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An Investigation of the Transient Response of an RC Circuit with an Unknown Capacitance Value Using Probability Theory

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Abstract: In this research, we investigate a resistor capacitor electric circuit that exhibits an exponentially decaying transient response. Due to uncertainty in the precise capacitance value, we treat the capacitance as a continuous uniformly distributed random variable. Using this approach, we derive the desired transient current response of the circuit as a function of the capacitance. Subsequently, we develop a probability model for the response current, expressed in terms of probability density function and cumulative distribution function. The model's validity and correctness are verified, and it is further utilized for probabilistic analysis of the transient current. We demonstrate the application of the model for determining the probability of the transient current response reaching a specific value. By following the same procedure used to derive the probability model of the transient current, probability distributions for other circuit parameters, such as voltages and currents, can also be obtained. Furthermore, for parameters that are functions of the transient current, the probability model can also be obtained from the already derived probability model. To illustrate this, we derive the probability models of three other parameters in the circuit from the already obtained models. We also present examples to demonstrate the usage of the developed probability models.

Keywords: current probability distribution; probabilistic circuit analysis; transient response; electric current prediction; capacitance random variable; RC circuit

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1. Introduction

Among different types of electric circuits, a circuit consisting of resistors and a capacitor, i.e., an RC circuit, is an important type of circuit which has numerous applications. RC circuits find use in almost all types of low- or high-voltage and alternating-current (AC) or direct-current (DC) circuits [1–4]. The most frequent uses of RC circuits include timing applications and filtering applications corresponding to the time domain and frequency domain, respectively. When used in an application involving timing, an RC circuit is usually employed for controlling the rise or fall timing of a circuit variable, such as a voltage or a current [5]. The rise or fall of the circuit parameter is used for the control or triggering of an event. Likewise, in frequency-domain applications, an RC circuit may be deployed for filtering a certain band of frequencies [6]. These are the most frequent applications of an RC circuit. However, these are not the only applications where RC circuits are used. RC circuits may find numerous other uses as well. An RC circuit, for instance, can serve as a differentiator or an integrator, or it can be used for surge suppression purposes. When used in a surge suppression circuit [7,8], the capacitor in the circuit absorbs the energy due to the surge, whereas the resistance determines the time rate at which this energy is absorbed and dissipated. An RC surge suppressor may also be connected to a circuit breaker or relay to disconnect the circuit while the surge is being absorbed.

The work in [9] proposes a two-stage RC polyphase filter with low loss for use in vector sum phase shifters. The authors show that the phase shifter performance can be improved by allowing some gain error while simultaneously decreasing the phase error. A vector sum phase shifter is improved by utilizing the proposed RC polyphase filter as it reduces the area and the loss without affecting the phase shift performance. An investigation of the impact of an RC trigger circuit on the performance of high-voltage Darlington electrostatic discharge protection is presented in [10]. For this purpose, activation sensitivity in electrostatic discharge conditions is analyzed and the conduction behaviour is examined. A design methodology to determine the RC value is also proposed. The work in [11] uses a parallel scheme of an RC circuit and a metal oxide arrestor to improve the cut-off current capacity of a direct-current solid-state circuit breaker. The RC circuit, which acts as a buffer, helps reduce the energy-absorbing pressure of the metal oxide arrestor while the fault clearing time is also shortened. For the suppression of false triggering oscillations in a gallium nitride half-bridge circuit, an RC snubber circuit is investigated in [12]. The proposed RC circuit can also be employed for the design of oscillation suppression circuits for higher-order systems. For the analysis of false triggering oscillation, a double-pulse circuit using a high-frequency equivalent circuit is obtained. Then, the characteristic equations are analyzed using the root locus method to establish the RC region. When the RC parameters are within the region, the RC snubber circuit provides a better oscillation suppression effect. An RC bridge oscillation chaotic circuit based on memristor is presented in [13]. The Lyapunov exponents spectrum, bifurcation diagram and phase portrait are used for the analysis of the dynamic behavior of the circuit. The dynamical behaviors which are analyzed include symmetric and asymmetric single scroll-coexistence, symmetric double-scroll coexistence and limit-cycle coexistence. The work in [14] presents an operational amplifier with a four-port RC network in the feedback as an active cell negative group delay circuit. Operational amplifier parameters and an RC cell are used to express the unity direct-chain feedback group delay frequency response. A low-voltage, fully differential, Butterworth fourth-order active RC low-pass filter is proposed in [15]. The RC filter has a maximum cut-off frequency of 160 MHz and is designed for communication systems which require a reconfigurable cut-off frequency at a supply voltage of 0.6 V. The work in [16] proposes an RC-type generator circuit using an n channel metal oxide semiconductor field effect transistor (MOSFET) for the stabilization of current by the prevention of its direction fluctuation. Time-fractional RC circuits are investigated and a convergence analysis of the Neumann–Neumann waveform relaxation method for these circuits is performed in [17]. In the classical waveform relaxation method, a system of large-scale ordinary differential equations is decoupled into small-scale subsystems. However, the classical waveform relaxation methods converge slowly for the strongly coupled systems. However, longitudinal waveform relaxation methods, such as the Neumann–Neumann waveform relaxation method, provide simple partitioning and have uniform convergence. Therefore, the work in [17] analyzes a time-fractional RC circuit using the Neumann–Neumann waveform relaxation method. The work in [18] exploits probability theory for the analysis of an RC circuit with a failed capacitor. The capacitance of the capacitor is treated as a random variable and the probability model of the exponentially rising transient response current of the circuit is then developed as a derived random variable. An RL circuit is investigated using the probability methods in [19]. The probability model of the transient response of the circuit is developed, treating the inductor as a random variable.

