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A Decision-Making Approach Based on a Multi Q-Hesitant Fuzzy Soft Multi-Granulation Rough Model

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Abstract: In this paper, we propose a new hybrid model, multi Q-hesitant fuzzy soft multi-granulation rough set model, by combining a multi Q-hesitant fuzzy soft set and multi-granulation rough set. We demonstrate some useful properties of these multi Q-hesitant fuzzy soft multi-granulation rough sets. Furthermore, we define multi Q-hesitant fuzzy soft (M^kQHFS) rough approximation operators in terms of M^kQHFS relations and M^kQHFS multi-granulation rough approximation operators in terms of M^kQHFS relations. We study the main properties of lower and upper M^kQHFS rough approximation operators and lower and upper M^kQHFS multi-granulation rough approximation operators. Moreover, we develop a general framework for dealing with uncertainty in decision-making by using the multi Q-hesitant fuzzy soft multi-granulation rough sets. We analyze the photovoltaic systems fault detection to show the proposed decision methodology.

Keywords: Q-hesitant fuzzy soft set; multi Q-hesitant fuzzy soft rough set; photovoltaic systems fault detection approach; decision-making method

1. Introduction

The notion of rough set theory was introduced by Pawlak in 1982 [1]. It is a mathematical approach concerning uncertainty that comes from noisy, inexact or incomplete information. In rough set theory, the equivalence relation plays a significant role in creating the upper and lower approximations of the set. Currently, rough set approximations [2] have been constructed into fuzzy sets [3], intuitionistic fuzzy sets [4], hesitant fuzzy sets [5] and covering sets [6]. The soft set theory, originally initiated by Molodtsov [7], is a general tool for dealing with uncertainty. Different from some traditional tools for dealing with uncertainties, such as the theory of fuzzy sets [3], the theory of probability and the theory of rough sets [1], the advantage of soft set theory is that it is free from the inadequacy of the parametrization tools of those theories. According to Molodtsov [7], the soft set theory applied successfully to many fields such as functions' smoothness, game theory, theory of measurement and so on. Maji and Roy [8] introduced the soft set into the decision-making problems with the help of the rough theory. Necessary and possible hesitant fuzzy sets, and probabilistic soft sets and dual probabilistic soft sets in decision-making have discussed in [9,10]. Moreover, many new rough set models have been established by combining the Pawlak rough set with other uncertainty theories such as soft set theory. Feng [11] provided a framework to combine fuzzy set, rough set, and soft set all together, which gave rise to several interesting new concepts such as rough soft set, soft rough set and soft rough fuzzy set [12]. Zhang et al. [13] proposed the notion of soft rough intuitionistic fuzzy sets and intuitionistic fuzzy soft rough sets, which are generalized soft rough set models. Akram et al. [14] presented a new hybrid model, a hesitant N-soft set model for group decision-making. Several research works have been done to solve different real life decision-making problems (see [15–19]). All of these models have always been described by the expression of a one-dimensional membership function that can not be able to deal with the information that appears in a two-dimensional universal set. From this point of view, the idea of Q-fuzzy sets was came out. Afterwards, the concept of multi Q-fuzzy soft sets [20–24] was established to combine the key feature of soft sets and Q-fuzzy sets with multi membership values. The notion of multi Q-hesitant fuzzy soft sets is the generalization of multi Q-fuzzy soft sets. This extension can easily handle the difficulty more objectively than other developed Q-fuzzy set approaches. The combination of multi Q-hesitant fuzzy soft sets and rough sets will be an improved model of hesitant fuzzy rough approaches that concern both areas theoretical and practical applications. Qian et al. [25] proposed the model of multi-granulation rough sets. The main idea of this model is based on defined multiple equivalence relations in a given universe that eliminated the restrictions that may occur through the single equivalence relations in classical rough sets [1] perfectly. The notions of multi-granulation fuzzy rough sets and multi-granulation hesitant fuzzy rough sets are presented by Sun et al. [26] and Zhang et al. [27], respectively, to solve decision-making problems. For other notations and terminologies not mentioned in this paper, the readers are referred to [28–33].

In the field of electrical engineering, photovoltaic systems fault detection is one of the challenging tasks that electrical experts have faced in recent years dealing with a substantial amount of uncertain information. Different experts would give their different judgments towards the systems fault detection data. Hence, by combining multi Q-hesitant fuzzy soft sets with multi-granulation rough sets, we constructed the concept of a multi Q-hesitant fuzzy soft multi-granulation rough set model and its application in photovoltaic systems fault detection through developing a new data analysis model in fault detection procedures under the framework of Q-hesitant fuzzy soft information. In this paper, we propose a new hybrid model, multi Q-hesitant fuzzy soft multi-granulation rough set model, by combining a multi Q-hesitant fuzzy soft set and a multi-granulation rough set. We present some of its fundamental properties. We develop a general framework for dealing with uncertainty decision-making by using the multi Q-hesitant fuzzy soft multi-granulation rough sets. We use the photovoltaic systems fault detection to indicate the principle steps of the decision methodology.

The presentation of the article is organized as follows: In Section 2, we recalled some basic concepts of rough sets, soft sets and hesitant fuzzy soft sets. In Section 3, we have presented multi Q-hesitant fuzzy soft sets and discussed some properties. In Section 4, we have introduced a rough set model based on multi Q-hesitant fuzzy soft relation and have examined some properties of this model. In Section 5, we have generalized the notion of multi Q-hesitant fuzzy soft rough sets into multi Q-hesitant fuzzy soft multi-granulation rough set model. In Section 6, we have established a general approach to decision-making based on multi Q-hesitant fuzzy soft multi-granulation rough sets and illustrated the principal steps of the proposed decision method by a numerical example. Finally, in Section 7, we have concluded the paper with a summary and outlook for further research.

2. Preliminaries

In this section, we recall some basic notions and definitions which will be used in this paper.

Definition 1 ([1])**.** *Let U be a non-empty finite universe and R be an equivalence relation on U. We use U/R to denote the family of all equivalence classes of R (or classifications of U), and $[x]_R$ to denote an equivalence class of R containing the element $x \in U$. The pair (U, R) is called an approximation space. For any $X \subseteq U$, we can define the lower and upper approximations of X as follows:*

$$\underline{R}(X) = \{x \in U : [x]_R \subseteq X\},$$

$$\overline{R}(X) = \{x \in U : [x]_R \cap X \neq \phi\}.$$

The pair $(\underline{R}(X), \overline{R}(X))$ is referred to as the rough set of X. The rough set $(\underline{R}(X), \overline{R}(X))$ gives rise to a description of X under the present knowledge, i.e., the classification of U.

Furthermore, the positive region, negative region, and boundary region of X about the approximation space (U, R) are defined as follows, respectively:

$$pos(X) = \underline{R}(X), neg(X) = \sim \overline{R}(X), bn(X) = \overline{R}(X) - \underline{R}(X),$$

where $\sim X$ stands for complementation of the set X.

Definition 2 ([7]). *Let E be the set of parameters with the connection to the objects in U. A pair (F, E) is called a soft set over U, where F is a mapping given by $F : E \longrightarrow P(U)$, $P(U)$ is a set of all subsets of U.*

This definition shows that a soft set over U is a parameterized family of subsets of the universe U. For $e \in E, F(e)$ is regarded as the set of e-approximate elements of the soft set (F, E).

Definition 3 ([5]). *Given a non-empty subset A of X, a hesitant fuzzy set $H_X = \{(x, h_X(x) : x \in X)\}$ on X satisfying the following condition:*

$$h_X(x) = \phi \text{ for all } x \notin A$$

is called a hesitant fuzzy set related to A (briefly, A-hesitant fuzzy set) on X and is represented by $H_A = \{(x, h_A(x) : x \in X)\}$, where h_A is a mapping from X to $p([0, 1])$ with $h_A(x) = \phi$ for all $x \notin A$.

Definition 4 ([34]). *Let $\tilde{H}(U)$ be the set of all hesitant fuzzy sets in U. A pair $(\tilde{F}, \tilde{A})$ is called a hesitant fuzzy soft set over U, where $\tilde{F}$ is a mapping given by*

$$\tilde{F} : A \longrightarrow \tilde{H}(U).$$

A hesitant fuzzy soft set is a mapping from parameters to $\tilde{H}(U)$. It is a parameterized family of hesitant fuzzy subsets of U. For $e \in A$, $\tilde{F}(e)$ may be considered as the set of e-approximate elements of the hesitant fuzzy soft set $(\tilde{F}, A)$.

