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Article

Emergent Dynamics of Five-Colour QCD Due to Dimensional Frustration

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Abstract: The consequences for five-colour QCD of a novel symmetry-breaking mechanism, published in an earlier paper, are further explored. In addition to the emergence of QED and three-colour QCD, there is also a candidate for the Z^0_μ. The representation theory of $SU(N)$ is applied to the matter sector and yields the quark and electron charge ratios, and a mechanism for generating fermion particle masses.

Keywords: higgsless symmetry breaking; monopole condensate; abelian dominance; representation theory

1. Introduction

The breaking of gauge symmetry in both the standard model and grand unified theories (GUTs) is typically based on the Higgs mechanism. However the Higgs mechanism suffers from various drawbacks, which include the hierarchy problem but worse still is the contrived nature of the potential needed to generate the symmetry breaking. The model outlined in this paper is based on a Higgs-free symmetry breaking mechanism that arises inevitably in $SU(N)$ gauge theories when $N > 4$ [1].

It is based on the long known but generally ignored result of Flyvbjerg [2] that QCD with five or more colours has an altered vacuum state due to the limited dimensionality of space, which has been dubbed "dimensional frustration". It also relies on studies by Kondo and coworkers [3,4] of the Wilson loop and non-Abelian Stokes' theorem which found that how both quarks and gluons feel the QCD monopole background depends on the corresponding roots/weights.

The author's earlier work [1] identified the monopole condensate of five-colour QCD, or at least a good candidate for it. It is essentially the Flyvbjerg model [2] with the third and

fourth Abelian components antiparallel in real space, which was shown to follow the physically motivated ansatz of minimising the cross-terms between Abelian vacuum components, and a straightforward energy-minimisation calculation. Gauge invariance was ensured by using the Cho–Faddeev–Niemi–Shabanov decomposition [5–8] to identify the internal Abelian directions, which also identifies the monopole degrees of freedom unambiguously [9]. It is therefore highly suitable for studies of the monopole condensate [10–15] and Abelian dominance [26].

The Flyvbjerg vacuum for $SU(N > 4)$ is a monopole condensate asymmetric with respect to its Abelian components, leading to the result that different colours are confined with different strengths. The argument rests on a result of Kondo's analyses [16,17] of the non-Abelian Stokes' theorem which found that how each Abelian subgroup contributes to the Wilson loop, and therefore the confinement, of a coloured object varies according to which representation it belongs to. For example, gluons in $SU(N)$ QCD are confined by the monopole degrees of freedom of the maximal Abelian subgroup $U(1)^{\otimes(N-1)}$, while quarks feel only the effects of $SU(N-1) \otimes U(1)$. This result follows in turn from representation theory, where the Abelian components' contribution to the confinement of a given colour source can be read off from the roots or weights of the gluon or quark respectively. With the asymmetry between these components mentioned above [1,18], different confinement strengths for different colours can be demonstrated when the number of colours is five or more.

One of the interesting consequences of unequally confined gluons is that the coupling will run differently for different gluon couplings, so that the full $SU(5)$ symmetry is reduced to an intact $SU(3)$ subgroup. It is argued in Section 5 of this paper that a state which, together with its conjugate, is neutral with respect to this subgroup but not the full $SU(5)$ group, has an extremely weak confinement compared to that of the fundamental colour charges, and might be considered "effectively white". The specifics of the dimensionally frustrated vacuum lead also to one massless and one massive photon-like state [1], which can be correlated with the photon and Z^0_μ, respectively [18].

New to this paper, which also concentrates on $SU(5)$, is that the resultant coupling of the final two Cartan components imposes an unconventional quantisation condition which can only be satisfied by certain subsets of the colour combinations simultaneously. Those charges whose flux does not meet this quantisation condition will suffer an effective mass due to flux that is "left over" forming a high energy region around them. Pleasingly, the particles corresponding to realistic quarks and electrons gain masses in ratios highly suggestive of, though a little higher than, experimentally determined ones.

An important comment on the nature of this paper is that it derives most of its results from the roots and weights of the $SU(N)$ representations with remarkably little calculation. In fact a surprising amount can be read off from the roots and weights with only back-of-the-envelope calculations, although future development will of course require more effort.

Section 2 describes the CFNS decomposition of $SU(N)$ QCD [5–8], which allows the unambiguous, mathematically consistent identification of the Abelian monopole components. A brief overview of dimensional frustration and a derivation of the resulting effective vacuum state is presented in Section 3. Section 4 reviews [1,18] the derivation of the existence of photon- and Z^0_μ-like particles, while Section 5 describes the matter content of the theory. Section 6 demonstrates the generation of apparently fundamental masses and even predicts quark and electron mass ratios suggestive of experiment. We conclude with a short discussion in Section 7.

2. The Cho–Faddeev–Niemi–Shabanov Decomposition

Our treatment of the monopole condensate rests on the CFNS decomposition [5–8] and uses the following notation: The Lie group $SU(N)$ has N^2-1 generators $\lambda^{(j)}$, of which $N-1$ are Abelian generators $\Lambda^{(i)}$. For simplicity, the gauge transformed Abelian directions (Cartan generators) are specified by

$$\hat{n}_i = U^\dagger \Lambda^{(i)} U \tag{1}$$

Similarly, the standard raising and lowering operators, $E_{\pm\alpha}$ for the root vectors α, are replaced by the gauge transformed ones,

$$E_{\pm\alpha} \to U^\dagger E_{\pm\alpha} U \tag{2}$$

where $E_{\pm\alpha}$ refers to the gauge transformed operator throughout the rest of this chapter.

