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Extension of the TODIM Method to Intuitionistic Linguistic Multiple Attribute Decision Making

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Abstract: Practical decision situations are becoming increasingly complicated. It is common for a person to select or rank alternatives with respect to multiple attributes, and the TODIM (an acronym in Portuguese of interactive and multiple attribute decision making) method, which is one of the first multiple attribute decision making (MADM) methods based on prospect theory, has received more attention due to its great performance in considering the bounded rationality of decision makers (DMs). However, the classical TODIM method can only handle the MADM problems with crisp numbers. In this paper, considering that intuitionistic linguistic variables are convenient to describe uncertain or imprecise information, we propose the intuitionistic linguistic TODIM (IL-TODIM) method and intuitionistic uncertain linguistic TODIM (IUL-TODIM) method to solve uncertain MADM problems with IL and IUL variables, respectively. Additionally, a novel distance measure for IUL numbers is developed, based on which we can obtain the corresponding dominance degree of one alternative over another. Finally, examples are provided to show the validity of the proposed methods, and we also conduct a comparison of the results between the IL-TODIM method and the existing intuitionistic fuzzy MADM methods to illustrate the effectiveness of our proposed methods.

Keywords: TODIM; intuitionistic linguistic variables; intuitionistic uncertain linguistic variables; MADM

1. Introduction

Real decision-making situations are increasingly complicated, and it is common for decision makers (DMs) to select alternatives with respect to multiple attributes. Note that the existing multiple attribute decision making (MADM) methods are mostly derived from the premise that the DMs always look for the solution corresponding to the highest expected utility [1–4], but some human behavioral studies have found that DMs are not completely rational under many practical decision situations [5–9]. Typically, people are more sensitive to losses than to gains. Based on a series of experiments and surveys, Kahneman and Tversky proposed the prospect theory, which was defined for decisions under risk and individual preferences [7]. This is a descriptive model, and the value is determined by the gains and losses from a reference point. The value function is described to be S-shaped. The concave part above the horizontal axis reflects risk aversion in case of gains, while the convex part in the negative quadrant is relatively steep, which implies risk-seeking in the face of losses.

Based on prospect theory, the TODIM (an acronym in Portuguese of interactive and multiple attribute decision making) method is one of the first MADM methods considering individual behavior whose principal idea is to calculate the dominance of one alternative over another by establishing the value function so that the ranking orders can be obtained according to the global dominance degree of each alternative. Considering DMs' behavior, the TODIM method is helpful to handle the MADM problems but it can only deal with crisp numbers. Due to limited time or incomplete information, the decision-making information provided by the DMs is often uncertain or imprecise. In order to solve uncertain MADM problems, some researchers extended the classical TODIM method to handle uncertain and imprecise information. Krohling and Souza proposed a fuzzy TODIM which can deal with the MADM problems represented by triangular or trapezoidal fuzzy numbers [10]. Fan et al. introduced a hybrid TODIM approach to handle the MADM problems with crisp numbers, interval-valued numbers, and fuzzy numbers [11]. Since the crisp numbers and type-1 fuzzy sets are not sufficient to evaluate the multi-criteria in some practical decision situations, Qin et al. presented an interval type-2 fuzzy TODIM method and applied it to a green supplier selection [12]. Although the fuzzy set (FS) can effectively depict the uncertainty and vagueness, it cannot consider the hesitation degree of DMs in the decision-making processes. As an extension of FS theory, the intuitionistic fuzzy set (IFS), which was characterized by a membership degree and a non-membership degree, was first developed by Atanassov [13]. Since the IFS can express the fuzzy information more flexibly and accurately, it has gained the increasing attention of researchers. Later, some studies were conducted to enrich the IFS theory. The interval-valued IFS, triangular IFS, intuitionistic trapezoidal fuzzy number (ITFN), and interval ITFN are proposed and applied to the MADM problems [14–17]. Furthermore, Krohling et al. extended the TODIM method to the intuitionistic fuzzy (IF) and interval-valued IF environment [18,19]. Lourenzutti and Krohling generalized the TODIM approach to consider the IF information and underlying random vectors [20]. Considering risk aversion and uncertainty, Li et al. presented a decision model based on IF-TODIM for distributor selection and evaluation [21]. To solve the multi-attribute group decision making (MAGDM) problems, Li et al. defined the interval IFS and used the entropy method to calculate the weight of each attribute [22]. Qin et al. presented an extended TODIM method to the triangular IF environment [23]. Considering that the Pythagorean FS, which is an extension of IFS, is superior in describing the uncertain MADM problems, Ren et al. extended the TODIM approach to attribute values taking the form of Pythagorean fuzzy information [24].