3. Multi Q-Hesitant Fuzzy Soft Sets

We first introduce the notion of Q-hesitant fuzzy soft sets as a generalization of Q-fuzzy soft sets.

Definition 5. *Let U be a universal set and Q be non-empty set. A Q-hesitant fuzzy set A_Q is a set given by*

$$A_Q = \{\langle (uq), h_{A_Q}(uq)\rangle : u \in U, q \in Q\},$$

where $h_{A_Q} : U \times Q \longrightarrow [0, 1]$. The function $h_{A_Q}(uq)$ is called the membership function of Q-hesitant fuzzy set, and the set of all Q-hesitant fuzzy sets over $U \times Q$ will be denoted by $QHF(U \times Q)$.

Definition 6. *Let U be a non-empty finite universe and Q be a non-empty set. For any $A_Q, B_Q \in QHF(U \times Q)$, then, for all $u \in U, q \in Q$, we have*

1. $h_{A_Q^c}(uq) = \sim h_{A_Q}(uq) = \bigcup_{\gamma \in h_{A_Q^c}(uq)} \{1 - \gamma\}$.
2. $A_Q \cup B_Q = \{\langle (uq), h_{A_Q}(uq) \vee h_{B_Q}(uq)\rangle, u \in U, q \in Q\}$.
3. $A_Q \cap B_Q = \{\langle (uq), h_{A_Q}(uq) \wedge h_{B_Q}(uq)\rangle, u \in U, q \in Q\}$.
4. $A_Q \oplus B_Q = \bigcup_{\gamma_1 \in h_{A_Q}(uq), \gamma_2 \in h_{B_Q}(uq)} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$.
5. $A_Q \otimes B_Q = \bigcup_{\gamma_1 \in h_{A_Q}(uq), \gamma_2 \in h_{B_Q}(uq)} \{\gamma_1\gamma_2\}$.

Definition 7. *Let U be a universal set and Q be non-empty set, I be a unit interval $[0, 1]$ and k be a positive integer. A multi Q-hesitant fuzzy set $\tilde{H}_Q$ in $U \times Q$ is a set defining by*

$$\tilde{H}_Q = \{\langle (uq), h^i_{\tilde{H}_Q}(uq)\rangle : u \in U, q \in Q \text{ for all } i = 1, 2, \cdots, k\},$$

where $h^i_{\tilde{H}_Q} : U \times Q \longrightarrow I^k$ *for all* $i = 1, 2, \cdots, k$. *The function* $h^1_{\tilde{H}_Q}(uq), h^2_{\tilde{H}_Q}(uq), \cdots, h^k_{\tilde{H}_Q}(uq)$ *is called the membership function of multi Q-hesitant fuzzy set and k is called the dimension of* $h^i_{\tilde{H}_Q}$. *The set of all multi Q-hesitant fuzzy set of dimension k in* $U \times Q$ *is denoted by* $M^kQHFS(U \times Q)$.

Definition 8. *Let* A_Q, B_Q *be a multi Q-hesitant fuzzy sets over* $U \times Q$. *Then,* A_Q *is said to be a multi Q-hesitant fuzzy subset of* B_Q *if*

$$h^i_{A_Q}(uq) \leq h^i_{B_Q}(uq)$$

holds for any $u \in U, q \in Q, i = i, 2, \cdots, k$ *and it is denoted by* $A_Q \subseteq B_Q$.

Definition 9. *Let U be a universal set and be non-empty set, E be the set of parameters and* $M^kQHF(U \times Q)$ *be the set of all multi Q-hesitant fuzzy sets on* $U \times Q$ *with the dimension k. Let* $A \subseteq E$ *the pair* (H_Q, A) *is called a multi Q-hesitant fuzzy soft set* (M^kQHFSS) *over U, where* (H_Q, A) *is given by the form*

$$(H_Q, A) = \{(e, h^i_Q(e)) : e \in A, h^i_Q(e) \in M^kQHFS(U \times Q)\},$$

where $h^i_Q : A \longrightarrow M^kQHF(U \times Q)$ *such that* $h^i_Q(e) \neq \phi$ *if* $e \notin A$. *The set of all multi Q-hesitant fuzzy soft sets over* $U \times Q$ *will be denoted by* $M^kQHFSS(U \times Q)$.

Example 1. *Suppose that a company wants to buy three types of products from two brands and wants to take the opinion of two specialists about these products (k= 2). Let* $U = \{u_1, u_2, u_3\}$ *be a set of products,* $Q = \{p, q\}$ *be a set of brands, and* $E = \{e_1 = \textit{easy to use}, e_2 = \textit{quality}, e_3 = \textit{price}\}$ *is the set of decision parameters. Then we can define the multi Q-hesitant fuzzy soft sets*(H_Q, A) *as follows:*

$$(H_Q, A) = \{\langle e_1, (\tfrac{u_1p}{(0.2,0.3)(0.1)}), (\tfrac{u_1q}{(0.1,0.3)(0.4,0.8)}), (\tfrac{u_3p}{(0.6,0.5)(0.2,0.2)}), (\tfrac{u_3q}{(0.2,0.4)(0.1)})\rangle,$$

$$\langle e_2, (\tfrac{u_2p}{(0.3,0.1)(0.2,0.3,0.6)}), (\tfrac{u_2q}{(0.5,0.3)(0.5,0.5,0.2)}), (\tfrac{u_3p}{(0.2,0.2)(0.4)}), (\tfrac{u_3q}{(0.7,0.3)(0.2,0.9)})\rangle,$$

$$\langle e_3, (\tfrac{u_1p}{(0.1,0.1)(0.4,0.4)}), (\tfrac{u_1q}{(0.1,0.3)(0.7,0.6)}), (\tfrac{u_2p}{(0.4,0.3)(0.4,0.1)}), (\tfrac{u_2q}{(0.2,0.6)(0.7,0.3)})\rangle\}.$$

Definition 10. *Let* (H_Q, A) *and* (F_Q, B) *be two multi Q-hesitant fuzzy soft sets,* (H_Q, A) *is said to be multi Q-hesitant fuzzy soft subset of* (F_Q, B) *if* $A \subseteq B$ *and* $H_Q(e) \subseteq F_Q(e)$ *for all* $e \in E$ *and denoted by* $(H_Q, A) \subseteq (F_Q, B)$.

Proposition 1. *Let* (H_Q, A), (F_Q, B) *and* (G_Q, C) *be three multi Q-hesitant fuzzy soft sets. Then,*

1. $(H_Q, A) \subseteq (U, E)$,
2. $(\phi, A) \subseteq (H_Q, B)$,
3. If $(H_Q, A) \subseteq (F_Q, B)$ and $(F_Q, B) \subseteq (G_Q, C)$, then $(H_Q, A) \subseteq (G_Q, C)$.

Definition 11. *A multi Q-hesitant fuzzy soft set* (H_Q, A) *of dimension k over* $U \times Q$ *is called the null multi Q-hesitant fuzzy soft set if* $H_Q(e) = \phi_k$ *for all* $e \in A$ *and it is denoted by* ϕ^k_A.

Definition 12. *A multi Q-hesitant fuzzy soft set* (H_Q, A) *of dimension k over* $U \times Q$ *is called the absolute multi Q-hesitant fuzzy soft set if* $H_Q(e) = 1_k$ *for all* $e \in A$ *and it is denoted by* U^k_A.

Definition 13. *Let* (H_Q, A) *be a multi Q-hesitant fuzzy soft set of dimension k over* $U \times Q$. *Then, the complement of* (H_Q, A) *is denoted by* $(H_Q, A)^c$ *and defined by* $(H_Q, A)^c$=(H^c_Q, A), *where* $H^c_Q : A \longrightarrow M^kQHFS(U \times Q)$ *is mapping given by* $H^c_Q(e) = (H_Q(e))^c$ *for all* $e \in A$.

Remark 1. *Clearly,* $((H_Q, A)^c)^c = (H_Q, A)$ *and* $(\phi_A^k)^c = U_A^k$, $(U_A^k)^c = \phi_A^k$.

Definition 14. *The union of two multi Q-hesitant fuzzy soft sets of dimension k over U,* (H_Q, A) *and* (F_Q, B) *is the multi Q-hesitant fuzzy soft set* (G_Q, C)*, where* $C = A \cup B$*, and for all* $e \in C$, $G_Q(e) = H_Q(e) \cup F_Q(e)$. *We write* $(H_Q, A) \cup (F_Q, B) = (G_Q, C)$.