Conventional techniques, which involve time- or frequency-domain analysis, can be employed for the investigation of RC circuits. However, these techniques have limited usage when the capacitor has malfunctioned and the operating value of its capacitance is no more the nominal value. The capacitor in these different RC circuits may malfunction due to a variety of reasons. In particular, the nominal value of the capacitance may drift due to various factors, such as temperature, low insulation resistance, dielectric degradation and high dissipation factor. While the exact value of the capacitance, which is a characteristic parameter, may not be determined in such a situation, it is possible to establish the range of values that the parameter can fall within. Probability methods along with regular circuit analysis theorems can be used for investigating various circuit parameters under these circumstances. Probability theory is a valuable mathematical tool that is used for the analysis and exploration of solutions to a number of engineering problems [20–24]. In this paper, we extend the work in [18,19] and utilize probability theory to analyze the exponentially decreasing transient current response of an RC electric circuit, where the precise value of the capacitance is unknown and is only known to exist within a specific interval of values. First, an expression for the decaying response of the circuit is derived using conventional circuit analysis techniques. The capacitance of the capacitor is considered as a uniformly distributed continuous random variable. Afterward, by adopting a probabilistic approach, we obtain the probability model for the intended parameter of the circuit, which, in our example, corresponds to a branch current that decays exponentially. The probability model, which is obtained in terms of cumulative distribution function (CDF) and probability density function (PDF), is verified for correctness using a validity check. The probability model is also exploited to investigate the probability that the value of the current being investigated falls in a particular interval.

2. Determination of Circuit Response

This section comprises the preliminary work, in which we perform deterministic analysis to derive an expression for the circuit response using basic circuit theorems. To best demonstrate our technique, we use a numerical example of the circuit shown in Figure 1 and find the circuit response in terms of the current $i_o(t)$ in Figure 1. We use this circuit for ease of demonstration and also due to the fact that the derivation of the probability model of the parameters of other circuit elements can also be explained easily. The demonstrated technique can easily be generalized for a simple RC circuit.

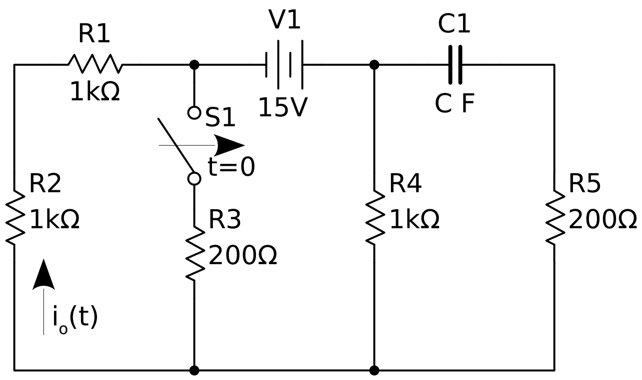

Figure 1. Response of the RC electric circuit is examined once the switch $S1$ is closed at time $t = 0$. The precise value of the capacitor's capacitance is unknown, except for the fact that it is restricted to a specific fixed interval of values.

The circuit is previously in a steady state. The circuit enters a transient state when the switch $S1$ closes at time $t = 0$. Afterward, the circuit returns to a steady state once again. The parameter values of all the circuit components are known as labeled in Figure 1. However, the precise value of the capacitor's capacitance C is not ascertainable, but it is known to fall within a continuous interval of values. The capacitance can take any of the values in the range with equal probability. After the capacitor takes a capacitance value at random from the interval, the value remains fixed and does not vary during the entire period of the transient response. Therefore, we assume that the capacitance is C farads (F), which is a uniformly distributed continuous random variable. It is to be noted that the capacitance can take on any other probability distribution, such as Gaussian distribution, depending upon the application and reason for its variation. It is also possible that the parameters of more than one circuit element are random variables. In this work, we assume that only the capacitance is a random variable and has continuous uniform distribution.

In Figure 1, a first-order RC circuit is shown. We are interested in the investigation of the response current through the resistor R2. The general form of the current through the resistor R2 is as below:

$$i_o(t) = k_1 + k_2 e^{-\frac{t}{\tau}}, \tag{1}$$

where k_1, k_2 and τ are constants, such that

$$k_1 = i_o(\infty), \tag{2}$$

$$k_2 = i_o(0) - i_o(\infty), \tag{3}$$

$$\tau = R_e C, \tag{4}$$

where $i_o(0)$ represents the initial current at time $t = 0$, while $i_o(\infty)$ represents the current at steady state when $t = \infty$. The Thevenin equivalent resistance as seen across the capacitor terminals is denoted by R_e.

The circuit response in terms of $i_o(t)$ can be determined using a step-wise approach. Figure 2 depicts the circuit during $t < 0$, when the switch has been open for an extended period, allowing the circuit to reach a steady-state condition. The initial voltage $v_c(0)$ across the capacitor before the closure of the switch $S1$ is found to be

$$v_c(0) = v_c(0^+) = v_c(0^-) = 5\,\text{V}. \tag{5}$$

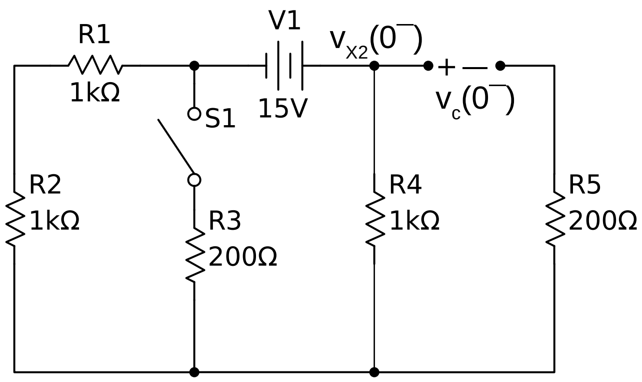

Figure 2. Prior to $t = 0$, the switch $S1$ had been open for a considerable amount of time, causing the circuit to settle into a steady state. At this point, the capacitor is fully charged and the voltage across the charged capacitor is $v_c(0^-)$.

