.15	10	3.3894	1.4609
	0.3	4	3.4160	1.2080
0.05	0.10	51	2.4917	1.4637
	0.15	19	3.4008	1.3423
	0.2	11	3.0401	1.2516
	0.25	8	3.9895	1.1674
	0.50	4	3.1974	0.7173

Table 2. The plan parameters when $\alpha = 0.10$; $\beta = 0.10$ and $d_1 = 5$.

p_1	p_2	n	c_1	c_2
0.001	0.002	151	3.3638	2.9841
	0.003	60	3.7240	2.9205
	0.004	40	3.8650	2.8610
	0.006	22	3.7215	2.7962
	0.008	17	3.7168	2.7429
	0.01	12	3.9341	2.7035
0.0025	0.005	126	3.6859	2.6923
	0.01	30	3.3694	2.5623
	0.015	18	3.8893	2.4770
	0.02	13	3.9230	2.4410
	0.025	10	3.9381	2.3863
0.005	0.01	110	3.2076	2.4493
	0.015	42	3.3055	2.3762
	0.02	26	3.6578	2.3116
	0.03	15	3.8236	2.2259
	0.04	10	3.7691	2.1604
	0.05	9	3.8312	2.0858
0.01	0.02	93	2.7234	2.1894
	0.03	35	3.7009	2.1060
	0.04	22	3.0012	2.0323
	0.05	15	3.5220	1.9866
	0.1	7	3.8554	1.8123
0.03	0.06	64	3.1131	1.7165
	0.09	23	3.5835	1.6088
	0.12	14	2.9151	1.5326
	0.15	10	1.4655	1.4598
	0.3	6	1.2389	1.1066
0.05	0.1	51	3.9700	1.4617
	0.15	18	2.4733	1.3395
	0.2	11	3.6075	1.2512
	0.25	8	3.0504	1.1416
	0.5	6	3.3012	0.7054

Table 3. The plan parameters when $\alpha = 0.10$; $\beta = 0.10$ and $d_1 = 10$.

p_1	p_2	n	c_1	c_2
0.001	0.002	155	3.9299	2.9855
	0.003	57	3.9924	2.9178
	0.004	40	3.8441	2.8771
	0.006	22	2.8000	2.7765
	0.008	15	3.8966	2.7442
	0.01	13	3.9976	2.7261
0.0025	0.005	129	3.5904	2.6924
	0.01	30	3.6197	2.5654
	0.015	18	3.7502	2.4766
	0.02	14	3.8565	2.4436
	0.025	11	2.4078	2.3604
0.005	0.01	111	3.1776	2.4529
	0.015	41	3.3770	2.3720
	0.02	26	3.4368	2.3179
	0.03	14	3.7240	2.2241
	0.04	11	3.6457	2.1551
	0.05	11	3.7843	2.1435

Table 3. Cont.

p_1	p_2	n	c_1	c_2
0.01	0.02	91	3.7666	2.1904
	0.03	34	3.3960	2.1032
	0.04	21	3.8900	2.0445
	0.05	16	3.6782	2.0048
0.03	0.1	11	1.7125	1.6840
	0.06	64	3.4675	1.7205
	0.09	24	2.8968	1.6181
	0.12	14	3.2757	1.5203
	0.15	11	3.8574	1.4644
	0.3	11	3.8097	1.2932
0.05	0.1	51	2.4851	1.4639
	0.15	18	3.6275	1.3396
	0.2	11	3.8537	1.2565
	0.25	11	3.0288	1.1701
	0.5	11	3.0181	1.1003

Table 4. The plan parameters when $\alpha = 0.05$; $\beta = 0.05$ and $d_1 = 3$.

