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Forecasting Based on High-Order Fuzzy-Fluctuation Trends and Particle Swarm Optimization Machine Learning

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Abstract: Most existing fuzzy forecasting models partition historical training time series into fuzzy time series and build fuzzy-trend logical relationship groups to generate forecasting rules. The determination process of intervals is complex and uncertain. In this paper, we present a novel fuzzy forecasting model based on high-order fuzzy-fluctuation trends and the fuzzy-fluctuation logical relationships of the training time series. Firstly, we compare each piece of data with the data of theprevious day in a historical training time series to generate a new fluctuation trend time series (FTTS). Then, we fuzzify the FTTS into a fuzzy-fluctuation time series (FFTS) according to the up, equal, or down range and orientation of the fluctuations. Since the relationship between historical FFTS and the fluctuation trend of the future is nonlinear, a particle swarm optimization (PSO) algorithm is employed to estimate the proportions for the lagged variables of the fuzzy AR (n) model. Finally, we use the acquired parameters to forecast future fluctuations. In order to compare the performance of the proposed model with that of the other models, we apply the proposed method to forecast the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) time series datasets. The experimental results and the comparison results show that the proposed method can be successfully applied in stock market forecasting or similarkinds of time series. We also apply the proposed method to forecast Shanghai Stock Exchange Composite Index (SHSECI) and DAX30 index to verify its effectiveness and universality.

Keywords: fuzzy forecasting; fuzzy-fluctuation trend; particle swarm optimization; fuzzy time series; fuzzy logical relationship

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1. Introduction

It is well known that historic time series imply the behavior rules of a given phenomenon and can be used to forecast the future of the same event [1]. Many researchers have developed time series models to predict the future of a complex system, e.g., regression analysis [2], the autoregressive moving average (ARIMA) model [3], the autoregressive conditional heteroscedasticity (ARCH) model [4], the generalized ARCH (GARCH) model [5], and so on. However, these methods require some premise hypotheses,such as a normality postulate [6], etc. Meanwhile, models that satisfied the constraints precisely can miss the true optimum design within the confines of practical and realistic approximations. Therefore, Song and Chissom proposed the fuzzy time series forecasting model [7–9]. Since then, the FTS model has been applied for forecasting in many nonlinear and complicated forecasting problems, e.g., stock market [10–13], electricity load demand [14,15], project cost [16], and the enrollment at Alabama University [17,18], etc.

A vast majority of FTS models are first-order and high-order fuzzy AR (autoregressive) models. These models can be considered as an equivalent version of AR (n) based on fuzzy lagged variables of time series. Most of these fuzzy time series models follow the basic steps as Chen proposed [19]:

Step 1: Define the universe U and the number and length of the intervals;
Step 2: Fuzzify the historical training time series into fuzzy time series;
Step 3: Establish fuzzy logical relationships (FLR) according to the historical fuzzy time series and generate forecasting rules based on fuzzy logical groups (FLG);
Step 4: Calculate the forecast values according to the FLG rules and the right-hand side (RHS) of the forecasted point.

In order to improve the accuracy of such kinds of FTS models, researchers have proposed other improved models based on Chen's model. For example, concerning the determination of suitable intervals, Huarng [20] proposed averages and distribution methods to determine the optimal interval length. Huarng and Yu [21] proposed an unequal interval length method based on ratios of data. Since then, many studies [20,22–27] have been carried out for the determination of the optimal interval length using statistical theory. Some authors even employed PSO techniques to determine the length of the intervals [12]. In fact, in addition to the determination of intervals, the definition of the universe of discourse also has an effect on the accuracy of the forecasting results. In these models, minimum data value, maximum data value, and two suitable positive numbers must be determined to make a proper bound of the universe of discourse.