Gluon fluctuations in the $\hat{n}_i$ directions are described by $c_\mu^{(i)}$. The gauge field of the covariant derivative which leaves the $\hat{n}_i$ invariant is

$$g\mathbf{V}_\mu \times \hat{n}_i = -\partial_\mu \hat{n}_i \tag{3}$$

In general this is

$$\mathbf{V}_\mu = c_\mu^{(i)} \hat{n}_i + \mathbf{B}_\mu, \quad \mathbf{B}_\mu = g^{-1} \partial_\mu \hat{n}_i \times \hat{n}_i \tag{4}$$

where summation is implied over i. $\mathbf{B}_\mu$ can be a attributed to non-Abelian monopoles, as indicated by the $\hat{n}_i$ describing the homotopy group $\pi_2[SU(N)/U(1)^{\otimes(N-1)}] \approx \pi_1[U(1)^{\otimes(N-1)}]$. The monopole field strength

$$\mathbf{H}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\mathbf{B}_\mu \times \mathbf{B}_\nu \tag{5}$$

has only Abelian components, *i.e.*

$$H_{\mu\nu}^{(i)} \hat{n}_i = \mathbf{H}_{\mu\nu} \tag{6}$$

where $H_{\mu\nu}^{(i)}$ has the eigenvalue $H^{(i)}$. Since only the magnetic backgrounds are of relevance to this study, $H^{(i)}$ is considered the magnitude of a background magnetic field $\mathbf{H}^{(i)}$. The field strength of the Abelian components $c_\mu^{(i)}$ also lies in the Abelian directions as expected and is shown by

$$\mathbf{F}_{\mu\nu} = F_{\mu\nu}^{(i)} \hat{n}_i \tag{7}$$

where

$$F_{\mu\nu}^{(i)} = \partial_\mu c_\nu^{(i)} - \partial_\nu c_\mu^{(i)} \tag{8}$$

The Lagrangian of the Abelian and monopole components is

$$-\frac{1}{4}(F_{\mu\nu}^{(i)} \hat{n}_i + \mathbf{H}_{\mu\nu})^2 \tag{9}$$

Using the CFNS decomposition facilitates the study of the infrared QCD dynamics by extracting the relevant degrees of freedom. This is so because the infrared dynamics of QCD exhibit Abelian dominance, as evidenced by a variety of studies [19–28].

In maximal Abelian gauge (MAG) the monopole potential, $\mathbf{B}_\mu$, obviously vanishes. This was treated in two-colour QCD by Cho and Pak [11] who imposed the MAG by gauge transforming $\hat{n}$ to point in the

$\hat{e}_3$ direction. This gauge transform automatically generates a nontrivial contribution to c_μ that replaces the'lost' magnetic potential. The monopole field strength $\mathbf{H}_{\mu\nu}$ reappears in the electric field strength $\mathbf{F}_{\mu\nu}$ in the same way. The interested reader is referred to [11] for the details, and the extension to higher numbers of colours is straightforward. Unless otherwise specified, this work shall be gauge invariant rather than using the MAG.

The dynamical degrees of freedom (DOF) perpendicular to $\hat{n}_i$ are denoted by $\mathbf{X}_\mu$, so if $\mathbf{A}_\mu$ is the gluon field then

$$\mathbf{A}_\mu \;=\; \mathbf{V}_\mu + \mathbf{X}_\mu = c_\mu^{(i)}\hat{n}_i + \mathbf{B}_\mu + \mathbf{X}_\mu \tag{10}$$

where

$$\mathbf{X}_\mu \perp \hat{n}_i, \;\; \mathbf{X}_\mu = g^{-1}\hat{n}_i \times \mathbf{D}_\mu \hat{n}_i, \;\; \mathbf{D}_\mu = \partial_\mu + g\mathbf{A}_\mu \times \tag{11}$$

$\mathbf{X}_\mu$ is orthogonal to all Abelian directions it can be expressed as a linear combination of the raising and lowering operators $E_{\pm\alpha}$, which leads to the definition

$$X_\mu^{(\pm\alpha)} \equiv E_{\pm\alpha}\mathrm{Tr}[\mathbf{X}_\mu E_{\pm\alpha}] \tag{12}$$

so

$$X_\mu^{(-\alpha)} = X_\mu^{(+\alpha)\dagger} \tag{13}$$

$\mathbf{H}_{\mu\nu}^{(\alpha)}$, defined by

$$\mathbf{H}_{\mu\nu}^{(\alpha)} = \alpha_j H_{\mu\nu}^{(j)} \tag{14}$$

is the monopole field strength tensor felt by $\mathbf{X}_\mu^{(\alpha)}$. The background magnetic field is defined by

$$\mathbf{H}^{(\alpha)} = \alpha_j \mathbf{H}^{(j)} \tag{15}$$

whose magnitude $H^{(\alpha)}$ is $\mathbf{H}_{\mu\nu}^{(\alpha)}$'s non-zero eigenvalue. Since both $\mathbf{B}_\mu, \mathbf{X}_\mu$ contain off-diagonal degrees of freedom, it is worth clarifying that $\mathbf{X}_\mu$ contains the quantum fluctuations taking place on a generally non-trivial background whose topology is contained in the monopole field $\mathbf{B}_\mu$. $\mathbf{B}_\mu$ has been strongly argued [11,24], though not proven, to generate confinement, while the dynamical components of the gluon field are confined. Indeed, it has been shown that after a rigorous quantisation of the CFNS decomposition that removes the excess gauge degrees of freedom, the off-diagonal field $\mathbf{X}_\mu$ must have the gauge transform

$$\delta_G \mathbf{X}_\mu = \theta \times \mathbf{X}_\mu \tag{16}$$

whose striking similarity to that of the fundamental representation indicates that the $\mathbf{X}_\mu$ field should be interpreted as a source of colour charge that is itself confined [5,11] by the same monopole condensate, via the dual-Meissner effect, as the quark fields. Indeed, confinement of both quarks and gluons requires the infrared area law of the Wilson loop, which has in turn been argued to follow from the monopole background [19–26,38]. Taken at face value, this implies that the stronger the monopole background felt by a coloured source, the stronger the confinement. Important studies by Kondo *et al* have shown that the Wilson loop for quarks and gluons receives different contributions from the monopole background [16,17,29,30]. The mathematical basis for this will be seen in section 3 to form an important part of dimensional frustration.