In real decision-making, since linguistic variables are convenient for describing uncertain or imprecise information, especially for qualitative information, studies on the TODIM approach—in which the attribute values in the form of linguistic variables/uncertain linguistic variables have attracted much attention—have made many achievements [25–28]. Furthermore, motivated by the effectiveness of the IFS and linguistic variables, Wang and Li proposed intuitionistic linguistic sets (ILS), intuitionistic linguistic numbers (ILN), and calculation methods [29]. Liu and Jin proposed intuitionistic uncertain linguistic (IUL) variables and introduced operational laws [30]. Then, a series of methods for solving MADM problems with IL/IUL information were been developed. Liu and Wang presented the IL power generalized weighted average operator and the IL power generalized ordered weighted average operator. Based on the two operators, they introduced two new methods for MAGDM problems [31]. Wang et al. developed three aggregation operators, including the IL weighted geometric averaging (ILWGA) operator, the IL ordered weighted geometric operator, and IL hybrid geometric operator, then they applied the new operators to solve MAGDM problems [32]. Liu introduced an IL generalized weighted average (ILGWA) operator, an IL generalized dependently-ordered weighted average operator, and an IL generalized dependent hybrid weighted aggregation operator [33]. Additionally, the IUL weighted geometric average operator, IUL ordered weighted geometric operator, interval-valued IULWGA operator, and the interval-valued IUL ordered weighted geometric operator were also developed [34,35]. Note that all of the methods mentioned above for solving MADM problems are based on aggregation operators, which may ignore the differences among the alternatives according to different attributes. In this paper, we first propose an intuitionistic linguistic TODIM

(IL-TODIM) method. Then, a novel distance measure for intuitionistic uncertain linguistic numbers (IULN) is developed, so that the extended TODIM method can deal with the MADM problems where all the attribute values are expressed in IULNs. Finally, a case study is applied to verify the feasibility and validity of the proposed methods. In addition, we make a comparison of the ranking orders of the alternatives between our proposed method and the existing intuitionistic fuzzy MADM method.

The remainder of this paper is organized as follows: In Section 2, some preliminary background on IL variables, IUL variables, and the classical TODIM method are provided. In Section 3, an extended TODIM method is developed to deal with MADM problems with IL numbers. In Section 4, the intuitionistic uncertain linguistic TODIM (IUL-TODIM) method is proposed. In Section 5, a case study is used to illustrate the validity of the proposed methods, and a comparison with other intuitionistic fuzzy MADM methods is also conducted. Finally, some conclusions and directions for future work are presented in Section 6.

2. Preliminaries

2.1. The Intuitionistic Fuzzy Set (IFS)

Definition 1. *[13] An IFS A in* Ω *is a mathematical object of the form* $= \{\langle \omega, \mu_A(\omega), v_A(\omega)\rangle : \omega \in \Omega\}$ *where* $\mu_A, v_A : \Omega \to [0,1]$, *with the condition* $0 \le \mu_A(\omega) + v_A(\omega) \le 1$, $\forall \omega \in \Omega$. *Here,* μ_A *and* v_A *represent the membership function and the non-membership function, respectively. The hesitancy degree can be calculated by* $\pi_A(\omega) = 1 - \mu_A(\omega) - v_A(\omega)$.

2.2. The Linguistic Set and Uncertain Linguistic Set

In practical decision-making situations, especially for solving vague problems, human-like expression of their views in natural language is less precise than numerical measurements, but closer to human cognitive behaviors. Hence, Zadeh presented the linguistic variable whose values are words or sentences [36].

Definition 2. *[36] Establish a finite and fully-ordered discrete linguistic term set* $S = (s_\alpha | \alpha = 0, 1, \ldots, L-1)$, *where L is an odd number. For instance,* $S = (s_0, s_1, s_2, s_3, s_4, s_5, s_6)$ *= {very low, low, slightly low, fair, slightly high, high, very high}, when L = 7. The fundamental property of the scale terms is ordered* $s_i \prec s_j$ *iff* $i < j$. *Then, Herrera et al. extended the discrete linguistic term to a continuous linguistic label* $\overline{S} = (s_\alpha | \alpha \in R)$ *[37]. For any linguistic variables* $s_i, s_i \in \overline{S}$, *the negation operator is calculated by*

$$neg(s_i) = s_{L-1-i}, \tag{1}$$

Definition 3. *[38] Suppose* $\widetilde{s} = [s_l, s_u]$, $s_l, s_u \in \overline{S}$, $l \le u$, s_l *and* s_u *are the lower limit and the upper limit of* $\widetilde{s}$, *respectively.* $\widetilde{s}$ *is called an uncertain linguistic variable.*

2.3. The Intuitionistic Linguistic Set (ILS) and Intuitionistic Linguistic Number (ILN)

Definition 4. *[29] An ILS A in* Ω *is defined as:* $A = \left\{\left\langle \omega\left[h_{\theta(\omega)}, (\mu_A(\omega), v_A(\omega))\right]\right\rangle | \omega \in \Omega\right\}$, *where* $h_{\theta(\omega)} \in \overline{S}$, $\mu_A, v_A : \Omega \to [0,1]$ *with the condition* $0 \le \mu_A(\omega) + v_A(\omega) \le 1$, $\forall \omega \in \Omega$. *Here,* $\mu_A(\omega)$ *and* $v_A(\omega)$ *stands for the membership degree and the non-membership degree of* ω *to the linguistic index* $h_{\theta(\omega)}$, *respectively. For each ILS in* Ω, *the hesitancy degree of* ω *to the linguistic index* $h_{\theta(\omega)}$ *is given by* $\pi(\omega) = 1 - \mu_A(\omega) - v_A(\omega)$, $\forall \omega \in \Omega$. *Obviously,* $0 \le \pi_A(\omega) \le 1$, $\forall \omega \in \Omega$.