p_1	p_2	n	c_1	c_2
0.001	0.002	246	3.4198	2.9845
	0.003	94	3.9443	2.9183
	0.004	59	3.9073	2.8757
	0.006	33	3.8889	2.8009
	0.008	26	3.8108	2.7363
	0.01	20	3.8911	2.6958
0.0025	0.005	209	3.1764	2.6898
	0.01	48	3.7530	2.5660
	0.015	28	3.5286	2.4824
	0.02	20	3.6565	2.4335
0.005	0.025	17	2.3806	2.3588
	0.01	182	3.3640	2.4511
	0.015	68	3.0169	2.3706
	0.02	43	3.0483	2.3093
	0.03	23	3.7964	2.2268
	0.04	17	3.5539	2.1648
0.01	0.05	13	3.9996	2.1168
	0.02	149	3.4811	2.1909
	0.03	56	3.2557	2.1026
	0.04	34	3.2229	2.0347
	0.05	25	3.9023	1.9816
	0.1	11	1.8231	1.8222
0.03	0.06	105	2.2099	1.7189
	0.09	39	3.7363	1.6065
	0.12	22	2.6545	1.5285
	0.15	16	3.5869	1.4569
	0.3	6	3.7337	1.1991
0.05	0.1	86	3.0352	1.4622
	0.15	31	3.8744	1.3461
	0.2	17	3.2382	1.2445
	0.25	12	3.0924	1.1606
	0.5	5	3.0085	0.7865

Table 5. The plan parameters when $\alpha = 0.05$; $\beta = 0.05$ and $d_1 = 5$.

p_1	p_2	n	c_1	c_2
0.001	0.002	255	3.6970	2.9822
	0.003	98	3.4399	2.9157
	0.004	60	3.8186	2.8705
	0.006	34	3.8556	2.7987
	0.008	25	3.7824	2.7450
	0.01	21	3.7976	2.6856
0.0025	0.005	205	3.8492	2.6909
	0.01	50	3.3888	2.5714
	0.015	27	3.7268	2.4869
	0.02	20	3.9461	2.4335
	0.025	17	3.6876	2.3707
0.005	0.01	177	3.4134	2.4501
	0.015	69	3.9088	2.3771
	0.02	42	3.3285	2.3109
	0.03	23	3.7813	2.2288
	0.04	17	3.4296	2.1601
	0.05	14	3.4711	2.1121
0.01	0.02	150	3.1164	2.1902
	0.03	57	3.7079	2.1052
	0.04	34	3.1515	2.0345
	0.05	24	3.4513	1.9841
	0.1	11	3.4608	1.7974
0.03	0.06	103	2.9333	1.7177
	0.09	39	3.6868	1.6049
	0.12	23	1.5333	1.5276
	0.15	16	3.4043	1.4619
	0.3	7	3.4607	1.2496
0.05	0.1	86	3.9006	1.4638
	0.15	30	2.6066	1.3374
	0.2	18	3.1795	1.2451
	0.25	12	2.9045	1.1554
	0.5	6	2.7580	0.7237

Table 6. The plan parameters when $\alpha = 0.05$; $\beta = 0.05$ and $d_1 = 10$.

p_1	p_2	n	c_1	c_2
0.001	0.002	247	3.7059	2.9842
	0.003	96	3.6896	2.9223
	0.004	58	3.6879	2.8696
	0.006	35	3.9449	2.8052
	0.008	25	2.7456	2.7453
	0.01	20	3.9529	2.7031
0.0025	0.005	206	3.7122	2.6918
	0.01	48	3.7121	2.5667
	0.015	30	3.4956	2.4778
	0.02	21	3.9908	2.4181
	0.025	16	3.7084	2.3739
0.005	0.01	178	3.9578	2.4499
	0.015	70	3.5863	2.3693
	0.02	41	3.4823	2.3166
	0.03	24	2.2205	2.2183
	0.04	17	3.6664	2.1543
	0.05	13	3.5100	2.1068

Table 6. Cont.