The initial current at $t = 0^+$ can be determined by considering the circuit in Figure 3, and is given by

$$i_o(0) = \frac{65}{23} = 2.83\,\text{mA}. \tag{6}$$

Similarly, the steady-state current after closing switch $S1$ is determined by using Figure 4. This can be calculated to be

$$i_o(\infty) = \frac{15}{13} = 1.154\,\text{mA}. \tag{7}$$

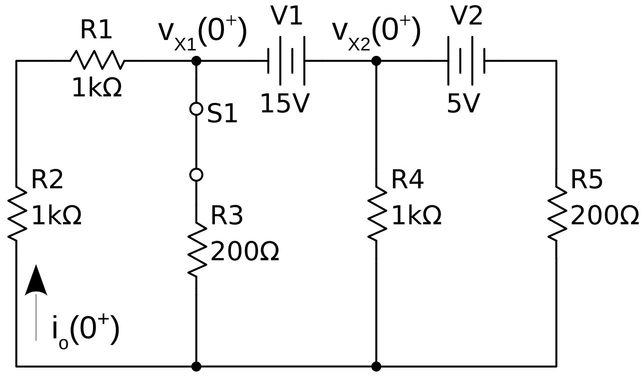

Figure 3. At $t = 0^+$ when the switch $S1$ is closed. The circuit is depicted in the figure. The capacitor has been substituted with a voltage source, the voltage of which is equivalent to the initial voltage of the capacitor.

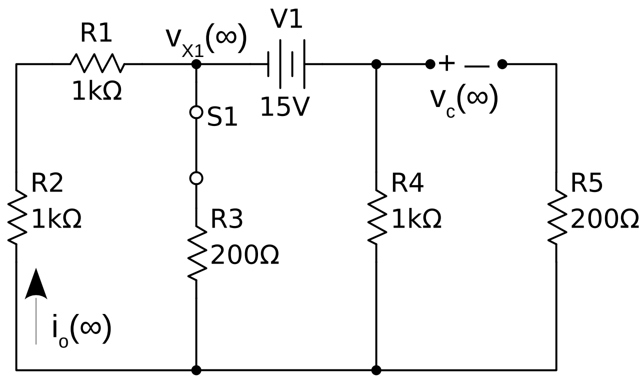

Figure 4. Once the switch $S1$ is closed, and ample time has elapsed, the circuit attains a steady-state status at $t > 0$, as shown in the figure. The capacitor is now charged to a different value.

To calculate the circuit time constant using (4), we must initially determine the Thevenin equivalent resistance across the capacitor terminals when the circuit has achieved steady-state status at $t > 0$. To determine the Thevenin equivalent resistance, we turn off the independent voltage source V1 in the circuit of Figure 4. The resulting circuit is shown in Figure 5. The Thevenin equivalent resistance R_e across the capacitor terminals is obtained as

$$R_e = \frac{4600}{13} = 353.85\,\Omega. \tag{8}$$

Therefore, the time constant is given by

$$\tau = 353.85C\text{ s}. \tag{9}$$

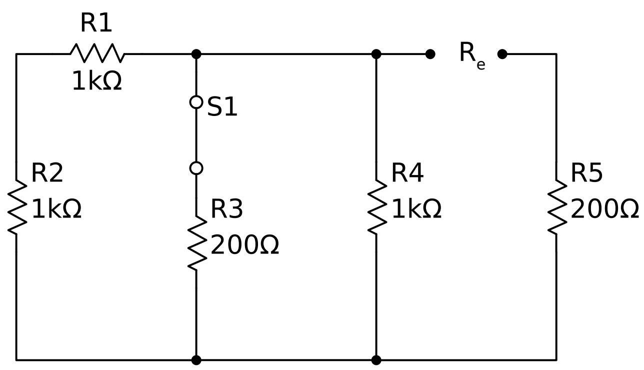

Figure 5. Circuit at $t > 0$ after turning off voltage source V1 for the calculation of the Thevenin resistance across the capacitor terminals.

From (2) and (7), we obtain

$$k_1 = \frac{15}{13} = 1.154. \tag{10}$$

Similarly, from (3), (6) and (7), we obtain

$$k_2 = \frac{65}{23} - \frac{15}{13} = \frac{500}{299} = 1.67. \tag{11}$$

Substituting (9)–(11) in (1), we have

$$i_o(t) = \frac{15}{13} + \frac{500}{299} e^{-\frac{t}{353.85C}} \text{ mA}, \tag{12}$$

which can also be written as

$$i_o(t) = 1.154 + 1.67 e^{-\frac{t}{353.85C}} \text{ mA}. \tag{13}$$

From (12) and (13), we can note that $i_o = \frac{65}{23} = 2.83$ mA at $t = 0$, and $i_o = \frac{15}{13} = 1.154$ mA at $t = \infty$. This suggests that the current $i_o(t)$ recedes during the transient phase, making the transient response in terms of $i_o(t)$ an exponential decay. It is important to note that the response's initial and final values at $t = 0$ and $t = \infty$ do not rely on the capacitance C's value. Nevertheless, the rapidity with which the response decays, i.e., the time constant, is dependent on the capacitance C. We plot the response in Figure 6 for $C = 15$ mF.

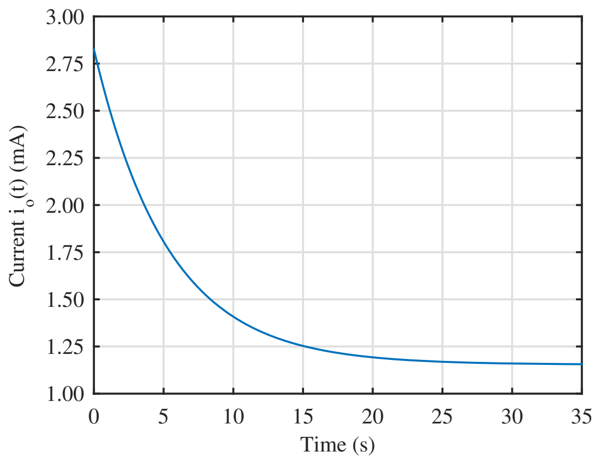

Figure 6. The plot illustrates the transient response with a capacitance value of 15 mF. At $t = 0$, the initial current i_o is 2.83 mA, whereas at $t = \infty$, it decreases to 1.154 mA. The current $i_o(t)$ displays an exponential decay from $t = 0$ to $t = \infty$.