Definition 5. *[29] Suppose* $A = \left\{ \left\langle x \left[h_{\theta(\omega)}, (\mu_A(\omega), v_A(\omega)) \right] \right\rangle | \omega \in \Omega \right\}$ *be an ILS, and* $\left\langle h_{\theta(\omega)}, (\mu_A(\omega), v_A(\omega)) \right\rangle$ *an be called an ILN. The ILS can be viewed as a collection of the ILNs, so A can also be expressed as =* $\left\{ \left\langle h_{\theta(\omega)}, (\mu_A(\omega), v_A(\omega)) \right\rangle | \omega \in \Omega \right\}$.

Definition 6. *[39] Let* $m = \left\langle h_{\theta(m)}, (\mu(m), v(m)) \right\rangle$ *and* $n = \left\langle h_{\theta(n)}, (\mu(n), v(n)) \right\rangle$ *be two ILNs, the operations of ILNs are defined as*

1. $m \oplus n = \left\langle h_{\theta(m)+\theta(n)}, \frac{\theta(m)\mu(m)+\theta(n)\mu(n)}{\theta(m)+\theta(n)}, \frac{\theta(m)v(m)+\theta(n)v(n)}{\theta(m)+\theta(n)} \right\rangle$
2. $m \otimes n = \left\langle h_{\theta(m)\theta(n)}, \mu(m)\mu(n), v(m) + v(n) \right\rangle$
3. $\lambda m = \left\langle h_{\lambda\theta(m)}, \mu(m), v(m) \right\rangle, \lambda \geq 0$
4. $m^{\lambda} = \left\langle h_{\theta(m)^{\lambda}}, \mu(m)^{\lambda}, 1 - (1 - v(m))^{\lambda} \right\rangle, \lambda \geq 0$

Definition 7. *[31] Let* $m = \left\langle h_{\theta(m)}, (\mu(m), v(m)) \right\rangle$ *and* $n = \left\langle h_{\theta(n)}, (\mu(n), v(n)) \right\rangle$ *be two ILNs, the normalized Hamming distance between them is defined as*

$$d(m, n) = \frac{1}{2(L-1)}(|(1 + \mu(m) - v(m))\theta(m) - (1 + \mu(n) - v(n))\theta(n)|), \tag{2}$$

2.4. The Intuitionistic Uncertain Linguistic Set (IULS) and Intuitionistic Uncertain Linguistic Number (IULN)

Definition 8. *[30] Let* $[s_{\theta(x)}, s_{\tau(x)}] \in \overline{S}$, *and X is a given domain,* $A = \left\{ \left\langle x \left[\left[s_{\theta(x)}, s_{\tau(x)} \right], (\mu_A(x), v_A(x)) \right] \right\rangle | x \in X \right\}$ *can be called an IULS, where* $\mu_A, v_A : X \to [0, 1]$, *with the condition* $0 \leq \mu_A(x) + v_A(x) \leq 1, \forall x \in X$. *Here,* $\mu_A(x)$ *and* $v_A(x)$ *represent the membership function and non-membership function, respectively. For each IULS in X, the indeterminacy degree is given by* $\pi(x) = 1 - \mu_A(x) - v_A(x), \forall x \in X$. *Obviously,* $0 \leq \pi_A(x) \leq 1, \forall x \in X$.

Definition 9. *[30] Suppose* $A = \left\{ \left\langle x \left[\left[s_{\theta(x)}, s_{\tau(x)} \right], (\mu_A(x), v_A(x)) \right] \right\rangle | x \in X \right\}$ *is an IULS, and* $\left\langle \left[s_{\theta(x)}, s_{\tau(x)} \right], (\mu_A(x), v_A(x)) \right\rangle$ *is called an IULN. IULS can be viewed as a collection of the IULNs, so A can also be expressed as* $A = \left\{ \left\langle \left[s_{\theta(x)}, s_{\tau(x)} \right], (\mu_A(x), v_A(x)) \right\rangle | x \in X \right\}$.

Definition 10. *[30] Let* $\widetilde{m} = \left\langle \left[s_{\theta(m)}, s_{\tau(m)} \right], (\mu(m), v(m)) \right\rangle$ *and* $\widetilde{n} = \left\langle \left[s_{\theta(n)}, s_{\tau(n)} \right], (\mu(n), v(n)) \right\rangle$ *be two IULNs, the operational rules of IULNs are defined as*

1. $\widetilde{m} \oplus \widetilde{n} = \left\langle \left[s_{\theta(m)+\theta(n)}, s_{\tau(m)+\tau(n)} \right], (1 - (1 - \mu(m))(1 - \mu(n)), v(m)v(n)) \right\rangle$
2. $\widetilde{m} \otimes \widetilde{n} = \left\langle \left[s_{\theta(m)\times\theta(n)}, s_{\tau(m)\times\tau(n)} \right], (\mu(m)\mu(n), v(m) + v(n) - v(m)v(n)) \right\rangle$
3. $\lambda\widetilde{m} = \left\langle \left[s_{\lambda\times\theta(m)}, s_{\lambda\times\tau(m)} \right], \left(1 - (1 - \mu(m))^{\lambda}, (v(m))^{\lambda}\right) \right\rangle, \lambda \geq 0$
4. $\widetilde{m}^{\lambda} = \left\langle \left[s_{(\theta(m))^{\lambda}}, s_{(\tau(m))^{\lambda}} \right], \left((\mu(m))^{\lambda}, 1 - (1 - v(m))^{\lambda}\right) \right\rangle, \lambda \geq 0$