Concerning the establishment of fuzzy logical relationships, many researchers utilize artificial neural networks to determine fuzzy relations [28–30]. The study of Aladag et al. [28] is considered as a basic high-order method for forecasting based on artificial neural networks. Meanwhile, fuzzy AR models are also widely used in many fuzzy time series forecasting studies [11,12,31–35]. In order to reflect the recurrence and the weights of different FLR in fuzzy AR models, Yu [36] used a chronologically-determined weight in the defuzzification process. Cheng et al. [37] used the frequencies of different right-hand sides (RHS) of FLG rules to determine the weight of each LHS. Furthermore, many studies employed the adaptive fuzzy inference system (ANFIS) method [38] for time series forecasting. For example, Primoz and Bojan [39] defined soft constraints based on ANFIS to discrete optimization for obtaining optimal solutions. Egrioglu et al. [40] proposed a model named the modified adaptive network based fuzzy inference system (MANFIS). Sarica et al. [41] developed a model based on an autoregressive adaptive network-based fuzzy inference system (AR-ANFIS), etc. Since 2013, considering the impacts of specification errors, fuzzy auto regressive and moving average (ARMA) time series forecasting models were proposed [42,43]. The initial first-order ARMA fuzzy time series forecasting model was proposed by Egrioglu et al. [42] based on the particle swarm optimization method. Kocak [43] developed a high-order ARMA fuzzy time series model based on artificial neural networks. Kocak [44] used both fuzzy AR variables and fuzzy MA variables to increase the performance of the forecasting models.

A forecasting model is used to predict the future fluctuation of a time series based on current values. Therefore, we present a novel method to forecast the fluctuation of a stock market based on a high-order AR (n) fuzzy time series model and particle swarm optimization (PSO) arithmetic. Unlike existing models, the proposed model is based on the fluctuation values instead of the exact values of the time series. Firstly, we calculate the fluctuation for each datum by comparing it with the data of its previous day in a historical training time series to generate a new fluctuation trend time series (FTTS). Then, we fuzzify the FTTS into fuzzy-fluctuation time series (FFTS) according to the up, equal, or down range of each fluctuation data value. Since the relationship between historical FFTS and future fluctuation trends is nonlinear, a PSO algorithm is employed to estimate the proportion of each AR and MA parameter in the model. Finally, we use these acquired parameters to forecast future fluctuations. The advantages provided by the proposed method are as follows.

The remaining content of this paper is organized as follows: Section 2 introduces some preliminaries of fuzzy-fluctuation time series based on Song and Chissom's fuzzy time series [7–9]. Section 4 introduces the process of the PSO machine learning method. Section 4 describes a novel approach for forecasting based on high-order fuzzy-fluctuation trends and the PSO heuristic learning process. In Section 5, the proposed model is usedto forecast the stock market using TAIEX datasets from 1997 to 2005, SHSECI from 2007 to 2015, and the year 2015 of the DAX30 index. Conclusions and potential issues for future research are summarized in Section 6.

2. Preliminaries

Song and Chissom [7–9] combined fuzzy set theory with time series and presented the following definitions of fuzzy time series. In this section, we will extend fuzzy time series to fuzzy-fluctuation time series (FFTS) and propose the related concepts.

Definition 1. *Let* $L = \{l_1, l_2, ..., l_g\}$ *be a fuzzy set in the universe of discourse* U *; it can be defined by its membership function,* $\mu_L : U \rightarrow [0, 1]$ *, where* $\mu_L(u_i)$ *denotes the grade of membership of* u_i*,* $U = \{u_1, u_2, ...u_i, ..., u_l\}$.

The fluctuation trends of a stock market can be expressed by a linguistic set $L = \{l_1, l_2, ..., l_g\}$, e.g., let $g = 3$, $L = \{l_1, l_2, l_3\}$ = *{down, equal, up}*. The element l_i and its subscript i is strictly monotonically increasing [45], so the function can be defined as follows: $f : l_i = f(i)$. To preserve all of the given information, the discrete $L = \{l_1, l_2, ..., l_g\}$ also can be extended to a continuous label $\overline{L} = \{l_a | a \in R\}$, which satisfies the above characteristics.

