

Article

Reliability-Centered Preventive Maintenance Optimization for a Single-Component Mechanical Equipment

Yaojun Liu ^{1,2}, Yuhua Tang ³, Ping Wang ^{2,*}, Xiaolin Song ¹ and Meilin Wen ^{4,5}

¹ Wuhu State-Owned Factory of Machining, Wuhu 241000, China; liuyaojun@easycac.cn (Y.L.); songxiaolin@easycac.cn (X.S.)

² College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210002, China

³ Lunar Exploration and Space Engineering Center (LESEC), Beijing 100097, China; tangyh@dsel.cc

⁴ School of Reliability and Systems Engineering, Beihang University, Beijing 100191, China; wenmeilin@buaa.edu.cn

⁵ The Key Laboratory on Reliability and Environmental Engineering Technology, Beihang University, Beijing 100191, China

* Correspondence: wangping@easycac.cn

Abstract: Due to the high failure rates of mechanical equipment with complex structures and numerous moving parts, devising an effective preventive maintenance (PM) plan and avoiding the influence brought by failure is crucial. However, some PM efforts are disorganized, unpractical, and unscientific, leading to prolonged downtime and significant cost losses. The challenge in creating PM plans is exacerbated by the asymmetry between maintenance and failure data. Therefore, focusing on single-unit mechanical equipment, the reliability-centered maintenance (RCM) idea is put forward to find out the key parts to implement preventive maintenance, and PM models are built to draw up a more reasonable PM plan. Such strategies aim to lower maintenance costs and enhance economic performance. Data on past maintenance and failures are analyzed to determine the life distribution and maintenance effect functions, helping to quantify the uncertainty caused by data asymmetry. Two PM optimization models considering time-varying failure rates are proposed: one focuses on minimizing costs, while the other aims to maximize availability. A PM plan example is demonstrated using a component from a tire-building machine including six parts, which proves the validity of the models. The availability results of two parts corresponding to the maintenance strategy obtained by the availability maximization model are above 0.99, and the results of total costs per unit time of the remaining four parts obtained by the cost minimization model are under 5.69.

Keywords: reliability-centered maintenance; preventive maintenance optimization model; mechanical equipment; failure rate; reliability



Citation: Liu, Y.; Tang, Y.; Wang, P.; Song, X.; Wen, M. Reliability-Centered Preventive Maintenance Optimization for a Single-Component Mechanical Equipment. *Symmetry* **2024**, *16*, 16. <https://doi.org/10.3390/sym16010016>

Academic Editor: Alexander Zaslavski

Received: 2 December 2023

Revised: 18 December 2023

Accepted: 20 December 2023

Published: 21 December 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The organized, practical, and scientific preventive maintenance work of mechanical equipment can result in a lower failure rate. And according to a reasonable PM plan, spare part production and supply can also be arranged properly, which avoids the incidents that spare parts are often still in transit when machines break down. This prevents delays in replacing failed parts, potentially reducing the risk of prolonged machine downtime and significant cost losses. Therefore, to improve the efficiency and quality of safe operation and extend the service life, minimizing the influence on mechanical systems brought by failure and working out a preventive maintenance (PM) plan have become urgent problems to be solved.

After years of development and evolution of preventive maintenance models, there are many kinds of PM policies, including the age-dependent PM policy [1–4], the periodic PM policy [5–7], the failure-limit PM policy [8], the sequential PM policy [9], the repair-limit

PM policy for single-unit systems [10], and the on-condition PM policy for single and multi-unit systems [11–15], etc. Where the practical application in engineering is concerned, considering the difficulties in the implementation of preventive maintenance, the failure-limit PM policy and the repair-limit PM policy are not widely used alone. Comparatively speaking, the age-dependent PM policy, the periodic PM policy, and the sequential PM policy are simpler and easier to carry out, which reduces the complexity of the process of preventive maintenance work and the determination of time between PMs. The equal time between PMs in age-dependent PM policy and periodic PM policy leads to a decrease in reliability with increasing maintenance times. So, a more effective method named reliability-centered maintenance (RCM) [16–19] is offered to save maintenance resources and costs.

In the related research of RCM, the PM optimization model considering the failure rate is getting more and more attention. Stringer et al. [20] point out that failure rate trends in maintenance planning decisions may reduce the risk of premature failure and prevent unnecessary costs associated with repairing or replacing equipment. The papers [21–23] also point to the importance of considering time-varying failure rates in reliability-centered maintenance decisions. However, few RCM studies consider the variation in failure rates caused by both maintenance and precession work periods. In actual engineering, the uncertainty caused by the asymmetry of data on past maintenance and failures is also an important factor affecting the PM optimization results, and how to take it into account in the optimization model is a problem that needs to be solved.