3. The Vacuum State of Five-Colour QCD

The one-loop effective energy for N-colour QCD was found by a detailed calculation in [2] and is consistent with corresponding calculations for the two-colour theory by [10,31,32]. It is straightforward to read off the expression for five-colour QCD, which is [1]

$$\mathcal{H} = \sum_{\alpha>0} \|\mathbf{H}^{(\alpha)}\|^2 \left[\frac{1}{5g^2} + \frac{11}{48\pi^2}\ln\frac{H^{(\alpha)}}{\mu^2}\right] \tag{17}$$

It also follows from Flyvbjerg's calculations that this is minimal when

$$H^{(\alpha)} = \mu^2 \exp\left(-\frac{1}{2} - \frac{48\pi^2}{55g^2}\right) \tag{18}$$

This neglects an alleged imaginary component [32] which recent work [11–15,33,34] has found to be a mere artifact of the quadratic approximation. Taking this to be the case, the Savvidy vacuum is employed. This can be criticised for lacking Lorentz covariance but can be reasonably expected to match the true vacuum at least locally.

Since

$$\begin{aligned}
\|\mathbf{H}^{(1,0,0,0)}\| &= \|\mathbf{H}^{(1)}\|,\\
\left\|\mathbf{H}^{\left(\pm\frac{1}{2},\frac{\sqrt{3}}{2},0,0\right)}\right\|^2 &= \frac{1}{4}\|\mathbf{H}^{(1)}\|^2 + \frac{3}{4}\|\mathbf{H}^{(2)}\|^2 \pm \frac{\sqrt{3}}{2}\mathbf{H}^{(1)}\cdot\mathbf{H}^{(2)},\\
\left\|\mathbf{H}^{\left(\pm\frac{1}{2},\frac{1}{\sqrt{12}},\frac{2}{\sqrt{6}},0\right)}\right\|^2 &= \frac{1}{4}\|\mathbf{H}^{(1)}\|^2 + \frac{1}{12}\|\mathbf{H}^{(2)}\|^2 + \frac{2}{3}\|\mathbf{H}^{(3)}\|^2 \pm \sqrt{\frac{2}{3}}\mathbf{H}^{(1)}\cdot\mathbf{H}^{(2)}\\
&\quad \pm\frac{1}{2\sqrt{3}}\mathbf{H}^{(1)}\cdot\mathbf{H}^{(3)} + \frac{\sqrt{2}}{3}\mathbf{H}^{(2)}\cdot\mathbf{H}^{(3)},\\
\left\|\mathbf{H}^{\left(0,-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{6}},0\right)}\right\|^2 &= \frac{1}{3}\|\mathbf{H}^{(2)}\|^2 + \frac{2}{3}\|\mathbf{H}^{(3)}\|^2 - \frac{2\sqrt{2}}{3}\mathbf{H}^{(2)}\cdot\mathbf{H}^{(3)},\\
\left\|\mathbf{H}^{\left(0,0,-\frac{\sqrt{3}}{\sqrt{8}},\frac{\sqrt{5}}{\sqrt{8}}\right)}\right\|^2 &= \frac{3}{8}\|\mathbf{H}^{(3)}\|^2 + \frac{5}{8}\|\mathbf{H}^{(4)}\|^2 - \frac{\sqrt{15}}{4}\mathbf{H}^{(3)}\cdot\mathbf{H}^{(4)},\\
\left\|\mathbf{H}^{\left(0,-\frac{\sqrt{3}}{\sqrt{8}},\frac{1}{\sqrt{24}},\frac{\sqrt{5}}{\sqrt{8}}\right)}\right\|^2 &= \frac{3}{8}\|\mathbf{H}^{(2)}\|^2 + \frac{1}{24}\|\mathbf{H}^{(3)}\|^2 + \frac{\sqrt{5}}{\sqrt{8}}\|\mathbf{H}^{(4)}\|^2 - \sqrt{\frac{1}{16}}\mathbf{H}^{(2)}\cdot\mathbf{H}^{(3)}\\
&\quad -\frac{\sqrt{15}}{4}\mathbf{H}^{(2)}\cdot\mathbf{H}^{(4)} + \frac{\sqrt{5}}{\sqrt{48}}\mathbf{H}^{(3)}\cdot\mathbf{H}^{(4)},\\
\left\|\mathbf{H}^{\left(\pm\frac{1}{2},\frac{1}{\sqrt{12}},\frac{1}{\sqrt{24}},\frac{\sqrt{5}}{\sqrt{8}}\right)}\right\|^2 &= \frac{1}{4}\|\mathbf{H}^{(1)}\|^2 + \frac{1}{12}\|\mathbf{H}^{(2)}\|^2 + \frac{1}{24}\|\mathbf{H}^{(3)}\|^2 + \frac{\sqrt{5}}{\sqrt{8}}\|\mathbf{H}^{(4)}\|^2\\
&\quad \pm\sqrt{\frac{2}{3}}\mathbf{H}^{(1)}\cdot\mathbf{H}^{(2)} \pm \frac{1}{\sqrt{24}}\mathbf{H}^{(1)}\cdot\mathbf{H}^{(3)} + \frac{1}{\sqrt{16}}\mathbf{H}^{(2)}\cdot\mathbf{H}^{(3)}\\
&\quad \pm\frac{\sqrt{5}}{\sqrt{8}}\mathbf{H}^{(1)}\cdot\mathbf{H}^{(4)} + \frac{\sqrt{5}}{\sqrt{24}}\mathbf{H}^{(2)}\cdot\mathbf{H}^{(4)} + \frac{\sqrt{5}}{\sqrt{48}}\mathbf{H}^{(3)}\cdot\mathbf{H}^{(4)}
\end{aligned} \tag{19}$$

