Supplementary Materials:

1. Introduction

In this supplement of [1]. A Mathematica routine is provided that was used to verified Table 1 in [1]. Another Mathematica routine is also provided where it is evident how the variation of the parameters change the dynamics of the soliton solutions.

2. Families of NLS with Varible coefficients

In this section, we provide the Mathematica routine that was used to verify Table 1 in [1].

TraditionalForm[

Column[{

Style["Ecuaciones de Riccati", 14, Bold, Italic, "Tahoma"],

Manipulate[

Column[{

Column[{

b[t_]:=$\frac{\text{b1}}{4*l_0}$;

β[t_]:=1;

γ[t_]:=t;

τ[t_]:=t;

ϵ[t_]:=0;

c[t_]:=c1;

d[z_]:=d1;

f[z_]:=f1;

c[z_]:=c2;

α[t_]:=$l_0 * \left(\frac{\text{c1}}{4}\right)$;

δ[t_]:=$-l_0 * \left(\frac{g[t]}{2}\right)$;

h[t_]:=$-l_0 * \lambda * \mu[t]$;

κ[t_]:=$\kappa_0 - \frac{l_0}{4} * \int_0^t (g[z])^2\,dz$;

μ[t_]:=$\mu_0 * \text{Exp}\left[\int_0^t (2*d[z] - c[z])\,dz\right]$;

(*g[t_]:=$g_0 - 2*l_0*\text{Exp}\left[-\int_0^t c[z]\,dz\right] * \int_0^t \text{Exp}\left[\int_0^z c[y]\,dy\right] f[z]dz$;*)

g[t_]:=0;

y1[x_, t_]:=$\frac{1}{\sqrt{\mu[t]}} * \text{Exp}\left[I*\left(\alpha[t]*x^2 + \delta[t]*x + \kappa[t]\right)\right] * \sqrt{v} * \text{Sech}\left[\sqrt{v}*x\right] *$

Exp$[-I * v * t]$;

y2[x_, t_]:= $\frac{1}{\sqrt{\mu[t]}} * \text{Exp}\left[I * \left(\alpha[t] * x^2 + \delta[t] * x + \kappa[t]\right)\right] * A * \text{Tanh}[A * x] *$

Exp$[-2 * I * (A\hat{}2) * t]$;

(*N1[x_, t_]:= $- I * D[w[x,t],t] + l_0 * D[w[x,t],\{x,2\}] + b[t] * x^2 * w[x,t] - I * c[t] * x * D[w[x,t],x] -$
$I * d[t] * w[x,t] - f[t] * x * w[x,t] + I * g[t] * D[w[x,t],x] + h[t] * w[x,t] * (\text{Abs}[w[x,t]])^2$;*)

Grid[{{"b(t)", "c(t)", "c(z)"}, {$b[t]$, c1, c2}}, Frame → All],

Space,

Grid [{{"l_0", "λ", "μ_0", "g_0", "κ_0"}, {$l_0, \lambda, \mu_0, g_0, \kappa_0$}}, Frame → All],

Space,

Grid[{{"f(z)", "d(z)", "β(t)", "γ(t)", "τ(t)", "ϵ(t)"}, {f1, d1, $\beta[t], \gamma[t], \tau[t], \epsilon[t]$}},

Frame → All],

Space,

Assuming [{{$\gamma, \mu_0, g_0, \kappa_0, x, t, z$} ∈ Reals, $t > 0, m \in \mathbb{Z}, m > -1$},

Grid[{{"α(t)", "δ(t)", "h(t)", "κ(t)", "μ(t)", "g(t)"},

{$\alpha[t], \delta[t], h[t], \kappa[t], \mu[t], g[t]$}}, Frame → All]],

Assuming [{{$\gamma, \mu_0, g_0, \kappa_0, x, t, z, v, A$} ∈ Reals, $v > 0, t > 0, m \in \mathbb{Z}, m > -1$},

Grid[{{"ψ(x,t)", "ψ(x,t)"}, {y1$[x,t]$, y2$[x,t]$}}, Frame → All]],

Assuming [{{$\gamma, \mu_0, g_0, \kappa_0, x, t, z, v, A$} ∈ Reals, $v > 0, t > 0, m \in \mathbb{Z}, m > -1$},

Grid[{{"μ(t)"}, {InputForm[$\mu[t]$]}}, Frame → All]],

(*Assuming [{{$\gamma, \mu_0, g_0, \kappa_0, x, t, z, v, A$} ∈ Reals, $v > 0, t > 0, m \in \mathbb{Z}, m > -1$},