Accordingly, focusing on single-component mechanical equipment, a reliability-centered PM optimization model is put forward in this paper. Failure rates over time due to both factors are considered simultaneously in this model, and the uncertainty in the failure data is measured using a Weibull distribution. The contributions of this paper include the following: (i) Theoretically, the proposed reliability-centered PM optimization model takes both the sudden change in reliability and the changes in the failure rate into account to describe the maintenance effect and considers the variation in failure rates to quantify the uncertainty of data on past maintenance and failures. (ii) Practically, the preliminary optimization of preventive maintenance for mechanical equipment has been achieved, and a more scientific and reasonable preventive maintenance plan has been formulated, providing support for the adjustment of preventive maintenance plans and a basis for future preventive maintenance work in mechanical equipment company. The structure of this paper is organized as follows: Section 2 describes the preventive maintenance problem to be addressed. In Section 3, a model of failure rate variation considering the effects of both maintenance and precession work periods is presented to provide a basis for reliability-centered maintenance optimization models. Section 4 introduces two types of PM optimization models: minimizing costs or maximizing availability. Section 5 illustrates the validity of the proposed model using a type of tire-building machine in MESNAC as a case study.

2. Problem Description for PM Optimization

Considering the operability of preventive maintenance, a combined PM policy is examined. Under this type of policy, as shown in Figure 1, we suggest that at a fixed time iT , preventive maintenance is taken. If failure happens in preventive maintenance, corrective maintenance is taken. A unit is replaced when the n th preventive maintenance needs to be taken. At each preventive maintenance, the reliability of a working unit is no less than the minimum reliability of a working unit and assuming the following:

1. A unit is put into operation at a new state from the beginning;
2. The preventive maintenance is imperfect, which means a unit cannot be as good as its new condition after the preventive maintenance and will obey the combined model of age-declining model and failure-rate-increasing model;
3. The corrective maintenance is minimal, which means the failure rate remains the same before and after the corrective maintenance;

4. Replacement is perfect maintenance, which means a unit becomes as good as new after replacement.

These assumptions regarding our model are detailed and explained in the remainder of this section. The state of a unit is assumed to be new from the beginning of its operation. This is a commonly used assumption when developing maintenance plans. The development of maintenance plans often occurs before the equipment is put into operation. At this time, neglecting the defective equipment that may have not been detected, the equipment is generally in a new state. Moreover, there are three assumptions related to the effect of maintenance. It is somewhere between as good as new and as bad as old. There are many possible causes for imperfect maintenance, such as the repair of the wrong part, only partial repair of the faulty part, repair of the faulty part but damage of an adjacent part, or incorrect assessment of the inspected unit condition [24]. Therefore, the preventive maintenance, the corrective maintenance, and the replacement are assumed to be imperfect, minimal, and perfect, respectively, which is more realistic. One or several of these assumptions are also made in the research mentioned earlier, for example, [1–7].

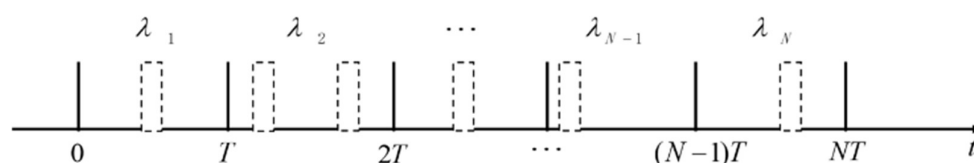


Figure 1. Preventive maintenance policy.

3. Failure Rate Model for Reliability-Centered Maintenance Optimization Models

To establish the reliability model, the failure rate is used to describe the reliability of a unit before and after preventive maintenance through the life distribution function and the maintenance effect function. Therefore, a method is provided to determine the reliability index and solve the failure frequency in the PM optimization model.

In reliability, common procedures are usually based on the assumption that the data follow a Weibull distribution [25]. For mechanical equipment, mainly the declining type of failure, considering the asymmetry of the failure data, the Weibull distribution is used to describe the failure rate in a typical way [26]:

$$\lambda(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} e^{\left(\frac{t}{\theta}\right)^{\beta}}, (\theta > 0, \beta > 0, t \geq 0). \quad (1)$$

In practice, the reliability of a unit is not always the same, especially after preventive maintenance; the unit cannot be restored to its original. So, based on the life distribution function, we need to further describe the effect of restoration after preventive maintenance to show the reliability of a unit. The restoration degree of a unit's reliability caused by maintenance can be understood as a maintenance effect and expressed as a function.

During the running of a unit, preventive maintenance is mostly imperfect. The condition of a unit can be improved after preventive maintenance but cannot totally recover back to its original condition. Therefore, the age-declining model [27] is used to describe the reliability of the moment after maintenance.