Definition 11. *Let* $\widetilde{m} = \left\langle \left[s_{\theta(m)}, s_{\tau(m)} \right], (\mu(m), v(m)) \right\rangle$ *and* $\widetilde{n} = \left\langle \left[s_{\theta(n)}, s_{\tau(n)} \right], (\mu(n), v(n)) \right\rangle$ *be two IULNs; the normalized Hamming distance between* $\widetilde{m}$ *and* $\widetilde{n}$ *is defined by*

$$d(\widetilde{m}, \widetilde{n}) = \frac{1}{4(L-1)} \begin{pmatrix} |(1 + \mu(m) - v(m))\theta(m) - (1 + \mu(n) - v(n))\theta(n)| + \\ |(1 + \mu(m) - v(m))\tau(m) - (1 + \mu(n) - v(n))\tau(n)| \end{pmatrix}, \tag{3}$$

2.5. The Classical TODIM Method

The TODIM method was firstly proposed by Gomes and Lima [40,41]. The mathematical formulations of the TODIM method are shown as

Step 1: Define the decision matrix $X = [x_{ic}]n \times m$, which are the evaluations of alternatives A_i according to criterion C_c. x_{ic} is a crisp number, $i = 1, \ldots, n$, $c = 1, \ldots, m$. n and m represent the number of alternatives and the number of criteria, respectively

$$X = \begin{array}{c c} & \begin{array}{ccc} C_1 & \cdots & C_m \end{array} \\ \begin{array}{c} A_1 \\ \cdots \\ A_n \end{array} & \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix} \end{array}$$

Step 2: Normalize the decision matrix $X = [x_{ic}]n \times m$ into $Y = [y_{ic}]n \times m$.

Step 3: Let $w = (w_1, w_2, \cdots, w_m)$ be the weight vector of the criteria $C_1, C_2, \cdots, C_m$, where $0 \leq w_i \leq 1$ and $\Sigma_{i=1}^{m} w_i = 1$. It is necessary that the DM defines a reference criterion C_r, $1 \leq r \leq m$, usually the reference criterion with the highest weight. Calculate relative weight $w_{rc} = w_c / w_r$, where C_c is a generic criterion.

Step 4: Calculate the dominance of A_i over A_j using the following expression. The term $\varphi_c(A_i, A_j)$ represents the partial dominance. θ is the attenuation factor of the losses, and the choice of θ has an influence on the shape of the prospect value function.

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \varphi_c(A_i, A_j), \qquad i, j = 1, \ldots, n \tag{4}$$

where

$$\varphi_c(A_i, A_j) = \begin{cases} \sqrt{w_{rc}(y_{ic} - y_{jc}) / \sum_{c=1}^{m} w_{rc}} & (y_{ic} - y_{jc} > 0) \\ 0 & (y_{ic} - y_{jc} = 0) \\ -\frac{1}{\theta} \sqrt{(y_{jc} - y_{ic})(\sum_{c=1}^{m} w_{rc}) / w_{rc}} & (y_{ic} - y_{jc} < 0) \end{cases}$$

Step 5: Normalize the dominance measurements

$$\varepsilon_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)}{\max \sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)} \tag{5}$$

Step 6: Sort the alternatives according to the value ε_i.

3. IL-TODIM—An Intuitionistic Linguistic TODIM Method

In this section, based on the TODIM method and ILS, we proposed the intuitionistic linguistic TODIM (IL-TODIM) method which considers the DM's behavioral characteristics and can deal with the ILNs directly. The mathematical formulations of the IL-TODIM method is described in the following steps:

Step 1: The criteria are normally classified into two types: benefit criteria and cost criteria, and the DM needs to evaluate the alternatives $A_i (i = 1, \ldots, n)$ with respect to criteria $C_c (c = 1, \ldots, m)$. Each evaluation value can be expressed by IL variable $x = \left\langle h_{\theta(x)}, (\mu(x), v(x)) \right\rangle$. Then we can obtain the IL decision matrix $X = [x_{ic}]n \times m$ with $i = 1, \ldots, n$, and $c = 1, \ldots, m$, where

$x = \left\langle h_{\theta(x)}, (\mu(x), v(x)) \right\rangle$. Firstly, the IL decision matrix X should be normalized into $R = [r_{ic}]n \times m$, where $r = \left\langle r_{\theta(x)}, (\mu(x), v(x)) \right\rangle$. The linguistic index $r_{\theta(x)}$ of the normalized value r_{ic} is calculated as

$$r_{\theta(x)} = neg(h_{\theta(x)}) = h_{L-1-\theta(x)}, \text{ where } h_{\theta(x)} \in \overline{S} \text{ for the cost criteria,} \tag{6a}$$

$$r_{\theta(x)} = h_{\theta(x)}, \text{ where } h_{\theta(x)} \in \overline{S} \text{ for the benefit criteria,} \tag{6b}$$