3. Probability Model

Since the capacitor's capacitance is merely determined to exist within a range of values, we can use probability techniques to explore the response further. By considering the capacitance as a uniform continuous random variable, the circuit response can be regarded as a random variable that is dependent on the capacitance. In this section, we perform a probabilistic analysis of the circuit response and obtain the probability model in terms of the cumulative distribution function (CDF) and probability density function (PDF) of the current I_o.

To explain the notation, we denote a random variable with a capital letter and any of its realizations are represented using the corresponding lower-case letter. Therefore, x is any one of the numerical values of the random variable X.

Suppose we know that the capacitance C lies in a continuous interval (a, b), where $b > a > 0$, and each value within the range has the same likelihood of occurrence. In that case, we can treat the capacitance C as a uniform random variable over the interval (a, b). This implies that the probability density function (PDF) $f_C(c)$ of C can be expressed as follows:

$$f_C(c) = \begin{cases} \frac{1}{b-a} & a \leq c < b, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Next, the CDF $F_C(c)$ is then given by the following:

$$F_C(c) = \begin{cases} 0 & c \leq a, \\ \frac{c-a}{b-a} & a < c \leq b, \\ 1 & c > b. \end{cases} \tag{15}$$

During the interval $(0, 5\tau)$, the RC circuit is in the transient state. Consider a time t such that

$$t = \frac{n\tau}{C}, \tag{16}$$

where the number n fulfills the condition

$$0 < t \leq 5\tau,$$
$$0 < \frac{n\tau}{C} \leq 5\tau,$$
$$0 < n \leq 5C.$$

During the transient state, the following relation holds true for the random variable C, with $a < c \leq b$:

$$0 < n \leq 5a. \tag{17}$$

Using the value of τ from (9) in (16), we obtain

$$t = \frac{n\tau}{C} = 353.83n. \tag{18}$$

Substituting this value in (12), we obtain an expression for the random variable I_o, as below:

$$I_o = \frac{15}{13} + \frac{500}{299}e^{-\frac{n}{C}}. \tag{19}$$

The random variable I_o is a derived random variable that depends on the random variable C. From (19), we obtain

$$\frac{500}{299}e^{-\frac{n}{C}} = I_o - \frac{15}{13},$$

$$C = \frac{-n}{\ln\left[\frac{299}{500}\left(I_o - \frac{15}{13}\right)\right]}. \tag{20}$$

If (20) is to be valid, then $I_o - \frac{15}{13} > 0$, i.e., $I_o > \frac{15}{13}$. Since C is a uniform random variable over the interval (a, b) with $b > a > 0$, it can be established that $C > 0$. Therefore, the denominator in (20) meets the following condition:

$$\ln\left[\frac{299}{500}\left(I_o - \frac{15}{13}\right)\right] < 0. \tag{21}$$

As a result, $C > 0$.

3.1. Probability Distribution of I_o

Now, we will obtain the probability distribution of the random variable I_o. For this, we first derive the cumulative distribution function (CDF) $F_{I_o}(i_o)$, which is given by

$$F_{I_o}(i_o) = P[I_o \leq i_o]. \tag{22}$$

The expression $I_o \leq i_o$ can be evaluated using (19), as below:

$$\frac{15}{13} + \frac{500}{299} e^{-\frac{n}{C}} \leq i_o,$$

$$\frac{n}{C} \geq -\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]. \tag{23}$$

We can conclude from (20) and (21) that $-\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right] > 0$ in (23). Therefore, the reciprocal of (23) gives us the following:

$$C \leq \frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]}. \tag{24}$$

The CDF can now be obtained from (22) and (24), as below:

$$F_{I_o}(i_o) = P\left[C \leq \frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]}\right], \tag{25}$$

This gives us the following:

$$F_{I_o}(i_o) = F_C\left(\frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]}\right). \tag{26}$$

Hence, from (15), we obtain the following:

$$\frac{c-a}{b-a} = \frac{\frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]} - a}{b-a},$$

$$\frac{c-a}{b-a} = \frac{-n}{(b-a)\left[\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]\right]} - \frac{a}{b-a}. \tag{27}$$

For a complete specification of the CDF $F_{I_o}(i_o)$, we need to determine three intervals for i_o corresponding to $c \leq a$, $a < c \leq b$ and $c > b$ as is evident from (15). The expressions for these three intervals are derived in the following:

$$c \leq a$$

Using (20), we obtain the following:

$$\frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]} \leq a,$$

$$i_o \le \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{a}}. \tag{28}$$

Therefore, the interval for $F_{I_o}(i_o)$ corresponding to the interval $c \le a$ for $F_C(c)$ is given by (28). We now derive an expression for the interval which corresponds to $c > b$ in the following by using (20):

$$c > b$$
$$\frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]} > b,$$
$$i_o > \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{b}}. \tag{29}$$

Analogously, the expression for the third interval is obtained through the following derivation:

$$a < c \le b$$
$$a < \frac{-n}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]} \le b,$$
$$\frac{15}{13} + \frac{500}{299} e^{-\frac{n}{a}} < i_o \le \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{b}}. \tag{30}$$

We can now obtain the complete CDF $F_{I_o}(i_o)$ using (27)–(30), as follows:

$$F_{I_o}(i_o) = \begin{cases} 0 & i_o \le \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{a}}, \\ \frac{-n}{(b-a)\left[\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]\right]} - \frac{a}{b-a} & \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{a}} < i_o \le \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{b}}, \\ 1 & i_o > \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{b}}. \end{cases} \tag{31}$$

We obtain the PDF by differentiating the CDF in (31):

$$f_{I_o}(i_o) = \frac{n}{(b-a)(i_o - \frac{15}{13})\left[\ln\left[\frac{299}{500}(i_o - \frac{15}{13})\right]\right]^2}. \tag{32}$$

The PDF $f_{I_o}(i_o)$ can now be completely specified, as below:

$$f_{I_o}(i_o) = \begin{cases} \frac{n}{(b-a)(i_o - \frac{15}{13})\left[\ln\left[\frac{299}{500}(i_o - \frac{15}{13})\right]\right]^2} & \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{a}} \le i_o < \frac{15}{13} + \frac{500}{299} e^{-\frac{n}{b}}, \\ 0 & \text{otherwise.} \end{cases} \tag{33}$$

Either the CDF presented in (31) or the PDF given in (33) provides a complete description of the probability distribution of I_o and, consequently, the transient response of the circuit.