In the age-declining model, an age-declining factor is introduced, and the changes in failure rate describe the maintenance effect, which is expressed as follows:

$$\lambda_{k+1}(t) = \lambda_k(b_k \cdot T + t). \quad (2)$$

The age of a unit is declined to be b_k ($0 < b_k < 1$) times lower than before. At the same time, the initial failure rate after preventive maintenance is $\lambda_k(b_k \cdot T)$ instead of zero, as shown in Figure 2.

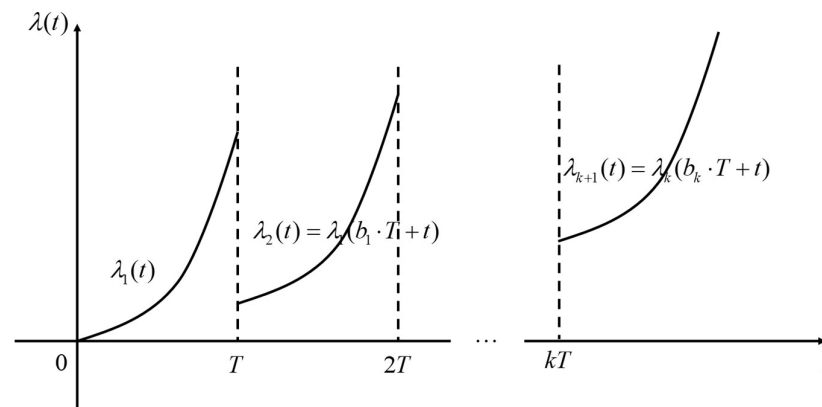


Figure 2. The age-declining model.

However, after preventive maintenance, a unit's change in condition is not only reflected in the moment but also in the whole process of work after maintenance. Therefore, the failure-rate-increasing model [28] is used to describe the reliability during the subsequent working period after maintenance.

In the failure-rate-increasing model, a failure-rate-declining factor is introduced, and the changes in failure rate describe the maintenance effect as well, which is expressed as follows:

$$\lambda_{k+1}(t) = a_k \cdot \lambda_k(t). \quad (3)$$

The initial failure rate after preventive maintenance is still zero, but the rate of change in the failure rate is increased to be a_k ($a_k > 1$) times higher than before, as shown in Figure 3.

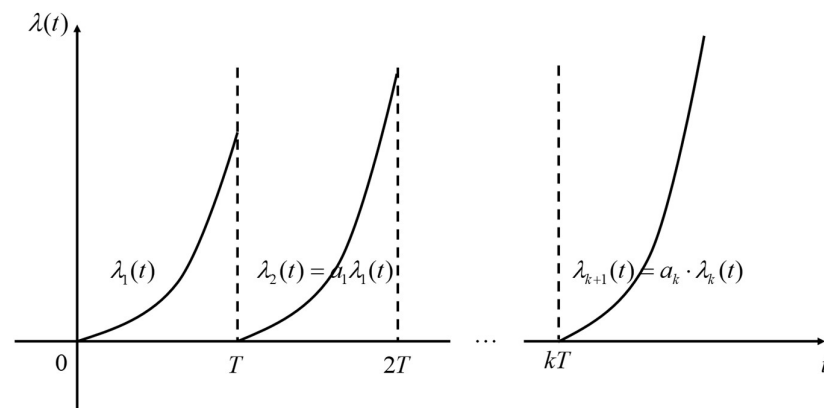


Figure 3. The failure-rate-increasing model.

To be closer to the actual condition of a unit after the preventive maintenance, the two basic models mentioned above are combined. This approach takes into account both the sudden change in reliability immediately after maintenance and the overall change in reliability compared to the previous working period. The failure rate distribution function in each PM is further described.

In the combining model, the changes in failure rate describe the maintenance effect as well, which is expressed as follows:

$$\lambda_{k+1}(t) = a_k \cdot \lambda_k(b_k \cdot T + t). \quad (4)$$

The age of a unit declined, the initial failure rate after preventive maintenance was not zero anymore, and the speed of the failure rate increased, as shown in Figure 4.

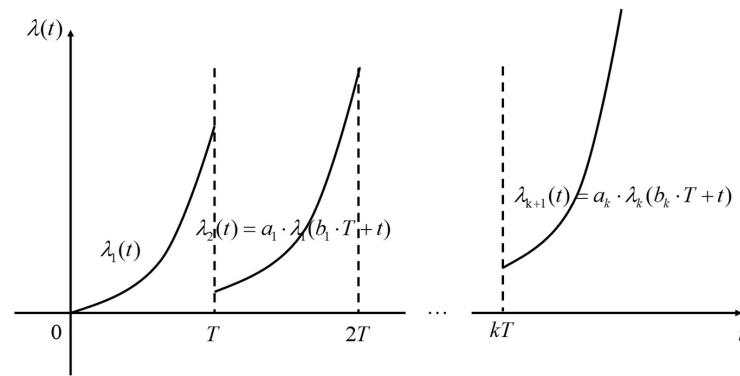


Figure 4. The combining model.

