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Geometric Properties of Certain Analytic Functions Associated with the Dziok-Srivastava Operator

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Abstract: The objective of the present paper is to derive certain geometric properties of analytic functions associated with the Dziok–Srivastava operator.

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1. Introduction

Throughout this paper, we assume that:

$$n, p \in \mathbb{N}, \ -1 \leq B < A \leq 1, \ \alpha > 0 \ \text{ and } \ \beta < 1. \tag{1}$$

Let $A_n(p)$ denote the class of functions of the form:

$$f(z) = z^p + \sum_{k=n+p}^{\infty} a_k z^k \tag{2}$$

which are analytic in the open unit disk $U = \{z : |z| < 1\}$. If $f(z) = z^p + \sum_{k=n+p}^{\infty} a_k z^k \in A_n(p)$ and $g(z) = z^p + \sum_{k=n+p}^{\infty} b_k z^k \in A_n(p)$, then the Hadamard product (or convolution) of f and g is defined by:

$$(f * g)(z) = z^p + \sum_{k=n+p}^{\infty} a_k b_k z^k$$

For:

$$\alpha_j \in \mathbb{C} \ (j = 1, 2, \cdots, l) \text{ and } \beta_j \in \mathbb{C} \setminus \{0, -1, -2, \cdots\} \ (j = 1, 2, \cdots, m)$$

the generalized hypergeometric function ${}_lF_m(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m; z)$ is defined by:

$${}_lF_m(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \cdots (\alpha_l)_k}{(\beta_1)_k \cdots (\beta_m)_k} \frac{z^k}{k!}$$

$$(l \leq m+1; l, m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; z \in U)$$

where $(x)_k$ is the Pochhammer symbol given by $(x)_k = x(x+1)\cdots(x+k-1)$ for $k \in \mathbb{N}$ and $(x)_0 = 1$. Corresponding to the function $z^p \ {}_lF_m(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m; z)$, the well-known Dziok–Srivastava operator [1] $H(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m) : A_n(p) \to A_n(p)$ is defined by:

$$H(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m) f(z) = (z^p {}_lF_m(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m; z) * f(z)$$

$$(l \leq m+1; l, m \in \mathbb{N}_0; z \in U)$$

If $f \in A_n(p)$ is given by (2), then we have:

$$H(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m) f(z) = z^p + \sum_{k=n+p}^{\infty} \frac{(\alpha_1)_k \cdots (\alpha_l)_k}{(\beta_1)_k \cdots (\beta_m)_k} \frac{a_k}{k!} z^k$$

For convenience, we write:

$$H_m^l(\alpha_1) = H(\alpha_1, \cdots, \alpha_l; \beta_1, \cdots, \beta_m) \quad (l \leq m+1; l, m \in \mathbb{N}_0)$$

It is noteworthy to mention that the Dziok–Srivastava operator is a generalization of certain linear operators considered in earlier investigations.

Next, we consider the function $h(A, B; z) = (1 + Az)/(1 + Bz)$ for $z \in U$. It is known that the function $h(A, B; z)$ is the conformal map of U onto a disk, symmetrical with respect to the real axis, which is centered at the point $(1 - AB)/(1 - B^2)$ $(B \neq \pm 1)$ and with its radius equal to $(A - B)/(1 - B^2)$ $(B \neq \pm 1)$. Furthermore, the boundary circle of this disk intersects the real axis at the points $(1 - A)/(1 - B)$ and $(1 + A)/(1 + B)$ with $B \neq \pm 1$.

Let $P[A, B]$ denote the class of functions of the form $p(z) = 1 + p_1 z + \cdots$, which are analytic in U and satisfy the subordination $p(z) \prec h(A, B; z)$. It is clear that $p \in P[A, B]$ if and only if:

$$\left| p(z) - \frac{1 - AB}{1 - B^2} \right| < \frac{A - B}{1 - B^2} \quad (-1 < B < A \leq 1; z \in U)$$

and:

$$\mathrm{Re}\, p(z) > \frac{1 - A}{2} \quad (B = -1; z \in U)$$

For two functions f and g analytic in U, f is said to be subordinate to g, written by $f(z) \prec g(z) \quad (z \in U)$, if there exists a Schwarz function w in U such that:

$$|w(z)| \leq |z| \quad \text{and} \quad f(z) = g(w(z)) \quad (z \in U)$$

Furthermore, if the function g is univalent in U, then:

$$f(z) \prec g(z) \quad (z \in U) \Longleftrightarrow f(0) = g(0) \quad \text{and} \quad f(U) \subset g(U)$$

Many properties of analytic functions have been investigated by several authors(see [1–11]). In this paper, we derive certain geometric properties of analytic functions associated with the well-known Dziok–Srivastava operator.