With the help of the techniques developed in this section, a probability model of a general RC circuit can be developed in a similar manner starting from (1). In addition, higher-order circuits can also be investigated using a similar approach.

3.2. Validity Check

We can confirm the validity of (33) if the following can be confirmed to hold:

$$\int_{-\infty}^{+\infty} f_{I_o}(i_o) \mathrm{d}i_o = 1. \tag{34}$$

Therefore, from (33), we have

$$\int_{-\infty}^{+\infty} f_{I_o}(i_o)\mathrm{d}i_o = \int_{\frac{15}{13}+\frac{500}{299}e^{-\frac{n}{a}}}^{\frac{15}{13}+\frac{500}{299}e^{-\frac{n}{b}}} \frac{n}{(b-a)(i_o-\frac{15}{13})\left[\ln\left[\frac{299}{500}(i_o-\frac{15}{13})\right]\right]^2}\mathrm{d}i_o. \tag{35}$$

This can be solved using substitution. Therefore, let

$$u = \ln\left[\frac{299}{500}(i_o - \frac{15}{13})\right]. \tag{36}$$

From this, we can obtain the following:

$$\mathrm{d}i_o = (i_o - \frac{15}{13})\mathrm{d}u. \tag{37}$$

By the substitution in (36), we obtain the limits for u in (35) as follows:

$$u = -\frac{n}{a}. \tag{38}$$

$$u = -\frac{n}{b}. \tag{39}$$

Substituting (36) and (37) in (35), and making use of the limits (38) and (39), we obtain

$$\int_{-\infty}^{+\infty} f_{I_o}(i_o)\mathrm{d}i_o = \frac{n}{(b-a)}\int_{-\frac{n}{a}}^{-\frac{n}{b}} \frac{\mathrm{d}u}{u^2}, \tag{40}$$

$$\int_{-\infty}^{+\infty} f_{I_o}(i_o)\mathrm{d}i_o = \frac{n}{(b-a)}\left| -\frac{1}{u}\right|_{-\frac{n}{a}}^{-\frac{n}{b}} = 1. \tag{41}$$

Hence, (33) satisfies the condition in (34). Therefore, it is concluded that it is a valid PDF.

3.3. Utilization of the Probability Model

To illustrate the usage of the probability model, we provide an example where we take $a = 10$ mF and $b = 20$ mF, resulting in an expected value of C as $E[C] = 15$ mF. By substituting these values into (14) and (15), we acquire the ensuing PDF and CDF of C, respectively.

$$f_C(c) = \begin{cases} \frac{1}{10} & 10 \leq c < 20, \\ 0 & \text{otherwise}, \end{cases} \tag{42}$$

$$F_C(c) = \begin{cases} 0 & c \leq 10, \\ \frac{c-10}{10} & 10 < c \leq 20, \\ 1 & c > 20. \end{cases} \tag{43}$$

The unit mF for C is used in (42) and (43). In accordance with (17), let us now assume $n = a = 10$ mF. Therefore, from (18), we obtain $t = 353.83 \times 10 \times 10^{-3} = 3.538$ s. As $10 < c \leq 20$, therefore, using (9), we note that $t = 3.538 < \tau$, which is within the transient state. By substituting $a = 10$, $b = 20$ and $n = a = 10$ into (31), we can obtain the following CDF:

$$F_{I_o}(i_o) = \begin{cases} 0 & i_o \leq 1.77, \\ \frac{-1}{\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]} - 1 & 1.77 < i_o \leq 2.17, \\ 1 & i_o > 2.17. \end{cases} \tag{44}$$

Similarly, the PDF for this specific example can be obtained from (33), as below:

$$f_{I_o}(i_o) = \begin{cases} \frac{1}{\left(i_o - \frac{15}{13}\right)\left[\ln\left[\frac{299}{500}\left(i_o - \frac{15}{13}\right)\right]\right]^2} & 1.77 \leq i_o < 2.17, \\ 0 & \text{otherwise.} \end{cases} \tag{45}$$

The probability model for the scenario where $C \sim \text{uniform}(10, 20)$ can be obtained from the CDF in (44) and the PDF in (45). In the CDF and the PDF, the unit mA is used for the current i_o. The CDF and PDF for this example are plotted in Figures 7 and 8, respectively.

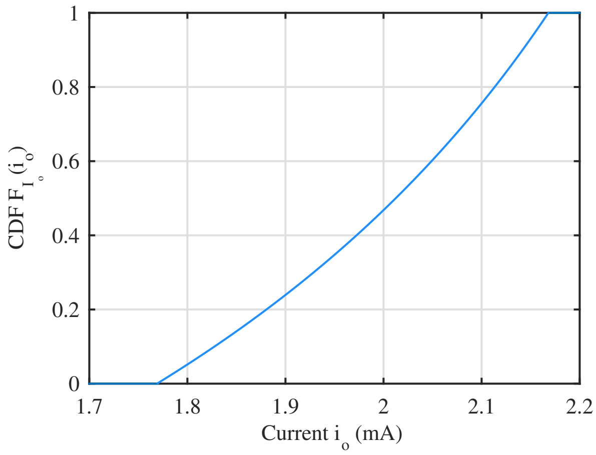

Figure 7. The cumulative distribution function (CDF) $F_{I_o}(i_o)$ when $a = 10$ mF, $b = 20$ mF and C is a uniform random variable over the interval (a, b).

