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Review

Chaos in the Real World: Recent Applications to Communications, Computing, Distributed Sensing, Robotic Motion, Bio-Impedance Modelling and Encryption Systems

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Abstract: Most of the papers published so far in literature have focused on the theoretical phenomena underlying the formation of chaos, rather than on the investigation of potential applications of chaos to the real world. This paper aims to bridge the gap between chaos theory and chaos applications by presenting a survey of very recent applications of chaos. In particular, the manuscript covers the last three years by describing different applications of chaos as reported in the literature published during the years 2018 to 2020, including the matter related to the symmetry properties of chaotic systems. The topics covered herein include applications of chaos to communications, to distributed sensing, to robotic motion, to bio-impedance modelling, to hardware implementation of encryption systems, to computing and to random number generation.

Keywords: applications of chaos; nonlinear dynamics; chaotic circuits and systems

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1. Introduction

Referring to chaotic dynamical systems, most of the papers published so far in literature have focused on the theoretical phenomena underlying the formation of chaos, rather than on the investigation of potential applications of chaos to the real world. An interesting attempt to deal with chaos applications has been done in a book published in 2016, where a number of contributions have been presented [1]. However, since five years have passed from the publication of reference [1], it would be interesting to analyse the most recent results regarding the application of chaos to the real world. This paper aims to bridge the gap between chaos theory and chaos applications by presenting a survey of very recent applications of chaos. In particular, the manuscript covers the last three years, i.e., it illustrates different applications of chaos as reported in the papers published in the years 2018–2020, including the matter related to the symmetry properties of chaotic systems. The manuscript is organized as follows. In Section 2 the applications of chaos to communications are illustrated [2–12], including the use of chaos in radar-based systems and underwater acoustic channels. Section 3 highlights how chaos may represent a promising approach in sensing applications [13], whereas Section 4 illustrates different applications of chaos in robotic motion [14–16]. In Section 5 it is shown how chaos can give significant insights to the quality assessments of fruits and vegetables [17]. Section 6 presents some applications of chaos to the hardware implementation of encryption systems [18–26] and to random number generation [27–35], including the use of chaos for secure transfer of medical images. Finally, Section 7 illustrates how chaos-based logic gates can be used for computing [36], whereas Section 8 highlights how chaos can be exploited in neuromorphic applications [37,38].

2. Applications of Chaos to Communications

Since the applications of chaos to communications cover several topics, this section is organized in three subsections: applications for chaos-based secure communications,

applications of chaos for modulation efficiency and applications by virtue of the noise-like properties of chaos.

2.1. Applications for Chaos-Based Secure Communications

Some properties of chaos, such as complex behaviour, noise-like dynamics and spread spectrum can be used to encode data so that the transmitted message is inaccessible to possible eavesdroppers. To this purpose, a number of interesting papers have been published in the years 2018–2020. For example, in [5], new architectures of perturbed digital chaos for image permutation and diffusion have been proposed. The chaotic dynamics is perturbed at bit level using perturbation on state variables and/or on control parameters [5]. Under persistent perturbations, the dynamics of the chaotic map prove to be nonstationary, thus providing advantages in image permutation and diffusion based on cryptography. Simulation results using a logistic map have clearly demonstrated the effectiveness of the conceived approach [5].

In [7], an encryption-based method for region of interest (ROI) security in high-efficiency video coding (HEVC) standards has been illustrated. The selective approach exploits a chaos-based stream cipher to encrypt some HEVC syntax elements. In particular, a standard HEVC decoder is used to decode the bit stream, whereas a secret key is necessary to decrypt the ROI [7]. The suggested chaos-based ROI encryption exploits the tile property of HEVC, which enables each video frame to be split into independent rectangular areas. Experimental measurements have highlighted that the conceived approach enables secure video encryption to be achieved in real-time, with satisfying bit rate and complexity overheads [7].

In [6], the security of some steganagraphic methods (analysed in the spatial and frequency domain) has been improved by exploiting the properties of chaos. In particular, a statistical steganalysis system has been built in order to assess the robustness of three chaos-based steganographic methods [6]. Experimental results have shown that the conceived steganalysis system, which exploits multi-dimensional feature vectors, is able to effectively detect hidden information, regardless of the message size [6].