Definition 2. *Let* $X(t)(t = 1, 2, ..., T)$ *be a time series of real numbers, where* T *is the number of the time series.* $Y(t)$ *is defined as a fluctuation time series, where* $Y(t) = X(t) - X(t-1), (t = 2, 3, ..., T)$*. Each element of* $Y(t)$ *can be represented by a fuzzy set* $S(t)(t = 2, 3, ..., T)$ *as defined in Definition 1. Then we call time series* $Y(t)$ *to befuzzified into a fuzzy-fluctuation time series (FFTS)* $S(t)$*.*

Definition 3. *Let* $S(t)$ $(t = 2, 3, ..., T)$ *be a FFTS. If* $S(t)$ *is determined by* $S(t-1), S(t-2), ..., S(t-n)$*, then the fuzzy-fluctuation logical relationship is represented by*:

$$S(t) \leftarrow S(t-1), S(t-2), ..., S(t-n) \tag{1}$$

and it is called the nth-order fuzzy-fluctuation logical relationship (FFLR) of the fuzzy-fluctuation time series, where $S(t)$ *is called the left-hand side(LHS) and* $S(t-n), ..., S(t-2)S(t-1)$ *is called the right-hand side(RHS) of the FFLR. This model can be considered as an equivalent of the auto-regressive model of AR (n), defined in Equation (2):*

$$\overline{S}(t) = \phi_1 S(t-1) + \phi_2 S(t-2), ..., \phi_n S(t-n) + \varepsilon_t \tag{2}$$

where $\phi_k(k = 1, 2, .., n)$ *represents the portion of* $S(t-k)$ *for calculating the forecast is* ϕ_k *,* ε_t *is the calculation error, and* $\overline{S}(t)$ *is introduced to preserve more information, as described in Definition 1.*

3. PSO-Based Machine Learning Method

In this paper, the particle swarm optimization (PSO) is employed to estimate the parameters in Equation (2). The PSO method was introduced as an optimization method for continuous nonlinear functions [46]. It is a stochastic optimization technique, which is similar to social models, such as birds flocking or fish schooling. During the optimization process, particles are distributed randomly in the design space and their location and velocities are modified according to their personal best and global best solutions. Let *m+1* represent the current time step, $x_{i,m+1}$, $v_{i,m+1}$, $x_{i,m}$, $v_{i,m}$ indicate the current position, current velocity, previous position, and previous velocity of particle i, respectively. The position and velocity of particle i are manipulated according to the following equations:

$$x_{i,m+1} = x_{i,m} + v_{i,m+1} \tag{3}$$

$$v_{i,m+1} = w \times v_{i,m} + c_1 \times Rand() \times (p_{i,m} - x_{i,m}) + c_2 \times Rand() \times (p_{g,m} - x_{i,m}) \tag{4}$$

where w is an inertia weight which determines how much the previous velocity is preserved [47], c_1 and c_2 are the self-confidence coefficient and social confidence coefficient, respectively, $Rand() \in [0,1]$ is a random number, and $p_{i,m}$ and $p_{g,m}$ are the personal best position found by particle i and the global best position found by all particles in the swarm up to time step m, respectively.

Let the design space be defined by $[x_{min}, x_{max}]$. If the position of particle i exceeds the boundary, then $v_{i,m+1}$ is modified as follows:

$$x_{i,m+1} = \begin{cases} x_{\max} - (0.5 \times Rand() \times (x_{\max} - x_{\min})), & if \quad x_{i,m+1} > x_{\max} \\ x_{\min} + (0.5 \times Rand() \times (x_{\max} - x_{\min})), & if \quad x_{i,m+1} < x_{\min} \end{cases} \tag{5}$$

4. A Novel Forecasting Model Based on High-Order Fuzzy-Fluctuation Trends

In this paper, we propose a novel forecasting model based on high-order fuzzy-fluctuationtrends and a PSO machine learning algorithm. In order to compare the forecasting results with other researchers' work [10,11,27,36,48–51], the authentic TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) is employed to illustrate the forecasting process. The data from January 1999 to October 1999 are used as training time series and the data from November 1999 to December 1999 are used as testing dataset. The basic steps of the proposed model are shown in Figure 1.