In general, the initial failure rate which expresses the unit without any preventive maintenance can be obtained by the life distribution function. Then, the relationship between the failure rate after repeated preventive maintenance and the initial failure rate can be derived from the combining model:

According to Equation (4),

$$\lambda_2(b_2 \cdot T + t) = a_1 \cdot \lambda_1[b_1 \cdot T + (b_2 \cdot T + t)] = a_1 \cdot \lambda_1[(b_1 + b_2) \cdot T + t]. \quad (5)$$

And $\lambda_3(t) = a_2 \cdot \lambda_2(b_2 \cdot T + t)$; then,

$$\lambda_3(t) = a_2 \cdot \lambda_2(b_2 \cdot T + t) = a_1 \cdot a_2 \cdot \lambda_1[(b_1 + b_2) \cdot T + t]. \quad (6)$$

So, the failure rate after the k th preventive maintenance is

$$\lambda_{k+1}(t) = \prod_{i=1}^k a_i \cdot \lambda_1\left(\sum_{i=1}^k b_i \cdot T + t\right). \quad (7)$$

4. Reliability-Centered PM Optimization Models

Considering the maintenance cost, other maintenance-related indexes, and reliability-related indexes, an optimization model of preventive maintenance work is established to control maintenance costs and improve reliability and maintainability performance. This chapter will elaborate on the detailed contents of the model.

4.1. Indexes in PM Models

1. Total cost:

The total cost of the PM optimization model consists of the cost of CM, PM, and replacement, and they can be expressed as

The CM cost of one replacement:

$$C_1 = C_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt, \quad (8)$$

which is the product of the cost of one corrective maintenance and the failure frequency (equal to the number of CM) calculated by the integral of the failure rate.

The PM cost of one replacement:

$$C_2 = C_p(N - 1), \quad (9)$$

which is the product of the cost of one preventive maintenance and the number of PMs;

The cost of one replacement:

$$C_3 = C_R. \quad (10)$$

According to Equations (8)–(10), the total cost is expressed as follows:

$$C_z = C_1 + C_2 + C_3 = C_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt + C_p(N-1) + C_R. \quad (11)$$

2. Total time:

The total time of the PM optimization model consists of the time of CM, PM, and replacement, and the working time can be expressed as

The CM time of one replacement:

$$T_1 = T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt, \quad (12)$$

which is the product of the time of one corrective maintenance plan and the failure frequency (equal to the number of CM) calculated by the integral of the failure rate.

The PM time of one replacement:

$$T_2 = T_p(N-1), \quad (13)$$

which is the product of the time of one preventive maintenance plan and the number of PMs;

The cost of one replacement:

$$T_3 = T_R. \quad (14)$$

The working time of one replacement:

$$T_4 = NT. \quad (15)$$

According to Equations (12)–(15), the total time is expressed as

$$T_z = T_1 + T_2 + T_3 + T_4 = NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt + T_p(N-1) + T_R. \quad (16)$$

3. The total cost of one replacement per unit of time:

From the cost perspective, when making a PM plan, a proper scheme needs to be arranged so that the total cost is minimized, so we have $\bar{C}$ express the total cost of one replacement per unit of time according to Equations (11) and (16) as follows:

$$\bar{C} = \frac{C_z}{T_z} = \frac{C_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt + C_p(N-1) + C_R}{NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt + T_p(N-1) + T_R}. \quad (17)$$

4. Availability:

From the performance perspective, when making a PM plan, a proper scheme needs to be arranged so that the percentage of working time is the highest in total time, we have $A_u = \frac{NT}{T_z}$ express the ratio of working time to total working time, and according to Equation (16), A_u is expressed as

$$A_u = \frac{NT}{T_z} = \frac{NT}{NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t) dt + T_p(N-1) + T_R}. \quad (18)$$

5. Reliability:

The aim of preventive maintenance is to reduce the failure frequency and reach a higher level of reliability. At each preventive maintenance, the reliability should not be lower than the minimum reliability of a working unit, so we have

$$R_k(T) = e^{-\int_0^T \lambda_k(t)dt} > R_m. \quad (19)$$

4.2. Optimization Models

Considering the actual usage, the goals of preventive maintenance are different for the products with higher reliability requirements or those that have a greater impact on economic performance. Therefore, two PM optimization models are built to meet the maintenance policies under different goals.