where $\pm$ is used to avoid repetition of roots whose only difference is the $+$ or $-$ sign, it follows that

$$\|\mathbf{H}^{(i)}\| = \|\mathbf{H}^{(j)}\|, \quad \mathbf{H}^{(i)} \perp \mathbf{H}^{(j)}, \quad i \neq j \tag{20}$$

which means that the chromomagnetic field components must be equal in magnitude but mutually orthogonal in the lowest energy state, as found by Flyvbjerg [2]. However, three dimensional space

can only accommodate three mutually orthogonal vectors. Since the number of Cartan components, *i.e.* components corresponding to Abelian generators, is always $N-1$ in $SU(N)$ it follows that QCD with more than four colours cannot achieve such an arrangement. (The reader should note that this line of argument still holds in the MAG since the electric field strength is of the corresponding form Equation (7). The MAG shall be ignored from now on.)

Note that the form of the Flyvbjerg vacuum and its role in colour confinement requires infrared Abelian monopole dominance, while the conservation of colour symmetry is required for the QCD vacuum to be well-defined. These have been studied and confirmed in both analytic and numerical [19–28] studies. References [27,28] in particular use a lattice formalism that reduces to the CFNS decomposition in the continuum limit.

Substituting the Cartan basis $\mathbf{H}^{(i)}$ leads to intractable equations that cannot be solved analytically, but it is reasonable to expect that the lowest attainable energy state is only slightly different from (17) and that this difference is due to the failure of mutual orthogonality. One proposed ansatz [1,18] is that all Cartan components are equal in magnitude to what they would be in the absence of dimensional frustration, and that their relative orientations in real space are chosen so as to minimise the energy. This leaves three of the four mutually orthogonal and the remaining one a linear combination of those three. This remainder will increase the effective energy through its scalar products with the mutually orthogonal vectors but not all scalar products contribute equally, which follows from the form of the root vectors in Equation (19). Hence the orientation of the remaining real space vector in relation to the mutually orthogonal ones impacts the vacuum energy.

It is clear that the lowest energy state should receive the smallest available scalar product contribution. The corresponding scalar product is one of

$$\begin{aligned}&\mathbf{H}^{(1)}\cdot\mathbf{H}^{(2)},\mathbf{H}^{(1)}\cdot\mathbf{H}^{(3)},\mathbf{H}^{(1)}\cdot\mathbf{H}^{(4)},\\&\mathbf{H}^{(2)}\cdot\mathbf{H}^{(3)},\mathbf{H}^{(2)}\cdot\mathbf{H}^{(4)},\mathbf{H}^{(3)}\cdot\mathbf{H}^{(4)}\end{aligned}\tag{21}$$

As can be seen from Table 1, $\mathbf{H}^{(3)}=-\mathbf{H}^{(4)}$ (antiparallel) yields the lowest effective energy when all other scalar products are zero.

Substituting this result into (19) finds that all $\mathbf{H}^{(\alpha)}$ have the same magnitude except for those that couple to $\mathbf{H}^{(4)}$, namely $\mathbf{H}^{\left(?,?,?,\sqrt{\frac{5}{8}}\right)}$, where ? indicates that there are several possible values. The other background field strengths are

$$\|\mathbf{H}^{(\alpha)}\|^2=H^2\tag{22}$$

while the strongest is

$$\|\mathbf{H}^{\left(0,0,-\sqrt{\frac{3}{8}},\sqrt{\frac{5}{8}}\right)}\|^2=H^2\left(1+\frac{\sqrt{15}}{4}\right)\tag{23}$$

and the weakest are

$$\|\mathbf{H}^{\left(?,?,\frac{1}{\sqrt{24}},\sqrt{\frac{5}{8}}\right)}\|^2=H^2\left(1-\sqrt{\frac{5}{48}}\right)\tag{24}$$

It should be remembered that the negative signs are affected by the antiparallelism of $\mathbf{H}^{(3)},\mathbf{H}^{(4)}$.

Table 1. Candidate parallel components for vacuum condensate. The column on the left is for parallel vectors, the column on the right is for antiparallel vectors. $\Delta\mathcal{H}$ should be multiplied by $H^2\frac{11}{96\pi^2}$.

$\mathbf{H}^{(i)} = +\mathbf{H}^{(j)}$	$\Delta\mathcal{H}$	$\mathbf{H}^{(i)} = -\mathbf{H}^{(j)}$	$\Delta\mathcal{H}$
$\mathbf{H}^{(1)} = +\mathbf{H}^{(2)}$	1.06381	$\mathbf{H}^{(1)} = -\mathbf{H}^{(2)}$	1.06381
$\mathbf{H}^{(1)} = +\mathbf{H}^{(3)}$	0.857072	$\mathbf{H}^{(1)} = -\mathbf{H}^{(3)}$	0.857072
$\mathbf{H}^{(1)} = +\mathbf{H}^{(4)}$	0.715651	$\mathbf{H}^{(1)} = -\mathbf{H}^{(4)}$	0.715651
$\mathbf{H}^{(2)} = +\mathbf{H}^{(3)}$	1.01655	$\mathbf{H}^{(2)} = -\mathbf{H}^{(3)}$	0.656584
$\mathbf{H}^{(2)} = +\mathbf{H}^{(4)}$	0.882589	$\mathbf{H}^{(2)} = -\mathbf{H}^{(4)}$	0.577976
$\mathbf{H}^{(3)} = +\mathbf{H}^{(4)}$	1.00042	$\mathbf{H}^{(3)} = -\mathbf{H}^{(4)}$	0.540983