p_1	p_2	n	c_1	c_2
0.01	0.02	146	2.9641	2.1899
	0.03	56	3.8422	2.1011
	0.04	33	3.6253	2.0392
	0.05	25	3.8764	1.9772
	0.1	11	3.8363	1.7778
0.03	0.06	104	2.6316	1.7185
	0.09	38	3.1868	1.6112
	0.12	22	3.7694	1.5272
	0.15	17	3.0419	1.4531
	0.3	11	3.3751	1.0947
0.05	0.1	84	2.9680	1.4625
	0.15	30	3.5914	1.3393
	0.2	17	3.2897	1.2421
	0.25	12	2.8807	1.1583
	0.5	11	0.8640	0.6545

From Tables 1–6, we note the following trends in plan parameters:

1. For the same values of α and β , as d_1 increases from 3 to 10, the value of n increases, which means that a larger sample size is needed from the lot of the product when the total number of consecutive items increases between two consecutive non-conforming items.
2. We also note that for the same values of d_1 , the value of the sample size increases as α and β decrease, which means that as the producer's confidence level about the acceptance of a good lot increases and the consumer's risk decreases, they are willing to inspect a larger sample from the lot of the product.

2.2. Plan When the Standard Deviation Is Unknown

In this section, we derive the OC function of the proposed plan when the population standard deviation of the normal distribution is unknown but estimable as a sample standard deviation.

Step 1: Select a random sample of size n from an inspection point and calculate $\hat{\mu}_Y$.

Step 2: The lot will be accepted and considered conforming if $v_s = (U - \hat{\mu}_Y)/S$, where $S = \sqrt{\sum_{i=1}^n (X_i - \bar{X})^2 / (n-1)}$, is larger than c_1 ; if smaller than c_2 , the lot will be rejected and declared as non-conforming. If $c_2 \leq v_s \leq c_1$, go to the next step.

Step 3: Count the number of inspected units between the current and the last nonconforming units. The number is taken as CRL.

Step 4: Accept the lot if $CRL \geq d$; otherwise, reject the lot.

The OC function of the proposed plan when the population standard deviation is unknown is derived as follows:

$$P_a(p) = P\{v_s > c_1\} + P\{c_2 < v_s < c_1\}P\{CRL \geq d\}. \quad (14)$$

Letting $B_1 = P\{v_s > c_1\}$, $B_2 = P\{c_2 < v_s < c_1\}$ and $B_3 = \{CRL \geq d\}$.

The lot acceptance probability can be written as

$$P_a(p) = B_1 + B_2B_3, \quad (15)$$

where $B_1 = P\{(U - \hat{\mu}_Y)/S > c_1\}$,

So, $B_1 = P\{c_1S + \hat{\mu}_Y < U\}$. Now, according to Duncan [24],

$$c_1S + \hat{\mu}_Y \sim N(\mu_Y + c_1E(S), \text{Var}(\hat{\mu}_Y) + c_1^2\text{Var}(S)) \quad (16)$$

since $E(S) = c_4\sigma$ and $V(S) = \sigma^2(1 - c_4^2)$, where, $c_4 = [2/(n-1)]^{1/2}\Gamma(n/2)/\Gamma[n-1/2]$.

So, the normal distribution of $c_1S + \hat{\mu}_Y$ is given as $c_1S + \hat{\mu}_Y \sim N(\mu_Y + c_1c_4\sigma_Y, \sigma_Y^2/n + c_1^2\sigma_Y^2(1 - c_4^2))$:

$$B_1 = P\left\{ \frac{c_1\sigma_Y + \hat{\mu}_Y - (\mu_Y + c_1c_4\sigma_Y)}{\sigma_Y\sqrt{1/n + c_1^2(1 - c_4^2)}} < \frac{U - (\mu_Y + c_1c_4\sigma_Y)}{\sigma_Y\sqrt{1/n + c_1^2(1 - c_4^2)}} \right\}. \quad (17)$$