2. Main Results

Theorem 1. *Let f belong to the class $A_n(p)$. Furthermore, let:*

$$\frac{H_m^l(\alpha_1) f(z)}{z^p} \in P[A, B]. \tag{3}$$

Then:

$$\mathrm{Re}\left\{ \frac{H_m^l(\alpha_1) f(z)}{z^p} + \alpha z \left(\frac{H_m^l(\alpha_1) f(z)}{z^p} \right)' \right\}$$
$$\leq \begin{cases} \frac{1 + (A + B + n\alpha(A - B)) r^n + ABr^{2n}}{(1 + Br^n)^2} & \text{if } M_n(A, B, \alpha, r) \leq 0, \quad (4) \\[2ex] \frac{L_n^2 - 4\alpha^2 K_A K_B}{4\alpha(A - B) r^{n-1} (1 - r^2) K_B} & \text{if } M_n(A, B, \alpha, r) \geq 0, \quad (5) \end{cases}$$

where:

$$\begin{cases} K_A = 1 - A^2 r^{2n} + nAr^{n-1}(1-r^2), \\ K_B = 1 - B^2 r^{2n} + nBr^{n-1}(1-r^2), \\ L_n = 2\alpha(1 - ABr^{2n}) + n\alpha(A+B)r^{n-1}(1-r^2) + (A-B)r^{n-1}(1-r^2), \\ M_n(A,B,\alpha,r) = 2\alpha K_B(1+Ar^n) - L_n(1+Br^n). \end{cases} \tag{6}$$

The result is sharp.

Proof. For $z = 0$, the equality in (4) holds true. Thus, we assume that $0 < |z| = r < 1$. From (3), we can write:

$$\frac{H_m^l(\alpha_1)f(z)}{z^p} = \frac{1 + Az^n\varphi(z)}{1 + Bz^n\varphi(z)} \quad (z \in U), \tag{7}$$

where $\varphi(z)$ is analytic and $|\varphi(z)| \le 1$ in U. From (7), we have:

$$\begin{aligned} &\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z\left(\frac{H_m^l(\alpha_1)f(z)}{z^p}\right)' \\ &= \frac{H_m^l(\alpha_1)f(z)}{z^p} + \frac{\alpha(A-B)z^n(n\varphi(z) + z\varphi'(z))}{(1+Bz^n\varphi(z))^2} \\ &= \frac{H_m^l(\alpha_1)f(z)}{z^p} + \frac{n\alpha}{A-B}(A - BH_m^l(\alpha_1)f(z)/z^p)(H_m^l(\alpha_1)f(z)/z^p - 1) \\ &+ \frac{\alpha(A-B)z^{n+1}\varphi'(z)}{(1+Bz^n\varphi(z))^2} \end{aligned} \tag{8}$$

By using the Carathéodory inequality:

$$|\varphi'(z)| \le \frac{1 - |\varphi(z)|^2}{1 - r^2},$$

we get:

$$\begin{aligned} \mathrm{Re}\left\{\frac{z^{n+1}\varphi'(z)}{(1+Bz^n\varphi(z))^2}\right\} &\le \frac{r^{n+1}(1 - |\varphi(z)|^2)}{(1-r^2)|1+Bz^n\varphi(z)|^2} \\ &= \frac{r^{2n}|A - BH_m^l(\alpha_1)f(z)/z^p|^2 - |H_m^l(\alpha_1)f(z)/z^p - 1|^2}{(A-B)^2 r^{n-1}(1-r^2)} \end{aligned} \tag{9}$$

Set $\frac{H_m^l(\alpha_1)f(z)}{z^p} = u + iv \quad (u, v \in \mathbb{R})$. Then, (8) and (9) give:

$$\begin{aligned} &\mathrm{Re}\left\{\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z\left(\frac{H_m^l(\alpha_1)f(z)}{z^p}\right)'\right\} \\ &\le \left(1 + n\alpha\frac{A+B}{A-B}\right)u - \frac{n\alpha A}{A-B} - \frac{n\alpha B}{A-B}(u^2 - v^2) \\ &+ \alpha\frac{r^{2n}((A-Bu)^2 + (Bv)^2) - ((u-1)^2 + v^2)}{(A-B)r^{n-1}(1-r^2)} \\ &= \left(1 + n\alpha\frac{A+B}{A-B}\right)u - \frac{n\alpha}{A-B}(A + Bu^2) \\ &+ \alpha\frac{r^{2n}(A-Bu)^2 - (u-1)^2}{(A-B)r^{n-1}(1-r^2)} + \frac{\alpha}{A-B}\left(nB - \frac{1 - B^2r^{2n}}{r^{n-1}(1-r^2)}\right)v^2 \end{aligned} \tag{10}$$

