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Degree Approximation-Based Fuzzy Partitioning Algorithm and Applications in Wheat Production Prediction

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Received: 15 November 2018; Accepted: 30 November 2018; Published: 18 December 2018

Abstract: Recently, prediction modelling has become important in data analysis. In this paper, we propose a novel algorithm to analyze the past dataset of crop yields and predict future yields using regression-based approximation of time series fuzzy data. A framework-based algorithm, which we named DAbFP (data algorithm for degree approximation-based fuzzy partitioning), is proposed to forecast wheat yield production with fuzzy time series data. Specifically, time series data were fuzzified by the simple maximum-based generalized mean function. Different cases for prediction values were evaluated based on two-set interval-based partitioning to get accurate results. The novelty of the method lies in its ability to approximate a fuzzy relation for forecasting that provides lesser complexity and higher accuracy in linear, cubic, and quadratic order than the existing methods. A lesser complexity as compared to dynamic data approximation makes it easier to find the suitable de-fuzzification process and obtain accurate predicted values. The proposed algorithm is compared with the latest existing frameworks in terms of mean square error (MSE) and average forecasting error rate (AFER).

Keywords: wheat production prediction; fuzzy rules; time series; fuzzy regression

1. Introduction

Currently, time series having indecision observations are called fuzzy time series, a term originally defined by Song and Chissom [1,2]. The interpretation obtained from time series is then transformed into fuzzy sets. There is a need for data available in numerous forms multiplied over time. Forecasting is suitable for circumstances where vacillation linked to the outcome is tangible. Time series exploration is an essential mechanism for forecasting the unknown on the basis of its past history. The two significant methods that fit this category are time series and regression. Modern approaches to time series forecasting are influenced by the repetition of history itself. Time series include the recorded values of the variable in the past and also include the present value. This method supports the discovery of arrangements and the inference of future events based on the patterns established as the chief focus material of time series analysis. Solutions to various practical problems related to finance, economics, marketing, and business as well as prediction in economic and sales forecasting, information systems forecasting, stock market prediction, the number of outpatient visits, etc., can be determined using time series.

The idea for the exploratory work on this topic came from an extensive study of work previously done in the niche field of extrapolative demonstration using fuzzy logic. In an agrarian country with a primarily tropical climate, a tropical plant like wheat presents itself as a very lucrative and justifiable topic. To put this into perspective, Asia on its own harvests and ingests more than three quarters of the global wheat production. If economists are to be believed, this dominance by Asia in the global wheat market leads to a reduction in poverty in the region. With an improvement in production and quality of yield, wheat becomes more accessible to people from all walks of life at a lower price, which in turn pushes farmers to invest in sophisticated and valued crops. These crops bring additional income and prosperity to the farmers' families and improve consumer food products. The sustainable computing and management of natural resources has therefore become an imperative field of study [3]. The assessment and forecasting of wheat manufacture certainly require much effort [4].

Various experimental results have been published where prediction has been shown on different datasets based on time series forecasting. Forecasting and predictions help in combating decision-making problems. Askar [5] proposed an autoregressive moving average model to predict wheat crop yield. Sachin [6,7] worked specifically on predicted rice yield for inventory management using a fuzzy time series model. Narendra [8] proposed a model for a terse-period agricultural protraction estimate. Eğrioğlu et al. [9,10] and Wangren et al. [11], on the other hand, implemented a generalized equivalent length breaks implanted for improvement. The former used a genetic approach.

In contrast to the above discussed methods, the proposed method in this study focused on diverse and finer levels of partitions with respect to fuzzy series data. Using this degree approximation method based fuzzy portioning, a higher prediction accuracy was observed. The method of fuzzy partitioning involved the creation of newly generated fuzzy sets based on the underlying data. The time series wheat data undertaken consisted of dynamic data whose feature value changes as a function of time. In partitioning, elements that are more similar than others form members of one set, whereas dissimilar elements form different fuzzy sets. Prediction was done under a fuzzy environment that consisted of ambiguity, improbability, and inaccuracy. The fuzzy intervals were divided based on the frequency of number of times series data. Later, historical time series data analysis was performed by computation of higher order logical fuzzy relations based on the universe of discourse. The novelty of this paper is explained below.

The proposed method used the first 9th and 11th interval time series fuzzy partitioning for wheat production prediction. Based on the interval-based fuzzy partition degree, approximation was applied for real-time wheat produce forecasting. De-fuzzified outputs obtained from approximations were estimated for error and compared with four existing methods. The decision to use fuzzy partitioning in comparison to a regression model was due to the fact that relationships become more complex when dealing with time series data. As proposed in our case, the wheat dataset was dynamic as a function of time, and the use of regression would not produce compact sets. Fuzzy partitioning was a better approach that used degrees of memberships rather than a strict rule as in case of regression. Because the relationship in our time series dataset was not sufficient to apply regression, fuzzy partitioning was a better choice. The method of fuzzy partitioning was closer to human observation behavior as compared to a linear regression model. Furthermore, the new method for forecasting wheat production with a fuzzy time series using degree approximation as a fuzzy relation for forecasting provided lesser complexity in the linear order. Such simplicity was extended to cubic and quadratic polynomial approximation which minimized the time needed to generate relational equations based on complex min-max composition operations, as well as the various hits and trials of the defuzzification process that might be required to achieve better accuracy as used in [6–9,12] as well as by Singh [13]. Two-set partitioning with lower and higher approximation performed over regression analysis finally helped in selecting a best fit line/values that represents the average across all points in graph [14].

The rest of the paper is organized as follows: Section 2 provides the literature overview about the use and progress of time series-based fuzzy partition for prediction problems. Section 3 gives the complete explanation of the proposed framework for the algorithm formulated. Section 3.1 gives a diagram workflow representation of the framework followed by a numerical example explaining the methodology in brief. In Section 3.2, a detailed explanation of the proposed methodology, which we named data algorithm for degree approximation-based fuzzy partitioning (DAbFP), is given with intermediate results. The fuzzy logic relation (FLR) for different intervals is calculated using the wheat yield dataset for different years. Thereafter, average forecasting error rate (AFER) and mean square error (MSE) formulas are also mentioned. Section 4 lists experiments using different degree polynomials and calculating the AFER and MSE for the corresponding polynomials with their respective plots. The proposed algorithm is compared with the existing methods in terms of AFER and MSE with respect to other algorithms. Finally, Section 5 depicts the final conclusions about the method and its implications over the wheat dataset and also emphasizes its future application and scope.

2. Related Works

2.1. Literature Review

Fuzzy-based time series forecasting is used to examine information which is neither explicit nor precise. Researchers have developed fuzzy time series perceptions and definitions to deal with imprecise and vague information systems where decisions or predictions could be carried out. This was later proposed by Song and Chissom, who also portrayed a special dynamic forecasting process with linguistic values [1,2,15]. Fuzzy forecasting to predict links in social networks has been described by authors in [16]. Later, the authors in [12] formally defined a fuzzy time series model described in Section 2.2.