To reduce the possibility of failure and avoid unnecessary loss, the total cost of one replacement per unit of time is chosen as the objective function. We proposed a cost-minimization model:

$$\left\{ \begin{array}{l} \text{Min } \bar{C}(T, N) = \frac{C_z}{T_z} = \frac{C_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t)dt + C_p(N-1) + C_R}{NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t)dt + T_p(N-1) + T_R} \\ \text{s.t.} \\ R_k(T) = e^{-\int_0^T \lambda_k(t)dt} > R_m, \\ A_u = \frac{NT}{T_z} = \frac{NT}{NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t)dt + T_p(N-1) + T_R} \geq A_0, \\ F(T, N) \geq 0, \\ T \geq 0, \\ N \in \mathbb{Z}. \end{array} \right. \quad (20)$$

where $F(T, N)$ means other constraints in practical applications.

As for the failed product which may cause a long-time machine halt, resulting in a great loss in cost and even endanger the safety of life, to ensure the operation of the system, availability is chosen as the objective function. We proposed an availability maximization model:

$$\left\{ \begin{array}{l} \text{Max } A_u = \frac{NT}{T_z} = \frac{NT}{NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t)dt + T_p(N-1) + T_R} \\ \text{s.t.} \\ R_k(T) = e^{-\int_0^T \lambda_k(t)dt} > R_m, \\ \bar{C}(T, N) = \frac{C_z}{T_z} = \frac{C_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t)dt + C_p(N-1) + C_R}{NT + T_f \cdot \sum_{k=1}^N \int_0^T \lambda_k(t)dt + T_p(N-1) + T_R} \leq C_0, \\ F(T, N) \geq 0, \\ T \geq 0, \\ N \in \mathbb{Z}. \end{array} \right. \quad (21)$$

where $F(T, N)$ means other constraints in practical applications.

5. Case Study

As the failure rate of a type of tire-building machine in MESNAC is at a high level, devising an effective preventive maintenance (PM) plan has become an urgent problem to be solved. The preventive maintenance work of this type of tire-building machine can cause a long-time machine halt and result in a huge loss of cost. Therefore, this paper focuses on the type of tire-building machine in MESNAC. An application of the PM optimization model is presented, and a process of working out the optimizing PM plan of a type of tire building machine and the preference for carrying out preventive maintenance is provided by our models.

Combined with the experience estimation of maintenance and the abstraction of actual data, the parameters of the Weibull distribution are given. Failure rate modeling-related parameters are given based on historical data as well as experience. Repair-related costs and durations are obtained from the repair records of the actual product. The parameters of the PM optimization model were determined, as shown in Table 1.

Table 1. PM optimization model parameters.

Part Number	β	θ	a	b	C_f	C_p	C_R	C_0	A_0	R_m	T_f	T_p	T_R
001	2.88	4767	1.2	0.35	300	120	8816	5.22	/	0.89	0.53	0.26	1.33
002	1.10	7416	1.2	0.35	300	120	5747.05	5.15	/	0.87	0.49	0.24	1.22
019	1.08	2973	1.2	0.35	300	120	287.56	/	0.79	0.88	0.40	0.20	1.00
020	1.21	90,277	1.2	0.35	300	120	6667.86	/	0.80	0.86	0.32	0.16	0.80
027	1.33	75,818	1.2	0.35	300	120	558.68	/	0.80	0.87	0.08	0.04	0.20
028	1.04	136,409	1.2	0.35	300	120	550.42	/	0.81	0.89	0.44	0.22	1.10

Both component 001 and component 002 are optimized to maximize availability, and the rest are optimized to minimize cost. Therefore, they differ in model parameters A_0 and C_0 .

Because the maintenance effect of different maintenance personnel is similar, the failure-rate-increasing factor and the age-declining factor of each component are estimated by experience, which are $a_k = a = 1.20$ and $b_k = b = 0.35$. So, according to Equation (7), the failure rate after the k th preventive maintenance is

$$\begin{aligned}\lambda_{k+1}(t) &= \prod_{i=1}^k a_i \cdot \lambda_1\left(\sum_{i=1}^k b_i \cdot T + t\right) = a^k \cdot \lambda_1(kb \cdot T + t) \\ &= 1.20^k \cdot \lambda_1(0.35k \cdot T + t).\end{aligned}\quad (22)$$

Since the labor cost and service cost of preventive maintenance are nearly identical for all components, the parameters of the cost of one corrective maintenance and the cost of one preventive maintenance for each component are taken to be the same as $C_f = 300$, $C_p = 120$.

Combined with the requirements of the enterprise, the characteristics of the components themselves, the working environment, and the safety requirements, we decided to choose one of the models (Equations (20) and (21)) to calculate the time between two preventive maintenance and the number of preventive maintenance before replacement. The results of solving the optimization models, using the aforementioned parameters, are presented in Table 2.