The discussion is facilitated by a notation inspired by Dynkin diagrams. The root vectors implicitly specified in Equation (19) are all linear combinations of a few basis vectors, which according to Lie algebra representation theory can be chosen for convenience. We take the basis vectors

$$(1,0,0,0), \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0\right) \left(0, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}, 0\right) \left(0, 0, -\sqrt{\frac{3}{8}}, \sqrt{\frac{5}{8}}\right) \tag{25}$$

which are each represented by

$$\text{OXXX}, \text{XOXX}, \text{XXOX}, \text{XXXO} \tag{26}$$

respectively. The remaining root vectors are sums of these basis vectors. In this notation their representation contains an "O" if the corresponding basis vector is included and "X" if it is not. For example the root vector

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0\right) = (1,0,0,0) + \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0\right) \tag{27}$$

is represented by

$$\text{OOXX} = \text{OXXX} + \text{XOXX} \tag{28}$$

When convenient, a "?" is used to indicate that either "O" or "X" might be substituted.

Assuming the dual superconductor model of confinement [5,35–38], it follows that different valence gluons and even different quarks (in the fundamental representation) will be confined with different strengths and therefore at different length scales. Indeed, the previous papers on this subject explored the phenomenology that might arise from this.

The end of this section describes, but does not rederive, an important result of reference [1] on this subject, the separation of conventional $SU(3)$ QCD dynamics at intermediate energy scales. From Kondo *et al*'s finding that the confinement, specifically the contribution to the Wilson loop, depends on the root, it follows that the gluons that feel the background $H^{\left(0,0,-\frac{\sqrt{3}}{\sqrt{8}},\frac{\sqrt{5}}{\sqrt{8}}\right)}$ (XXXO), are confined the most strongly, while those that feel the backgrounds of the form $H^{\left(?,?,\frac{1}{\sqrt{24}},\frac{\sqrt{5}}{\sqrt{8}}\right)}$ (??OO), are confined least strongly. By considering the effect on the running coupling it can be shown that couplings among the ??XX gluons run faster than the other remaining couplings, so that the couplings within that subgroup are stronger at intermediate energies than those involving any other gluons, effectively providing an intact $SU(3)$ dynamic that shall be seen to correlate well with conventional QCD.

4. The Emergence of Massless and Massive Photons

Neglecting off-diagonal gluons, the equality $\mathbf{H}^{(3)} = -\mathbf{H}^{(4)}$ allows the change in variables

$$c_\mu^{(3)}\hat{n}_3 \to \frac{1}{2}(c_\mu^{(3)}\hat{n}_3 + c_\mu^{(4)}\hat{n}_4) + \frac{1}{2}(c_\mu^{(3)}\hat{n}_3 - c_\mu^{(4)}\hat{n}_4) = \frac{1}{\sqrt{2}}(A_\mu + Z_\mu),$$
$$c_\mu^{(4)}\hat{n}_4 \to \frac{1}{2}(c_\mu^{(3)}\hat{n}_3 + c_\mu^{(4)}\hat{n}_4) - \frac{1}{2}(c_\mu^{(3)}\hat{n}_3 - c_\mu^{(4)}\hat{n}_4) = \frac{1}{\sqrt{2}}(A_\mu - Z_\mu) \quad (29)$$

Substituting Equation (29) into the Abelian dynamics (9) finds that the antisymmetric combination Z_μ couples to the background

$$H(\hat{n}_3 - \hat{n}_4) \quad (30)$$

but the symmetric combination A_μ decouples from it.

The initial analysis argued that Z_μ (E_μ in that paper), was therefore confined [1], but there has been further consideration [39] of the interaction of Abelian gluons with the monopole condensate. Since the Abelian gluons are not colour sources they need not generate flux tubes as quarks do. Instead, their coupling to the monopole condensate, Equation (30) in this case, can be viewed as a sink whose characteristic time imposes a mass-gap expected to approximate the deconfinement energy. This was argued [39] to generate interesting phenomenology in conventional QCD and perhaps even nuclear physics. In this case, it provides a mass for the Z_μ, rather than confining it, so that it is now regarded as a massive photon- or Z_μ^0-like excitation[40]. The massless A_μ is photon-like.

The symmetry of the effective infrared dynamics is therefore reduced to $SU(3) \otimes U(1) \otimes U(1)$. The symmetry of the remaining generators is spoilt by the different confinement strengths (as given by the different magnitudes of $H^{(\alpha)}$) and runnings of the coupling constant as described above and in [1,18]. It is important to note that this does NOT mean that the corresponding gluons decouple. It only means that the effective action is not invariant under those generators. There is an analogous situation in the electroweak theory where the massive $W^\pm$, corresponding to the broken off-diagonal $SU(2)$ generators, nonetheless couples to the photon.

5. Matter Field Representations

The dynamics of the fundamental representation are also determined by its couplings to the monopole condensate, which is itself determined by the fundamental weights.