Definition 15. *The intersection of of two multi Q-hesitant fuzzy soft sets of dimension k over U,* (H_Q, A) *and* (F_Q, B) *with* $A \cap B \neq \phi$ *is the multi Q-hesitant fuzzy soft set* (G_Q, C)*, where* $C = A \cap B$*, and for all* $e \in C$,

$$G_Q(e) = \begin{cases} H_Q(e) & for \quad e \in A - B, \\ F_Q(e) & for \quad e \in B - A, \\ H_Q(e) \cup F_Q(e) & for \quad e \in A \cap B. \end{cases}$$

In this case, we write $(H_Q, A) \cap (F_Q, B) = (G_Q, C)$.

Theorem 1. *Let* (H_Q, A) *and* (F_Q, B) *be two multi Q-hesitant fuzzy soft sets of dimension k over* $U \times Q$*. Then,*

1. $(H_Q, A) \cup (H_Q, A) = (H_Q, A)$,
2. $(H_Q, A) \cap (H_Q, A) = (H_Q, A)$,
3. $(H_Q, A) \cup \phi_A^k = (H_Q, A)$,
4. $(H_Q, A) \cap \phi_A^k = \phi_A^k$,
5. $(H_Q, A) \cup U_A^k = U_A^k$,
6. $(H_Q, A) \cap U_A^k = (H_Q, A)$,
7. $(H_Q, A) \cup (F_Q, B) = (F_Q, B) \cup (H_Q, A)$,
8. $(H_Q, A) \cap (F_Q, B) = (F_Q, B) \cap (H_Q, A)$.

4. Multi Q-Hesitant Fuzzy Soft Rough Set

Definition 16. *Let* (H_Q, A) *be a multi Q-hesitant fuzzy soft set over* $U \times Q$*. A multi Q-hesitant fuzzy subset of* $(U \times Q) \times (E \times Q)$ *is called a multi Q-hesitant fuzzy soft relation* (M^kQHFSR) *from* $(U \times Q)$ *to* $(E \times Q)$ *given by*

$$R_Q = \{\langle (uq, eq), h^i_{R_Q}(uq, eq)\rangle, uq \in U \times Q, eq \in E \times Q, i = 1, 2, \cdots, k\},$$

where $h^i_{R_Q} : (U \times Q) \times (E \times Q) \longrightarrow [0,1]^k$.

Definition 17. *Let U be nonempty universe, Q be a nonempty set and E be the set of parameters.* R_Q *is a multi Q-hesitant fuzzy soft relation* $R_Q \in M^kQHFSR((U \times Q) \times (E \times Q))$ *and the triple* $((U, Q), (E, Q), R_Q)$ *is multi Q-hesitant fuzzy soft approximation space. For any* $A_Q \in M^kQHFS(E)$*, the lower and upper approximations of* A_Q *with respect to* (U, E, Q, R_Q) *denoted by* $\underline{R_Q}(A_Q)$ *and* $\overline{R_Q}(A_Q)$*, are two multi Q-hesitant fuzzy soft sets, respectively, defined as follows:*

$$\underline{R_Q}(A_Q) = \{\langle (uq), h_{\underline{R_Q}(A_Q)}(uq)\rangle : (uq) \in U \times Q\},$$
$$\overline{R_Q}(A_Q) = \{\langle (uq), h_{\overline{R_Q}(A_Q)}(uq)\rangle : (uq) \in U \times Q\},$$

where

$$h_{\underline{R_Q}(A_Q)}(uq) = \{\langle \bigwedge_{e \in E} \{(1 - h^i_{R_Q}(uq, eq)) \vee h^i_{A_Q}(eq)\}\rangle : (uq) \in U \times Q, i = 1, 2, ..., k\},$$
$$h_{\overline{R_Q}(A_Q)}(uq) = \{\langle \bigvee_{e \in E} \{h^i_{R_Q}(uq, eq) \wedge h^i_{A_Q}(eq)\}\rangle : (uq) \in U \times Q, i = 1, 2, ..., k\}.$$

$\underline{R_Q}(A_Q)$ *and* $\overline{R_Q}(A_Q)$ *are, respectively, called the lower and upper Q-hesitant fuzzy soft rough approximations' operators. The pair* $(\underline{R_Q}(A_Q), \overline{R_Q}(A_Q))$ *is called the multi Q-hesitant fuzzy soft rough set of* A_Q *with respect to* (U, E, Q, R_Q)*. Moreover, if* $\underline{R_Q}(A_Q) = \overline{R_Q}(A_Q)$*, then* A_Q *is called definable.*

Example 2. *Suppose that* $U = \{u_1, u_2, u_3\}$ *is the set of cars that Mr X wants to buy and* $Q = \{q_1, q_2\}$ *represents the companies of the different cars. They form the universe (U,Q) and let* $E = \{e_1 = size, e_2 = price, e_3 = colour\}$ *be the set of parameters. Consider a multi Q-hesitant fuzzy soft relation* $R_Q : U \times Q \longrightarrow E \times Q$ *with dimension k = 2 is given by Table 1.*

Table 1. Multi Q-hesitant fuzzysoft relation R_Q.

R_Q	e_1q_1	e_1q_2	e_2q_1	e_2q_2
(u_1q_1)	{(0.2)(0.6,0.4)}	{(0.3,0.7)(0.6)}	{(0.5,0.4,0.6)(0.6,0.5)}	{(0.4,0.2)(0.1,0.3)}
(u_1q_2)	{(0.8,0.5)(0.2)}	{(0.6,0.9)(0.2,0.9)}	{(0.3)(0.2,0.7)}	{(0.5,0.2,0.1)(0.1,0.5)}
(u_2q_1)	{(0.1,0.3)(0.9,0.7,0.2)}	{(0.5,0.1,)(0.6,0.2)}	{(0.4)(0.5)}	{(0.2,0.4)(0.2,0.8)}
(u_2q_2)	{(0.5)(0.6)}	{(0.9,0.5)(0.6,0.7,0.4)}	{(0.6)(0.3,0.1)}	{(0.2)(0.6,0.1)}

Now, if Mr X gives the optimum decision object $A_Q \in M^kQHF(E)$*, which is a Q-hesitant fuzzy subset defined as follows:*

$$A_Q = \{\langle((e_1q_1), \{(0.1, 0.3)(0.4, 0.5)\}), ((e_1q_2), \{(0.2, 0.4)(0.5, 0.6)\})\rangle, \langle((e_2q_1), \{(0.3, 0.6)(0.6, 0.7)\}), ((e_2q_2), \{(0.2, 0.5), (0.2, 0.8)\})\rangle\}.$$

Then, by Definition 17, we have

$$h_{\underline{R_Q}}(u_1q_1) = \bigwedge_{e\in E}\{(1 - h^2_{R_Q})(u_1q_1, eq) \vee h^2_{A_Q}(eq)\}$$
$$= (\{(0.8), (0.4, 0.6)\} \vee \{(0.1, 0.3)(0.4, 0.5)\}) \wedge (\{(0.7, 0.3), (0.4)\} \vee \{(0.2, 0.4)(0.5, 0.6)\})$$
$$\wedge (\{(0.5, 0.6, 0.4), (0.4, 0.5)\} \vee \{(0.3, 0.6)(0.6, 0.7)\}) \wedge (\{(0.6, 0.8), (0.9, 0.7)\} \vee \{(0.2, 0.5), (0.2, 0.8)\})$$
$$= \{(0.8, 0.8), (0.4, 0.6)\} \wedge \{(0.7, 0.4), (0.5, 0.6)\} \wedge \{(0.5, 0.6, 0.6), (0.6, 0.7)\} \wedge \{(0.6, 0.8), (0.9, 0.8)\}$$
$$= \{(0.5, 0.4, 0.4), (0.4, 0.6)\}.$$

Similarly, we have

$$h_{\underline{R_Q}}(u_1q_2) = \{(0.2, 0.4, 0.4)(0.8, 0.6)\},$$
$$h_{\underline{R_Q}}(u_2q_1) = \{(0.5, 0.6)(0.4, 0.5, 0.7)\},$$
$$h_{\underline{R_Q}}(u_2q_2) = \{(0.2, 0.5)(0.4, 0.5, 0.5)\},$$
$$h_{\overline{R_Q}}(u_1q_1) = \{(0.3, 0.4, 0.6)(0.6, 0.6)\},$$
$$h_{\overline{R_Q}}(u_1q_2) = \{(0.3, 0.4, 0.4)(0.2, 0.7)\},$$
$$h_{\overline{R_Q}}(u_2q_1) = \{(0.3, 0.4)(0.5, 0.8, 0.8)\},$$
$$h_{\overline{R_Q}}(u_2q_2) = \{(0.3, 0.6)(0.5, 0.6, 0.5)\}.$$