In [3], a piecewise-linear function is exploited as a dynamic selector for dynamics editing. The approach, which is based on offset boosting, enables any chaotic dynamics from any nonlinear system to be extracted without modifying the system parameters. The authors of reference [3] suggest the use of dynamic selectors as novel keys for chaos-based secure communications, particularly when very complex dynamical systems are involved.

In [4], two robust keyed hash functions comprising a chaotic system and a chaotic neural network have been proposed. The input message M is hashed to a hash value h with a fixed number of 256 or 512 bits. In particular, the chaotic system generates pseudo-chaotic samples, which become parameter values for the neural structure [4]. This combined architecture ensures that the generated hash functions possess excellent statistical properties, including high message and key sensitivity [4]. Thus, the proposed keyed chaos-based hash functions are suitable for digital signature, data integrity and message authentication [4].

In [11], a matrix projective synchronization scheme for fractional-order maps is presented. A novel synchronization error is introduced, and a control algorithm is designed to stabilize the error dynamics at the origin. The synchronization of fractional maps is exploited for a secure communication application, which includes speech encryption over a public channel [11].

In reference [12], S-boxes generated by using chaotic functions are analysed to measure their actual resistance to cryptanalysis. By using two heuristic methods and starting from pseudo-random S-boxes, the author of reference [12] has designed S-boxes that significantly outperform all previously published chaos-based S-boxes. Moreover, the approach in [12] enables to reach near-optimal average coordinate nonlinearity values, which are significantly greater than those known in literature.

2.2. Applications of Chaos for Modulation Efficiency

When chaos is used to modulate a waveform in communication systems, the bit error ratio can be improved with respect to the classical half-sine pulse frequency-modulated waveform. To this purpose, some promising results have been recently published in reference [2]. In particular, in [2] the issue related to the search for efficient solutions for the coexistence of radar and communication systems has been discussed. Specifically, the authors of reference [2] have proposed a new approach to control chaotic trajectories, which assures the coexistence of both radar and communication systems. By selecting a state variable for controlling the trajectory, it is possible to produce a controlled chaos-based frequency modulated (CCBFM) waveform for transmitting a signal in joint radar-communication systems [2]. Moreover, a communication receiver and a radar receiver have been designed in order to decode the information and extract the signature of the target, respectively. Analyses of the conceived chaos-based approach have been conducted [2]. In particular, the bit error ratio highlights that the CCBFM waveform has better performances than the classical half-sine pulse frequency-modulated waveform. Moreover, when the signal-to-noise ratio is about -30 dB, it can be shown that the adoption of CCBFM waveforms assures good target detection and classification [2]. The conducted analysis has clearly demonstrated that chaos-based frequency-modulated waveforms can be effectively exploited for joint radar-communication systems in a shared spectrum [2].

2.3. Applications of Chaos by Virtue of the Noise-Like Properties

Communications can benefit from the noise-like properties of chaotic systems. Some recent manuscripts have addressed this topic. For example, in [8], a key issue in radio wave communication systems has been discussed, i.e., how to exploit chaos with minimum modifications of conventional systems. In particular, the author of reference [8] have proposed the adoption of chaotic shape-forming filters as substitutes for conventional shape-forming filters. By using a suboptimal threshold, Bit Error Rate (BER) in both multipath channel and system performance has been improved. Additionally, the removal of the complex algorithm for channel equalization has led to simply the software of the overall architecture [8]. The authors of reference [8] also suggest to readily use chaotic shape-forming filters in 5G or 6G communication systems, given that the shape-forming of baseband signals is of paramount importance in radio wave systems.

In [9], suitable chaotic signals and matched filters are proposed for digital communications in wireless channels characterized by severe physical constraints. In particular, the behaviour of underwater acoustic channels is studied. The novelty in the communication system illustrated in [9] is twofold. On the one hand, hybrid systems are used as chaotic signal generators, including matched filters for maximizing the signal to noise ratio at the receiver side. On the other hand, matched filters are designed in order to reduce the noise effect when the information is decoded. The approach in [9] yields good communication capabilities, mostly if we consider the challenges posed by underwater acoustic channels. Moreover, the method is characterized by many advantages related to the properties of chaos, including robustness to narrowband interference. The method in [9] is also environment-friendly, given that noise-like chaotic signals do not disturb the whale's navigation system [10].