Qiu, Liu, and Li [17] proposed a particle swarm optimization technique for similar forecasting. Primarily, data dealing with time series from the University of Alabama [18] were used. An average autocorrelation function was framed to give high forecasting accuracy. In order to analyze time series using computed fuzzy logical relations of higher order, Garg et al. [19], Son [20], Hunrag [21,22], Hwang and Chen [23], Lee Wang and Chen [24], Chu and Kim [25], and Sheta [26] developed extensive fuzzy as well as decision based forecasting methods in order to augment forecasting accuracy, each having minor variations. Lee [27] proposed a fuzzy candlestick [28] pattern to store financial expertise. To obtain highly mosaic matrix computations, a multivariate heuristic model was modelled and implemented in [29]. A determination of the interval over varying length was given by Hiemstra [30]. A number of repetitions of fuzzy relationships were used to determine the weights in fuzzy time series data in [31–33]. Regular increasing Monotone (RIM) quantifiers were used by Garg et al. [34,35] to design a priority matrix.

Several distinguished and relative works have been done by Klir et al. [36] and Dostal [37] with some native approaches for prediction. The use of optimization techniques in commercial and communal sector was also demonstrated by Dostal [38]. Li et al. [39] introduced fuzzy logic linking to chaos theory. Peters [40,41] extended it to fractal market analysis in capital markets. Trippi [42] represented fuzzy logic to chaos and non-linear dynamics in financial markets. Altroc [43] applied to business and finance using neuro-fuzzy. Hamam et al. [44] evaluated superiority of understanding of haptic centered uses based on fuzzy logic. Alreshoodi [45] researched an experiential learning established on a fuzzy logic method to measure the QOS/QOE correlation for covered video streaming. Doctor et al. [46] entrenched agent-based method for comprehending ambient intellect. Wang et al. [47] generated fuzzy instructions by learning from instances. In [48] a high order approximation for forecasting tourism demands in turkey using fuzzy time series data and artificial neural network is proposed. Another, new approach using fuzzy type-2 logic and fractal theory was given by Castillo and Melin [49]. The experimental study was done to establish the span of breaks with fuzzy time series [50]. A non-linear optimization with polynomial time series is another work presented by authors [51]. The forecasting models based on Event discretization function were placed forward.

In this paper, the dataset used for forecasting wheat production is taken from a source [52]. Son et al. [53] established a fuzzy clustering method for weather forecasting. Also, a neuro-fuzzy system has been designed and evaluated for insurance forecasting [54]. In [54], the authors have used an ensemble learning technique with limited fuzzy weights. Adaptive neuro-fuzzy [55] framework is another work by [56] in field of wheat production forecasts.

In [57], a different dataset for wheat production forecasts using soil properties has been used. Some of the properties of soil like shear strength has been predicted in [58]. Similarly, an adaptive fuzzy rule-based technique with automatic parameter updating has been used to model financial time series in [59]. A systematic approach has been discussed in [60] for detection of structural breaks in time series, namely the fuzzy transform and other method of fuzzy natural logic. It is based on F-transform to calculate slope of time-series. Another problem of the separable verification of fuzzy binary relations has been addressed in [61] providing necessary conditions and a well-organized algorithm for checking the same.

2.2. Mathematical Preliminary

This section presents the preliminaries needed to understand any problem of time series forecasting

Definition 1 [62]**.** *Given F(t) as the group of all possible values of fuzzy time series at time t, F(t − 1) is group of all possible values at t − 1 having Z as a fuzzy relation between F(t) and F(t − 1) where Z is a union of all fuzzy relations defined as:*

$$\begin{gathered} Z = Z(t, t-1) = f_{i1}(t-1)Xfjo(t) \cup fi2(t-2)X \\ fj1(t-1) \cup \ldots\ldots \cup fin(t-n)Xfjn-1(t-n+1) \end{gathered} \tag{1}$$

Then a first order time invariant series model is expressed as

$$F(t) = F(t-1)^{\circ}Z(t, t-1) \tag{2}$$

Definition 2 [62]**.** *Let be U the universe of discourse, U = {u1, u2, u3 . . . } and U be a finite set A fuzzy set F of U can be expressed as follows:*

$$\sum_{i=1}^{n} \mu_A(u_i)/u_i \tag{3}$$

$$= \frac{\mu_A(u_1)}{u_1} + \frac{\mu_A(u_2)}{u_2} + \frac{\mu_A(u_3)}{u_3} + \frac{\mu_A(u_4)}{u_4} \ldots\ldots + \frac{\mu_A(u_n)}{u_n}. \tag{4}$$

where "+" is operator ∪ and "/" is separator.

Definition 3 [63]**.** *Assume that F(t) is a fuzzy time series, and Z (t, t − 1) is a first order model of time series F(t). If*

$$\begin{gathered} Z(t, t-1) \in Z(t-1, t-2) \forall \text{ time } (t) \\ \text{then } F(t) \rightarrow \text{Time invariant fuzzy time series} \end{gathered} \tag{5}$$

$$\begin{gathered} Z(t, t-1) \in t \;\&\; \neq Z(t-1, t-2) \;\forall \text{ time } (t) \\ \text{then } F(t) \rightarrow \text{Time variant fuzzy time series} \end{gathered} \tag{6}$$

Definition 4 [**64**]. *Given F(t) as the time series data D, with Ft(I) as fuzzy set, then defuzzified value Fd is defined as the z-value with the highest membership degree.*

$$\mu_A(z_o) \geq \mu_A(z) \;\forall\; z \in F \tag{7}$$

$$z_0 = \max(deg(\mu_A))in\ F \tag{8}$$

Definition 5 [64]**.** *Given F(t) as the time series data D, with Ft(I) as fuzzy set, a quasi-arithmetic mean for fuzzified output is:*

$$F(t)[\mathrm{I} = 1 \ldots n] = \left[1/n \sum_{i=1}^{n} x_i\right] 1/\alpha \tag{9}$$

α = 1 for arithmetic means
Forecasting models are categorized as follows:
AR (Autoregressive) Models,
MA (Moving Average) Models,
ARMA (Autoregressive Moving Average) Models,
ARIMA (Autoregressive Integrated Moving Average) Models.

Definition 6 [65]**.** *AR model of a given order r is defined as:*

$$A_t = \rho_1 W_{t-1} + \ldots\ldots + \rho_r W_{t-r} + noise(\epsilon) \tag{10}$$

where W_{t-1} .... are independent variables and $\rho_1 \ldots \rho_r$ are model parameters.
AR(r) model =
(a) $A_2 = \rho_1 W_1 + e_2$
(b) $A_3 = \rho_1 W_2 + \rho_2 W_1 + e_3$
(c) $A_n = \rho_1 W_{n-1} + \rho_{n-1} W_1 + e_n$

$$Y_{((n-1)w1)} = W_{((n-1)w(n-1))} \rho_{((n-1)w)} + e_{((n-1)w1)} \tag{11}$$

Substituting values for ρ parameter aids in prediction.

3. The Proposed Framework

3.1. The Need of This Framework

Recent studies on wheat production forecasts have been conducted in [56–58]. Here, the later of an artificial neural network with fuzzy systems have been used for predicting forecast for a 5-degree polynomial in only two periods. In another work, ensemble learning with limited fuzzy weights was used. While the former uses another artificial neural network to forecast production based on energy inputs, another decision making analysis has been done in [66]. Several prediction procedures on case basis has been done by authors in [67–70]. The above stated method provided prediction using support vector machines based on soil properties. A similar prediction was performed in [7–9] where data are not partitioned and fuzzified as per time series.

The proposed method in this paper will take the yield data in reference to time series fuzzified in diverse partitions and give precise prediction. The precision comes from the 9 or 11-level linguistic partition carried out over large time series scale. Our method outperforms the existing 4 methods in terms of RMSE and AFER. Hence, a consolidated framework to perform predictions over multiple and diverse linguistic partitions is needed.