Thus, we conclude that:

$$\underline{R_Q}(A_Q) = \{\langle(u_1q_1), \{(0.5, 0.4, 0.4), (0.4, 0.6)\}\rangle, \langle(u_1q_2), \{(0.2, 0.4, 0.4)(0.8, 0.6)\}\rangle, \langle(u_2q_1), \{(0.5, 0.6)(0.4, 0.5, 0.7)\}\rangle, \langle(u_2q_2), \{(0.2, 0.5)(0.4, 0.5, 0.5)\}\rangle\},$$

$$\overline{R_Q}(A_Q) = \{\langle(u_1q_1), \{(0.3, 0.4, 0.6)(0.6, 0.6)\rangle, \langle(u_1q_2), \{(0.3, 0.4, 0.4)(0.2, 0.7)\}\rangle, \langle(u_2q_1), \{(0.3, 0.4)(0.5, 0.8, 0.8)\}\rangle, \langle(u_2q_2), \{0.3, 0.6)(0.5, 0.6, 0.5)\}\rangle\}.$$

The pair $(\underline{R_Q}(A_Q), \overline{R_Q}(A_Q))$ *is called a multi Q-hesitant fuzzy soft rough set with dimension 2.*

Theorem 2. *Let (U,E,Q,R_Q) be multi Q-hesitant fuzzy soft approximation space. The lower and upper Q-hesitant fuzzy soft rough approximations operators $\underline{R_Q}(A_Q)$ and $\overline{R_Q}(A_Q)$, respectively, for any $A_Q, B_Q \in M^kQHF(E)$ satisfy the following properties:*

1. $\underline{R_Q}(A_Q^c) = (\overline{R_Q}(A_Q))^c$, $\overline{R_Q}(A_Q^c) = (\underline{R_Q}(A_Q))^c$,
2. $A_Q \subseteq B_Q \Rightarrow \underline{R_Q}(A_Q) \subseteq (\underline{R_Q}(B_Q))$, $A_Q \subseteq B_Q \Rightarrow \overline{R_Q}(A_Q) \subseteq (\overline{R_Q}(A_Q))$,
3. $\underline{R_Q}(A_Q \cap B_Q) = \underline{R_Q}(A_Q) \cap (\underline{R_Q}(B_Q))$, $\overline{R_Q}(A_Q \cup B_Q) = \overline{R_Q}(A_Q) \cup (\overline{R_Q}(B_Q))$,
4. $\underline{R_Q}(A_Q \cup B_Q) \supseteq \underline{R_Q}(A_Q) \cup (\underline{R_Q}(B_Q))$, $\overline{R_Q}(A_Q \cap B_Q) \subseteq \overline{R_Q}(A_Q) \cap (\overline{R_Q}(B_Q))$.

Proof. 1. By Definition 17, we have

$$\underline{R_Q}(A_Q^c) = \{\langle (uq), h^i_{\underline{R_Q}(\sim A_Q)}(uq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \bigwedge_{e\in E}\{h^i_{\sim R_Q}(uq,eq) \vee h^i_{\sim A_Q}(eq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \sim (\bigvee_{e\in E}\{h^i_{R_Q}(uq,eq) \wedge h^i_{A_Q}(eq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \sim h^i_{\overline{R_Q}(A_Q)}(uq)\rangle : (uq) \in U \times Q, i = 1,2,...,k\}$$
$$= (\overline{R_Q}(A_Q))^c.$$

Similarly, we can obtain that $\overline{R_Q}(A_Q^c) = (\underline{R_Q}(A_Q))^c$.

2. If $A_Q \subseteq B_Q$, by Definition 8, $h^i_{A_Q}(uq) \leq h^i_{B_Q}(uq)$ for all $u \in U, q \in Q$. Therefore, $\bigwedge_{e\in E}\{(1 - h^i_{R_Q})(uq,eq) \vee h^i_{A_Q}(eq)\} \leq \bigwedge_{e\in E}\{(1 - h^i_{R_Q})(uq,eq) \vee h^i_{B_Q}(eq)\}$, thus $h^i_{\underline{R_Q}(A_Q)}(uq) \leq h^i_{\underline{R_Q}(B_Q)}(uq)$. It follows that $\underline{R_Q}(A_Q) \subseteq \underline{R_Q}(B_Q)$.

3. $$\underline{R_Q}(A_Q \cap B_Q) = \{\langle (uq), h^i_{\underline{R_Q}(A_Q\cap B_Q)}(uq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \bigwedge_{e\in E}(1 - h^i_{R_Q})(uq,eq) \vee h^i_{A_Q\cap B_Q}(eq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \bigwedge_{e\in E}(1 - h^i_{R_Q})(uq,eq) \vee (h^i_{A_Q}(eq) \wedge h^i_{B_Q}(eq))\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \Big(\bigwedge_{e\in E}((1 - h^i_{R_Q})(uq,eq) \vee h^i_{A_Q}(eq))\Big) \wedge \Big(\bigwedge_{e\in E}((1 - h^i_{R_Q})(uq,eq) \vee h^i_{B_Q}(eq))\Big)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), h^i_{\underline{R_Q}(A_Q)}(uq) \wedge h^i_{\underline{R_Q}(B_Q)}(uq)\rangle : uq \in U \times Q\}$$
$$= \underline{R_Q}(A_Q) \cap \underline{R_Q}(B_Q).$$

Hence, $\underline{R_Q}(A_Q \cap B_Q) = \underline{R_Q}(A_Q) \cap \underline{R_Q}(B_Q)$.

Similarly, we can prove that $\overline{R_Q}(A_Q \cap B_Q) = \overline{R_Q}(A_Q) \cap \overline{R_Q}(B_Q)$.

4. $$\underline{R_Q}(A_Q \cup B_Q) = \{\langle (uq), h^i_{\underline{R_Q}(A_Q\cup B_Q)}(uq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \bigwedge_{e\in E}(1 - h^i_{R_Q})(uq,eq) \vee h^i_{A_Q\cup B_Q}(eq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \bigwedge_{e\in E}(1 - h^i_{R_Q})(uq,eq) \vee (h^i_{A_Q}(eq) \vee h^i_{B_Q}(eq))\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), \Big(\bigwedge_{e\in E}((1 - h^i_{R_Q})(uq,eq) \vee h^i_{A_Q}(eq))\Big) \vee \Big(\bigwedge_{e\in E}((1 - h^i_{R_Q})(uq,eq) \vee h^i_{B_Q}(eq))\Big)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \{\langle (uq), h^i_{\underline{R_Q}(A_Q)}(uq) \vee h^i_{\underline{R_Q}(B_Q)}(uq)\rangle : uq \in U \times Q, i = 1,2,...,k\}$$
$$= \underline{R_Q}(A_Q) \cup (\underline{R_Q}(B_Q)).$$

Hence, $\underline{R_Q}(A_Q \cup B_Q) = \underline{R_Q}(A_Q) \cup \underline{R_Q}(B_Q)$.

Similarly, we can prove that $\overline{R_Q}(A_Q \cup B_Q) = \overline{R_Q}(A_Q) \cup \overline{R_Q}(B_Q)$.

□

Theorem 3. *Let R_Q, S_Q be multi Q-hesitant fuzzy soft relations from $(U \times Q)$ to $(E \times Q)$, if $R_Q \subseteq S_Q$, for any $A \in QHF(E)$, then:*

1. $\underline{R_Q}(A_Q) \supseteq \underline{S_Q}(A_Q)$,
2. $\overline{R_Q}(A_Q) \subseteq \overline{S_Q}(A_Q)$.

Proof. 1. If $R_Q \subseteq S_Q$, then, by Definition 8, we have $h^i_{R_Q}(uq, eq) \leq h^i_{S_Q}(uq, eq)$ for all $uq \in U \times Q$, $eq \in E \times Q$, then
$\underline{R_Q}(A_Q) = \{\langle (uq), h^i_{\underline{R_Q(A_Q)}}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \{\langle (uq), \bigwedge_{e\in E}\{(1 - h^i_{R_Q})(uq, eq) \vee h^i_{A_Q}(eq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$\geq \{\langle (uq), \bigwedge_{e\in E}\{(1 - h^i_{S_Q})(uq, eq) \vee h^i_{A_Q}(eq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \{\langle (uq), h^i_{\underline{S_Q(A_Q)}}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \underline{S_Q}(A_Q)$.