Confinement of the fundamental representation is determined by the maximal stability group [16, 17,29,30], which for $SU(5)$ is $U(4) \approx SU(4) \otimes U(1)$, where for any given element $(\cdots \psi \cdots)^T$, the $SU(4)$ acts only on the remaining orthogonal elements while the $U(1)$ causes it inconsequential phase changes. This latter $U(1)$ describes the monopole condensate contributing to the confinement and is given by the corresponding weight of the fundamental representation. Indeed, the above-cited studies of the Wilson loop in quark confinement by Kondo *et al* have shown that the Wilson loop of the fundamental representation feels an effect equal to that of only the monopoles generated by $\hat{n}_{N-1}$ (remember that N-colour QCD has $N-1$ Abelian directions). They defined the complete monopole potential defined in Section 2 to be the maximal representation, and the monopole potential generated by $\hat{n}_{N-1}$ alone to the minimal representation. Since the Wilson loop works with potentials rather than fields, this and similar

studies use the V_μ field defined in Equation 4, from which the electric and magnetic field strengths are generated.

The weights of the fundamental representation of $SU(5)$ are

$$\begin{aligned}
(1\,0\,0\,0\,0)^T &: \left(\frac{1}{2}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{24}}, \frac{1}{\sqrt{40}}\right) \\
(0\,1\,0\,0\,0)^T &: \left(-\frac{1}{2}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{24}}, \frac{1}{\sqrt{40}}\right) \\
(0\,0\,1\,0\,0)^T &: \left(0, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{24}}, \frac{1}{\sqrt{40}}\right) \\
(0\,0\,0\,1\,0)^T &: \left(0, 0, -\sqrt{\frac{3}{8}}, \frac{1}{\sqrt{40}}\right) \\
(0\,0\,0\,0\,1)^T &: \left(0, 0, 0, -\sqrt{\frac{2}{5}}\right)
\end{aligned} \tag{31}$$

If all Abelian components of the chromomonopole condensate were of equal magnitude and mutually orthogonal in real space with all cross-terms equal to zero then they would all be confined at equal length scales and nothing remarkable would happen. However, we already know that such is not the case.

The first three lines in Equation (31) all have identical (small) dependence on $\mathbf{H}^{(3)}, \mathbf{H}^{(4)}$, and the same dependencies on $\mathbf{H}^{(1)}, \mathbf{H}^{(2)}$ as in $SU(3)$ QCD. Since the last two lines show no dependence on $\mathbf{H}^{(1)}, \mathbf{H}^{(2)}$, it is only natural to associate the first three elements with the quark colours of the standard model. Indeed, the two additional weight entries provide little additional confinement because the cross terms between $\mathbf{H}^{(3)}, \mathbf{H}^{(4)}$ almost cancel the sum of their squares, so that the total contribution squared of $\mathbf{H}^{(3)}, \mathbf{H}^{(4)}$ is

$$\left(\frac{1}{\sqrt{24}}\mathbf{H}^{(3)} + \frac{1}{\sqrt{40}}\mathbf{H}^{(4)}\right)^2 = H^2\frac{1}{60}(4 - \sqrt{15}) \tag{32}$$

As can be seen from the bracket on the right-hand-side, the cross terms almost cancel this contribution. According to the naïve interpretation of the dual superconductor model employed here this corresponds to extremely weak confinement. Since it is inconsequential compared to the QCD confinement and might very well scale to zero at lower energies anyway (although this has not been proven!) It is assumed that this corresponds to an effectively unconfined state, whose colour combination has been dubbed [1,18] "effectively white" to distinguish it from truly white states.

The remaining weights have non-zero elements only in the third and fourth position. The final weight corresponds to a colour charge, referred to as infrared ($\dot{r}$), whose condensate coupling is slightly stronger than that of the QCD colours discussed above, while the penultimate one corresponds to the colour charge ultraviolet ($\dot{w}$), whose condensate coupling is nearly twice as strong as that of the QCD colours. (These charges are henceforth referred to as the invisible colours.) This occurs because there are positively contributing cross terms between $\mathbf{H}^{(3)}, \mathbf{H}^{(4)}$ (remember their antiparallelism). Previous works [1,18] have argued that ultraviolet must therefore be combined into some neutral combination with infrared at a very small length scale, since infrared only forms unconfined physical states in combination with ultraviolet. Of course, neutral quark-antiquark mesons are also possible.

The weight entries also determine the coupling to dynamic degrees of freedom, both diagonal and off-diagonal. That the first two entries of the first three weights shown in Equation (31) mirror the

fundamental weights of $SU(3)$ suggests that the corresponding quark colours inherit a minimal coupling to the emergent $SU(3)$ gluon dynamics [1]. Their coupling to the A_μ, Z_μ fields is similarly determined by the symmetric and antisymmetric combinations respectively of the third and fourth entries. That these entries are equal in the first three weights implies that the corresponding $SU(5)$ colour charges have identical couplings to these fields.

The fundamental representation can be shown as

$$[1] = (r\, b\, g\, \dot{w}\, \dot{r})^T \tag{33}$$

where the electric charge of the QCD down quarks is implicit in the colour, as discussed above. At highest energies the dynamics are those of $SU(5)$ in the extreme weak-coupling limit.

We now consider the antisymmetric representation

$$[2] = [1] \otimes_{AS} [1] = \begin{bmatrix} 0 & r/b & r/g & r/\dot{w} & r/\dot{r} \\ -r/b & 0 & b/g & b/\dot{w} & b/\dot{r} \\ -r/g & -b/g & 0 & g/\dot{w} & g/\dot{r} \\ -r/\dot{w} & -b/\dot{w} & -g/\dot{w} & 0 & \dot{w}/\dot{r} \\ -r/\dot{r} & -b/\dot{r} & -g/\dot{r} & -\dot{w}/\dot{r} & 0 \end{bmatrix} \tag{34}$$