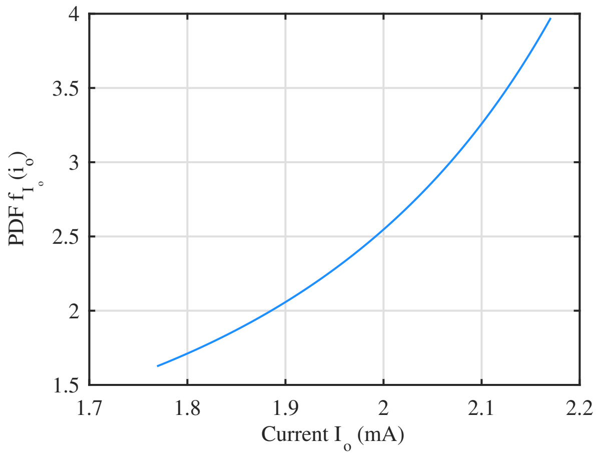

Figure 8. The probability density function (PDF) $f_{I_o}(i_o)$ when $a = 10$ mF, $b = 20$ mF and C is uniform (a, b) random variable.

Utilizing the CDF and PDF, probabilities can be calculated for varying values of the current i_o, which in turn helps in analyzing the transient response. As an example, let us find $P[I_o \leq 2.0]$. This can be calculated using either the CDF or the PDF. As the former is convenient, we use the CDF from (44) to obtain

$$P[I_o \leq 2.0] = F_{I_o}(2.0) = 0.468. \tag{46}$$

Therefore, it can be stated that the probability that i_o is below or equal to 2.0 mA at $t = 3.538$ s is 46.8%. It can be observed from (44) and (45) that $F_{I_o}(i_o) = 0$ and $f_{I_o}(i_o) = 0$ for $i_o < 1.77$. Therefore, $P[I_o \leq 2.0]$ implies $P[1.77 < I_o \leq 2.0]$. In other words, the probability of i_o falling between 1.77 and 2.0 mA is 46.8%. This also implies that the probability of i_o falling between 2.0 and 2.17 mA is $100 - 46.8 = 53.2\%$. All these probability values for this specific example are calculated at time $t = 3.538$ s, which falls within the transient state. The probability for any other time can be calculated by choosing an appropriate value of n.

The probability models derived in this article can be leveraged for the analysis and investigation of an electric circuit with a failed component. In artificial intelligence and machine learning, probability theory is a key technique [25,26]. Designing an intelligent system that can carry out a desired task after learning from the environment and the data is the goal of artificial intelligence and machine learning applications. Numerous outcomes may be possible while carrying out a task. As a result, the machine learning model uses data to make predictions, approximations and estimations. In carrying out these important processes, probability theory is crucial [27–30]. The probability models created in our research can also be used to analyze and forecast how a circuit will behave when a component fails.

4. Probability Models of Other Parameters

The probability model for any voltage or current in the circuit can be obtained by following a similar approach used for deriving the probability model of I_o. The circuit parameter of interest is represented as a function of C, and its corresponding CDF and PDF are obtained using a similar procedure to that employed for deriving the probability model of I_o. Alternatively, if a circuit variable is a function of I_o, then its probability distribution can also be determined from the probability model of I_o. To further illustrate this methodology, we derive the probability models of the following three parameters of the circuit:

1. Voltage at node V_{x1};
2. Current I_1 through the resistor R3;
3. Voltage at node V_{x2}.

4.1. Voltage V_{x1}

Let us consider the voltage across the branch containing resistors $R1$ and $R2$. We designate this voltage as V_{x1}, and it is given by

$$V_{x1} = -(R1 + R2)I_o. \tag{47}$$

The unit of i_o is mA in the CDF (31) and PDF (33); therefore, we can use $R1 + R2 = 2$ kΩ, and

$$V_{x1} = -2I_o. \tag{48}$$

It is easy to notice that V_{x1} is a random variable that is derived from I_o and is a constant multiple of I_o. Therefore, we can establish the relationships between the CDFs $F_{V_{x1}}(v_{x1})$ and $F_{I_o}(i_o)$ and the PDFs $f_{V_{x1}}(v_{x1})$ and $f_{I_o}(i_o)$ as follows:

$$F_{V_{x1}}(v_{x1}) = P[V_{x1} \leq v_{x1}], \tag{49}$$

Using (48), we have

$$F_{V_{x1}}(v_{x1}) = P[-2I_o \leq v_{x1}], \tag{50}$$

$$F_{V_{x1}}(v_{x1}) = P[I_o > -\frac{v_{x1}}{2}], \tag{51}$$

$$F_{V_{x1}}(v_{x1}) = 1 - P[I_o \leq -\frac{v_{x1}}{2}], \tag{52}$$

Therefore,

$$F_{V_{x1}}(v_{x1}) = 1 - F_{I_o}(-\frac{v_{x1}}{2}). \tag{53}$$

Differentiating this, we obtain the PDF as shown below:

$$f_{V_{x1}}(v_{x1}) = \frac{1}{2} f_{I_o}(-\frac{v_{x1}}{2}). \tag{54}$$

From (31) and (53), we obtain the CDF $F_{V_{x1}}(v_{x1})$, as shown below:

$$F_{V_{x1}}(v_{x1}) = \begin{cases} 0 & v_{x1} \leq -\frac{30}{13} - \frac{1000}{299} e^{-\frac{n}{b}}, \\ \frac{n}{(b-a)\left[\ln\left[-\frac{299}{500}\left(\frac{v_{x1}}{2} + \frac{15}{13}\right)\right]\right]} + \frac{b}{b-a} & -\frac{30}{13} - \frac{1000}{299} e^{-\frac{n}{b}} < v_{x1} \leq -\frac{30}{13} - \frac{1000}{299} e^{-\frac{n}{a}}, \\ 1 & v_{x1} > -\frac{30}{13} - \frac{1000}{299} e^{-\frac{n}{a}}. \end{cases} \tag{55}$$

In a similar manner, we obtain the PDF $f_{V_{x1}}(v_{x1})$ from (33) and (54) as follows:

$$f_{V_{x1}}(v_{x1}) = \begin{cases} \frac{-n}{2(b-a)(\frac{v_{x1}}{2} + \frac{15}{13})\left[\ln\left[-\frac{299}{500}(\frac{v_{x1}}{2} + \frac{15}{13})\right]\right]^2} & -\frac{30}{13} - \frac{1000}{299} e^{-\frac{n}{b}} \leq v_{x1} < -\frac{30}{13} - \frac{1000}{299} e^{-\frac{n}{a}}, \\ 0 & \text{otherwise.} \end{cases} \tag{56}$$