3. Applications of Chaos in Distributed Sensing

Since the measurement of physical variables over extended regions requires several sensors that can result in impractical bandwidth and power consumption, there is the need for distributed sensing approaches where readouts are directly obtained at the ensemble level. Chaos can face this challenging issue via a nonlinear analog approach based on chaotic synchronization. For example, in [13], a precursory study is presented, which brings together two separate research fields, i.e., wireless sensor networks and electronic chaotic oscillators. The authors of [13] demonstrate that it is possible to build distributed sensing systems by implementing the nodes with simple analog chaotic circuits. In particu-

lar, [13] presents the first physical evidence that chaotic circuits can be coupled remotely in extended networks. A remarkable result is that several emergent phenomena (including synchronization, desynchronization and chaotic transitions) can be obtained even in the case of weak coupling [13]. Experimental measurements have shown that chaotic single-transistor oscillators are suitable for wireless coupling via mutual induction [13]. This has enabled a form of telemetry for luminous flux to be implemented. The results in [13] highlight the capability to generate accurate estimations of the average of distributed physical magnitudes from complex ensemble dynamics, indicating that the use of chaotic circuits represents a promising approach in sensing applications.

4. Applications of Chaos in Robotic Motion

In applications of robotic motion, there is the need to combine an efficient area coverage with unpredictable movements of the robot. Chaos enables both goals to be achieved, as shown in some recent manuscripts [14,16]. For example, robotic systems can be used in the military for patrol missions or to find explosives [14]. In accomplishing these tasks, the robots should recognize their initial positions and those of the targets, in order to instantly update the map of the workspace [14]. Thus, efficient path planning methods for creating a trajectory in the workspace are required. Since in patrolling missions it is crucial for the robot to move randomly, the use of chaotic systems is proposed in [14] to generate unpredictable trajectories for the motion of the robots. In particular, the considered chaotic path generator is a modified logistic May map, which is exploited for implementing the robot's movement in four and eight directions [14]. Simulations results are carried out to show the capabilities of the method. In particular, the results in [14] highlight that the use of eight directions improves the behaviour of the robot, both in coverage percentage and in the reduction of the number of visits of the same cells in the square workspace.

In [15], an efficient chaotic path planning algorithm for autonomous mobile robots is illustrated. The objective is to cover a given terrain using chaotic motion. The method in [15] exploits a logistic map with a chaotic tactic, which uses a modulo function to generate a sequence of eight directions on a grid. Using extensive simulations, it is shown that the approach is effective to guarantee path coverage, as the number of iterations increases [15]. In addition, the efficiency of the algorithm is further improved with a pheromone inspired memory technique [15].

Referring to chaotic path planning, in [16], chaotic motion patterns are generated using chaotic pseudo-random bit generators based on modified logistic maps. Namely, the bitstream produced by the generator is converted into motion commands in different directions. Successively, the chaotic logistic map is combined with an inverse pheromone approach in order to get a reduction in the number of revisits in each cell of the workspace [16]. The considered robot is able to move in four or eight directions, even though the conducted analysis highlights that the motion in eight directions provides better results [16]. Simulation results show that the use of the pheromone can significantly increase the coverage percentage, leading to better performances in different scenarios, which include the presence of obstacles in the workspace [16].

5. Applications of Chaos to Bio-Impedance Modelling

Chaotic maps have been recently used with the aim of increasing the efficacy and efficiency of some optimization algorithms. For example, in reference [17] it has been shown that, in order to assess the chemical and physiological changes of plant tissues, it is of paramount importance to carry out parameter extractions of the bio-impedance models using chaos-based optimization algorithms [17]. Namely, impedance measurements can give significant insights in the field of quality assessments of fruits and vegetables, with the aim to better characterize agricultural products. In [17] the flower pollination algorithm (FPA), the grey wolf optimizer (GWO) and a number of their chaotic variants are utilized to extract the parameters of bio-impedance models. By using experimental data for a number of different vegetables, the performances of some chaotic maps applied to FPA and GWO

are evaluated over some benchmark functions [17]. The obtained results are compared using fitting errors and execution times, in order to highlight the trade-off between accuracy and speed of convergence [17]. For each presented model, the objective is to choose the best algorithm for parameter extraction. For the simplified Hayden model, both the chaotic FPA and the chaotic GWO achieve satisfying results [17]. For the double-shell model, the chaotic GWO achieves more accurate and consistent results with respect to the chaotic FPA. Finally, the fractional-order versions of the considered models are analysed, with the aim to investigate new approaches in bio-impedance parameter extractions [17].