Table 2. The results of solving the optimization models.

Part Number	$T(h)$	N	$\bar{C}/A_u$
001	109.5788	20	-/0.9929
002	139.4297	10	-/0.9926
019	78.3502	8	1.9588/-
020	56.0123	33	5.6856/-
027	197.4413	25	0.7202/-
028	135.1919	25	1.0451/-

From Table 2, it can be found that the availability optimization results of component 001 and component 002 are similar, both are higher than 0.992, and the preventive maintenance optimization of component 020 has the largest total cost of one replacement per unit time.

Concentrated on the parts with similar time between maintenance strategies, the first preventive maintenance of the parts with the longest time between maintenance strategies is taken as a cycle unit. The results of the preventive maintenance policy are expressed in Table 3.

Table 3. The results of PM policy.

Part Number	The First PM	The Second PM	The Third PM
001	-	129 h	-
002	-	129 h	-
019	67 h	-	200 h
020	67 h	129 h	200 h
027	-	-	200 h
028	-	129 h	-

After adjustment, the preventive maintenance plan flow of a basic cycle unit (400 h) is expressed as a PM time axis in Figure 5.

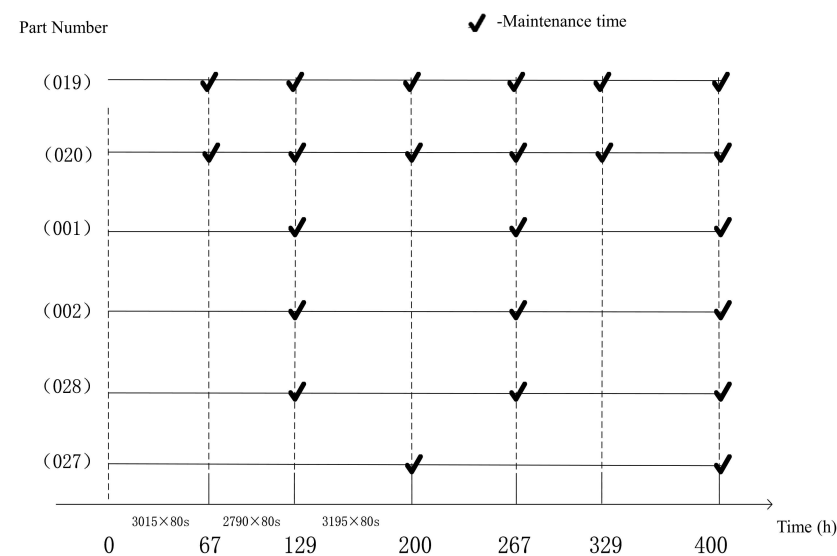
**Figure 5.** The preventive maintenance plan of a basic cycle unit for each part.

Figure 5 shows that the PM optimization model proposed in this paper is capable of solving the preventive maintenance decision-making problem for the tire-building machine. For minimizing the total cost of maintenance, component 019 and component 020 have more frequent preventive maintenance schedules, and component 027 has a longer preventive maintenance interval. The preventive maintenance optimization strategy is the same for component 001 and component 002 under the maximizing availability objective.

6. Discussion and Conclusions

Tables 2 and 3 and Figure 5 show that after the optimization of the preventive maintenance decision, both component 019 and component 020 have short preventive maintenance intervals. Combined with Table 1, it can be seen that the causes of the short preventive maintenance intervals for component 019 and component 020 are different. For component 019, it is due to its own fast-growing failure rate, so it is necessary to shorten the preventive maintenance intervals to ensure that the cost is minimized; for component 020, it is mainly due to its high replacement cost and low preventive maintenance cost, and shortening the preventive maintenance intervals can effectively reduce the total cost. Component 027 has a long preventive maintenance interval, mainly because of its slow-growing failure rate and low replacement cost, so a long preventive maintenance interval can effectively reduce the maintenance cost. In contrast to component 027, component 028 has a longer downtime for replacement and, therefore, should not have an excessively long preventive maintenance time. From Table 2, component 001 and component 002 have similar optimal availability results and similar preventive maintenance intervals during the optimization process. The main reason is that they have similar cost constraints as well as reliability

constraints. These optimization results are in line with the qualitative results, which, side by side, confirm the validity of the proposed models in Section 4.

In summary, this paper studies the optimization of reliability-centered preventive maintenance for single-unit mechanical equipment. By analyzing the preventive maintenance policy applicable to single-unit mechanical equipment, the life distribution function and maintenance effect function are obtained. On this basis, two optimization models of preventive maintenance are established, which are the cost minimization model and the availability maximization model. These models contribute to the development of a more rational preventive maintenance plan through comprehensive analysis.

