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Editorial

Mathematical Aspects in Non-Equilibrium Thermodynamics

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Introduction

Prof. Csaba Asszonyi, D.Sc. (1941–2022): The present Special Issue is dedicated to the memory of our beloved, respected friend, colleague and teacher, the late Professor Csaba Asszonyi.

The research field of professor Asszonyi was continuum mechanics and irreversible thermodynamics. He played a pioneering role in establishing the thermodynamical background of rock rheology and engineering rock mechanics. He was educated as a mechanical engineer, started his career as research engineer, and then performed coordinated mining research in Hungary. Later on, he went into the industry and became a company group leader. Seventeen years ago, he established the Montavid Thermodynamic Research Group.

His thinking focused on thermodynamic concepts, connecting an application-oriented, engineering attitude with deep theoretical ideas. He developed several industrial applications of thermodynamic rheology. His contributions included the extension of linear viscoelasticity with internal variables and the unification of classical rheological bodies in a thermodynamic framework. He was the author of more than two hundred articles, dozens of patents, and ten books. He refused honours and distinctions, and only at the end of his life became the honorary president of the Society for the Unity of Science and Technology.

The Special Issue "Mathematical Aspects in Non-equilibrium Thermodynamics" consists of five original research papers. Although the current topic has a long history, there are still numerous open questions regarding the structure of evolution equations, the corresponding thermodynamically compatible initial and boundary conditions, and also their relation to experimental and practical aspects. These five papers actually cover various recent and relevant topics such as optimization, finite time thermodynamics, the role of the second law in continuum physics, multi-component mixtures, and boundary conditions. We hope that this Special Issue will be able to play a role in further progress to come in the future.

In the paper "The Role of the Second Law of Thermodynamics in Continuum Physics: A Muschik and Ehrentraut Theorem Revisited" by V. A. Cimmelli and P. Rogolino [1], the authors revisited the second law of thermodynamics and how the entropy inequality plays a crucial role in the derivation of evolution equations, also providing local and global formulations of the second law. The classical results of Muschik and Ehrentraut are reformulated in the present modern mathematical context of second law, thus highlighting a few geometric aspects as an outcome. They also emphasized that the non-equilibrium concept of temperature and entropy far from equilibrium is not necessarily identical to the one close to equilibrium, and how these notions need further investigation.

The paper "Integrability of the Multi-Species TASEP with Species-Dependent Rates", written by Eunghyun Lee [2], is related to totally asymmetrical simple exclusion processes; it

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was shown that the Bethe ansatz method is applicable to processes in the present N-particle mixtures with species-dependent rates, providing transition probabilities for all possible compositions of species. Despite the limitations detailed in [2], this study serves as a basis for future investigation to see if the methods can be used to study the species inhomogeneity of other multi-species models.

In the paper “Shock Structure and Relaxation in the Multi-Component Mixture of Euler Fluids” by Madjarevic et al. [3], an important benchmark study related to shock structures is presented. Here, the authors utilized a multi-component mixture of Euler fluids, whose evolution equations possess a hyperbolic structure, originating from extended thermodynamics. The present study is concerned with a three-component mixture of polyatomic gases inheriting the kinetic theory formulation for the phenomenological coefficients. The the quantitative characteristics of the shock profiles, such as the temperature overshoot, the shock thickness, and the resulting relaxation times were investigated, thus providing a deeper insight into a complex, coupled phenomenon.

The paper “Cyclic Control Optimization Algorithm for Stirling Engines” by Raphael Paul and Karl Heinz Hoffmann [4] deals with an optimization problem related to non-equilibrium Stirling engines. The authors focused their attention on the optimization of both the power and efficiency, using an indirect iterative gradient algorithm. The problem formulation led to a particular Hamiltonian system, describing attractive and repulsive limit cycles, with periodic boundary conditions. They provided detailed insight into the problem formulation and optimization algorithm, and therefore their results are of high importance in dealing with similar optimization tasks for other thermodynamic cycles.

The last paper of the present Special Issue, titled “Recent Advances on Boundary Conditions for Equations in Nonequilibrium Thermodynamics” [5], is written by Wen-An Yong and Yizhou Zhou. They focused on linearized systems obeying the hyperbolic structure originating from extended thermodynamics and reviewed the possible (proper) boundary conditions in the light of uniform and generalized Kreiss conditions. The structural stability of the studied PDEs was also satisfied. As these conditions are strongly related to the suitability of hyperbolic equations, the present results could serve as a future basis for the comparison of various thermodynamic approaches and provide hints to extend the present formalism to nonlinear problems.

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References

1. Cimmelli, V.A.; Rogolino, P. The Role of the Second Law of Thermodynamics in Continuum Physics: A Muschik and Ehrentraut Theorem Revisited. *Symmetry* **2022**, *14*, 763. [CrossRef]
2. Lee, E. Integrability of the Multi-Species TASEP with Species-Dependent Rates. *Symmetry* **2021**, *13*, 1578. [CrossRef]
3. Madjarević, D.; Pavić-Colić, M.; Simić, S. Shock Structure and Relaxation in the Multi-Component Mixture of Euler Fluids. *Symmetry* **2021**, *13*, 955. [CrossRef]
4. Paul, R.; Hoffmann, K.H. Cyclic Control Optimization Algorithm for Stirling Engines. *Symmetry* **2021**, *13*, 873. [CrossRef]
5. Yong, W.-A.; Zhou, Y. Recent Advances on Boundary Conditions for Equations in Nonequilibrium Thermodynamics. *Symmetry* **2021**, *13*, 1710. [CrossRef]