Step 2: Calculate the dominance degree of alternative A_i over A_j using the expression

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \varphi_c(A_i, A_j), \qquad i, j = 1, \ldots, n, \tag{7}$$

where

$$\varphi_c(A_i, A_j) = \begin{cases} \sqrt{d(r_{ic}, r_{jc}) w_{rc} / \sum_{c=1}^{m} w_{rc}} & (d(r_{ic}, r_{jc}) \geq 0) \\ -\frac{1}{\theta}\sqrt{d(r_{ic}, r_{jc}) (\sum_{c=1}^{m} w_{rc}) / w_{rc}} & (d(r_{ic}, r_{jc}) < 0) \end{cases}$$

The term $d(r_{ic}, r_{jc})$ standards for the distance between two ILNs r_{ic} and r_{jc}, calculated by Equation (2). $d(r_{ic}, r_{jc}) \geq 0$ represents a gain or nil, while $d(r_{ic}, r_{jc}) < 0$ denotes a loss. The global matrix of dominance $\delta(A_i, A_j)$ is obtained through summing up the partial dominance measurements $\varphi_c(A_i, A_j)$.

Step 3: Sort the alternatives A_i by the normalized values ε_i, calculated by Equation (5). The best alternative is the one which has the highest value ε_i.

4. IUL-TODIM—An Intuitionistic Uncertain Linguistic TODIM Method

In order to solve the MADM problems where all the attribute values are expressed in IULNs, we presented the intuitionistic uncertain linguistic TODIM (IUL-TODIM) method, and developed a novel distance measures for IULNs, based on which we can obtain the corresponding dominance degree of one alternative over another.

Step 1: Normalize the IUL decision matrix $\widetilde{X} = [\widetilde{x}_{ic}]n \times m$ with $i = 1, \ldots, n$ and $c = 1, \ldots, m$, where $\widetilde{x} = \left\langle [s_{\theta(x)}, s_{\tau(x)}], (\mu(x), v(x)) \right\rangle$. The normalized uncertain linguistic index $\left[r_{\theta(x)}, r_{\tau(x)}\right]$ is calculated as

$$\begin{gathered}[\mathrm{r}_{\theta(x)}, r_{\tau(x)}] = neg([s_{\theta(x)}, s_{\tau(x)}]) = [s_{L-1-\tau(x)}, s_{L-1-\theta(x)}], \\ \text{where } [s_{\theta(x)}, s_{\tau(x)}] \in \overline{S}. \text{ for the cost criteria;}\end{gathered} \tag{8a}$$

$$[\mathrm{r}_{\theta(x)}, r_{\tau(x)}] = [s_{\theta(x)}, s_{\tau(x)}], \text{ where } [s_{\theta(x)}, s_{\tau(x)}] \in \overline{S} \text{ for the benefit criteria,} \tag{8b}$$

Then, the normalized IUL decision matrix $\widetilde{R} = [\widetilde{r}_{ic}]n \times m$ is constructed, where $\widetilde{r} = \left\langle [r_{\theta(x)}, r_{\tau(x)}], (\mu(x), v(x)) \right\rangle$.

Step 2: Calculate the dominance degree of alternative A_i over A_j

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \varphi_c(A_i, A_j), \qquad i, j = 1, \ldots, n \tag{9}$$

where

$$\varphi_c(A_i, A_j) = \begin{cases} \sqrt{d(\widetilde{r}_{ic}, \widetilde{r}_{jc}) w_{rc} / \sum_{c=1}^{m} w_{rc}} & (d(\widetilde{r}_{ic}, \widetilde{r}_{jc}) \geq 0) \\ -\frac{1}{\theta}\sqrt{d(\widetilde{r}_{ic}, \widetilde{r}_{jc}) (\sum_{c=1}^{m} w_{rc}) / w_{rc}} & (d(\widetilde{r}_{ic}, \widetilde{r}_{jc}) < 0) \end{cases}$$

The term $d(\widetilde{r}_{ic}, \widetilde{r}_{jc})$ represents the distance between two IULNs $\widetilde{r}_{ic}$ and $\widetilde{r}_{jc}$, calculated by Equation (3).

Step 3: Sort the alternatives A_i by the normalized values ε_i, calculated by Equation (5).

5. Numerical Example

In order to illustrate the feasibility and validity of the IL-TODIM and IUL-TODIM methods, we carry out the computational experiments which is cited from [33]. Consider the problem of choosing the most appropriate strategy for an investment company; the alternatives, attributes, and other detailed knowledge of this example can be obtained in [33].

5.1. The IL-TODIM Method Decision Process and Results

As mentioned above, we can utilize the proposed IL-TODIM method to solve this MADM problem and obtain the most desirable alternative. It is known that C_2 (growth index) and C_3 (social-political impact) are the benefit criteria, while C_1 (risk index) and C_4 (environmental impact) are the cost criteria. The attenuation factor of losses θ is set to 1, which means the loss contributes with its real value to the global value.