Note that:

$$\frac{1-B^2r^{2n}}{r^{n-1}(1-r^2)} \geq \frac{1-r^{2n}}{r^{n-1}(1-r^2)} = \frac{1}{r^{n-1}}(1+r^2+r^4+\cdots+r^{2(n-2)}+r^{2(n-1)})$$
$$= \frac{1}{2r^{n-1}}[(1+r^{2(n-1)})+(r^2+r^{2(n-2)})+\cdots+(r^{2(n-1)}+1)] \tag{11}$$
$$\geq n \geq nB$$

Using (10) and (11), we obtain:

$$\mathrm{Re}\{\frac{H_m^l(\alpha_1)f(z)}{z^p}+\alpha z(\frac{H_m^l(\alpha_1)f(z)}{z^p})'\} \leq \left(1+n\alpha\frac{A+B}{A-B}\right)u - \frac{n\alpha}{A-B}(A+Bu^2)$$
$$+\alpha\frac{r^{2n}(A-Bu)^2-(u-1)^2}{(A-B)r^{n-1}(1-r^2)} = \psi_n(u) \tag{12}$$

It is known that for $|\xi| \leq \sigma \ (\sigma < 1)$,

$$\left|\frac{1+A\xi}{1+B\xi}-\frac{1-AB\sigma^2}{1-B^2\sigma^2}\right| \leq \frac{(A-B)\sigma}{1-B^2\sigma^2} \tag{13}$$

and:

$$\frac{1-A\sigma}{1-B\sigma} \leq \mathrm{Re}\left\{\frac{1+A\xi}{1+B\xi}\right\} \leq \frac{1+A\sigma}{1+B\sigma} \tag{14}$$

Furthermore, (7) and (14) show that:

$$\frac{1-Ar^n}{1-Br^n} \leq \mathrm{Re}\{\frac{H_m^l(\alpha_1)f(z)}{z^p}\} \leq \frac{1+Ar^n}{1+Br^n}.$$

Now, we calculate the maximum value of $\psi_n(u)$ on the segment $\left[\frac{1-Ar^n}{1-Br^n}, \frac{1+Ar^n}{1+Br^n}\right]$. Obviously,

$$\psi_n'(u) = 1+n\alpha\frac{A+B}{A-B}-\frac{2n\alpha B}{A-B}u+2\alpha\frac{(1-ABr^{2n})-(1-B^2r^{2n})u}{(A-B)r^{n-1}(1-r^2)}$$

$$\psi_n''(u) = -\frac{2\alpha}{A-B}\left(nB+\frac{1-B^2r^{2n}}{r^{n-1}(1-r^2)}\right) < 0 \quad \text{(see (11))} \tag{15}$$

and $\psi_n'(u) = 0$ if and only if:

$$u = u_n = \frac{2\alpha(1-ABr^{2n})+n\alpha(A+B)r^{n-1}(1-r^2)+(A-B)r^{n-1}(1-r^2)}{2\alpha[1-B^2r^{2n}+nBr^{n-1}(1-r^2)]}$$
$$= \frac{L_n}{2\alpha K_B} \quad \text{(see (6))} \tag{16}$$

Since:

$$2\alpha K_B(1-Ar^n)-L_n(1-Br^n)$$
$$= 2\alpha[(1-Ar^n)(1-B^2r^{2n})-(1-Br^n)(1-ABr^{2n})]$$
$$-n\alpha r^{n-1}(1-r^2)[(A+B)(1-Br^n)-2B(1-Ar^n)]-(A-B)r^{n-1}(1-r^2)(1-Br^n)$$
$$= -2\alpha(A-B)r^n(1-Br^n)-n\alpha(A-B)r^{n-1}(1-r^2)(1+Br^n)$$
$$-(A-B)r^{n-1}(1-r^2)(1-Br^n) < 0$$

we see that:

$$u_n > \frac{1-Ar^n}{1-Br^n} \tag{17}$$

However, u_n is not always less than $\frac{1+Ar^n}{1+Br^n}$. The following two cases arise.