3.2. The Workflow Diagram

In this section, an overview of the proposed framework with simulation steps is given in Figure 1. Table 1 gives the linguistic fuzzy set partitioning while Table 2 gives the frequency distribution over 9 interval partitioning.

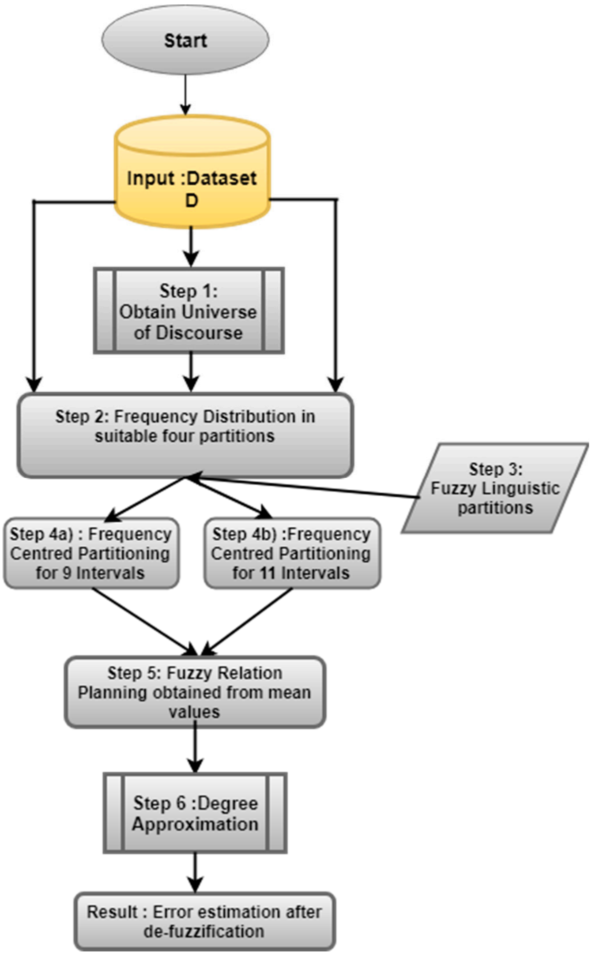


Figure 1. Degree Approximation-Based Fuzzy Partitioning Algorithm and Applications DAbFP simulation Workflow.

Table 1. Fuzzy linguistic partitions.

F1	very meagre produce
F2	meagre produce
F3	better than poor produce
F4	not so quality produce
F5	average production
F6	superior produce
F7	very superior produce
F8	Very very superior produce
F9	tremendous produce

Table 2. Frequency Distribution (9 Interval).

Fuzzy Sets	Upper	Lower	Frequency
F1	233,200	343,355	3
F2	233,355	343,511	2
F3	233,511	343,666	2
F4	233,666	343,822	3
F5	233,822	344,133	4
F7	234,133	344,288	3
F8	234,288	344,444	2
F9	234,444	344,600	1

3.3. DAbFP Algorithm

The proposed algorithm is performed on the source dataset taken under following steps:
Step 1: Let D denotes the source dataset variable.

$$U = [D_{min} - x, D_{max} + y] \tag{12}$$

Using Definition 2, Universe of Discourse (U) is defined as

$$D_{min}, D_{max} \in Max, Min\{dataset(D)\} \tag{13}$$

$$x, y \in R+, \text{ given } R \text{ as real numbers.} \tag{14}$$

Step 2: Partition the dataset D into suitable four frequencies to perform subsequent forecasting steps to each group:

$$D \to \frac{D}{fi}, \{i \in 1, 2 \ldots 4\} \tag{15}$$

Step 3: Using above partitioned data as D_{new}, we define fuzzy sets as F1, F2 ... F7 linguistically mapped over the universe of discourse U defined as follows:

$$\text{F1} = \frac{1}{q1} + \frac{0.5}{q2} + \frac{0}{q3} + \frac{0}{q4} + \frac{0}{q5} + \frac{0}{q6} + \frac{0}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{16}$$

$$\text{F2} = \frac{0.5}{q1} + \frac{1}{q2} + \frac{0.5}{q3} + \frac{0}{q4} + \frac{0}{q5} + \frac{0}{q6} + \frac{0}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{17}$$

$$\text{F3} = \frac{0}{q1} + \frac{0.5}{q2} + \frac{1}{q3} + \frac{0.5}{q4} + \frac{0}{q5} + \frac{0}{q6} + \frac{0}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{18}$$

$$\text{F4} = \frac{0}{q1} + \frac{0}{q2} + \frac{0.5}{q3} + \frac{1}{q4} + \frac{0.5}{q5} + \frac{0}{q6} + \frac{0}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{19}$$

$$\text{F5} = \frac{0}{q1} + \frac{0}{q2} + \frac{0}{q3} + \frac{0.5}{q4} + \frac{1}{q5} + \frac{0.5}{q6} + \frac{0}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{20}$$

$$\text{F6} = \frac{0}{q1} + \frac{0}{q2} + \frac{0}{q3} + \frac{0}{q4} + \frac{0.5}{q5} + \frac{1}{q6} + \frac{0.5}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{21}$$

$$\text{F7} = \frac{0}{q1} + \frac{0}{q2} + \frac{0}{q3} + \frac{0}{q4} + \frac{0.5}{q5} + \frac{1}{q6} + \frac{0.5}{q7} + \frac{0}{q8} + \frac{0}{q9} \tag{22}$$

$$\text{F8} = \frac{0}{q1} + \frac{0}{q2} + \frac{0}{q3} + \frac{0}{q4} + \frac{0}{q5} + \frac{0.5}{q6} + \frac{1}{q7} + \frac{0.5}{q8} + \frac{0}{q9} \tag{23}$$

$$\text{F9} = \frac{0}{q1} + \frac{0}{q2} + \frac{0}{q3} + \frac{0}{q4} + \frac{0}{q5} + \frac{0}{q6} + \frac{0.5}{q7} + \frac{1}{q8} + \frac{0.5}{q9} \tag{24}$$

Every partition obtained in the partitioning based on frequency is represented by F(I), where (I) indicates the intervals inside its value exist. The value of the outcome increases on increasing the value of "I". The same taxonomy helps to provide an evocative vision to the researchers. For instance, every interval can be signified by fuzzy partitions if we are operating on 9 partitions, as presented beneath:

Hence, growth in the suffix (I) is evidently related through greater harvest in the production of wheat and having the same taxonomy. Subsequently, Fuzzy Logic Relationships (FLR) is recognized for the specified group of values. It can be elucidated over the particular instance. Here, $q1, q2 \ldots q7 \in$ fixed length intervals.

Step 4: From above partitioned data as D_{new}, we define 11 fuzzy sets as F1, F2 ... F11 over U. Similar equations (as 5 to 13) are observed for 11 intervals. Here, $q1, q2 \ldots q11 \in$ fixed length intervals.