where $[1]$ is the fundamental representation and $\otimes_{AS}$ indicates an antisymmetric cross-product. The top-left-hand corner has the same interpretation as in conventional GUT theories [41]. Red/blue is effectively antigreen, and the $U(1)$ (in this case electric) charge is double that of the QCD quarks in the fundamental representation. In other words the 3×3 block matrix in the top-left-hand corner can be associated with the anti-up quark, *i.e.*, one conventional colour charge and twice the electric charge of the down quark, when the fundamental representation [1] contains the down quark. The remaining entries of $[2]$, except for one $\dot{w}/\dot{r}$, contain one invisible colour and one visible colour charge. They shall be referred to as shadow colours. $\dot{w}/\dot{r}$ contains both invisible colour charges, and its weight is therefore the sum of the last two weights in 31, namely

$$\left(0, 0, -\sqrt{\frac{3}{8}}, -\frac{3}{\sqrt{40}}\right) \tag{35}$$

which describes an $SU(3)$ colourless state with an electric charge that is negative three times that of the down quark, *i.e.* a positron. The existence of this charge ratio between effectively white states is a pleasing prediction, as it is simply an unexplained coincidence in the standard model.

Finally, by the same reasoning, the Wilson loop and consequently the confinement of the off-diagonal gluons $\mathbf{X}_\mu^{(\alpha)}$ is determined by the monopole backgrounds $\mathbf{H}^{(\alpha)}$ (see the discussion of 19). As with the weights of the fundamental representation, the entries of the corresponding root determine the coupling to dynamic Abelian fields. Consistent with the above discussion of the fundamental $[1]$ representation, the first two elements of the first two roots are identical to the roots of $SU(3)$, while the latter two entries are zero. Hence the corresponding gluons take part in the emergent $SU(3)$ dynamics but do not couple to either of A_μ, Z_μ. The remaining gluons are confined at slightly different strengths to the $SU(3)$ gluons according to the corresponding root, interact with at least one of the invisible colours mentioned above, and also with both of the A_μ, Z_μ fields. In particular, they have electric charge.

The reader may have noticed that the predicted electric charges, *i.e.*, minimal couplings to A_μ, of the invisible and shadow quarks, and of the gluons that couple to them, are not multiples of the down quark's charge. This is a dangerous prediction, although these states are neither white nor effectively white but confined into states that do have conventional quantisation. Only future quantitative analysis can determine if experimental probes have yet excluded this possibility to a small enough scale.

6. The Origin of Mass

The link between the roots and confinement strengths arises in the derivation of the non-Abelian Stokes' theorem [16,17] in the context of the Wilson loop. We have seen that Abelian flux of the form $(c_\mu^{(3)}\hat{n}_3 + c_\mu^{(4)}\hat{n}_4)$ is unconfined and fails to contribute to either confinement or the Wilson loop. It is effectively ignored by the monopole condensate. The antisymmetric combination, by contrast, does contribute to the Wilson loop and its square is proportional to the non-QCD flux felt, and quantised by, the condensate. However, while squares of individual weight entries are trivially quantised, those of antisymmetric combinations of them are not.

This flux quantisation issue is illustrated by the dual analogy in a type II superconductor. In that case magnetic flux trying to enter the superconductor can only do so in multiples of $\frac{\hbar}{2e}$. Any amount of flux less than that amount, or left-over from a multiple of that amount, remains within a penetration depth of the surface.

Applying this to the dimensionally frustrated theory suggests that the antisymmetric flux will form tubes of multiples of a certain amount of flux, while the remainder falls off exponentially within the London penetration depth. Since electric fields of any gauge group have an energy density, this restricted flux creates an effective mass for the quarks and gluons.

There is, however, an additional complication due to the Dirac quantisation principle. This requires that the charges all be quantised and forbids the existence of non-quantised values. This would seem to eliminate all but a subset of the quarks and gluons. (Which subset is addressed below.) Alternately, different quarks and gluons may be restricted to different domains in spacetime. However the condensation of monopoles into monopole-antimonopole pairs leaves room for all colours. If the non-quanta of flux are restricted to a region significantly smaller than the correlation length then they become imperceptible to the monopole condensate which perceives the corresponding charge as quantised and massive.

An effective mass suppresses the quantum loops of the corresponding particle. Since gluon loops effectively lower the vacuum energy, energy minimization requires that the chosen vacuum suppress the gluon loops as little as possible.

To facilitate the discussion, we shall use units such that the flux due to the antisymmetric combination of the last two root/weight entries, henceforth referred to as antisymmetric flux, is given by

$$\frac{1}{4}(v_3 - v_4)^2$$

the square of the antisymmetric combination of v_3, v_4, the third and fourth entries of the root/weight. The restricted flux is the portion of the antisymmetric flux that cannot be accommodated by flux tubes. Table 2 shows the antisymmetric flux, the number of antisymmetric flux quanta, and the restricted flux, corresponding to each root and weight.

Table 2. The antisymmetric flux, the number of flux quanta, and the restricted flux for each root and weight.

	AS flux	Nb flux quanta	Restricted flux
??XX	0	0	0
??OX	$1/6$	9	0.0119
XXXO	$\left(4+\sqrt{15}\right)/16$	28	0.0105
??OO	$\left(8-\sqrt{15}\right)/48$	5	0
down (rbg)	$\left(4+\sqrt{15}\right)/80$	5	0.0124
$\dot{w}$	$\left(8-\sqrt{15}\right)/80$	3	0
$\dot{r}$	$1/10$	5	0.0140
$\dot{w}$-shadow	$\left(8+10\sqrt{\frac{3}{5}}\right)/120$	7	0.0108
$\dot{r}$-shadow	$\left(32+30\sqrt{\frac{3}{5}}\right)/120$	26	0.0132
up	$\left(4+\sqrt{15}\right)/40$	11	0.0077
$e^{\pm}$ ($\dot{w}\,\dot{r}$)	$3\left(4+\sqrt{15}\right)/80$	17	0.0029

For the purpose of minimising gluon masses, the correct choice corresponds to the smallest gluon antisymmetric flux, which not only leaves a significant proportion of the gluons unsuppressed, but suppresses the remaining gluons minimally, if no other factors act to favour one group of gluons over another.