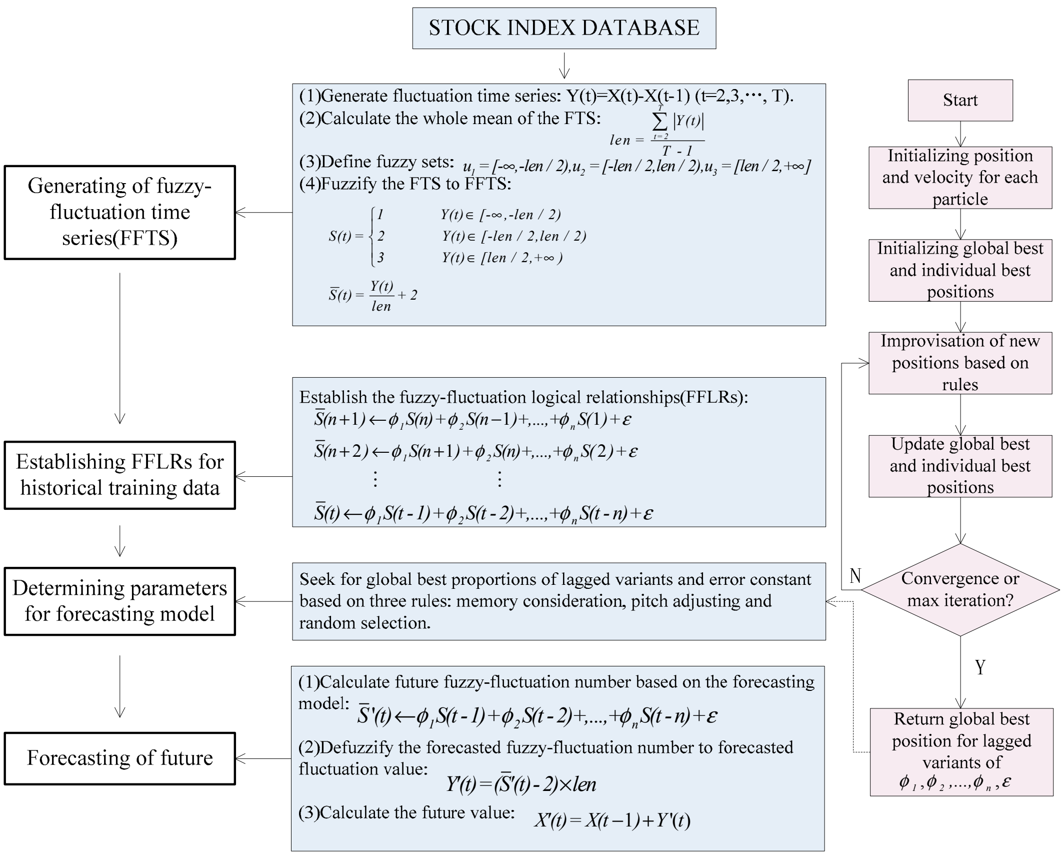

Figure 1. Flowchart of our proposed forecasting model.

Step 1: Construct FFTS for historical training data

For each element $\mathrm{X(t)(t = 1, 2, \ldots, T)}$ in the historical training time series, its fluctuation trend is determined by $\mathrm{Y(t) = X(t) - X(t-1), (t = 2, 3, \ldots, T)}$. According to the range and orientation of the fluctuations, $\mathrm{Y(t)(t = 2, 3, \ldots, T)}$ can be fuzzified into a linguistic set {down, equal, up}. Let *len* be the whole mean of all elements in the fluctuation time series $\mathrm{Y(t)(t = 2, 3, \ldots, T)}$, define $u_1 = [-\infty, -len/2)$, $u_2 = [-len/2, len/2)$, $u_3 = [len/2, +\infty]$, then $\mathrm{Y(t)(t = 2, 3, \ldots, T)}$ can be fuzzified into a fuzzy-fluctuation time series $\mathrm{S(t)(t = 2, 3, \ldots, T)}$. It is can also be extended to a continuous labeled time series $\mathrm{\overline{S}(t)(t = 2, 3, \ldots, T)}$, which preserves the accurate original information of $\mathrm{Y(t)(t = 2, 3, \ldots, T)}$.

Step 2: Establish nth-order FFLRs for the forecasting model

According to Equation (2), each $\mathrm{\overline{S}(t)(t \geq n + 2)}$ can be represented by its previous n days' fuzzy-fluctuation number. Therefore, the total of FFLRs for historical training data is $pn = T - n - 1$.