Let $z = \frac{c_1\sigma_Y + \hat{\mu}_Y - (\mu_Y + c_1c_4\sigma_Y)}{\sigma_Y\sqrt{1/n + c_1^2(1 - c_4^2)}}$, where z is a standard normal random variable:

$$B_1 = P\left\{ z < \frac{U - \mu_Y - c_1c_4\sigma_Y}{\sigma_Y\sqrt{1/n + c_1^2(1 - c_4^2)}} \right\}. \quad (18)$$

Let $(U - \mu_Y)/\sigma_Y = z_p$, where z_p is the p -th percentile under the standard normal distribution, then we have

$$B_1 = \Phi\left((z_p - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right). \quad (19)$$

Similarly, the other factors can be written as

$$\begin{aligned} B_2 &= P\{c_2 < v < c_1\} = P(v > c_2) - P(v > c_1) \\ &= \Phi\left((z_p - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_2^2(1 - c_4^2)}}\right) - \Phi\left((z_p - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right). \end{aligned} \quad (20)$$

Now, we derive this probability for B_3 as follows

$$B_3 = P\{CRL \geq d\} = 1 - P\{CRL < d\} = 1 - 1 + (1 - p)^{CRL} = (1 - p)^{CRL}. \quad (21)$$

Finally, the OC function can be rewritten as

$$\begin{aligned} P_a(p) &= \Phi\left((z_p - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right) + \left\{ \Phi\left((z_p - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_2^2(1 - c_4^2)}}\right) - \right. \\ &\quad \left. \Phi\left((z_p - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right) \right\} ((1 - p)^{CRL}). \end{aligned} \quad (22)$$

(Minimizing n .)

Subject to

$$\begin{aligned} P_a(p_1) &= \Phi\left((z_{p_1} - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right) + \left\{ \Phi\left((z_{p_1} - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_2^2(1 - c_4^2)}}\right) - \right. \\ &\quad \left. \Phi\left((z_{p_1} - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right) \right\} ((1 - p_1)^{CRL}) \geq 1 - \alpha \end{aligned} \quad (23)$$

$$\begin{aligned} P_a(p_2) &= \Phi\left((z_{p_2} - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right) + \left\{ \Phi\left((z_{p_2} - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_2^2(1 - c_4^2)}}\right) - \right. \\ &\quad \left. \Phi\left((z_{p_2} - c_1c_4)\sqrt{\frac{1}{\frac{1}{n} + c_1^2(1 - c_4^2)}}\right) \right\} ((1 - p_2)^{CRL}) < \beta. \end{aligned} \quad (24)$$

The plan parameters are determined through the grid search method. It is important to note that, using the above non-linear optimization solution, there may exist multiple combinations of plan

parameters that satisfy the constraints given in Equations (23) and (24). The plan parameters with smaller values of sample size n will be selected for the proposed plan.

The parameters of the proposed plan when the population standard deviation is unknown are shown in Tables 7–12.

Table 7. The plan parameters when $\alpha = 0.10$; $\beta = 0.10$ and $d_1 = 3$.

p_1	p_2	n	c_1	c_2
0.001	0.008	153	2.9792	2.9754
	0.01	122	2.9510	2.9277
0.0025	0.015	151	3.9044	2.7319
	0.02	106	3.9143	2.6785
0.005	0.025	84	3.9496	2.6318
	0.03	113	3.8947	2.4941
	0.04	77	3.9536	2.4373
0.01	0.05	60	2.3960	2.3950
	0.04	148	3.9160	2.3188
	0.05	103	3.8683	2.2736
0.03	0.1	43	2.1212	2.1093
	0.09	139	3.7827	1.9360
	0.12	80	3.8701	1.8689
	0.15	55	3.5204	1.8113
0.05	0.3	21	1.6229	1.6169
	0.15	99	3.5967	1.7045
	0.2	56	3.3451	1.6266
	0.25	38	3.2364	1.5625
	0.5	14	1.3384	1.3055