Example for the Probability Model of V_{x1}

We now demonstrate the usage of the probability model of V_{x1} given by (55) and (56). For this, we use the probability model of C as described in (42) and (43). Therefore, $a = 10$ mF, $b = 20$ mF and $n = a = 10$ mF. Substituting these values in (55), we obtain the CDF for our example:

$$F_{V_{x1}}(v_{x1}) = \begin{cases} 0 & v_{x1} \leq -4.336, \\ \frac{1}{\ln\left[-\frac{299}{500}\left(\frac{v_{x1}}{2} + \frac{15}{13}\right)\right]} + 2 & -4.336 < v_{x1} \leq -3.538, \\ 1 & v_{x1} > -3.538. \end{cases} \tag{57}$$

Similarly, from (56), we obtain the PDF for the specific case, as shown below:

$$f_{V_{x1}}(v_{x1}) = \begin{cases} \frac{-1}{2(\frac{v_{x1}}{2} + \frac{15}{13})\left[\ln\left[-\frac{299}{500}(\frac{v_{x1}}{2} + \frac{15}{13})\right]\right]^2} & -4.336 \leq v_{x1} < -3.538, \\ 0 & \text{otherwise.} \end{cases} \tag{58}$$

Let us now calculate the probability $P[V_{x1} > -4]$. This can be calculated using either (57) or (58). Using (57), we have

$$P[V_{x1} > -4] = 1 - P[V_{x1} \leq -4] = 1 - F_{V_{x1}}(-4) = 0.468. \tag{59}$$

It is to be noted that $v_{x1} = -4$ V when $i_o = 2$ mA. Therefore, the result in (59) is in harmony with (46). As a further example, let us calculate the probabilities $P[-4 < V_{x1} \leq -3.7]$ and $P[-4.3 < V_{x1} \leq -4]$ and compare the results. Again, from (57), we obtain the probabilities as follows:

$$P[-4 < V_{x1} \leq -3.7] = F_{V_{x1}}(-3.7) - F_{V_{x1}}(-4) = 0.327, \tag{60}$$

$$P[-4.3 < V_{x1} \leq -4] = F_{V_{x1}}(-4) - F_{V_{x1}}(-4.3) = 0.462. \tag{61}$$

So, the likelihood of V_{x1} falling between -4.3 V and -4 V at time $t = 3.538$ s, given that C follows a uniform distribution between 10 and 20, is greater than the likelihood of V_{x1} falling between -4 V and -3.7 V.

4.2. Current through Resistor R3

We now derive the probability model of the current through the resistor R3 in the direction from the node V_{x1} to the bottom node. Let us denote this current using I_1. The current I_1 is given by

$$I_1 = \frac{V_{x1}}{200}\,\text{A} \tag{62}$$

To keep the model derived for I_o consistent, let us express I_1 also in mA. Therefore,

$$I_1 = 5V_{x1}\,\text{mA} \tag{63}$$

This expresses the current I_1 as a function of V_{x1}. Therefore, we can leverage the already derived probability model of V_{x1} to develop the probability model of I_1. The CDF $F_{I_1}(i_1)$ of I_1 is given by

$$F_{I_1}(i_1) = P[I_1 \le i_1], \tag{64}$$

$$F_{I_1}(i_1) = P[5V_{x1} \le i_1], \tag{65}$$

$$F_{I_1}(i_1) = P[V_{x1} \le \frac{i_1}{5}], \tag{66}$$

$$F_{I_1}(i_1) = F_{V_{x1}}(\frac{i_1}{5}), \tag{67}$$

Differentiating this, we obtain the PDF as follows:

$$f_{I_1}(i_1) = \frac{1}{5} f_{V_{x1}}(\frac{i_1}{5}), \tag{68}$$

From (55) and (68), we obtain the CDF $F_{I_1}(i_1)$, as follows:

$$F_{I_1}(i_1) = \begin{cases} 0 & i_1 \le -\frac{150}{13} - \frac{5000}{299}e^{-\frac{n}{b}}, \\ \dfrac{n}{(b-a)\left[\ln\left[-\frac{299}{500}\left(\frac{i_1}{10}+\frac{15}{13}\right)\right]\right]} + \frac{b}{b-a} & -\frac{150}{13} - \frac{5000}{299}e^{-\frac{n}{b}} < i_1 \le -\frac{150}{13} - \frac{5000}{299}e^{-\frac{n}{a}}, \\ 1 & i_1 > -\frac{150}{13} - \frac{5000}{299}e^{-\frac{n}{a}}. \end{cases} \tag{69}$$

We can obtain the PDF by differentiating (69) or from (56) and (68) as follows:

$$f_{I_1}(i_1) = \begin{cases} \dfrac{-n}{10(b-a)(\frac{i_1}{10}+\frac{15}{13})\left[\ln\left[-\frac{299}{500}(\frac{i_1}{10}+\frac{15}{13})\right]\right]^2} & -\frac{150}{13} - \frac{5000}{299}e^{-\frac{n}{b}} \le i_1 < -\frac{150}{13} - \frac{5000}{299}e^{-\frac{n}{a}}, \\ 0 & \text{otherwise.} \end{cases} \tag{70}$$

Example for the Probability Model of I_1

For our example, let us assume the values of the parameters a, b and n are the same as those used previously, i.e., $a = 10$ mF, $b = 20$ mF and $n = a = 10$. Therefore, the probability model of C is the same as that given by (42) and (43). We obtain the CDF for this example by substituting the values of the parameters a, b and n in (69) as follows:

$$F_{I_1}(i_1) = \begin{cases} 0 & i_1 \le -21.681, \\ \dfrac{1}{\left[\ln\left[-\frac{299}{500}\left(\frac{i_1}{10}+\frac{15}{13}\right)\right]\right]} + 2 & -21.681 < i_1 \le -17.690, \\ 1 & i_1 > -17.690. \end{cases} \tag{71}$$

Similarly, using the parameter values in (70), we obtain the PDF as follows:

$$f_{I_1}(i_1) = \begin{cases} \dfrac{-1}{10(\frac{i_1}{10}+\frac{15}{13})\left[\ln\left[-\frac{299}{500}(\frac{i_1}{10}+\frac{15}{13})\right]\right]^2} & -21.681 \leq i_1 < -17.690, \\ 0 & \text{otherwise.} \end{cases} \tag{72}$$

Using either the CDF in (71) or the PDF in (72), we can obtain the probability for a specific range of the current I_1. For example, we obtain $P[I_1 \leq -19]$ from the PDF as follows:

$$P[I_1 \leq -19] = \int_{-21.681}^{-19} f_{I_o}(i_o)\mathrm{d}i_o = 0.761, \tag{73}$$

We can also solve this using the CDF, as $P[I_1 \leq -19] = F_{I_1}(-19) = 0.761$.