Grid[{{"ψ(x,t)", "ψ(x,t)"}, {InputForm[y1$[x,t]$], InputForm[y2$[x,t]$]}}, Frame → All]],*)

(*Assuming [{{$\gamma, \mu_0, g_0, \kappa_0, x, t, z, v, A$} ∈ Reals, $v > 0, t > 0, m \in \mathbb{Z}, m > -1$},

Grid[{{"NASψ(x,t)"}, {InputForm[N1$[x,t]$]}}, Frame → All]*)}]}],

{{b1, 1, "b(t)"}},

{{c1, 1, "c(t)"}},

{{c2, 1, "c(z)"}},

{{d1, 0, "d(z)"}},

{{f1, 0, "f(z)"}},

{{l_0, 1, "l_0"}},

{{λ, 1, "λ"}},

{{κ_0, κ0, "κ_0"}},

{{g_0, g0, "g_0"}},

{{μ_0, μ0, "μ_0"}}]}]]

2.1. Dynamics of Peregrine and Dark Solitons Animations in Mathematica

In this section, for the benefit of the interested reader of [1], we provide a Mathematica routine that illustrates the change of the dynamics of Peregrine and Dark soliton solutions when the parameters change.

Manipulate[Column[{Style["Peregrine Soliton Solution", 14, Bold, Italic, "Tahoma"], Plot3D[

$$\mathbf{Abs}\left[\frac{1}{\sqrt{\mathrm{Cosh}[t]}} * A *\right.$$

$$\left(3 + 16 * I * A^2 * \left(-\frac{\beta 0^2}{4\left(\frac{1}{2c2t} + \alpha 0\right)} + \gamma 0\right) - 16 * A^2 * \left(-\frac{\beta 0^2}{4\left(\frac{1}{2c2t} + \alpha 0\right)} + \gamma 0\right)^2 -\right.$$

$$\left.4 * A^2 * \left(\frac{x\beta 0}{2c2t\left(\frac{1}{2c2t} + \alpha 0\right)} - \frac{\beta 0\delta 0}{2\left(\frac{1}{2c2t} + \alpha 0\right)} + \epsilon 0\right)^2\right) \Big/$$

$$\left.\left(1 + 16 * A^2 * \left(-\frac{\beta 0^2}{4\left(\frac{1}{2c2t} + \alpha 0\right)} + \gamma 0\right)^2 + 4 * A^2 * \left(\frac{x\beta 0}{2c2t\left(\frac{1}{2c2t} + \alpha 0\right)} - \frac{\beta 0\delta 0}{2\left(\frac{1}{2c2t} + \alpha 0\right)} + \epsilon 0\right)^2\right)\right],$$

{x, −10, 5},

{t, −5, 5},

PlotRange → {0, Range},

ImageSize → 300,

AxesLabel → {"Distance x", "Time t", Abs["ψ(x,t)"]}

]}], {{A, 2}, 1, 10}, {c2, 1, 10}, {α0, 0, 10}, {β0, −1, 10}, {δ0, −1, 10}, {ϵ0, −1, 10},

{γ0, 1, 10}, {{Range, 5}, 1, 20}, ControlPlacement → Left]

Manipulate[Column[{Style["Dark Soliton Solution", 14, Bold, Italic, "Tahoma"], Plot3D[

$$\mathbf{Abs}\left[\frac{1}{\sqrt{2c2t\left(\frac{1}{2c2t} + \alpha 0\right)\mu 0}} * A * \mathrm{Tanh}\left[A * \left(\frac{x\beta 0}{2c2t\left(\frac{1}{2c2t} + \alpha 0\right)} - \frac{\beta 0\delta 0}{2\left(\frac{1}{2c2t} + \alpha 0\right)} + \epsilon 0\right)\right]\right],$$

{x, −10, 5},

{t, −5, 5},

PlotRange → {0, Range},

```
ImageSize → 300,
AxesLabel → {"Distance x", "Time t", Abs["ψ(x,t)"]}
]}], {A, 1, 10}, {c2, 1, 10}, {α0, 0, 10}, {β0, −1, 10}, {δ0, −1, 10}, {ϵ0, −1, 10},
{γ0, 1, 10}, {{μ0, −1}, −1, 10}, {{Range, 2}, 1, 20}, ControlPlacement → Left]
```

Reference

1. Amador, G.; Colon, K.; Luna, N.; Mercado, G.; Suazo, E. On Solutions for Linear and Nonlinear Schrödinger Equations with Variable Coefficients: A Computational Approach. *Symmetry* **2016**, *8*, doi:10.3390/sym8060038.