Table 8. The plan parameters when $\alpha = 0.10$; $\beta = 0.10$ and $d_1 = 5$.

p_1	p_2	n	c_1	c_2
0.001	0.008	151	2.9752	2.9701
	0.01	119	2.9468	2.9360
0.0025	0.015	149	3.8303	2.7309
	0.02	105	3.9921	2.6789
0.005	0.025	85	2.6362	2.6328
	0.03	111	3.8117	2.4938
	0.04	78	3.9747	2.4385
0.01	0.05	61	2.3968	2.3910
	0.04	148	3.3235	2.3180
	0.05	103	3.9378	2.2733
0.03	0.1	43	3.9939	2.1109
	0.09	140	3.9770	1.9352
	0.12	80	3.4821	1.8680
	0.15	55	3.3794	1.8108
0.05	0.3	21	1.6228	1.6172
	0.15	99	3.6847	1.7042
	0.2	56	3.9822	1.6253
	0.25	38	3.8038	1.5671
	0.5	14	1.3662	1.3353

Table 9. The plan parameters when $\alpha = 0.10$; $\beta = 0.10$ and $d_1 = 10$.

p_1	p_2	n	c_1	c_2
0.001	0.008	153	2.9792	2.9619
	0.01	118	2.9430	2.9382
0.0025	0.015	151	3.9947	2.7295
	0.02	108	3.9812	2.6723
	0.025	85	2.6500	2.6223
0.005	0.03	113	3.9430	2.4955
	0.04	78	3.9899	2.4336
	0.05	60	2.3948	2.3868
0.01	0.04	148	3.8961	2.3183
	0.05	104	3.6389	2.2731
	0.1	44	2.1334	2.0947
0.03	0.09	139	2.9529	1.9361
	0.12	80	3.4453	1.8684
	0.15	55	3.8115	1.8128
0.05	0.3	21	1.6131	1.6074
	0.15	99	3.0060	1.7038
	0.2	56	3.9726	1.6277
	0.25	38	3.8273	1.5637
	0.5	14	1.3177	1.3153

Table 10. The plan parameters when $\alpha = 0.05$; $\beta = 0.05$ and $d_1 = 3$.

p_1	p_2	n	c_1	c_2
0.001	0.006	131	3.0253	3.0253
	0.008	95	2.9752	2.9656
	0.01	72	2.9479	2.9429
0.0025	0.01	163	3.9527	2.8041
	0.015	92	2.7339	2.7337
	0.02	67	2.6762	2.6754
0.005	0.025	52	2.6549	2.6330
	0.02	124	3.7278	2.5722
	0.03	69	3.9674	2.4982
	0.04	48	2.4463	2.4367
	0.05	37	2.4016	2.3974
0.01	0.03	154	3.6499	2.3766
	0.04	91	3.6024	2.3221
	0.05	64	3.7575	2.2755
0.03	0.1	27	2.1415	2.1059
	0.09	85	3.8778	1.9386
	0.12	49	3.8849	1.8701
	0.15	34	3.9906	1.8202
0.05	0.3	13	1.6467	1.6274
	0.15	61	2.9675	1.7067
	0.2	35	3.9433	1.6332
	0.25	24	3.8601	1.5612
	0.5	9	1.3900	1.3796

Table 11. The plan parameters when $\alpha = 0.05$; $\beta = 0.05$ and $d_1 = 5$.

p_1	p_2	n	c_1	c_2
0.001	0.006	134	3.0210	3.0185
	0.008	93	2.9744	2.9712
	0.01	72	2.9494	2.9332
0.0025	0.01	164	3.8663	2.8040
	0.015	94	3.9907	2.7329
	0.02	66	2.6814	2.6751
	0.025	50	2.6487	2.6404
0.005	0.02	124	3.9426	2.5711
	0.03	69	2.5004	2.4900
	0.04	49	2.4509	2.4307
	0.05	38	2.3921	2.3907
0.01	0.03	154	3.7962	2.3770
	0.04	91	3.5615	2.3215
	0.05	64	3.9424	2.2756
	0.1	27	2.1368	2.1111
0.03	0.09	85	3.4555	1.9391
	0.12	49	3.9763	1.8716
	0.15	34	3.7792	1.8131
	0.3	13	1.6426	1.6363
0.05	0.15	61	3.3949	1.7081
	0.2	34	3.9277	1.6335
	0.25	24	3.6845	1.5605
	0.5	9	1.3449	1.3025