Step 5: Mean of middle values of fuzzy partitions on the Right-Hand Side of Fuzzy logic relation (FLR) is calculated. This calculation is performed for degree approximation. For instance, in the 2nd order FLR, F4 <- F2, F7. If P and Q are the centers of Interval F2 and F7 respectively then

$$\mathrm{R} = \mathrm{P} + \mathrm{Q}/2 \tag{25}$$

where for fuzzy partition F4, R is the center. Likewise, for 3rd order FLR:

If F <- F2, F7, F3 where P, Q, R are the centers for Interval F2, F7 and F3 respectively then

$$\mathrm{S} = (\mathrm{P} + \mathrm{Q} + \mathrm{R})/3 \tag{26}$$

Here, S is the mean fuzzy value for a particular forecast year. It is used in Linear Regression Model as a variable, for thorough de-fuzzification. From this, the results can be used to calculate the forecast value:

$$\text{Mean Fuzzy Value(MFV)} = \sum_{i=1}^{i=n} value(Fi(d))/ \tag{27}$$

Here, n is total number of values while $value(Fi(d))$ is the fuzzy value at degree.

As per the steps followed in the proposed algorithm, Tables 3–5 give the intermediate results. In this section, the concluding part of the devised algorithm is explained with the results presented in Tables 6–9.

Step 6: After degree approximation based on fuzzy logic relation, defuzzification is performed using regression analysis. On plotting the points, we select a Best Fit line that represents average across all points in graph. Thereafter, the equation of line is estimated which can be linear or polynomial of higher degrees 2, 3, 4, 5 or 6. In the consequent section, we use two important constraints to associate the outcome as stated below:

Average Forecasting Error Rate (AFER) =

$$\sum_{i=1}^{i=n} ((mod(X_I - Y_i)/X^i)/n) * 100 \tag{28}$$

Mean Square Error (MSE).

$$= \sum_{i=1}^{n} ((X - Y)2)/n \tag{29}$$

Here, X_i is the actual production cost whereas Y_i is the predicted value.

3.4. Numerical Example

In year 1981, Produce = 3552 (fits to F3).

In year 1982, Produce = 4177 (fits to F7).

In year 1983, Produce = 3372 (fits to F2).

In year 1984, Produce = ? (Assume this request to be forecast, let F be the partition where value is contained).

Hence, the above Logical Relationships can assist forecast for a specific year by means of the values obtained for earlier years and then creating a relationship amongst values. Fuzzy Logical Relationship of Order 3 is: F = F2, F7, F3, here F is the forecast partition for produce in year 1984.

Now, an appropriate defuzzification procedure can be functional on these values to forecast value of the harvest in year 1984 (conferred in step 5), agreed that appropriate calculations are done for fuzzy sets which resemble to the fuzzy partitions in the previous years.

By means of formulation stated above, the results are shown in Tables 3–9 is calculated for 9 and 11 partitions in together of order two and order three FLR.

Table 3. Frequency Centered Partitioning (9 Interval).

Fuzzy Sets	Upper	Lower	New Fuzzy Sets
AF1A	932,007	3252.76	Z1
	3253.76	3303.8	Z2
	3303.8	3356.66	Z3
AF2A	3356.66	3432.435	Z4
	3432.435	3512.2	Z5
AF3A	3512.2	3589.985	Z6
	3589.985	3677.75	Z7
AF4A	3677.75	3729.5	Z8
	3729.5	3771.45	Z9
	3771.45	3823.3	Z10
AF5A	3823.3	3862.1985	Z11
	3862.1985	3900.175	Z12
	3900.175	3949.9725	Z13
	3949.9725	3988.865	Z14
AF7A	4234.3	4285.25	Z15
	4285.25	4238	Z16
	4238	4289.95	Z17
AF8A	4289.95	4367.735	Z18
	4367.735	4445.5	Z19
AF9A	4445.5	4600	Z20

Table 4. Frequency Distribution (11 Interval).

Fuzzy Sets	Upper	LOWER	Frequency Uency
A1	3200	3327	3
A2	3327	3454	1
A3	3454	3581	2
A4	3581	3709	3
A5	3709	3836	1
A6	3836	4091	4
A7	3937	4120	3
A8	4091	4218	2
A9	4218	4345	2
A10	4345	4472	1
A11	4472	4600	1

Table 5. Frequency Centered Partitioning (11 Interval).

Fuzzy Sets	Upper	Lower	New Fuzzy Sets
A1	3200.000	3253.423	NF1
	3253.423	3295.847	NF2
	3295.847	3330.270	NF3
A2	3330.270	3460.540	NF4
A3	3460.540	3521.175	NF5
	3521.175	3579.810	NF6
A4	3579.810	3630.233	NF7
	3630.233	3670.657	NF8
	3670.657	3711.080	NF9
A5	3711.080	3841.350	NF10
A6	3841.350	3870.168	NF11
	3870.168	3900.985	NF12
A7	3900.985	3929.803	NF13
	3929.803	3970.720	NF14
A8	4091.990	4149.525	NF15
	4149.525	4220.160	NF16
A9	4220.160	4290.795	NF17
	4290.795	4351.430	NF18
A10	4351.430	4469.700	NF19
A11	4469.700	4600.000	NF20

Table 6. (9 INTERVALS, Fuzzy Logic Relation 2nd DEGREE).

Year	Product	Fuzzy Sets	FLR Relations	Avg.	Mid Fuzzy Value
1981	3552	Z6	-	-	3549.9875
1982	4177	Z15	-	-	4159.225
1983	3372	Z4	Z4<-Z15,Z6	3854.60625	3394.4375
1984	3455	Z5	Z5<-Z4,Z15	3776.83125	3472.2125
1985	3702	Z8	Z8<-Z5,Z4	3433.325	3692.575
1986	3670	Z8	Z8<-Z8,Z5	3582.39375	3692.575
1987	3865	Z12	Z12<-Z8,Z8	3692.575	3880.5315
1988	3592	Z7	Z7<-Z12,Z8	3786.55325	3627.7625
1989	3222	Z1	Z1<-Z7,Z12	3754.147	3225.925
1990	3750	Z9	Z9<-Z1,Z7	3426.84375	3744.425
1991	3851	Z11	Z11<-Z9,Z1	3485.175	3841.644
1992	3231	Z1	Z1<-Z11,Z9	3793.0345	3225.925
1993	4170	Z15	Z15<-Z1,Z11	3533.7845	4159.225
1994	4554	Z20	Z20<-Z15,Z1	3692.575	4522.2
1995	3872	Z12	Z12<-Z20,Z15	4340.7125	3880.5315
1996	4439	Z19	Z19<-Z12,Z20	4201.36575	4405.5125
1997	4266	Z17	Z17<-Z19,Z12	4143.022	4262.925
1998	3219	Z1	Z1<-Z17,Z19	4334.21875	3225.925
1999	4305	Z18	Z18<-Z1,Z17	3744.425	4327.7375
2000	3928	Z13	Z13<-Z18,Z1	3776.83125	3919.419

Table 7. (11 INTERVALS, Fuzzy Logic Relation 2nd DEGREE).