2. Similarly, it can be proved.

□

5. Multi Q-Hesitant Fuzzy Soft Multi-Granulation Rough Set

Definition 18. *Let U be a universal set and Q be non-empty set, and E be the set of parameters and R_{Q_j},(j=1,2,...,m) be multi Q_m-hesitant fuzzy soft relations over $(U \times Q) \times (E \times Q)$, and (U, E, Q, R_{Q_j}) be called multi Q-hesitant fuzzy soft multi-granulation approximation space, for any $A_Q \in M^kQHF(E)$, the optimistic lower and upper approximation of A_Q with respect to (U, E, Q, R_{Q_j}) are defined as follows:*

$$\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) = \{\langle (uq), h^i_{\underline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q)}(u, q)\rangle : uq \in U \times Q\},$$
$$\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) = \{\langle (uq), h^i_{\overline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q)}(u, q)\rangle : uq \in U \times Q\},$$

where

$$h_{\underline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q)}(uq) = \{\langle \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} \{(1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q}(eq)\}\rangle : uq \in U \times Q\},$$
$$h_{\overline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q)}(uq) = \{\langle \bigwedge_{j=1}^{m} \bigvee_{i=1}^{k} \{h^i_{R_{Q_i}}(uq, eq) \wedge h^i_{A_Q}(eq)\}\rangle : uq \in U \times Q\}.$$

The pair $(\underline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q), \overline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q))$ is called an optimistic multi Q-hesitant fuzzy soft multi-granulation rough set of A_Q with respect to (U, E, Q, R_{Q_j}).

Theorem 4. *Let (U, E, Q, R_{Q_j}) be multi Q-hesitant fuzzy soft multi-granulation approximation space and $R_{Q_j} \in M^kQHFSR((U \times Q) \times (E \times Q))$,(j =1,2,...,m) be multi Q_m hesitant fuzzy soft relations over $(U \times Q) \times (E \times Q)$, for any $A_Q, B_Q \in M^kQHF(E)$, the optimistic lower and upper approximation satisfy the following properties:*

1. $\underline{\sum_{j=1}^m R_{Q_j}}^{o}(A^c_Q) = (\overline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q))^c$,
   $\overline{\sum_{j=1}^m R_{Q_j}}^{o}(A^c_Q) = (\underline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q))^c$.
2. $A_Q \subseteq B_Q \Rightarrow \underline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q) \subseteq \underline{\sum_{j=1}^m R_{Q_j}}^{o}(B_Q)$,
   $A_Q \subseteq B_Q \Rightarrow \overline{\sum_{j=1}^m R_{Q_j}}^{o}(A_Q) \subseteq \overline{\sum_{j=1}^m R_{Q_j}}^{o}(B_Q)$.

3. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q) = \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cap \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$,
$\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cup B_Q) = \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cup \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$.
4. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cup B_Q) \supseteq \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cup \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$,
$\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q) \subseteq \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cap \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$.

Proof. 1. By Definition 18, we have,

$\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q^c) = \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(\sim A_Q)}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} \{\sim h^i_{R_{Q_j}}(uq, eq) \vee h^i_{\sim A_Q}(eq)\rangle : uq \in U \times Q\}$
$= \{\langle (uq), \sim (\bigwedge_{j=1}^{m} \bigvee_{i=1}^{k} \{h^i_{R_{Q_j}}(uq, eq) \wedge h^i_{A_Q}(eq))\rangle : uq \in U \times Q\}$
$= \{\langle (uq), \sim h^i_{\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= (\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q))^c$.

Similarly, we can obtain that $\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q^c) = (\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q))^c$.

2. If $A_Q \subseteq B_Q$, by Definition 8, $h^i_{A_Q}(u, q) \le h^i_{B_Q}(uq)$ for all $u \in U, q \in Q$, therefore, $\bigvee_{j=1}^{m} \bigwedge_{i=1}^{k}\{(1 - h^i_{R_{Q_j}})(uq, eq) \vee h_{A_Q}(e, q)\} \le \bigvee_{i=1}^{m} \bigwedge_{i=1}^{k}\{(1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{B_Q}(eq)\}$, thus $h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)}(uq) \le h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)}(uq)$ it follows that $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \subseteq \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$.

3. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q) = \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q)}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} (1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q \cap B_Q}(eq)\rangle : uq \in U \times Q\}$
$= \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} (1 - h^i_{R_{Q_j}})(uq, eq) \vee (h^i_{A_Q}(eq) \wedge h^i_{B_Q}(eq))\rangle : uq \in U \times Q\}$
$= \{\langle (uq), \left(\bigvee_{i=1}^{m} \bigwedge_{i=1}^{k}((1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q}(eq))\right) \wedge$
$\left(\bigvee_{j=1}^{m} \bigwedge_{i=1}^{k}((1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{B_Q}(eq))\right)\rangle : uq \in U \times Q\}$
$= \{\langle (uq), h^i_{\underline{\sum_{i=1}^{m} R_{Q_i}}^{o}(A_Q)}(uq) \wedge h_{\underline{\sum_{i=1}^{m} R_{Q_i}}^{o}(B_Q)}(uq)\rangle : uq \in U \times Q\}$
$= \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cap (\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q))$.

Hence, $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q) = \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cap \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$.

Similarly, we can prove that $\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q) = \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cap \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$.

4. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cup B_Q) = \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cup B_Q)}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} (1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q \cup B_Q}(eq)\rangle : uq \in U \times Q\}$
$= \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} (1 - h^i_{R_{Q_j}})(uq, eq) \vee (h^i_{A_Q}(eq) \vee h^i_{B_Q}(eq))\rangle : uq \in U \times Q\}$
$= \{\langle (uq), \left(\bigvee_{j=1}^{m} \bigwedge_{i=1}^{k}((1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q}(eq))\right) \vee$
$\left(\bigvee_{i=1}^{m} \bigwedge_{i=1}^{k}((1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{B_Q}(eq))\right)\rangle : (uq) \in U \times Q\}$
$= \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)}(uq) \vee h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cup (\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q))$.

Hence, $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cup B_Q) = \underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cup \underline{R_Q}(B_Q)$.

Similarly, we can prove that $\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cup B_Q) = \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \cup \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(B_Q)$.
□

Theorem 5. *Let $R_{Q_j}, S_{Q_j} \in M^kQHFSR((U \times Q) \times (E \times Q))$ $(j = 1, 2, ..., m)$ be multi Q_m hesitant fuzzy soft relations over $(U \times Q) \times (E \times Q)$, if $R_{Q_j} \subseteq S_{Q_j}$, for any $A_Q \in M^kQHF(E)$, the following properties are true:*

1. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \supseteq \underline{\sum_{j=1}^{m} S_{Q_j}}^{o}(A_Q)$,
2. $\overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \subseteq \overline{\sum_{j=1}^{m} S_{Q_j}}^{o}(A_Q)$.

Proof. 1. If $R_{Q_j} \subseteq S_{Q_j}$, then, by Definition 8, we have $h^i_{R_{Q_j}}(uq, eq) \leq h^i_{S_{Q_j}}(uq, eq)$ for all $(u, q) \in U \times Q$, $eq \in E \times Q$, then
$\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) = \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)}(uq)\rangle : uq \in U \times Q, i = 1, 2, ..., k\}$
$= \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} \{(1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q}(eq)\rangle : uq \in U \times Q\}$
$\geq \{\langle (uq), \bigvee_{j=1}^{m} \bigwedge_{i=1}^{k} \{(1 - h^i_{S_{Q_j}})(uq, eq) \vee h^i_{A_Q}(eq)\rangle : uq \in U \times Q\}$
$= \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} S_{Q_j}}^{o}(A_Q)}(uq)\rangle : uq \in U \times Q\}$
$= \underline{\sum_{j=1}^{m} S_{Q_j}}^{o}(A_Q)$.