6. Applications of Chaos to Encryption

In order to efficiently use chaos in cryptography, chaotic systems need to be implemented such that the generated entropy can produce confusion and diffusion properties. These features along with cryptographic primitives share unique characteristics, which enable chaos to be effectively applied to encryption. Herein two main topics are discussed. One issue regards the hardware implementations of encryption systems [18–26], whereas the other is related to the capability of chaotic systems to generate random numbers [27–35].

6.1. Applications of Chaos in the Hardware Implementation of Encryption Systems

Different technologies have been recently exploited for implementing cryptographic systems in hardware. For example, in [18], at first, a novel hyperchaotic map has been presented by taking a one-dimensional infinite collapse model as seed. Successively, since the conceived map highlights a wide hyperchaotic range, high sensitivity, good ergodicity and extreme complexity performance, a secure communication system has been designed and implemented in hardware [18]. The input message (that can be an image, a text, or a sound) is processed via a delta modulator and then encrypted by exploiting the conceived hyperchaotic map. At the receiver side, the map and a delta demodulator are employed to retrieve the original message. Experimental measurements have shown both efficiency and simplicity of the secure communication system when the information signal is transmitted over an optical channel [18].

In [19], a very simple chaotic system showing both odd and even symmetries is presented, along with an experimental set-up based on FPGA platforms. In particular, a chaos-based speech encryption scheme is designed and implemented on an FPGA using Verilog HDL [19]. The security of the encryption scheme is assessed, with the aim to show the effectiveness of the conceived approach [19]. Moreover, by virtue of good use of the hardware resources, the encrypted scheme is able to guarantee a throughput of 1.3 Gbit/sec via a Xilinx Kintex 7.

In [20], the FPGA implementation of a sound encryption system based on a fractional-order chaotic system is illustrated. A number of techniques are exploited to enhance the throughput as well as to reduce the hardware utilization. Security analysis techniques (including basic statistical aspects, the NIST tests, mean square error and key sensitivity) are carried out to highlight the robustness of the conceived approach [20].

In [21], a novel stream cipher based on a chaotic skew tent map is illustrated and realized in a 0.18 μm CMOS technology. The conceived ciphering algorithm exploits a linear feedback shift register, which modifies the chaotic trajectories produced by the skew tent map dynamics. This enables the randomness of the generated sequences to be improved [21]. The implemented stream cipher is able to achieve encryption speeds of 1 Gbps with power consumption as low as 24.1 mW. A security analysis of the cipher is carefully carried out [21]. In particular, the generated keystreams are analysed via NIST randomness tests, showing that they are undistinguishable from truly random sequences. Key sensitivity and key space size are analysed, with the aim to highlight the security of the conceived cipher [21].

In [22], a chaotic system without equilibrium points (i.e., with hidden attractors) is at first introduced and implemented in a real electronic circuit. Then, S-boxes are realized and employed in a new image encryption algorithm [22]. In particular, some S-boxes with

good cryptographic properties are exploited for sub-byte operations. The performances of both S-boxes and encryption procedures are analysed to highlight the capabilities of the method [22].

By virtue of their complex dynamic behaviours, chaotic systems with coexisting attractors [23–25] represent an interesting topic in view of their practical application to encryption. For example, in [26] an extended Lü system with coexisting attractors is at first introduced, along with analyses of bifurcation diagrams and phase portraits. Then, a hardware implementation of the conceived system is illustrated, along with an efficient chaotic image encryption algorithm [26].