Year	Product	Fuzzy Sets	FLR relation	Avg.	Fuzzy
1981	3552	F6	-	-	3549.9925
1982	4177	F16	-	-	4186.3425
1983	3372	F4	F4<-F16,F6	3868.1675	3390.905
1984	3455	F5	F5<-F4,F16	3788.62375	3486.3575
1985	3702	F9	F9<-F5,F4	3438.63125	3687.868325
1986	3670	F9	F9<-F9,F5	3587.112913	3687.868325
1987	3865	F11	F11<-F9,F9	3687.868325	3852.25875
1988	3592	F7	F7<-F11,F9	3770.063538	3603.021665
1989	3222	F1	F1<-F7,F11	3727.640208	3221.211665
1990	3750	F10	F10<-F1,F7	3412.116665	3772.714995
1991	3851	F11	F11<-F10,F1	3496.96333	3852.25875
1992	3231	F2	F2<-F11,F10	3812.486873	3263.634995
1993	4170	F16	F16<-F2,F11	3557.946873	4186.3425
1994	4554	F20	F20<-F16,F2	3724.988748	4536.35
1995	3872	F12	F12<-F20,F16	4361.34625	3884.07625
1996	4439	F19	F19<-F12,F20	4210.213125	4409.065
1997	4266	F17	F17<-F19,F12	4146.570625	4249.9775
1998	3219	F1	F1<-F17,F19	4329.52125	3221.211665
1999	4305	F18	F18<-F1,F17	3735.594583	4313.6125
2000	3928	F13	F13<-F18,F1	3767.412083	3915.89375

Table 8. 9 INTERVALS, Fuzzy Logic Relation 3rd DEGREE.

Year	Product	Fuzzy Sets	FLR Relations	Avg	Mid Fuzzy Value
1981	3552	Z6	-	-	3549.9875
1982	4177	Z15	-	-	4159.225
1983	3372	Z4	-	-	3394.4375
1984	3455	Z5	Z5<-Z4,Z15,Z6	3701.216667	3472.2125
1985	3702	Z8	Z8<-Z5,Z4,Z15	3675.291667	3692.575
1986	3670	Z8	Z8<-Z8,Z5,Z4	3519.741667	3692.575
1987	3865	Z12	Z12<-Z8,Z8,Z5	3619.120833	3880.5315
1988	3592	Z7	Z7<-Z12,Z8,Z8	3755.227167	3627.7625
1989	3222	Z1	Z1<-Z7,Z12,Z8	3733.623	3225.925
1990	3750	Z9	Z9<-Z1,Z7,Z12	3578.073	3744.425
1991	3851	Z11	Z11<-Z9,Z1,Z7	3532.704167	3841.644
1992	3231	Z1	Z1<-Z11,Z9,Z1	3603.998	3225.925
1993	4170	Z15	Z15<-Z1,Z11,Z9	3603.998	4159.225
1994	4554	Z20	Z20<-Z15,Z1,Z11	3742.264667	4522.2
1995	3872	Z12	Z12<-Z20,Z15,Z1	3969.1186667	3880.5315
1996	4439	Z19	Z19<-Z12,Z20,Z15	4187.318833	4405.5125
1997	4266	Z17	Z17<-Z19,Z12,Z20	4269.414667	4262.925
1998	3219	Z1	Z1<-Z17,Z19,Z12	4182.989667	3225.925
1999	4305	Z18	Z18<-Z1,Z17,Z19	3964.7875	4327.7375
2000	3928	Z13	Z13<-Z18,Z1,Z17	3938.8625	3919.419

Table 9. 11 INTERVALS, Fuzzy Logic Relation 3rd DEGREE.

Year	Product	Fuzzy Sets	FLR Relation	Avg	Fuzzy
1981	3552	F6	-	-	3549.9925
1982	4177	F16	-	-	4186.3425
1983	3372	F4	-	-	3390.905
1984	3455	F5	F5<-F4,F16,F6	3709.08	3486.3575
1985	3702	F9	F9<-F5,F4,F16	3687.868333	3687.868325
1986	3670	F9	F9<-F9,F5,F4	3521.710275	3687.868325
1987	3865	F11	F11<-F9,F9,F5	3620.69805	3852.25875
1988	3592	F7	F7<-F11,F9,F9	3742.665133	3603.021665
1989	3222	F1	F1<-F7,F11,F9	3714.382913	3221.211665
1990	3750	F10	F10<-F1,F7,F11	3558.830693	3772.714995
1991	3851	F11	F11<-F10,F1,F7	3532.316108	3852.25875
1992	3231	F2	F2<-F11,F10,F1	3615.395137	3263.634995
1993	4170	F16	F16<-F2,F11,F10	3629.536247	4186.3425
1994	4554	F20	F20<-F16,F2,F11	3767.412082	4536.35
1995	3872	F12	F12<-F20,F16,F2	3995.442498	3884.07625
1996	4439	F19	F19<-F12,F20,F16	4202.25625	4409.065
1997	4266	F17	F17<-F19,F12,F20	4276.497083	4249.9775
1998	3219	F1	F1<-F17,F19,F12	4181.039583	3221.211665
1999	4305	F18	F18<-F1,F17,F19	3960.084722	4313.6125
2000	3928	F13	F13<-F18,F1,F17	3928.267222	3915.89375

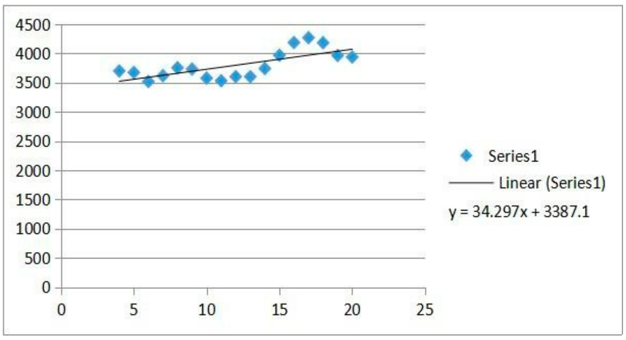
4. Results and Discussion

4.1. Linear Polynomial

A linear polynomial relation is defined as:

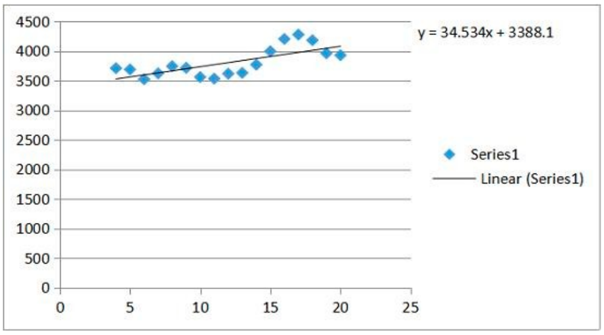
$$Y = mX + C \text{ where } C \in constant$$

Here, variable Y provides the value that is predicted. Output for year and the input variable X fed to equation using form (14, 15, 16, which relates to years 1981, 1982, 1983, 1984 . . .). By means of Figure 2a,b, one can calculate the yearly predicted results and then estimating the AFER and MSE as given in Table 10.