However, the optimization of preventive maintenance for mechanical equipment in this paper is still at the basic stage, and there are still many problems to be further studied. This also inspires us to carry out targeted and planned management in maintenance work. Preventive maintenance based on experience and subjective decision making is not reasonable enough, and the determination of preventive maintenance intervals also needs to be scientifically and systematically analyzed in conjunction with the use, operation, and maintenance of equipment. In addition, more research is needed to balance higher costs, and higher availability, while considering the actual difficulty of implementing maintenance plans at the same time. Therefore, in future work, we aim to incorporate more complex and accurate reliability models to bridge the gap between the optimization model for preventive maintenance and real-world scenarios. We also plan to consider the impact of epistemic uncertainty on maintenance decisions and extend the optimization models to multi-unit mechanical systems.

Author Contributions: Conceptualization, Y.L. and M.W.; methodology, Y.T. and P.W.; software, Y.L., X.S. and M.W.; formal analysis, Y.T.; resources, X.S.; writing—original draft preparation, Y.L.; writing—review and editing, P.W.; visualization, M.W. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Stable Supporting Project of Science and Technology on Reliability and Environmental Engineering Laboratory (WDZC20220102).

Data Availability Statement: Data are contained in the article.

Conflicts of Interest: The authors declare no conflicts of interest.

Symbols and Notations

T	the time between preventive maintenance strategies, a decision variable;
N	the number of preventive maintenance strategies, a decision variable, and positive integer;
k	the number of preventive maintenance strategies, $k = 1, 2, \dots, N$;
a_k	the failure-rate-increasing factor at the k th preventive maintenance plan, $k = 1, 2, \dots, N$;
b_k	the age-declining factor at the k th preventive maintenance plan, $k = 1, 2, \dots, N$;
$\lambda_k(t)$	the failure rate distribution function at the k th preventive maintenance plan, which means the failure rate distribution function between the $(k - 1)$ th preventive maintenance and the k th preventive maintenance plans, $k = 1, 2, \dots, N$;
C_f	the cost of one corrective maintenance plan, a constant;
C_p	the cost of one preventive maintenance plan, a constant;
C_R	the cost of one replacement, a constant;
C_z	the total cost of one replacement;
$\bar{C}$	the total cost of one replacement per unit time;
C_0	the maximum cost of one replacement per unit time;
T_f	the time of one corrective maintenance plan, a constant;
T_p	the time of one preventive maintenance plan, a constant;
T_R	the time of one replacement, a constant;
T_z	the total time of one replacement;
A_u	availability;
$R_k(t)$	the reliability function at the k th preventive maintenance plan, $k = 1, 2, \dots, N$;
R_m	the minimum reliability of a working unit.