6.2. Applications of Chaos to Random Number Generation

With the rapid development of communication technology, it is worth noting that random numbers are used in many cryptographic protocols, including key management, identity authentication and image encryption. Since randomness and unpredictability are required by encryption, it is clear that chaos has good features in this regard, by virtue of the sensitive dependence on initial conditions, periodicity and reproduction. Some interesting manuscripts have been recently published on the topic [27–35]. For example, in [27] the dynamics of some discrete-time chaotic systems in the digital (i.e., finite-precision) domain is studied. Differently from classical approaches treating a digital chaotic map as a black box, in [27] the dynamical properties of the logistic map and the tent map are explored with fixed-point arithmetic from a new perspective. Namely, the innovative approach is based on state-mapping networks, where each value in the digital domain is treated as a node, whereas the mapping between pairs of nodes is treated as a directed edge [27]. Scale-free properties of the logistic map under the state-mapping network approach are clearly highlighted. Moreover, extension of the technique to floating-point arithmetic is studied in detail [27]. Finally, the authors of reference [27] highlight that to understand the state-mapping network structure of chaotic discrete-time systems in digital computers, it may help to take into account unwelcome degenerations of chaos in finite-precision domains. This may help to enhance the randomness of pseudorandom number generators based on chaotic maps [27].

In [28], it is pointed out that low-power devices used in Internet-of-things networks currently lack security owing to the high power consumption of random number generators. Consequently, the authors of reference [28] present a low-power true random number generator that exploits a four-dimensional hyperchaotic system characterized by hidden attractors. A customized circuit is realized using 130 nm CMOS technology, with the aim to integrate it into low-power devices. Moreover, chaotic data post-processing via SHIFT-XOR is exploited to produce random bit outputs [28]. The consumption of the hyperchaotic circuit reaches the maximum value of 980μW to generate the chaotic signals, whereas the dissipation of the static current is about 623μA. Finally, the authors of reference [28] highlight that the approach guarantees ready-to-use random bit sequences that passed the randomness test NIST SP800-22.

A novel chaotic system, which exploits a recently proposed signum thermostat, is presented in [29]. The implemented circuit produces a signal whose probability distribution function is accurately Gaussian. Data from the implemented circuit are analysed. The experimental results prove to be in agreement with the theoretical analysis, indicating that the implemented chaotic circuit is effective in producing accurate Gaussian random numbers [29].

In [30], a simple method for designing pseudo-random bit generators is presented. The approach enables multiple bits per iteration to be generated from the decimal part of a chaotic map [30]. The objective is achieved by removing the decimal part of the state in each iteration and by comparing each digit with respect to a threshold value. By using such a technique, more than one bit can be generated in each iteration. This represents an advantage over most of the chaos-based generators available in the literature, since they can produce only one bit [30]. The final bitstream passes all NIST tests. Moreover,

simulation results show that for each iteration about 8 digits can be extracted for most of the chaotic maps, indicating the effectiveness of the conceived approach in designing innovative pseudo-random bit generators [30].

By using fuzzy triangular numbers, in [31] a modification of the well-known logistic map is proposed. The modified map, analysed via bifurcation diagrams and Lyapunov exponents, exhibits a high number of chaos-related phenomena, including antimonotonicity and crisis. The map is then exploited for generating pseudo-random bit sequences via an effective algorithm [31]. The conceived bit generator readily passes the NIST statistical tests. Note that the proposed technique enables the complexity of any chaotic map to be increased. Moreover, note that the method in [31] can be generalized and, consequently, utilized for pseudo-random bit generators based on chaotic maps different from the logistic one.

In [32], a novel chaotic system with hyperbolic sinusoidal function is introduced, where self-excited attractors and hidden attractors coexist. A chaotic circuit is designed and implemented. By varying its parameters, it is shown that the chaotic circuit presents two sets of hidden attractors (with line of equilibria and no equilibrium point), which provide additional complexity to the system dynamics [32]. On the basis of these results, a random number generator is designed, which successfully passes all the NIST statistical tests [32].

In [33], a modified logistic map that exhibits a number of different chaotic behaviours (i.e., antimonotonicity, crisis, and coexisting attractors) is proposed. The increased complexity of the map dynamics is then exploited to implement a chaos-based pseudo-random bit generator. The approach exploits techniques that include multiple map comparison, bit reversal and XOR [33]. The conceived generator passes all the NIST tests, confirming the validity of the approach.