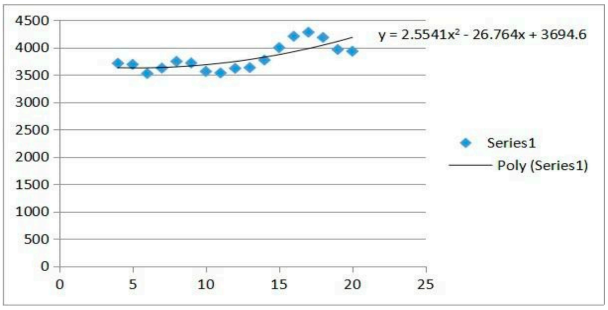


(a)

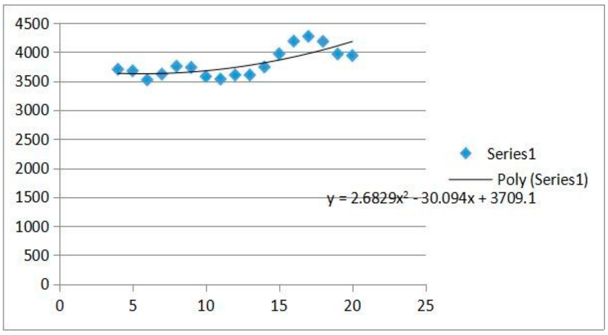
Figure 2. *Cont.*



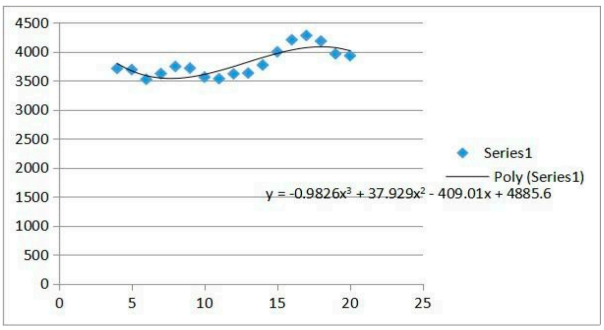
(b)



(c)

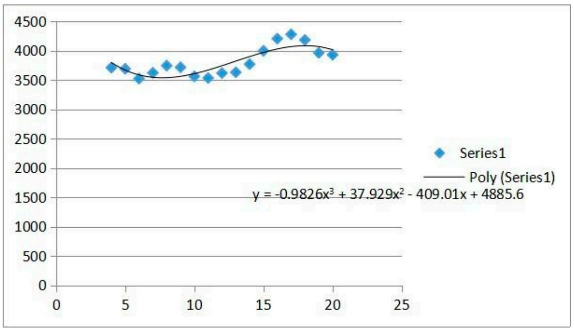


(d)



(e)

Figure 2. *Cont.*



(f)

Figure 2. (**a**–**f**): 9th to 11th Interval for fuzzified degree-based approximation AFER and MSE.

Table 10. MSE and AFER values for all intervals.

9th Interval		**11th Interval**	
FLR 2nd Degree	**FLR 3rd Degree**	**FLR 2nd Degree**	**FLR 3rd Degree**
-	-	-	-
-	-	-	-
42,986.55822	-	44,818.16021	-
22,492.80058	4800.826944	23,809.72442	5074.567696
5095.1044	20,567.86223	4501.739025	19,945.9129
188.677696	5947.185924	90.136036	5579.492416
33,522.68046	56,558.82804	32,002.70545	55,301.16624
13,352.2647	4826.914576	14,330.00526	5237.706384
261,321.3504	224,460.8555	265,543.3655	227,439.3328
78.1456	397.2049	166.6681	274.2336
4424.378256	7505.276689	3904.875121	6893.316676
335,389.2404	322,242.4169	340,019.6045	326,621.3941
111,708.356	113,595.2875	109,089.5024	110,861.0298
479,672.5971	471,614.5746	474,288.4066	465,702.5158
226.8036	873.498025	357.777225	1163.4921
276,987.4796	253,157.9099	272,989.5303	248,358.7027
107,355.8331	87,527.8142	104,900.1977	84,577.43568
555,013.0801	616,925.4189	560,578.6435	625,225.4669
99,454.4525	70,892.79005	97,145.04576	67,992.64852
7617.7984	21,036.6016	8266.4464	22,734.6084
MSE = 130,938.2001	MSE = 134,290.0745	MSE = 130,933.4741	MSE = 134,057.8249
AFER = 7.352165941	AFER = 7.50564575	AFER = 7.360701563	AFER = 7.497227115

4.2. Quadratic Polynomial

A linear polynomial relation is defined as:

$$Y = Ax^2 + Bx + C$$

Here, variable Y will give the value that is predicted Output for year and the input variable X fed to equation using form (1, 2, 3, 4 . . . ., which relates to years 1981, 1982, 1983, 1984 . . .). By means of Figure 2c,d, one can calculate the yearly predicted results and then estimating the AFER and MSE was calculated matching to figure as given in Table 11.

Table 11. MSE and AFER values for all intervals.

9th Interval		11th Interval	
FLR 2nd Degree	**FLR 3rd Degree**	**FLR 2nd Degree**	**FLR 3rd Degree**
-	-	-	-
-	-	-	-
86,872.72867	-	86,973.79553	-
42,656.78884	31,205.36382	43,329.25339	30,070.88937
1748.494225	5821.308506	1515.5449	5985.730056
53.41855744	2014.178496	11.20374784	1939.204525
38,394.91329	55,270.0822	36,517.22259	54,101.41093
7616.54162	2309.148473	8616.88906	2698.84406
222,169.1256	187,981.991	227,928.7106	192,375.311
1499.2384	5409.6025	1044.5824	4573.8169
13,907.90945	21,993.80947	12,407.55388	20,095.30221
278,480.7993	253,320.5535	285,381.6062	260,326.8975
145,760.7025	158,971.1792	141,014.3692	153,420.3511
536,451.9471	548,144.1831	527,917.3339	538,011.1009
174.636225	113.5823063	63.5209	17.53515625
290,677.1154	275,185.8551	285,894.6794	269,126.8781
103,181.132	85,931.00097	100,960.487	83,089.84966
599,950.6294	668,580.3041	603,535.0764	674,659.1905
66,975.2661	39,664.34711	66,480.0217	38,764.05762
30,520.09	63,695.6644	30,317.7744	63,988.7616
MSE = 137,060.6376	MSE = 141,506.5973	MSE = 136,661.6458	MSE = 140,779.1254
AFER = 7.687795338	AFER = 7.758800407	AFER = 7.653515775	AFER = 7.720197268

4.3. Cubic Polynomial

The cubic polynomial relation is given as:

$$Y = Ax^3 + Bx^2 + Cx + D$$

Here, variable Y will give the value that is predicted output for each year and input variable X fed to equation using form (1, 2, 3, 4 . . . ., which relates to years 1981, 1982, 1983, 1984 . . .). By means of Figure 2e,f, one can calculate the yearly predicted results and then estimating the AFER and MSE matching to figure as given in Table 12.

Table 12. MSE and AFER values for all intervals.

9th Interval		11th Interval	
FLR 2nd Degree	**FLR 3rd Degree**	**FLR 2nd Degree**	**FLR 3rd Degree**
-	-	-	-
-	-	-	-
290,632.1524	-	313,062.5185	-
77,523.93313	103,695.3347	81,762.1411	114,607.7066
7830.037656	1600	8025.920156	1299.6025
15,835.50426	6608.649401	17,384.10606	7268.858358
120,277.2455	98,004.56003	125,760.2382	103,028.674
4043.230265	2068.721482	4999.281871	2928.454871
119,470.9499	119,186.2386	116,370.5638	114,544.0704
14,713.69	19,909.21	14,859.61	20,793.64
21,494.40413	34,105.81594	20,329.99744	33,467.62901
309,726.0861	253,318.1376	319,561.3766	260,429.7685
89,440.41254	131,289.6959	81,819.51089	122,710.9307
367,945.1196	452,673.8343	348,552.3228	431,268.5495
18,985.39516	6037.29	24,176.36266	9254.44

Table 12. *Cont.*

9th Interval		11th Interval	
FLR 2nd Degree	**FLR 3rd Degree**	**FLR 2nd Degree**	**FLR 3rd Degree**
150,648.9335	185,762.3792	137,798.9429	170,127.8712
41,022.08703	46,271.79584	35,822.33797	39,841.27777
674,680.875	729,631.6726	684,819.8035	744,583.9843
109,253.5845	55,372.86685	112,612.3927	56,584.23018
4830.25	11,491.84	8172.16	7779.24
MSE = 135,464.105	MSE = 132,766.3554	MSE = 136,438.3104	MSE = 131,795.231
AFER = 7.752071496	AFER = 8.228273107	AFER = 7.744400101	AFER = 8.305847824