Table 12. The plan parameters when $\alpha = 0.05$; $\beta = 0.05$ and $d_1 = 10$.

p_1	p_2	n	c_1	c_2
0.001	0.006	132	3.0262	3.0120
	0.008	92	2.9814	2.9707
	0.01	72	2.9481	2.9467
0.0025	0.01	165	2.8045	2.8003
	0.015	93	2.7362	2.7267
	0.02	66	2.6876	2.6611
	0.025	51	2.6485	2.6192
0.005	0.02	125	3.9041	2.5699
	0.03	71	3.8810	2.4888
	0.04	50	2.4499	2.4475
	0.05	37	2.3963	2.3925
0.01	0.03	154	3.7925	2.3772
	0.04	91	3.5878	2.3196
	0.05	63	2.2770	2.2767
	0.1	26	2.1320	2.1235
0.03	0.09	85	3.3253	1.9381
	0.12	49	3.9748	1.8696
	0.15	34	1.8176	1.8100
	0.3	13	1.6231	1.6141
0.05	0.15	61	3.4726	1.7079
	0.2	35	3.7932	1.6326
	0.25	24	3.9281	1.5688
	0.5	11	1.3624	1.2874

From Tables 1–12, we note that for the same values of α and β , n is larger in the case of an unknown population standard deviation than in the known case, which means that when the population standard deviation is unknown, a larger sample size is required from a lot of the product to satisfy the given producer's risk and consumer's risk. Also, from Tables 7–12 we note that, for the same values of d , the values of the sample size increase as α and β decrease, which means that as the producer's

confidence level about the acceptance of a good lot increases and the consumer's risk decreases, they are willing to inspect a larger sample from a lot of the product.

3. The Advantages of the Proposed Plan

The sampling plans reported in McWilliams et al. [24] and in this study for the same specified parameters are shown in Table 13 when the population standard deviation is unknown.

Table 13. The comparison of the proposed plan with McWilliams et al. [24] when $\alpha = 0.10$; $\beta = 0.10$.

p_1	McWilliams et al. [24] Sampling Plan		Proposed Plan for $d = 10$	
			Known σ	Unknown σ
	p_2	n	n	n
0.001	0.008	485	15	153
	0.01	388	13	118
0.0025	0.015	354	18	151
	0.02	194	14	108
0.005	0.025	155	11	85
	0.03	176	14	113
	0.04	96	11	78
	0.05	77	11	60
0.01	0.04	166	21	148
	0.05	105	16	104
	0.1	38	11	44
0.03	0.09	101	24	139
	0.12	54	14	80
	0.15	34	11	55
	0.3	12	11	21
0.05	0.1	0	51	0
	0.15	60	18	99
	0.2	32	11	56
	0.25	20	11	38
	0.5	7	11	14

From Table 13, it can be noted that the plan proposed herein is more efficient at reducing the sample size for the inspection of the finished lot of product. For example, when $p_1 = 0.001$, $p_2 = 0.008$, $\alpha = 0.10$, $\beta = 0.10$ and $d = 10$, the sample size n from the McWilliams et al. [24] sampling plan is 485, while the sample size for the plan proposed in this study is 15 when the standard deviation is known and 153 when it is unknown. Therefore, the proposed plan requires a smaller sample than the existing plan for the inspection of a lot. So, the proposed plan minimizes the inspection time.