4.3. Voltage V_{x2}

Considering the voltage relationships in Figure 3, it can be readily observed that the voltage V_{x2} is given by

$$V_{x2} = V_{x1} + 15. \tag{74}$$

Therefore, we can once again leverage the already derived probability model of V_{x1} to develop the probability model of V_{x2}. The CDF $F_{V_{x2}}$ is given by

$$F_{V_{x2}}(v_{x2}) = P[V_{x2} \leq v_{x2}], \tag{75}$$

$$F_{V_{x2}}(v_{x2}) = P[V_{x1} + 15 \leq v_{x2}], \tag{76}$$

$$F_{V_{x2}}(v_{x2}) = P[V_{x1} \leq v_{x2} - 15], \tag{77}$$

$$F_{V_{x2}}(v_{x2}) = F_{V_{x1}}(v_{x2} - 15). \tag{78}$$

Differentiating this, we obtain the relationship for the PDF $f_{V_{x2}}(v_{x2})$ as follows:

$$f_{V_{x2}}(v_{x2}) = f_{V_{x1}}(v_{x2} - 15). \tag{79}$$

From (55) and (78), the CDF $F_{V_{x2}}(v_{x2})$ is obtained as follows:

$$F_{V_{x2}}(v_{x2}) = \begin{cases} 0 & v_{x2} \leq \frac{165}{13} - \frac{1000}{299}e^{-\frac{n}{b}}, \\ \dfrac{n}{(b-a)\left[\ln\left[-\frac{299}{1000}\left(v_{x2}-\frac{165}{13}\right)\right]\right]} + \frac{b}{b-a} & \frac{165}{13} - \frac{1000}{299}e^{-\frac{n}{b}} < v_{x2} \leq \frac{165}{13} - \frac{1000}{299}e^{-\frac{n}{a}}, \\ 1 & v_{x2} > \frac{165}{13} - \frac{1000}{299}e^{-\frac{n}{a}}. \end{cases} \tag{80}$$

We obtain the PDF $f_{V_{x2}}(v_{x2})$ from (56) and (79) as follows:

$$f_{V_{x2}}(v_{x2}) = \begin{cases} \dfrac{-n}{(b-a)(v_{x2}-\frac{165}{13})\left[\ln\left[-\frac{299}{1000}(v_{x2}-\frac{165}{13})\right]\right]^2} & \frac{165}{13} - \frac{1000}{299}e^{-\frac{n}{b}} \leq v_{x2} < \frac{165}{13} - \frac{1000}{299}e^{-\frac{n}{a}}. \\ 0 & \text{otherwise.} \end{cases} \tag{81}$$

Example for the Probability Model of V_{x2}

As used in the examples for V_{x1} and I_1, we now substitute the same values of the parameters, i.e., $a = 10$ mF, $b = 20$ mF and $n = a = 10$ in (80) and (81) to obtain the CDF and the PDF, respectively, for the example for V_{x2} as follows:

$$F_{V_{x2}}(v_{x2}) = \begin{cases} 0 & v_{x2} \leq 10.664, \\ \dfrac{1}{\left[\ln\left[-\frac{299}{1000}\left(v_{x2}-\frac{165}{13}\right)\right]\right]} + 2 & 10.664 < v_{x2} \leq 11.462, \\ 1 & v_{x2} > 11.462. \end{cases} \tag{82}$$

$$f_{V_{x2}}(v_{x2}) = \begin{cases} \dfrac{-1}{\left(v_{x2}-\frac{165}{13}\right)\left[\ln\left[-\frac{299}{1000}\left(v_{x2}-\frac{165}{13}\right)\right]\right]^2} & 10.664 \leq v_{x2} < 11.462. \\ 0 & \text{otherwise.} \end{cases} \tag{83}$$

For further demonstration, let us now calculate the probability, say, $P[V_{x2} \leq 11]$, using the CDF in (82) as follows:

$$P[V_{x2} \leq 11] = F_{V_{x2}}(11) = 0.532 \tag{84}$$

We can calculate the same probability using the PDF in (83) as follows:

$$P[V_{x2} \leq 11] = \int_{10.664}^{11} f_{V_{x2}}(v_{x2})\mathrm{d}v_{x2} = 0.532, \tag{85}$$

To summarize, the probability model of a circuit parameter provided by the CDF and the PDF can be developed directly or it can be obtained from an already derived model of another parameter. The probability model can then used for the probability estimation of circuit parameters.

5. Conclusions

In this work, we have conducted an investigation into the exponentially decaying transient response of an RC electric circuit. The capacitance of the circuit's capacitor is subject to uncertainty, with its true value only known to exist within a range. To account for this, we have treated the capacitance as a continuous uniform random variable. Utilizing conventional analysis techniques, we have derived a general expression for the circuit's transient response as a function of the capacitance. Subsequently, we have applied probability methods to establish a probability model for the circuit's decaying transient response in terms of a branch current, which has been further utilized to derive the probability distribution of the branch voltage. These models have been utilized to perform a probabilistic analysis of the RC circuit's decaying transient response. In future work, we aim to extend our analysis to consider more than one circuit parameter as random variables and consider employing other distributions in the derivation of the probability model. We also plan to derive probability models of general first-order and higher-order circuits using the techniques developed in this work.

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