Step 3: Determine the parameters for the forecasting model based on the PSO machine learning algorithm

In this paper, the PSO method is employed to determine the parameters and a general error ε in Equation (2). The personal best position and global best position are determined by minimizing the root of the mean squared error (RMSE) in the training process:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (forecast(t) - actual(t))^2}{n}} \tag{6}$$

where n denotes the number of values forecasted, *forecast(t)* and *actual(t)* denote the forecasting value and actual valueat time *t*in the training process, respectively. For determined $\phi_k(k = 1, 2, .., n)$ and ε, the forecast value at time t is as follows:

$$forevast(t) = actual(t-1) + len \times (\phi_1 S(t-1) + \phi_2 S(t-2), \ldots, \phi_n S(t-n) + \varepsilon - 2) \tag{7}$$

The pseudo-code for the PSO-based machine learning algorithm is shown in Appendix A.

Step 4: Forecast test time series

For each data in the test time series, its future number can be forecasted according to Equation (7), based on the observed data point $X\,(t-1)$, its n-order fuzzy-fluctuation trends, and the parameters generated from the training dataset.

5. EmpiricalAnalysis

5.1. Forecasting TAIEX

Many studiesuse TAIEX1999as an example to illustrate their proposed forecasting methods [10,11,27,36,48–51]. In order to compare the accuracy with their models, we also use TAIEX1999 to illustrate the proposed method.

Step 1: Calculate the fluctuation trend for each element in the historical training dataset of TAIEX1999. Then, we use the whole mean of the fluctuation numbers of the training dataset to fuzzify the fluctuation trends into FFTS.For example, the whole mean of the historical dataset of TAIEX1999 from January to October is 85. That is to say, *len* = 85. For *X(1)* = 6152.43 and *X(2)* = 6199.91, *Y(2)* = 47.48, *S(2)* = 3, and $\overline{S}(2) \approx 2.5586$. On the other hand, based on the previous data *X(1)* and the accurate fuzzy number $\overline{S}(2)$, *X(2)* can be obtained by: $X(1) + len \times (\overline{S}(2) - 2)$, that is $6152.43 + (2.5588 - 2) \times 85 \approx 6199.91$. In this way, the historical training dataset can be represented by a fuzzified fluctuation dataset as shown in Appendix B.

Step 2: Based on the FFTS from 5 January 1999 to 30 October shown in Table 1, establish the nth-order FFLRs for the forecasting model. For example, suppose n=6, the following FFLRs of FFTS can be generated:

$$\begin{array}{c}\overline{S}(7) = 1.082 = \phi_1 + \phi_2 + 2\phi_3 + 2\phi_4 + 3\phi_5 + 3\phi_6 + \varepsilon_7 \\ \overline{S}(8) = 4.5091 = \phi_1 + \phi_2 + \phi_3 + 2\phi_4 + 2\phi_5 + 3\phi_6 + \varepsilon_8 \\ \vdots \\ \overline{S}(221) = 3.7433 = 2\phi_1 + 2\phi_2 + 2\phi_3 + 2\phi_4 + 3\phi_5 + \phi_6 + \varepsilon_{221}\end{array} \tag{8}$$

Table 1. Global best parameters obtained using PSO for training dataset.

φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	E	RMSE
−0.1638	0.0803	0.1372	−0.0321	0.0433	0.2546	1.4408	115.73

Step 3: Replace each error ε_t in Equation (8) with one and the same ε. Let the number of iterations *itern* = 100, the inertia weight w = 0.7298, the self-confidence coefficient and social confidence coefficient $c_1 = c_2 = 1.4962$, and use the PSO algorithm listed in Figure 1 to determine the parameters and ε. In the PSO process, each element in the generalizedEquation (8) is a particle and their personal best and global best positions are determined by the RMSE of the actual values and forecast values. The obtained global best parameters are shown in Table 1.