2. It can be proved similarly to 1.
□

Definition 19. *Let U be a universal set and Q be a non-empty set, and E be the set of parameters and R_{Q_j},(j=1,2,...,m) are multi Q_m-hesitant fuzzy soft relations over $(U \times Q) \times (E \times Q)$, the triple$(U, E, Q, R_{Q_j})$ is called multi Q-hesitant fuzzy soft multi-granulation approximation space, for any $A_Q \in M^kQHF(E)$, and the pessimistic lower and upper approximation of A_Q with respect to (U, E, Q, R_{Q_j}) are defined as follows:*

$$\underline{\sum_{j=1}^{m} R^p_{Q_j}}(A_Q) = \{\langle (uq), h^i_{\underline{\sum_{j=1}^{m} R^p_{Q_j}}(A_Q)}(u, q)\rangle : uq \in U \times Q\},$$
$$\overline{\sum_{j=1}^{m}} R^p_{Q_j}(A_Q) = \{\langle (uq), h^i_{\overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)}(u, q)\rangle : uq \in U \times Q\},$$

where
$$h_{\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)}(uq) = \{\langle \bigwedge_{j=1}^{m} \bigwedge_{i=1}^{k} \{(1 - h^i_{R_{Q_j}})(uq, eq) \vee h^i_{A_Q}(eq)\}\rangle : uq \in U \times Q\},$$
$$h_{\overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)}(uq) = \{\langle \bigvee_{j=1}^{m} \bigvee_{i=1}^{k} \{h^i_{R_{Q_i}}(uq, eq) \wedge h^i_{A_Q}(eq)\}\rangle : uq \in U \times Q\}.$$

The pair $(\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q), \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q))$ is called an pessimistic multi Q-hesitant fuzzy soft multi-granulation rough set of A_Q with respect to (U, E, Q, R_{Q_j}).

Theorem 6. *Let (U, E, Q, R_{Q_j}) be multi Q-hesitant fuzzy soft multi-granulation approximation space and $R_{Q_j} \in M^kQHFSR((U \times Q) \times (E \times Q)$,(i=1,2,...,m) be multi Q_m hesitant fuzzy soft relations over $(U \times Q) \times (E \times Q)$, for any $A_Q, B_Q \in M^kQHF(E)$, the pessimistic lower and upper approximation satisfy the following properties:*

1. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q^c) = (\overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q))^c, \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q^c) = (\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q))^c.$
2. $A_Q \subseteq B_Q \Rightarrow \underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \subseteq \underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(B_Q)$,
   $A_Q \subseteq B_Q \Rightarrow \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \subseteq \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(B_Q).$
3. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q \cap B_Q) = \underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \cap \underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(B_Q),$
   $\overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q \cup B_Q) = \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \cup \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(B_Q).$
4. $\underline{\sum_{i=1}^{m} R_{Q_i}}^{p}(A_Q \cup B_Q) \supseteq \underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \cup \underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(B_Q),$
   $\overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q \cap B_Q) \subseteq \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \cap \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(B_Q).$

Proof. It can easily be proved by using Theorem 4 and Definition 19. □

Theorem 7. *Let (U, E, Q, R_{Q_j}) be multi Q-hesitant fuzzy soft multi-granulation approximation space and $R_{Q_j}, S_{Q_j} \in M^kQHFSR((U \times Q) \times (E \times Q)$,(i=1,2,...,m) be multi Q_m hesitant fuzzy soft relations over $(U \times Q) \times (E \times Q)$, if $R_{Q_j} \subseteq S_{Q_j}$, for any $A_Q \in M^kQHF(E)$, the following properties are true :*

1. $\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \supseteq \underline{\sum_{j=1}^{m} S_{Q_j}}^{p}(A_Q),$
2. $\overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \subseteq \overline{\sum_{j=1}^{m} S_{Q_j}}^{p}(A_Q).$

Proof. It can be easily proved by Theorem 5 and Definition 19. □

6. Photovoltaic Systems Fault Detection Approach

Fuzzy sets and rough sets are both mathematical tools to handle uncertainties, they have a wide applications in many practical problems, especially in the area of decision-making. In many instances, we can not successfully utilize these classical methods to deal with decision-making problems since various types of uncertainties involved in these problems which require that second dimension must be added to the expression of the membership value.

Inspired by this, we construct a new model to the decision-making problem of photovoltaic system fault detection depending on the notion of M^kQHFS multi-granulation rough set.

6.1. The Application Model

Photovoltaic systems (solar panel) can be explained as a piece of equipment converting sunlight (photons) to electric energy. Loss of power in photovoltaic systems can occur suddenly any time. Therefore, it is necessary to detect faults as early as possible. Unexpected power loss is usually detected by comparing the output to a reference figure.

By employing the model of multi Q-hesitant fuzzy soft multi-granulation rough sets, we can indicate the loss of power in photovoltaic systems expressed as multi Q-hesitant fuzzy soft elements.

Let $U = \{u_1, u_2, ..., u_v\}$ be the fault type set, $Q = \{q_1, q_2\}$ represents the set of condition degrees and $E = \{e_1, e_2, ..., e_s\}$ be the set of power measurement. Let $R_{Q_j} \in M^kQHFSR((U \times Q) \times (E \times Q))$ $(j = 1, 2, ..., m)$, which was employed to indicate the electrical information given by m experts via the membership degrees between the fault detected with condition degrees and the power measurement with condition degrees. In addition, $A_Q \in M^kQHF(E)$ represents the power measurements with the condition degree of each measurement. Then, we construct a multi Q-hesitant fuzzy soft decision information system (U, E, Q, R_{Q_j}) of the electrical detection procedure.

First, based on the the score function definition given by Xia and Xu [29], we define the score function of M^kQHFS element as follows:

Definition 20. *Let $h^i_Q(uq)$ be M^kQHFS element, then the score function can be fined as follows:*

$$S(h^i_Q(uq)) = \{\frac{1}{l(h^i_Q)} \sum_{\gamma \in h^i_Q} \gamma, i = 1, 2, ..., k\},$$

where $l(h^i_Q)$ is the number of values in $(h^i_Q(uq))$.

By Definition 20, we can define the sum of A_Q and B_Q as follows:

Definition 21. *Letting A_Q and B_Q be two M^kQHFSS in $U \times Q$, we define the sum of $h^i_{A_Q}(uq)$ and $h^i_{B_Q}(uq)$ such that $i = 1, 2, ..., k$ by*

$$h^i_{A_Q}(uq) \oplus h^i_{B_Q}(uq) = \{\langle h^1_{A_Q}(uq) + h^1_{B_Q}(uq) - h^1_{A_Q}(uq)h^1_{B_Q}(uq), h^2_{A_Q}(uq)h^2_{B_Q}(uq), ..., h^k_{A_Q}(uq)h^k_{B_Q}(uq)\rangle\}.$$

Based on the decision-making strategy developed in [14], we introduce the following three measurement indices which are denoted by:

$$T_1 = \left\{(S,T) | \max_{u_s q_t} S\left(\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)(u_s q_t) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)(u_s q_t)\right)\right\},$$
$$T_2 = \left\{(X,Y) | \max_{u_x q_y} S\left(\underline{\sum_{j=1}^m R_{Q_j}}^p(A_Q)(u_x q_y) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q)(u_x q_y)\right)\right\},$$
$$T_3 = \left\{(V,N) | \max_{u_v q_n} S\left((\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q) \oplus \overline{\sum_{i=1}^m R_{Q_j}}^o(A_Q)) \oplus (\underline{\sum_{i=1}^m R_{Q_j}}^p(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q))\right)\right\}.$$

Now, the decision rules for photovoltaic systems fault detection by using a multi Q-hesitant fuzzy soft multi-granulation rough set are given as follows :

1. If $T_1 \cap T_2 \cap T_3 \neq \phi$, then the decision maker will choose (m, n) as the optimal object, where $(m, n) \in T_1 \cap T_2 \cap T_3$.
2. If $T_1 \cap T_2 \cap T_3 = \phi$ and $T_1 \cap T_2 \neq \phi$, then the decision maker will choose (m, n) as the optimal object, where $(m, n) \in T_1 \cap T_2$.
3. If $T_1 \cap T_2 \cap T_3 = \phi$ and $T_1 \cap T_2 = \phi$, then $(m, n) \in T_3$ is the determined fault type in level.

In the following, we present our method in an Algorithm 1 for the photovoltaic systems fault detection model by using a multi Q-hesitant fuzzy soft multi-granulation rough set.