Firstly, the decision matrix is necessary to be calculated by Equation (6), and we can obtain the normalized matrix R:

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad \end{array} \\ \begin{pmatrix} \langle s_1, (0.2, 0.6) \rangle & \langle s_3, (0.3, 0.7) \rangle & \langle s_4, (0.4, 0.5) \rangle & \langle s_2, (0.2, 0.7) \rangle \\ \langle s_2, (0.3, 0.7) \rangle & \langle s_5, (0.3, 0.6) \rangle & \langle s_2, (0.1, 0.8) \rangle & \langle s_3, (0.4, 0.6) \rangle \\ \langle s_2, (0.2, 0.7) \rangle & \langle s_5, (0.3, 0.6) \rangle & \langle s_1, (0.1, 0.8) \rangle & \langle s_2, (0.2, 0.7) \rangle \\ \langle s_3, (0.2, 0.7) \rangle & \langle s_3, (0.1, 0.7) \rangle & \langle s_4, (0.3, 0.6) \rangle & \langle s_1, (0.4, 0.5) \rangle \end{pmatrix} \end{array}$$

After the implementation of Equation (7), the final dominance matrix can be obtained in Table 1. Furthermore, Table 2 shows the overall values and the final ordering of all the alternatives. As can be seen, the best option is A_2 (a computer company) followed by A_1 (a car company), and the last choice is A_3 (a TV company). In order to illustrate the influence of the parameter θ on decision-making of this example, we change the values of θ from 1 to 5 and the increment is 1. The ranking orders of the four alternatives obtained by applying the IL-TODIM method with different values of θ are always $\mathrm{A}_2 \succ \mathrm{A}_1 \succ \mathrm{A}_4 \succ \mathrm{A}_3$. In spite of increasing the attenuation factor of the losses from 1 to 5, the preferences are maintained, with the ranking orders not suffering any alteration either, and it indicates the robustness of the computational results based on the DM's preferences.

Table 1. Final dominance matrix, with θ = 1.

	A_1	A_2	A_3	A_4
A_1	0.000	−1.618	−0.839	−0.215
A_2	−0.694	0.000	0.307	−0.893
A_3	−0.941	−1.298	0.000	−1.169
A_4	−1.079	−1.310	−0.736	0.000

Table 2. Ranking orders and normalized values of alternatives (θ = 1).

Ranking	Alternatives	Performance	
		Gross	Normalized
1	A_2	−1.28	1.000
2	A_1	−2.672	0.346
3	A_4	−3.125	0.213
4	A_3	−3.408	0.000

5.2. The IUL-TODIM Method Decision Process and Results

In this sub-section, we will discuss the same problem mentioned above, but the DM evaluates this problem and constructs an IUL decision matrix. The decision matrix and other details can be obtained from [30]. In such a case, we can utilize the IUL-TODIM method presented in Section 4 to obtain the ranking orders of investments. As mentioned above, the original IUL decision matrix is necessary to be normalized into $\widetilde{R}$ by using Equation (8).

$$\widetilde{R} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad C_1 \quad\quad\quad & \quad\quad C_2 \quad\quad\quad & \quad\quad C_3 \quad\quad\quad & \quad\quad C_4 \end{array} \\ \begin{pmatrix} \langle [s_1, s_1], (0.2, 0.6) \rangle & \langle [s_3, s_4], (0.3, 0.7) \rangle & \langle [s_4, s_5], (0.4, 0.5) \rangle & \langle [s_2, s_2], (0.2, 0.7) \rangle \\ \langle [s_1, s_2], (0.3, 0.7) \rangle & \langle [s_5, s_5], (0.3, 0.6) \rangle & \langle [s_2, s_3], (0.1, 0.8) \rangle & \langle [s_2, s_3], (0.4, 0.6) \rangle \\ \langle [s_2, s_2], (0.2, 0.7) \rangle & \langle [s_5, s_5], (0.3, 0.6) \rangle & \langle [s_1, s_3], (0.1, 0.8) \rangle & \langle [s_2, s_2], (0.2, 0.7) \rangle \\ \langle [s_2, s_3], (0.2, 0.7) \rangle & \langle [s_3, s_4], (0.1, 0.7) \rangle & \langle [s_4, s_5], (0.3, 0.6) \rangle & \langle [s_1, s_1], (0.4, 0.5) \rangle \end{pmatrix} \end{array}$$

Then, we can get the final dominance matrix with different values of θ by Equation (9) (Table 3). Finally, sort the alternatives A_i $(i = 1, 2, 3, 4)$ by the normalized values ε_i $(i = 1, 2, 3, 4)$ calculated by Equation (5). Tables 4 and 5 present comparative results of the rankings with different values of θ. As we can see in Table 5, when we increase the value of θ from 1 to 5, A_2 is always the best choice according to the DM's preference, and the only change is in the ranking order of A_3 and A_4.

Table 3. Final dominance matrix.

	A_1	A_2	A_3	A_4
A_1	0.000	0.2225–1.5387/θ	0.2275–0.9926/θ	0.2841–0.4114/θ
A_2	0.405–1.236/θ	0.000	0.1888–0.1614/θ	0.3616–1.356/θ
A_3	0.2775–1.2638/θ	0.0516–0.8528/θ	0.000	0.258–1.3417/θ
A_4	0.1317–1.3055/θ	0.2863–1.4384/θ	0.2772–1.0067/θ	0.000

Table 4. Final gross dominance degree with different values of θ.