Case (I). $u_n \geq \frac{1+Ar^n}{1+Br^n}$, that is $M_n(A,B,\alpha,r) \leq 0$. In view of $\psi_n'(u_n) = 0$ and (15), the function $\psi_n(u)$ is increasing on the segment $\left[\frac{1-Ar^n}{1-Br^n}, \frac{1+Ar^n}{1+Br^n}\right]$. Thus, we deduce from (12) that, if $M_n(A,B,\alpha,r) \leq 0$, then:

$$\begin{aligned}
\operatorname{Re}\left\{\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z(\frac{H_m^l(\alpha_1)f(z)}{z^p})'\right\} &\leq \psi_n\left(\frac{1+Ar^n}{1+Br^n}\right) \\
&= \left(1 + n\alpha\frac{A+B}{A-B}\right)\left(\frac{1+Ar^n}{1+Br^n}\right) \\
&- \frac{n\alpha}{A-B}\left(A + B\left(\frac{1+Ar^n}{1+Br^n}\right)^2\right) \\
&= \frac{1+Ar^n}{1+Br^n} - \frac{n\alpha}{A-B}\left(1 - \frac{1+Ar^n}{1+Br^n}\right)\left(A - B\frac{1+Ar^n}{1+Br^n}\right) \\
&= \frac{1 + (A+B+n\alpha(A-B))r^n + ABr^{2n}}{(1+Br^n)^2}
\end{aligned}$$

This gives (4).

Next, we consider the function f defined by:

$$\frac{H_m^l(\alpha_1)f(z)}{z^p} = \frac{1+Az^n}{1+Bz^n}$$

which satisfies the condition (3). It is easy to check that:

$$\frac{H_m^l(\alpha_1)f(r)}{r^p} + \alpha r(\frac{H_m^l(\alpha_1)f(r)}{r^p})' = \frac{1 + (A+B+n\alpha(A-B))r^n + ABr^{2n}}{(1+Br^n)^2}$$

which implies that the inequality (4) is sharp.

Case (II). $u_n \leq \frac{1+Ar^n}{1+Br^n}$, that is $M_n(A,B,\alpha,r) \geq 0$. In this case, we easily have:

$$\operatorname{Re}\left\{\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z(\frac{H_m^l(\alpha_1)f(z)}{z^p})'\right\} \leq \psi_n(u_n) \tag{18}$$

In view of (6), $\psi_n(u)$ in (12) can be written as:

$$\psi_n(u) = \frac{-\alpha K_B u^2 + L_n u - \alpha K_A}{(A-B)r^{n-1}(1-r^2)} \tag{19}$$

Therefore, if $M_n(A,B,\alpha,r) \geq 0$, then it follows from (16), (18), and (19) that:

$$\begin{aligned}
\operatorname{Re}\left\{\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z(\frac{H_m^l(\alpha_1)f(z)}{z^p})'\right\} &\leq \frac{-\alpha K_B u_n^2 + L_n u_n - \alpha K_A}{(A-B)r^{n-1}(1-r^2)} \\
&= \frac{L_n^2 - 4\alpha^2 K_A K_B}{4\alpha(A-B)r^{n-1}(1-r^2)K_B}
\end{aligned}$$

To show the sharpness, we take:

$$\frac{H_m^l(\alpha_1)f(z)}{z^p} = \frac{1+Az^n\varphi(z)}{1+Bz^n\varphi(z)} \quad \text{and} \quad \varphi(z) = \frac{z-c_n}{1-c_n z}$$

where $c_n \in \mathbb{R}$ is determined by:

$$\frac{H_m^l(\alpha_1)f(r)}{r^p} = \frac{1+Ar^n\varphi(r)}{1+Br^n\varphi(r)} = u_n \in \left(\frac{1-Ar^n}{1-Br^n}, \frac{1+Ar^n}{1+Br^n}\right]$$

Clearly, $-1 < \varphi(r) \le 1, -1 \le c_n < 1, |\varphi(z)| \le 1$ $(z \in U)$, and so, f satisfies the condition (3). Since:

$$\varphi'(r) = \frac{1-c_n^2}{(1-c_nr)^2} = \frac{1-|\varphi(r)|^2}{1-r^2}$$

from the above argument, we find that:

$$\frac{H_m^l(\alpha_1)f(r)}{r^p} + \alpha r(\frac{H_m^l(\alpha_1)f(r)}{z^p})' = \psi_n(u_n)$$

The proof of the theorem is now completed. □

Corollary 1. *Let $f \in A_1(p)$, and satisfy* $\mathrm{Re}\{H_m^l(\alpha_1)f(z)/z^p\} > \beta$ $(\beta < 1; z \in U)$*. Then, for* $|z| = r < 1$,

$$\mathrm{Re}\left\{\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z(\frac{H_m^l(\alpha_1)f(z)}{z^p})'\right\} \le \beta + (1-\beta)\frac{1+2\alpha r - r^2}{(1-r)^2}$$

The result is sharp.