Aslam et al. [17] proposed a plan for when the population standard deviation is unknown. The efficiency of the proposed sampling plan is also compared with the sampling plan proposed by [17] in terms of the sample size required for the inspection of the product. The sample size of both sampling plans for the same values of the specified parameters is reported in Table 14.

Table 14. The comparison of the proposed plan with Aslam et al. [17] when $\alpha = 0.05$; $\beta = 0.05$.

p_1	Aslam et al. [17] Sampling Plan		Proposed Plan for $d = 3$
	p_2	n	Unknown σ
0.001	0.01	230	72
0.0025	0.015	155	92
	0.02	120	67
	0.025	95	52
0.005	0.03	105	69
	0.04	85	48
	0.05	80	37
0.01	0.04	115	91
	0.05	70	64
	0.1	55	27

The proposed sampling plan provides smaller values of sample size as compared to the plan proposed by Aslam et al. [17] for an unknown population standard deviation case. For example, when AQL = 0.001 and LQL = 0.01, the proposed sampling plan provides $n = 72$ while the existing sampling plan provided $n = 230$. So, it is concluded that the proposed sampling plan is more efficient than the existing sampling in minimizing the cost of inspection directly associated with the sample size.

4. Application of the Proposed Plan

An electronic company in Saudi Arabia is interested in inspecting/testing power distribution switches (PDS), which are used in heavy-capacity loads using the proposed sampling plan in Hsu et al. [25]. The capacity load is the variable of interest. The quality assurance department set some standard for the inspection of short circuit threshold with an upper specification limit $U = 1.66A$:

$$AQL = 0.05, LQL = 0.5, d = 2, \alpha = 0.10; \beta = 0.10 \text{ and } CRL = 3.$$

To devise the proposed plan, it is assumed that the population standard deviation of the normal distribution is unknown. The 14 preliminary observations are:

$$0.8876, 1.8885, 0.6295, 1.0903, 1.2847, 0.1734, 1.0356, 0.5426, 0.9104, 0.1951, 0.5043, 1.5512, 1.7691, 0.1523.$$

For this data, the necessary statistics are calculated as follows:

$$\hat{\mu}_Y = \sum_{i=1}^n Y_i / n = 0.9010, S = 0.5751 \text{ and } U = 1.66A. \text{ The values of } v_s = (U - \hat{\mu}_Y) / S = 1.3194.$$

Then, the plan parameters for these specified parameters from Table 7 are $n = 14$, $c_1 = 1.3384$, $c_2 = 1.3055$.

In this example, since v_s is between c_1 and c_2 , according to Step 2, the decision will be made using attribute inspection (Step 3). For this inspection, we will count the number of defective CRL to make a final decision about the submitted lot of product. Since $CRL = 3 > d = 2$ (Step 4), the lot should be accepted.

5. Conclusions

In this paper, a new sampling plan is proposed, wherein the structure of the plan is given for the normal distribution with known and unknown population standard deviation. Some tables when σ is known and unknown are given for practical use. In addition, a practical application of the proposed plan in the industry is suggested. The proposed plan is more efficient than the existing plan in terms

of the sample size required for the inspection of the submitted lot of product while keeping the risk low. It is recommended that the proposed sampling plan be applied in industry for inspections to save on costs and time. The proposed sampling plan using a cost model can be studied in future research. The proposed plan for some non-normal distribution can be studied as future research.

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Abbreviations and Symbols

$\hat{\mu}_Y$	mean of random variable $Y_1, Y_2, \dots, Y_n$ of sample of size n
σ_Y^2	variance of random variable $Y_1, Y_2, \dots, Y_n$ of sample of size n
U	upper specification limit (USL)
v	statistic
c_1 and c_2	acceptance numbers
CRL	number of inspected units between the current and the last nonconforming units
d	acceptance number
$F_p(\text{CRL})$	the distribution function of CRL
OC	operating characteristic
AQL	acceptable quality level
LQL	limiting quality level

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