4.4. Results

From the above analysis, we have computed the mean MSE and AFER values for final predicted values where the degree approximation is computed accordingly. The proposed algorithm is initially compared with baseline method such as Chissom [1,2] on benchmark data for forecasting the enrollments of University of Alabama. The superiority in values in terms of MSE and AFER marks it as a probable candidate for predicting wheat production in future as shown in Table 13. For further performance analysis, the proposed method is hereby compared with existing methods as shown in Tables 14 and 15 for both 9 and 11 intervals. The proposed algorithm outperforms the existing ones in terms of MSE and AFER; thereby proving to be a best fit for wheat produce prediction. The MSE and AFER of the proposed algorithm comes out to be 362,119.88 and 5,107,713.738 for 3rd degree and 2nd degree polynomial in 9th interval as compared to the MSE of 36,559.88 and AFER AS 11.92547975 for 3rd degree polynomial of Yalaz et al. [64]. Similarly, the values of MSE and AFER are compared in Tables 13 and 14 for 9th interval 2nd degree polynomial. Also, the evaluation statistics of our proposed algorithm outperforms in 11th interval.

In Figure 3, the FLR 3rd degree MSE is generally higher than FLR 2nd degree MSE except in case of polynomial degree 3. We can infer that Linear FLR 2nd degree polynomial has the lowest MSE among all the cases for 9th interval. It is convenient to estimate a particular case is the best among all others. As it can be inferred from the graph, total 10 cases for 9th interval has been monitored. We have also worked on 8 cases in 11th interval partitioning as shown in Figure 4.

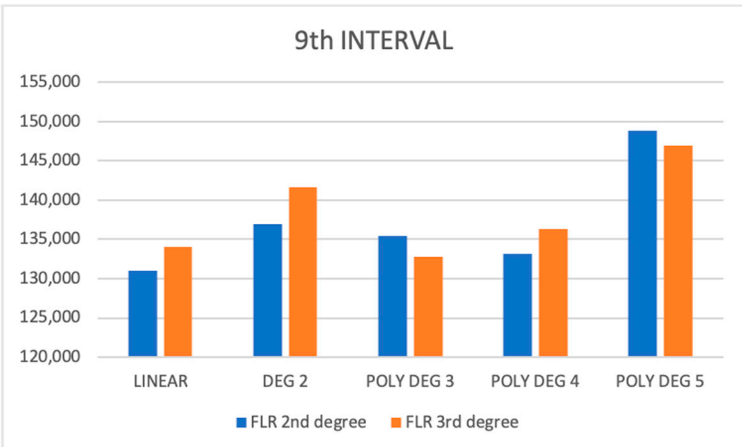


Figure 3. Comparison of MSE among all degrees in 9th interval.

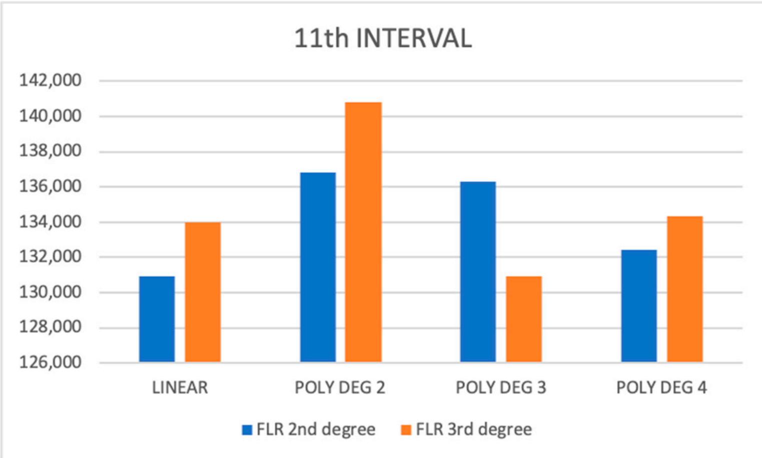


Figure 4. Comparison of MSE among all degrees in 11th interval.

Table 13. Comparison of MSE and AFER values for one set of intervals with Chissom [1,2] on enrollment data.

Year	Enrollement Data	Chissom [1,2]	Proposed Method (DAbFP)	
			2nd Degree	3rd Degree
1971	13,055	-	13,561	13,261
1972	13,563	14,000	13,756	13,786
1973	13,867	14,000	13,756	13,776
1974	14,696	14,000	14,451	14,431
1975	15,460	15,500	15,361	15,271
1976	15,311	16,000	15,361	15,661
1977	15,603	16,000	15,721	15,321
1978	15,861	16,000	15,900	15,887
1979	16,807	16,000	17,085	17,067
1980	16,919	16,813	17,085	17,067
1981	16,388	16,813	16,487	16,480
1982	15,433	16,789	15,385	15,371
1983	15,497	16,000	15,385	15,371
1984	15,145	16,000	15,029	15,012
1985	15,163	16,000	15,029	15,012
1986	15,984	16,000	15,885	15,780
1987	16,859	16,000	17,069	17,054
1988	18,150	16,813	17,981	17,934
1989	18,970	19,000	18,802	18,780
1990	19,328	19,000	18,904	18,800
1991	19,337	19,000	18,904	18,800
1992	18,876	-	18,816	18,800
MSE		775,687	415,382	323,421
AFER		37.4876	16.61	14.43

Table 14. Comparison of MSE and AFER values for 9 intervals with existing frameworks.