Proof. By considering $\frac{H_m^l(\alpha_1)f(z)/z^p - \beta}{1-\beta}$ instead of $H_m^l(\alpha_1)f(z)/z^p$, we only need to prove the corollary for $\beta = 0$. Putting $n = A = 1$ and $B = -1$ in (6), we get:

$$K_1 = 2(1-r^2), \quad K_{-1} = 0, \quad L_1 = 2\alpha(1+r^2) + 2(1-r^2)$$

and:

$$M_1(1,-1,\alpha,r) = -2(1-r)[1+\alpha-(1-\alpha)r^2] \le 0$$

Consequently, an application of (4) in Theorem 2.1 yields:

$$\mathrm{Re}\left\{\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z(\frac{H_m^l(\alpha_1)f(z)}{z^p})'\right\} \le \frac{1+2\alpha r - r^2}{(1-r)^2}$$

The sharpness follows immediately from that of Theorem 1. □

Theorem 2. *Let α_j $(j = 1, 2, \cdots, l)$ and β_s $(s = 1, 2, \cdots, m)$ be positive real numbers. Furthermore, let $f(z) = z^p + \sum_{k=n+p}^{\infty} a_k z^k \in A_n(p)$, and satisfy:*

$$\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z\left(\frac{H_m^l(\alpha_1)f(z)}{z^p}\right)' \in P[A,B] \tag{20}$$

Then:

$$|a_k| \le \frac{k!(A-B)(\beta_1)_k \cdots (\beta_m)_k}{(1+\alpha(k-p))(\alpha_1)_k \cdots (\alpha_l)_k} \quad (k \ge n+p) \tag{21}$$

The result is sharp for each $k \ge n+p$.

Proof. It is well known that if:

$$g(z) = \sum_{k=1}^{\infty} b_k z^k \prec \varphi(z) \quad (z \in U)$$

where $g(z)$ is analytic in U and $\varphi(z) = z + \cdots$ is convex univalent in U, then $|b_k| \leq 1$ $(k = 1, 2, 3, \cdots)$.

From (20), we have:

$$\frac{1}{A-B}\left(\frac{H_m^l(\alpha_1)f(z)}{z^p} + \alpha z\left(\frac{H_m^l(\alpha_1)f(z)}{z^p}\right)' - 1\right)$$
$$= \frac{1}{A-B}\sum_{k=n+p}^{\infty}\frac{(1+\alpha(k-p))(\alpha_1)_k\cdots(\alpha_l)_k \cdot a_k}{k!(\beta_1)_k\cdots(\beta_m)_k}z^{k-p} \tag{22}$$
$$\prec \frac{z}{1+Bz} \quad (z \in U)$$

In view of the function $\frac{z}{1+Bz}$ being convex univalent in U, it follows from (22) that:

$$\frac{(1+\alpha(k-p))(\alpha_1)_k\cdots(\alpha_l)_k}{k!(A-B)(\beta_1)_k\cdots(\beta_m)_k}|a_k| \leq 1 \quad (k \geq n+p)$$

which gives (21).

Next, we consider the function $f_{k-p}(z)$ defined by:

$$f_{k-p}(z) = z^p + (A-B)\sum_{q=1}^{\infty}\frac{(-B)^{q-1}(\beta_1)_{qk}\cdots(\beta_m)_{qk}(qk)!}{(1+\alpha q(k-p))(\alpha_1)_{qk}\cdots(\alpha_l)_{qk}}z^{q(k-p)+p} \quad (z \in U; k \geq n+p)$$

Since:

$$\frac{H_m^l(\alpha_1)f_{k-p}(z)}{z^p} + \alpha z\left(\frac{H_m^l(\alpha_1)f_{k-p}(z)}{z^p}\right)' = \frac{1+Az^{k-p}}{1+Bz^{k-p}} \prec \frac{1+Az}{1+Bz} \quad (z \in U)$$

and:

$$f_{k-p}(z) = z^p + \frac{k!(A-B)(\beta_1)_k\cdots(\beta_m)_k}{(1+\alpha(k-p))(\alpha_1)_k\cdots(\alpha_l)_k}z^k + \cdots$$

for each $k \geq n+p$, the proof of Theorem 2 is completed. □

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