	$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$	$\theta = 5$
A_1	−2.209	−0.737	−0.247	−0.002	0.146
A_2	−1.798	−0.421	0.038	0.267	0.415
A_3	−2.871	−1.142	-0.566	−0.277	−0.277
A_4	−3.055	−1.180	−0.555	−0.242	−0.055

Table 5. Ranking orders of alternatives with different values of θ.

Different Values of θ	Ranking Results
$\theta = 1$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$\theta = 2$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$\theta = 3$	$A_2 \succ A_1 \succ A_4 \succ A_3$
$\theta = 4$	$A_2 \succ A_1 \succ A_4 \succ A_3$
$\theta = 5$	$A_2 \succ A_1 \succ A_4 \succ A_3$

5.3. Discussion

5.3.1. A Comparison between IL-TODIM and Intuitionistic Fuzzy TODIM (IF-TODIM)

For validation purposes, we make a comparison of the computational results between our proposed IL-TODIM method and the existing intuitionistic fuzzy TODIM (IF-TODIM) method published in the literature [18]. Similar ranking orders of the alternatives are expected for the problem mentioned above. Suppose the linguistic variables transformed to trapezoidal fuzzy numbers in Table 6; the transformed decision matrix Y can be expressed as

$$Y = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} \langle(8,9,10,11),(0.2,0.6)\rangle & \langle(4,5,6,7),(0.3,0.7)\rangle & \langle(6,7,8,9),(0.4,0.5)\rangle & \langle(6,7,8,9),(0.2,0.7)\rangle \\ \langle(6,7,8,9),(0.3,0.7)\rangle & \langle(8,9,10,11),(0.3,0.6)\rangle & \langle(2,3,4,5),(0.1,0.8)\rangle & \langle(4,5,6,7),(0.4,0.6)\rangle \\ \langle(6,7,8,9),(0.2,0.7)\rangle & \langle(8,9,10,11),(0.3,0.6)\rangle & \langle(0,1,2,3),(0.1,0.8)\rangle & \langle(6,7,8,9),(0.2,0.7)\rangle \\ \langle(4,5,6,7),(0.2,0.7)\rangle & \langle(4,5,6,7),(0.1,0.7)\rangle & \langle(6,7,8,9),(0.3,0.6)\rangle & \langle(8,9,10,11),(0.4,0.5)\rangle \end{pmatrix} \end{array}$$

Table 6. The linguistic variables with their corresponding fuzzy numbers.

Linguistic Labels	Linguistic Terms	Trapezoidal Fuzzy Numbers
S_0	Very Poor	(0,0,0,1)
S_1	Poor	(0,1,2,3)
S_2	Slightly Poor	(2,3,4,5)
S_3	Fair	(4,5,6,7)
S_4	Slightly Good	(6,7,8,9)
S_5	Good	(8,9,10,11)
S_6	Very Good	(10,11,11,11)

The IF-TODIM method is applied to the decision matrix Y, and then we can obtain a normalized decision matrix as

$$\begin{array}{ccccc} & C_1 & C_2 & C_3 & C_4 \\ A_1 & \left\langle(0,\frac{1}{7},\frac{2}{7},\frac{3}{7}),(0.2,0.6)\right\rangle & \left\langle(0,\frac{1}{7},\frac{2}{7},\frac{3}{7}),(0.3,0.7)\right\rangle & \left\langle(\frac{6}{9},\frac{7}{9},\frac{8}{9},1),(0.4,0.5)\right\rangle & \left\langle(\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}),(0.2,0.7)\right\rangle \\ A_2 & \left\langle(\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}),(0.3,0.7)\right\rangle & \left\langle(\frac{4}{7},\frac{5}{7},\frac{6}{7},1),(0.3,0.6)\right\rangle & \left\langle(\frac{2}{9},\frac{3}{9},\frac{4}{9},\frac{5}{9}),(0.1,0.8)\right\rangle & \left\langle(\frac{4}{7},\frac{5}{7},\frac{6}{7},1),(0.4,0.6)\right\rangle \\ A_3 & \left\langle(\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}),(0.2,0.7)\right\rangle & \left\langle(\frac{4}{7},\frac{5}{7},\frac{6}{7},1),(0.3,0.6)\right\rangle & \left\langle(0,\frac{1}{9},\frac{2}{9},\frac{3}{9}),(0.1,0.8)\right\rangle & \left\langle(\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}),(0.2,0.7)\right\rangle \\ A_4 & \left\langle(\frac{4}{7},\frac{5}{7},\frac{6}{7},1),(0.2,0.7)\right\rangle & \left\langle(0,\frac{1}{7},\frac{2}{7},\frac{3}{7}),(0.1,0.7)\right\rangle & \left\langle(\frac{6}{9},\frac{7}{9},\frac{8}{9},1),(0.3,0.6)\right\rangle & \left\langle(0,\frac{1}{7},\frac{2}{7},\frac{3}{7}),(0.4,0.5)\right\rangle \end{array}$$

Finally, the ranking results obtained by the IL-TODIM method and IF-TODIM method are listed in Table 7. Note that the best alternative obtained by the two methods is the same, but the ranking orders are not completely consistent. The difference between them is the ranking of A_1 and A_4, and the discrepancy in the performance of the two alternatives, which is calculated by the IF-TODIM method, is only 0.002. We cannot exactly say that alternative A_4 is better than A_1 because of the approximate number processing. In the actual decision-making, trapezoidal fuzzy numbers cannot be given directly for the evaluation of alternatives, so there may be some errors existing in the process of transforming the linguistic variables to the trapezoidal fuzzy numbers. The linguistic variables can express fuzzy information more directly so that the IL-TODIM method can decrease the computational complexity, and has the advantages of simplicity and reliability.