It is clear from Table 2 that the relevant gluon group is ??OO, whose antisymmetric flux equals $\left(8-\sqrt{15}\right)/48$. The antisymmetric flux of $\dot{w}$, which corresponds to it, is $\left(8-\sqrt{15}\right)/80$. The highest common factor is

$$\left(8-\sqrt{15}\right)/240 \tag{36}$$

which shall be taken as the antisymmetric flux quantum. Of course it is not compulsory to assume that the $\dot{w}$ quark's antisymmetric flux is also an integer multiple of the quantised amount, but doing so seems natural and is consistent with a non-dimensionally frustrated theory. We shall see that it also leads to some suggestive results for the visible quark masses.

Gauge fields of any group are well-known to have an energy density proportional to the square of the field strength, which is proportional to the flux. Hence the effective mass generated by this mechanism is approximately proportional to the square of the restricted flux. Approximately, because the necessary changes to the condensate in this region, with additional complications due to the non-quantised flux, are expected to make nontrivial contributions that are not easy to calculate. Nevertheless the corresponding mass ratios are easily calculated. Note that a larger antisymmetric charge does not necessarily imply a larger mass. The crucial quantity is the remainder when this is divided by the antisymmetric flux quanta.

The masses of all particles as a multiple of the electron mass are presented in Table 3, but of real interest are the ratios of the quark and gluon masses with respect to the $e^{\pm}$ mass are compared to the experimental values in Table 4. Interestingly, they are of the same order as the experimental values, although a little higher than the experimental upper limit. This is pleasantly surprising given that it was not initially obvious that these masses would even be in the correct order of $m_e < m_u < m_d$.

Table 3. Quark and gluon masses in multiples of the $e^{\pm}$ mass.

Particle	mass/m_e
$e^{\pm}$	1
down (rbg)	18.3
$\dot{w}$	0
$\dot{r}$	23.3
$\dot{w}$-shadow	13.9
$\dot{r}$-shadow	20.7
up	6.96
??XX	0
??OX	16.8
XXXO	13.2
??OO	0

It is straightforward to calculate that this mechanism can never contribute a mass of more than 35.0 times the electron mass. It should be remembered, however, that other mechanisms may also contribute. Indeed, it has already been argued [14,34] that the first two Cartan components of the monopole condensate contribute enough mass to the valence gluons to stabilise the vacuum, so that these gluon masses should be regarded as the contribution due to antisymmetric flux quantization. Another example is the Z_μ which gains its effective mass through the monopole condensate acting as a sink.

Table 4. Predicted and experimentally determined [42] mass ratios.

	Experimental	Predicted
m_u/m_e	2.94 - 6.46	6.96
m_d/m_e	6.85 - 11.7	18.3

If a realistic unification were to employ such a mechanism, the effect would be an effective quark mass independent of any $SU(3)$ chiral symmetry breaking, consistent with the pion being a pseudo-Goldstone boson. Hence, in this theory, the up and down quarks are truly massless in the ultraviolet limit, gain an effective mass at unification energies, which deep inelastic scattering experiments record as a bare mass, and then suffer imperfect chiral symmetry breaking at the $SU(3)$ confinement scale, yielding the low-mass pion. It is an interesting fact that the mass of the pion (140 MeV) [42] is of the same order of magnitude as the deconfinement energy (151–195 MeV) [43]. Since the pion mass is sensitive to the quarks' "bare" mass, it seems that this "bare" mass has a highly coincidental value. It remains to be seen if this mechanism can be expected to yield such a coincidence on general principles.

Note that there should still be a truly massless Goldstone boson corresponding to the chiral symmetry breaking described in this section. Taking the standard model pion as an example, we expect a massless quarkonium state that is a quantum-mechanical mixture of all possible quark-antiquark pair states.

7. Conclusions

We have seen that dimensionally frustrated $SU(5)$ QCD possesses a sophisticated phenomenology. Its original attraction was spontaneous symmetry reduction without a Higgs mechanism, but a number of other features have also emerged, such as the generation of QED- and QCD-like dynamics and the spontaneous generation of quark and electron masses not inconsistent with experimental limits.

Important to this work is that the results and predictions chiefly follow from the group structure and representation theory. The important exception to this is the Flyvbjerg vacuum [2] which is crucial to the theory's development. One might worry that higher-order or non-perturbative corrections might spoil it away from the ultraviolet limit, but the vacuum's simplicity and symmetry suggest that its general form should be quite robust. The actual field strength, however, might be sensitive.

It is also important to remember that the only assumptions in this development are the ansatz, which is physically well-motivated, and that the quantum of antisymmetric flux is small enough to minimise the quark masses as well as the gluon masses, which is not unreasonable.

While the predicted electric charge and mass ratios for quarks and electrons are impressive, this theory still falls short of realistic physics. It lacks, among other things, both the $W^{\pm}$ and the neutrino, and has no obvious mechanism for matter generations or for $\mathcal{P}$ or $\mathcal{C}P$ violation. The experimental lack of baryons containing the invisible and shadow colours may not be as damning as one might first think. Invisible mesons would quickly decay via the electromagnetic force, and heavier baryons have decay channels to both visible and invisible colours. Sadly, this also eliminates the invisible and shadow sectors of this theory as candidates for dark matter.

In any event, the primary virtue of this theory is its naturalness. The symmetry breaking is compulsory rather than contrived, and the effective charges and masses arise spontaneously.

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