References

1. Richard, B.; Larry, H. Optimum Preventive Maintenance Policies. *Oper. Res.* **1960**, *8*, 90–100.
2. Nakagawa, T. Optimal policy of continuous and discrete replacement with minimal repair at failure. *Nav. Res. Logist. Q.* **1984**, *31*, 543–550. [\[CrossRef\]](#)
3. Sheu, S.; Kuo, C.; Nakagawa, T. Extended optimal age replacement policy with minimal repair. *RAIRO Rech. Oper.* **1993**, *27*, 337–351. [\[CrossRef\]](#)
4. Sanoubar, S.; Maillart, L.M.; Prokopyev, O.A. Age-replacement policies under age-dependent replacement costs. *IIE Trans.* **2021**, *53*, 425–436. [\[CrossRef\]](#)
5. Wang, H.; Pham, H. Some maintenance models and availability with imperfect maintenance in production systems. *Ann. Oper. Res.* **1999**, *91*, 305–318. [\[CrossRef\]](#)
6. Berg, M.; Epstein, B. A modified block replacement policy. *Nav. Res. Logist.* **1976**, *23*, 15–24. [\[CrossRef\]](#)
7. Nakagawa, T. A summary of periodic replacement with minimal repair at failure. *J. Oper. Res. Soc. Jpn.* **1981**, *24*, 213–228.
8. Laks, P.; Verhagen, W.J. Identification of optimal preventive maintenance decisions for composite components. *Transp. Res. Procedia* **2018**, *29*, 202–212. [\[CrossRef\]](#)
9. Wu, L.; Zhou, Q. Adaptive sequential predictive maintenance policy with nonperiodic inspection for hard failures. *Qual. Reliab. Eng. Int.* **2021**, *37*, 1173–1185. [\[CrossRef\]](#)
10. Bartholomew-Biggs, M.; Zuo, M.J.; Li, X. Modeling and optimizing sequential imperfect preventive maintenance. *Reliab. Eng. Syst. Saf.* **2009**, *94*, 53–62. [\[CrossRef\]](#)
11. Liu, X.; Li, J.; Al-Khalifa, K.N.; Hamouda, A.S.; Coit, D.W.; Elsayed, E.A. Condition-based maintenance for continuously monitored degrading systems with multiple failure modes. *IIE Trans.* **2013**, *45*, 422–435. [\[CrossRef\]](#)
12. Liu, B.; Wu, S.M.; Xie, M.; Kuo, W. A condition-based maintenance policy for degrading systems with age-and state-dependent operating cost. *Eur. J. Oper. Res.* **2017**, *263*, 879–887. [\[CrossRef\]](#)
13. Li, Y.; Ma, X.B.; Zhao, Y. A condition-based maintenance model for a three-state system subject to degradation and environmental shocks. *Comput. Ind. Eng.* **2017**, *105*, 210–226.
14. Liu, B.; Liang, Z.; Parlikad, A.K.; Xie, M.; Kuo, W. Condition-based maintenance for systems with aging and cumulative damage based on proportional hazards model. *Reliab. Eng. Syst. Saf.* **2017**, *168*, 200–209. [\[CrossRef\]](#)
15. Li, X.; Ran, Y.; Wan, F.; Zhang, G.; He, Y. Condition-based maintenance strategy optimization of meta-action unit considering imperfect preventive maintenance based on Wiener process. *Flex. Serv. Manuf. J.* **2022**, *34*, 204–233. [\[CrossRef\]](#)
16. Nowlan, F.S.; Heap, H.F. *Reliability-Centered Maintenance*; U.S. Department of Commerce: Springfield, VA, USA, 1978.
17. Okwuobi, S.; Ishola, F.; Ajayi, O.; Salawu, E.; Aworinde, A.; Olatunji, O.; Akinlabi, S.A. A Reliability-Centered Maintenance Study for an Individual Section-Forming Machine. *Machines* **2018**, *6*, 50. [\[CrossRef\]](#)
18. Song, M.; Zhang, X.; Lind, M. Automatic identification of maintenance significant items in reliability centered maintenance analysis by using functional modeling and reasoning. *Comput. Ind. Eng.* **2023**, *182*, 109409. [\[CrossRef\]](#)
19. Patil, S.S.; Bewoor, A.K.; Kumar, R.; Ahmadi, M.H.; Sharifpur, M.; PraveenKumar, S. Development of Optimized Maintenance Program for a Steam Boiler System Using Reliability-Centered Maintenance Approach. *Sustainability* **2022**, *14*, 10073. [\[CrossRef\]](#)
20. Stringer, A.D.; Thompson, C.C.; Barriga, C.I. Motor Trends: Effects of Era, Age, and Maintenance on Failure Rate. In Proceedings of the 2019 IEEE Energy Conversion Congress and Exposition (ECCE), Baltimore, MD, USA, 29 September 2019.
21. Alizadeh, A.; Fereidunian, A.; Moghimi, M.; Lesani, H. Reliability-Centered Maintenance Scheduling Considering Failure Rates Uncertainty: A Two-Stage Robust Model. *IEEE Trans. Power Deliv.* **2022**, *37*, 1941–1951. [\[CrossRef\]](#)
22. Thompson, C.C.; Barriga, C.I. Relationship Between Historical Trends, Equipment Age, Maintenance, and Circuit Breaker Failure Rates. *IEEE Trans. Ind. Appl.* **2019**, *55*, 5699–5707. [\[CrossRef\]](#)
23. Gong, Q.; Yang, L.; Li, Y.; Xue, B. Dynamic Preventive Maintenance Optimization of Subway Vehicle Traction System Considering Stages. *Appl. Sci.* **2022**, *12*, 8617. [\[CrossRef\]](#)
24. Pham, H.; Wang, H. Imperfect maintenance. *Eur. J. Oper. Res.* **1996**, *94*, 425–438. [\[CrossRef\]](#)
25. Ramos, P.L.; Nascimento, D.C.; Cocolo, C.; Nicola, M.J.; Alonso, C.; Ribeiro, L.G. Reliability-centered maintenance: Analyzing failure in harvest sugarcane machine using some generalizations of the weibull distribution. *Model. Simul. Eng.* **2018**, *2018*, 1241856. [\[CrossRef\]](#)
26. McCool, J.I. *Using the Weibull Distribution: Reliability, Modeling and Inference*; John Wiley and Sons: New York, NY, USA, 2012; Volume 950.
27. Nakagawa, T. Modified periodic replacement with minimal repair at failure. *IEEE Trans. Reliab.* **1981**, *30*, 165–168. [\[CrossRef\]](#)
28. Nakagawa, T. Periodic and sequential preventive maintenance policies. *J. Appl. Probab.* **1986**, *23*, 536–542. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.