Table 7. Rankings results by IL-TODIM and IF-TODIM (θ = 1).

Alternatives	IL-TODIM		IF-TODIM	
	Normalized Performance	Ranking Orders	Normalized Performance	Ranking Orders
A_1	0.346	2	0.035	3
A_2	1.000	1	1.000	1
A_3	0.000	4	0.000	4
A_4	0.213	3	0.037	2

5.3.2. A Comparison between IL-TODIM and Other Intuitionistic Linguistic MADM Methods

In order to show the validity and effectiveness of the proposed methods, we utilize other two existing intuitionistic linguistic MADM methods to solve the same problem described above. In this paper, we only consider that the evaluation is made by one decision-maker.

(1) We adopt the ILGWA operator proposed by Liu [33] to solve this problem. Firstly, the comprehensive evaluation value of each alternative is obtained: $(r_1, r_2, r_3, r_4) = \{\langle s_{4.06}, (0.73, 0.1)\rangle, \langle s_{4.38}(0.78, 0.15)\rangle, \langle s_{3.72}, (0.60, 0.22)\rangle, \langle s_{3.66}(0.73, 0.17)\rangle\}$, and the score function can be calculated as follows: $S(x_1) = \frac{4.06}{6}\left[0.73 + \frac{1}{2}(1 - 0.73 - 1)\right] = 0.5514$, $S(x_2) = 0.5949$, $S(x_3) = 0.4278$, and $S(x_4) = 0.4758$, then $S(x_2) > S(x_1) > S(x_4) > S(x_3)$. Hence, the ranking order is $A_2 \succ A_1 \succ A_4 \succ A_3$.

(2) In addition, Wang et al. [32] presented the ILWGA operator which is also an aggregation operator. Based on the normalized matrix R, we utilize the ILWGA operator to aggregate the evaluation values of the ith alternative. By the equation $r_i = \mathrm{ILWGA}(r_{i1}, r_{i2}, r_{i3}, r_{i4}) = {r_{i1}}^{0.32} \otimes {r_{i2}}^{0.26} \otimes {r_{i3}}^{0.18} \otimes {r_{i4}}^{0.24}$ $(i = 1, 2, 3, 4)$, we have $r_1 = \langle s_{2.0168}, (0.2518, 0.5394)\rangle$, $r_2 = \langle s_{2.7974}, (0.2638, 0.6780)\rangle$, $r_3 = \langle s_{2.2404}, (0.1961, 0.6995)\rangle$, and $r_4 = \langle s_{2.4272}, (0.2122, 0.6429)\rangle$. Then, the score $h(x_i)$ (i = 1, 2, 3, 4) is calculated by the new score function proposed in [32], and we have $h(x_1) = 1.4368$, $h(x_2) = 1.6387$, $h(x_3) = 1.1126$, and $h(x_4) = 1.3815$. We obtain that $h(x_2) > h(x_1) > h(x_4) > h(x_3)$, then $A_2 \succ A_1 \succ A_4 \succ A_3$.

We find that the ranking order is always $A_2 \succ A_1 \succ A_4 \succ A_3$, which further proves the feasibility of the IL-TODIM method. A similar comparison can be made to demonstrate the validity of the IUL-TODIM method, so we will not repeat it here. Compared with the two existing methods, the IL-TODIM method, which is based on prospect theory, can consider the DM's behavioral preference and risk attitude. In addition, the two existing methods, which are based on aggregation operators, can give the comprehensive value of each alternative, but they may also ignore the differences among the alternatives according to different attributes.

6. Conclusions

The MADM methods are widely applied in practical decision-making situations, and the TODIM method which fully considers DM's bounded rationality for decision-making has recently received much attention. Additionally, the IL variables and IUL variables more easily depict uncertain or fuzzy information. Therefore, it is meaningful and valuable to research uncertain MADM problems with the IL variables or IUL variables considering the behavior preferences of the DMs. In this paper, two methods named IL-TODIM and IUL-TODIM have been proposed, which are able to handle MADM problems affected by uncertainty represented by ILNs or IULNs, respectively. To verify the validity of the proposed methods, we have applied them to evaluate an investment problem. Furthermore, a comparison of the ranking orders between the IL-TODIM method and other intuitionistic fuzzy MADM methods have been conducted to demonstrate the validity and effectiveness of the proposed method.

It should be noted that since other parameters were explicitly given by Liu and Jin [30,33], the sensitivity analysis was only carried out by varying the value of θ, the attenuation factor of losses, after obtaining the ranking results through the implementation of the proposed methods. Further research related to the prospect theory should take into account the behavior of DMs, principally regarding the decision-making motivation and reference points.

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