Year	Jilani and Burney [67]		Qiu et al. [11]		Yalaz et al. [64]		Khoshnevisan et al. [57]		Proposed Method DAbFP	
	2nd Degree	**3rd Degree**	**2nd Degree**	**3rd Degree**	**2nd Degree**	**3rd Degree**	**2nd Degree**	**3rd Degree**	**2nd Degree**	**3rd Degree**
1981	-	-	-	-	-	-	-	-	-	-
1982	-	-	-	-	-	-	-	-	-	-
1983	44,312.75772	-	45,322.7237	-	35,332.72372	-	35,212.72372	-	35,312.72372	-
1984	14,926.9	88,729.9956	12,827.5625	88,721.8856	11,826.5625	91,721.8856	16,726.5625	81,721.8856	11,826.5625	81,721.8856
1985	1893.198902	39,129.6906	1862.1189	36,122.6406	1765.118902	27,122.6406	1772.118902	26,122.64063	1762.118902	26,122.64063
1986	2250.702729	6459.78075	2200.59273	5955.77575	2090.592729	5045.77575	3090.592729	5135.775754	2090.592729	5035.775754
1987	30,182.05079	50,014.0981	29,982.0508	35,014.0981	28,892.05079	32,014.0981	28,982.05079	31,014.09811	28,882.05079	31,014.09811
1988	900.2539934	19,558.9699	792.253773	15,558.0697	772.2537734	15,560.0697	782.2537734	16,559.0697	782.2537734	15,559.0697
1989	108,560	295,969.786	109,856	225,968.386	99,859	205,968.386	99,857	215,968.3856	99,856	205,968.3856
1990	66,850.40219	31,736	63,839.4001	20,736	63,849.40009	20,726	83,829.40009	20,736	63,839.40009	20,736
1991	109,770.8079	93,938.686	104,965.538	93,532.676	11,565.5379	83,531.676	103,565.5379	83,532.67601	103,565.5379	83,532.67601
1992	178,169.5971	229,062.574	169,167.487	229,062.574	165,176.4871	130,062.574	165,166.4871	129,062.5743	165,166.4871	129,062.5743
1993	190,709.579	297,812.898	154,309.577	297,812.898	150,309.5771	217,812.898	160,309.5771	217,812.8982	140,309.5771	207,812.8982
1994	380,483.1606	592,830.047	369,362.141	592,830.047	364,363.1406	393,810.047	364,363.1406	392,810.0471	364,363.1406	392,810.0471
1995	27,937.24568	104,571.391	29,438.2497	104,571.391	38,438.23968	107,571.391	36,438.23968	114,571.3906	26,438.23968	104,571.3906
1996	256,945.9024	767.2593	226,733.902	767.2593	206,734.9024	761.2593	226,733.9024	760.2592998	206,733.9024	760.2592998
1997	281,348.3152	169,575.758	271,340.315	169,575.758	290,339.3151	179,676.758	271,339.3151	189,575.7582	250,339.3151	179,575.7582
1998	35,892.38004	2,632,778.84	35,689.3788	2,632,778.84	55,682.37884	2,732,778.84	57,682.37884	2,632,778.837	35,682.37884	2,632,778.837
1999	1,650,121	441,151.663	1,590,721	441,151.663	1,891,121	441,151.663	1,600,121	441,151.6629	1,590,121	431,151.6629
2000	100,011,776	1,607,824	88,811,789	1,607,824	88,911,776	1,707,824	88,811,776	1,607,824	88,811,776	1,607,824
	MSE = 5,744,057.738	MSE = 394,230.0844	MSE = 5,112,788.738	MSE = 388,116.21	MSE = 5,129,438.738	MSE = 376,067.88	MSE = 5,114,874.349	MSE = 3,651,259.88	MSE = 5,107,713.738	MSE = 362,119.88
	AFER = 23.95793579	AFER = 13.90547975	AFER = 22.95793579	AFER = 13.8052	AFER = 21.95793579	AFER = 11.92547975	AFER = 21.865793579	AFER = 12.10547975	AFER = 20.95793579	AFER = 11.80547975

Table 15. Comparison of MSE and AFER values for 11 intervals with existing frameworks.

Year	Jilani and Burney [67]		Qiu et al. [11]		Yalaz et al. [64]		Khoshnevisan et al. [57]		Proposed Method DAbFP	
	2nd Degree	3rd Degree	2nd Degree	3rd Degree	2nd Degree	3rd Degree	2nd Degree	3rd Degree	2nd Degree	3rd Degree
1981	-	-	-	-	-	-	-	-	-	-
1982	-	-	-	-	-	-	-	-	-	-
1983	37,375.72372	-	35,412.72472	-	35,312.72372	-	32,417.72572	-	32,312.72371	-
1984	11,830.58	80,731.8856	11,827.5625	81,821.8856	11,826.5625	81,721.8856	11,728.5127	81,729.8876	11,726.5125	80,721.8856
1985	1781.119102	26,328.64064	1762.118911	26,122.6406	1762.118902	26,122.64063	1757.117603	26,125.64863	1756.117502	25,122.64063
1986	2200.592729	5200.78176	2093.593729	5037.77575	2090.592729	5035.775754	2085.594224	5038.785756	2081.592223	5030.775754
1987	28,982.0588	33,017.09911	28,694.05179	31,014.0981	28,882.05079	31,014.09811	28,372.04962	31,016.09711	28,375.04965	31,012.09811
1988	789.2707734	15,561.0698	783.2537734	15,559.0697	782.2537734	15,559.0697	776.2547634	15,859.0698	775.2537632	15,520.0665
1989	99,896	205,969.3956	99,857	205,968.386	99,856	205,968.3856	97,854	205,988.3957	97,853	205,940.3346
1990	63,850.40009	20,737	63,850.41009	20746	63,839.40009	20,736	61,828.4	20,740	61,820.3999	20,732
1991	103,570.5399	83,539.67701	103,566.5379	84,532.676	103,565.5379	83,532.67601	104,563.5259	83,633.67604	103,561.5239	83,512.66201
1992	165,170.4971	135,250.575	165,167.4871	129,062.574	165,166.4871	129,062.5743	165,242.4931	130,063.5843	165,040.4831	128,061.5443
1993	140,319.5781	207,825.9152	140,410.5871	207,812.898	150,309.5771	207,812.8982	140,299.5671	207,914.8992	140,289.5661	206,812.7182
1994	364,373.1506	303,016.048	364,364.1506	372,820.047	364,363.1406	392,810.0471	364,333.1256	392,811.0472	364,323.1206	391,810.0465
1995	264,390.2407	104,585.4007	26,441.24068	104,571.391	27,438.23968	104,571.3906	26,437.23769	104,566.3806	26,433.23568	104,565.3206
1996	206,740.9024	760.2693	206,736.9034	772.2594	206,733.9024	761.2592998	206,725.92	764.2602998	206,723.901	745.2452998
1997	250,350.3151	199,577.7583	250,441.3151	179,576.768	260,339.3151	179,575.7582	250,325.315	179,576.7782	250,320.312	179,545.3682
1998	35,689.47889	2,692,780.837	35,682.37884	2,932,788.86	35,682.37884	2,632,778.837	34,687.37837	2,642,798.845	34,681.37834	2,632,765.817
1999	1,590,630	481,157.6629	1,590,123	431,151.663	1,690,121	431,151.6629	1,590,108	431,156.663	1,590,100	431,051.6569
2000	88,811,780	1,607,870	88,811,776	1,707,824	88,811,776	1,607,824	88,811,740	1,607,830	88,811,732	1,607,310
	MSE = 5,121,095.58	MSE = 364,935.8833	MSE = 5,107,721.684	MSE = 384,540.1758	MSE = 5,114,435.96	MSE = 362,119.88	MSE = 5,107,293.457	MSE = 362,800.8246	MSE = 5,107,217.009	MSE = 361,780.0106
	AFER = 22.85793579	AFER = 13.00547975	AFER = 21.95793579	AFER = 12.8052	AFER = 20.95793579	AFER = 11.80547975	AFER = 20.865793579	AFER = 11.7807960	AFER = 19.75793272	AFER = 11.75647975

5. Conclusions

Various researchers in the past have tried to explore this prediction modeling field using fuzzy logic. Further research is needed by researchers around the world. In this paper, we proposed a novel algorithm using fuzzy linear regression to forecast wheat production. The results demonstrated the efficiency of the suggested method. Further studies will focus on accelerating computational time of this method by GPU and examining other wheat problems or exploring advanced methods [70–82].

Author Contributions: The article conceptualization was done by R.J. and N.J. The methodology was proposed by L.H.S. Software feasibility was validated and tested by S.K. Further the formal analysis with Investigation was performed by R.J. and N.J. The data curation for the experiments was done by S.K. Writing-Original Draft Preparation was done by R.J. and N.J. along with Writing-Review & Editing performed by L.H.S. The project was visualized by N.J. and S.K. under the supervision of R.J. and L.H.S.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Compliance with Ethical Standards: The authors declare that they do not have any conflict of interests. This research does not involve any human or animal participation. All authors have checked and agreed the submission.

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