In [34], it is pointed out that most of the chaotic maps used for cryptographic applications have a number of weaknesses, including limited chaotic ranges, small Lyapunov exponents and heavy computational costs. The authors of reference [34] make an attempt to overcome these drawbacks by constructing a new chaotic map with complex dynamic behaviour via geometric operations. In particular, Lyapunov exponents and some associated theorems are mathematically derived, so that the map is characterized by good chaotic performances [34]. Moreover, the design of a pseudo-random number generator is carried out using the conceived chaotic map. Performance evaluations show that the conceived generator can effectively produce high-quality random sequences [34].

In [35], a generalization of the double-humped logistic map is introduced and applied to pseudo-random number generation (PRNG) for the encryption of medical images. The approach offers secure communication transfer of medical MRI and X-ray images. Different tests are carried out (i.e., sensitivity test, histogram analysis, correlation coefficients and NIST analysis), with the aim to highlight the encryption efficiency in transmitting medical images [35].

7. Applications of Chaos to Computing

Modern digital computers perform computations using logic gates. On the other hand, the high sensitivity to initial conditions of chaotic systems leads their dynamics to switch fast between patterns. By combining logic gates and chaos properties, chaotic morphing logic gates can be implemented via nonlinear circuits that exhibit chaotic dynamics producing various patterns. Then, control mechanisms can be used to select patterns that correspond to different logic gates. Based on these considerations, interesting applications of chaos to computing have been recently presented [36]. In particular, the authors of reference [36] have presented an approach that combines two different research topics, i.e., chaos computing and logical stochastic resonance. On the one hand, the authors propose the implementation of fundamental logic operations by using the switch between chaotic attractors [36]. The idea is that the dynamics of some nonlinear systems can hop between attractors located in different parts of the phase space when system parameters and/or initial states are varied. Based on this idea, the implementation of the fundamental NOR gate is demonstrated [36]. The logic response can be switched from NOR to NAND via

small changes of the bias parameters, indicating that the circuits are readily reconfigurable. Another advantage is that complementary logic operations can be obtained in parallel using different circuit variables [36]. On the other hand, the authors of reference [36] present a generalized concept of logical stochastic resonance (LSR). Namely, they prove that the idea of LSR (to date based on the use of steady states) can be extended to chaotic attractors. Specifically, the authors of reference [36] show that the noise floor aids the reliability of the logic operations, even when they are based on the switch between states more complex than fixed points, such as chaotic attractors. Finally, experimental circuit realization of the conceived approach is reported, indicating the feasibility of the method in real-world applications [36].

8. Applications of Chaos to Neuron Modelling

Processing of data via neuromorphic nonlinear dynamics is a key element for brain-inspired computers. In [37] it is found that the Chua Corsage memristor has rich neuromorphic behaviours, with great potentials for hardware implementations of neuromorphic computing. In particular, in [37] the theory of local activity and edge of chaos are applied to analyse the neuromorphic dynamics of the second-order Chua Corsage memristor-based neuron model.

The authors of reference [38] report on the neuromorphic dynamics of nanowire networks (NWNs), a unique brain-inspired system with synapse-like memristive junctions embedded within a recurrent neural network-like structure. The paper shows that NWNs adaptively respond to time-varying stimuli, exhibiting different dynamic behaviours, ranging from order to chaos. Finally, the authors of reference [38] show that the dynamical state at the edge-of-chaos is optimal for learning, besides improving complex information processing tasks from the computational point of view.

9. Conclusions

As a Guest Editor of the Special Issue on Chaotic Systems and Nonlinear Dynamics, the author of the present manuscript has aimed to bridge the gap between chaos theory and chaos applications by presenting a survey of very recent applications of chaos. The manuscript has made a precise choice, i.e., to cover the last three years in the literature (2018 to 2020), so that only very recent applications of chaos have been taken into account. In this way, the manuscript would be considered a landmark for readers who seek fresh and novel applications of chaos to real-world issues. The covered topics have included applications of chaos to communications, to distributed sensing, to robotic motion, to bio-impedance modelling, to hardware implementation of encryption systems, to computing and to random number generation. The approach has made perceivable the great potential of the applications of chaos in solving complex real problems and, consequently, in improving our everyday life.

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