Algorithm 1. Photovoltaic systems fault detection

1. Input the universal set (U,Q).
2. Input the set (E,Q).
3. Construct multi Q-hesitant fuzzy soft relation according to m experts.
4. Give the testing set $A_Q \in M^kQHF(E)$.
5. Compute the M^kQHFS operators $\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)$, $\overline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)$,$\underline{\sum_{j=1}^m R_{Q_j}}^p(A_Q)$,$\overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q)$.
6. Calculate $\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)$,$\underline{\sum_{j=1}^m R_{Q_j}}^p(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q)$ and $\left((\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)) \oplus (\underline{\sum_{j=1}^m R_{Q_j}}^p(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q))\right)$.
7. Determine the score function values of $\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)$,$\underline{\sum_{j=1}^m R_{Q_j}}^p(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q)$ and $\left((\underline{\sum_{j=1}^m R_{Q_j}}^o(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^o(A_Q)) \oplus (\underline{\sum_{j=1}^m R_{Q_j}}^p(A_Q) \oplus \overline{\sum_{j=1}^m R_{Q_j}}^p(A_Q))\right)$.
8. Compute $T_1 \cap T_2 \cap T_3$ and $T_1 \cap T_2$, and confirm the determined fault type and its degree.

6.2. Example

For illustrating the efficiency of the proposed algorithm, we use a photovoltaic system fault diagnose problem with multi Q-hesitant fuzzy soft decision information.
Suppose that $U = \{u_1, u_2, u_3\}$ be the set of fault type where u_v stands for, partial shading, delamination, cracks in cells, respectively. $Q = \{q_1 = low, q_2 = high\}$ represent the set of status levels and $E = \{e_1, e_2, e_3\}$ be the set of power measurement where e_s stands for current, voltage, and series resistance, respectively. The photovoltaic system fault detection knowledge base with M^kQHFS information with dimension k = 1 is presented in Tables 2–4.

In photovoltaic system fault detection, assume that we take a fault testing sample, which is presented by the following multi Q-hesitant fuzzy soft information:

$$A_Q = \{\langle((e_1,q_1),0.9,0.4),((e_1,q_2),0.6,0.8,0.4)\rangle,\langle((e_2,q_1),0.1,0.9),((e_2,q_2),0.2,0.5)\rangle,$$
$$\langle((e_3,q_1),0.2,0.4,0.1),((e_3,q_2),0.3,0.7)\rangle\}.$$

Table 2. Knowledge given by expert 1.

R_{Q_1}	e_1q_1	e_1q_2	e_2q_1	e_2q_2	e_3q_1	e_3q_2
(u_1q_1)	{0.1,0.4,0.8}	{0.9,0.3}	{0.5,0.7,0.1}	{0.2,0.6,}	{0.9,0.3}	{0.1,0.2,0.4}
(u_1q_2)	{0.5,0.2}	{0.2,0.6}	{0.3,0.1,0.4}	{0.1,0.8}	{0.1,0.3}	{0.4,0.9,0.3}
(u_2q_1)	{0.8,0.1}	{0.1,0.8,0.7 }	{0.8,0.3}	{0.2,0.6,0.3}	{0.2,0.4,0.9}	{0.6,0.3}
(u_2q_2)	{0.3,0.4,0.5}	{0.8,0.3}	{0.5,0.4,0.3}	{0.1,0.6,0.7}	{0.2,0.9}	{0.3,0.1,0.6}
(u_3q_1)	{0.2,0.1}	{0.4,0.7,0.8}	{0.6,0.9,0.4}	{0.7,0.1}	{0.8,0.7,0.2}	{(0.4,0.5)}
(u_3q_2)	{0.1,0.2,0.4}	{0.7,0.2,0.5}	{0.5,0.6}	{0.1,0.2,0.8}	{0.4,0.2}	{0.7,0.3,0.1}

Table 3. Knowledge given by expert 2.

R_{Q_2}	e_1q_1	e_1q_2	e_2q_1	e_2q_2	e_3q_1	e_3q_2
(u_1q_1)	{0.6,0.2,0.7}	{0.3}	{0.4,0.8,0.2}	{0.1,0.4}	{0.2,0.7,0.3}	{0.5,0.9}
(u_1q_2)	{0.2,0.6}	{0.3,0.4}	{0.2,0.3}	{0.6,0.2}	{0.3,0.9}	{0.1,0.6,0.3}
(u_2q_1)	{0.4,0.2,0.6}	{0.1,0.2}	{0.7,0.5,0.7 }	{0.8,0.3,0.9}	{0.9,0.8,0.4}	{0.4,0.3}
(u_2q_2)	{0.9,0.6}	{0.4,0.8}	{0.3,0.1,0.9}	{0.6,0.5}	{0.7,0.3,0.6}	{0.1,0.7}
(u_3q_1)	{0.2,0.1,0.2}	{0.7,0.4}	{0.1,0.5,0.6}	{0.7,0.1,0.3}	{0.2,0.1}	{0.5,0.9,0.6}
(u_3q_2)	{0.7,0.8}	{0.3}	{0.4,0.8}	{0.1,0.2,0.4}	{0.2,0.7,0.3}	{0.4,0.5}

Table 4. Knowledge given by expert 3.

R_{Q_3}	e_1q_1	e_1q_2	e_2q_1	e_2q_2	e_3q_1	e_3q_2
(u_1q_1)	{0.6,0.2,0.1}	{0.2,0.3}	{0.1,0.2,0.9}	{0.2,0.8}	{0.8,0.5,0.6}	{0.7,0.3,0.6}
(u_1q_2)	{0.5,0.3}	{0.3,0.1,0.4}	{0.2,0.3}	{0.9,0.1,0.6}	{0.5,0.4}	{0.2,0.7,0.1}
(u_2q_1)	{0.4,0.6,0.5}	{0.5,0.1}	{0.2,0.8,0.7 }	{0.8,0.7}	{0.5,0.2,0.1}	{0.4,0.3}
(u_2q_2)	{0.3,0.4}	{0.8,0.2,0.5}	{0.4,0.9}	{0.1,0.2}	{0.8,0.5,0.3}	{0.5,0.3}
(u_3q_1)	{0.4,0.3,0.6}	{0.5,0.4}	{0.4,0.7,0.5}	{0.4,0.6}	{0.7,0.6,0.2}	{0.8,0.9,0.2}
(u_3q_2)	{0.8,0.2}	{0.3,0.1,0.3}	{0.9,0.1}	{0.4,0.6,0.7}	{0.3,0.8}	{0.6,0.4,0.7}

Now, by applying the steps of algorithm that we mentioned above, we first calculate the lower and upper approximation of optimistic and pessimistic multi Q-hesitant fuzzy soft multi-granulation rough sets of A_Q with respect to (U, E, Q, R_{Q_j}), respectively:

$$\underline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) = \{\langle(u_1q_1),\{0.5,0.5,0.4\}\rangle,\langle(u_1q_2),\{0.6,0.6,0.5\}\rangle,$$
$$\langle(u_2q_1),\{0.2,0.5,0.5\}\rangle,\langle(u_2q_2),\{0.5,0.5,0.5\}\rangle,\langle(u_3q_1),\{0.3,0.7,0.6\}\rangle,\langle(u_3q_2),\{0.6,0.7,0.5\}\rangle\},$$

$$\overline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) = \{\langle(u_1q_1),\{0.6,0.5,0.5\}\rangle,\langle(u_1q_2),\{0.3,0.6,0.4\}\rangle,$$
$$\langle(u_2q_1),\{0.4,0.5,0.3\}\rangle,\langle(u_2q_2),\{0.6,0.5,0.6\}\rangle,\langle(u_3q_1),\{0.4,0.7,0.5\}\rangle,\langle(u_3q_2),\{0.6,0.5,0.6\}\rangle\},$$

and

$\underline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) = \{\langle(u_1q_1), \{0.2, 0.4, 0.4\}\rangle, \langle(u_1q_2), \{0.2, 0.4, 0.1\}\rangle,$
$\langle(u_2q_1), \{0.2, 0.4, 0.1\}\rangle, \langle(u_2q_2), \{0.2, 0.4, 0.1\}\rangle, \langle(u_3q_1), \{0.2, 0.4, 0.4\}\rangle, \langle(u_3q_2), \{0.1, 0.4, 0.2\}\rangle\},$

$\overline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) = \{\langle(u_1q_1), \{0.6, 0.8, 0.9\}\rangle, \langle(u_1q_2), \{0.5, 0.7, 0.5\}\rangle,$
$\langle(u_2q_1), \{0.8, 0.8, 0.7\}\rangle, \langle(u_2q_2), \{0.9, 0.9, 0.9\}\rangle, \langle(u_3q_1), \{0.6, 0.9, 0.6\}\rangle, \langle(u_3q_2), \{0.8, 0.8, 0.8\}\rangle\}.$

Then, by Definition 21, we have:

$\underline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) = \{\langle(u_1q_1), \{0.8, 0.75, 0.7\}\rangle, \langle(u_1q_2), \{0.72, 0.84, 0.7\}\rangle,$
$\langle(u_2q_1), \{0.52, 0.75, 0.65\}\rangle, \langle(u_2q_2), \{0.8, 0.75, 0.8\}\rangle,$
$\langle(u_3q_1), \{0.58, 0.91, 0.8\}\rangle, \langle(u_3q_2), \{0.84, 0.85, 0.8\}\rangle\},$

$\underline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) = \{\langle(u_1q_1), \{0.68, 0.88, 0.94\}\rangle, \langle(u_1q_2), \{0.6, 0.82, 0.55\}\rangle,$
$\langle(u_2q_1), \{0.84, 0.88, 0.73\}\rangle, \langle(u_2q_2), \{0.92, 0.94, 0.91\}\rangle,$
$\langle(u_3q_1), \{0.86, 0.94, 0.76\}\rangle, \langle(u_3q_2), \{0.82, 0.88, 0.84\}\rangle\},$

$\left(\left(\underline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q)\right) \oplus \left(\underline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q)\right)\right)$
$= \{\langle(u_1q_1), \{0.936, 0.97, 0.982\}\rangle, \langle(u_1q_2), \{0.888, 0.9712, 0.865\}\rangle,$
$\langle(u_2q_1), \{0.9232, 0.97, 0.905\}\rangle, \langle(u_2q_2), \{0.984, 0.985, 0.982\}\rangle,$
$\langle(u_3q_1), \{0.8656, 0.9946, 0.952\}\rangle, \langle(u_3q_2), \{0.9712, 0.982, 0.968\}\rangle\}.$

In what follows, according to Definition 20, we calculate the score function values of multi Q-hesitant fuzzy soft elements

$S\left(\underline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q)\right) = \{\langle(u_1q_1), \{0.75\}\rangle, \langle(u_1q_2), \{0.753\}\rangle,$
$\langle(u_2q_1), \{0.64\}\rangle, \langle(u_2q_2), \{0.78\}\rangle, \langle(u_3q_1), \{0.76\}\rangle, \langle(u_3q_2), \{0.83\}\rangle\}.$

$S\left(\underline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q)\right) = \{\langle(u_1q_1), \{0.83\}\rangle, \langle(u_1q_2), \{0.65\}\rangle,$
$\langle(u_2q_1), \{(0.81\}\rangle, \langle(u_2q_2), \{0.92\}\rangle, \langle(u_3q_1), \{0.79\}\rangle, \langle(u_3q_2), \{0.84\}\rangle\}.$

$S\left(\left(\underline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{o}(A_Q)\right) \oplus \left(\underline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{3} R_{Q_j}}^{p}(A_Q)\right)\right)$
$= \{\langle(u_1q_1), \{0.96\}\rangle, \langle(u_1q_2), \{0.90\}\rangle, \langle(u_2q_1), \{0.93\}\rangle, \langle(u_2q_2), \{0.98\}\rangle,$
$\langle(u_3q_1), \{0.94\}\rangle, \langle(u_3q_2), \{0.97\}\rangle\}.$

Then, we obtain that

$$T_1 = \left\{(S,T) | \max_{u_sq_t} S\left(\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)(u_sq_t) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)(u_sq_t)\right)\right\} = (3,2),$$

$$T_2 = \left\{(X,Y) | \max_{u_xq_y} S\left(\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)(u_xq_y) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)(u_xq_y)\right)\right\} = (2,2),$$

$$T_3 = \left\{(V,N) | \max_{u_vq_n} S\left((\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)) \oplus (\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q))\right)\right\} = (2,2).$$

According to the above results, the decision maker will choose the type of fault u_2 and condition degree q_2. Thus, we find that the photovoltaic systems fault is initiated by a high degree of delamination.

6.3. Comparative Analysis and Discussion

To explore the effectiveness of the proposed model based on multi-Q hesitant fuzzy soft multi-granulation rough sets, we compare it with the method proposed in [27]. The method given in [27] deals with the decision-making problems of one-dimensional universal sets U and V with hesitant fuzzy information, while the model proposed in the present paper can handle the decision-making problems of two-dimensional universal sets $U \times Q$ and $E \times Q$ with multi hesitant fuzzy soft information that contains much more information to deal with uncertainties in data related to an object with parameter value and the information expressed more precisely and objectively during the decision-making process. Thus, the proposed method is more general and its application domain is wider than that of the method in [27]. Reference [27] proposed a decision-making method based on the TODIM approach, and the basic parts of the previous method compute the dominance degree $\zeta(p_i, p_k) = \sum_{j=1}^{n} \Phi_j(p_i, p_k)$ of each alternative p_i over each alternative p_k and the overall prospect values $\zeta(p_i)$ for alternative p_i according to the following expression, respectively:

$$\Phi_j(p_i, p_k) = \begin{cases} \sqrt{w_{jr}(h_{ij} - h_{kj})/(\sum_{j=1}^{n} w_{jr})} & \text{if} \quad h_{ij} - h_{kj} > 0, \\ 0 & \text{if} \quad h_{ij} - h_{kj} = 0, \\ -\frac{\sqrt{(\sum_{j=1}^{n} w_{jr})(h_{ij} - h_{kj})/w_{jr}}}{\theta} & \text{if} \quad h_{ij} - h_{kj} < 0, \end{cases}$$

and

$$\zeta(p_i) = \frac{\sum_{j=1}^{n} \Phi_j(p_i, p_k) - min_i\{\sum_{j=1}^{n} \Phi_j(p_i, p_k)\}}{max_i\{\sum_{j=1}^{n} \Phi_j(p_i, p_k)\} - min_i\{\sum_{j=1}^{n} \Phi_j(p_i, p_k)\}}.$$

As presented in [27], the optimistic decision criterion $\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)$, pessimistic decision criterion $\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)$ and the weighted decision criterion

$$\frac{1}{2}\left(\underline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{o}(A_Q)\right) \oplus \frac{1}{2}\left(\underline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q) \oplus \overline{\sum_{j=1}^{m} R_{Q_j}}^{p}(A_Q)\right)$$

are three alternatives, the fault types with condition degrees are the criteria, and the obtained evaluation values of the alternative with respect to the criterion are the elements in the decision matrix. The alternative with the largest overall prospect value is the optimal alternative. Then, in the optimal alternative, the fault type and condition degree with the largest score value are the determined fault type with its degree. Through utilizing the above procedure, we could obtain that $\zeta(p_1) = 0.22$, $\zeta(p_2) = 0.35$ and $\zeta(p_3) = 0.36$. Since the greater $\zeta(p_i)$ is, the better alternative p_i will be, the weighted decision criterion can be considered as the best alternative.

Then, we compute the score value of the fault types with condition degrees in the weighted decision criterion, which means the type of fault u_2 and condition degree q_2. Thus, we find that the photovoltaic systems fault is initiated by a high degree of delamination.

Discussion: Based on the above analysis, the results obtained by the proposed method in this paper are consistent with the one obtained using the compared method in [27], which further demonstrate the effectiveness and feasibility of the proposed model. There are two advantages of a multi Q-hesitant fuzzy soft multi-granulation rough set model in photovoltaic systems fault detection procedure. One advantage is that the hesitancy membership function in multi Q-hesitant fuzzy soft sets provides the electrical engineers with much more access to convey their understanding about the electrical knowledge base and another advantage is that the decision makers can control the size of the loss of information by adding another dimension to the universal sets. In light of the above, the greatness of the multi Q-hesitant fuzzy soft multi-granulation rough set model could decline the uncertainty to a great extent and enhance the accuracy and reliability of electrical detection effectively.

7. Conclusions

A multi Q-hesitant fuzzy soft multi-granulation rough set is a new hybrid model, which is a combination of powerful topics: multi Q-hesitant fuzzy soft sets and multi-granulation rough sets. We have defined M^kQHFS rough approximation operators in terms of M^kQHFS relations and M^kQHFS multi-granulation rough approximation operators in terms of M^kQHFS relations. We have investigated the properties of lower and upper M^kQHFS rough approximation operators and lower and upper M^kQHFS multi-granulation rough approximation operators. Finally, we have developed a general framework for dealing with uncertainty decision-making by using the multi Q-hesitant fuzzy soft multi-granulation rough sets. We have used the photovoltaic systems fault detection to indicate the principle steps of the decision methodology. In the future, we will mainly focus on investigating uncertain measures and knowledge reductions of the M^kQHFS rough sets.

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