Step 4: Use the obtained global best parameters in Table 1 to forecast the test dataset from 1 November 1999 to 30 December. For example, the forecasting value of the TAIEX on 8 November 1999 is calculated asfollows:

Firstly, according to the fuzzy-fluctuation trends (2,1,1,1,2,1) and the parameters in Table 1, the forecasted continuous labeled fuzzy-fluctuation number is:

$$2 \times (-0.1638) + 0.0803 + 0.1372 - 0.0321 + 2 \times 0.0433 + 0.2546 + 1.4408 = 1.6398$$

Then, the forecasted fluctuation from current value to next value can be obtained by defuzzifying the fluctuation fuzzy number:

$$(1.6398 - 2) \times 85 = -30.62$$

Finally, the forecasted value can be obtained by current value and the fluctuation value:

$$7376.56 - 30.62 = 7345.94$$

The other forecasting results are shown in Table 2 and Figure 2.

The forecasting performance can be assessed by comparing the difference between the forecasted valuesand the actual values. The widely used indicators in time series models comparisons are the mean squared error (MSE), root of the mean squared error (RMSE), mean absolute error (MAE), mean percentage error (MPE), etc.To compare the performance of different forecasting results, the Diebold-Mariano test statistic (S) is also widely used. These indicators are defined by Equations (9)–(13):

$$MSE = \frac{\sum\limits_{t=1}^{n} (forecast(t) - actual(t))^2}{n} \tag{9}$$

$$RMSE = \sqrt{\frac{\sum\limits_{t=1}^{n} (forecast(t) - actual(t))^2}{n}} \tag{10}$$

Table 2. Forecasting results from 1 November1999 to 30 December 1999.

Date (MM/DD/YYYY)	Actual	Forecast	(Forecast–Actual)^2	Date (MM/DD/YYYY)	Actual	Forecast	(Forecast–Actual)^2
11/1/1999	7814.89	7869.35	2965.89	12/1/1999	7766.20	7705.59	3673.57
11/2/1999	7721.59	7825.35	10,766.14	12/2/1999	7806.26	7790.48	249.01
11/3/1999	7580.09	7704.00	15,353.69	12/3/1999	7933.17	7824.29	11,854.85
11/4/1999	7469.23	7573.21	10,811.84	12/4/1999	7964.49	7967.96	12.04
11/5/1999	7488.26	7460.24	785.12	12/6/1999	7894.46	7965.87	5099.39
11/6/1999	7376.56	7468.50	8452.96	12/7/1999	7827.05	7897.62	4980.12
11/8/1999	7401.49	7345.94	3085.80	12/8/1999	7811.02	7806.25	22.75
11/9/1999	7362.69	7400.03	1394.28	12/9/1999	7738.84	7823.68	7197.83
11/10/1999	7401.81	7379.30	506.70	12/10/1999	7733.77	7701.12	1066.02
11/11/1999	7532.22	7410.86	14,728.25	12/13/1999	7883.61	7718.38	27,300.95
11/15/1999	7545.03	7553.82	77.26	12/14/1999	7850.14	7921.86	5143.76
11/16/1999	7606.20	7569.42	1352.77	12/15/1999	7859.89	7862.87	8.88
11/17/1999	7645.78	7631.90	192.65	12/16/1999	7739.76	7857.12	13,773.37
11/18/1999	7718.06	7667.91	2515.02	12/17/1999	7723.22	7750.49	743.65
11/19/1999	7770.81	7750.58	409.25	12/18/1999	7797.87	7733.15	4188.68
11/20/1999	7900.34	7800.66	9936.10	12/20/1999	7782.94	7815.10	1034.27
11/22/1999	8052.31	7936.55	13,400.38	12/21/1999	7934.26	7781.74	23,262.35
11/23/1999	8046.19	8079.43	1104.90	12/22/1999	8002.76	7953.13	2463.14
11/24/1999	7921.85	8072.42	22,671.32	12/23/1999	8083.49	8060.46	530.38
11/25/1999	7904.53	7908.83	18.49	12/24/1999	8219.45	8119.70	9950.06
11/26/1999	7595.44	7912.20	100,336.90	12/27/1999	8415.07	8246.57	28,392.25
11/29/1999	7823.90	7576.21	61,350.34	12/28/1999	8448.84	8462.94	198.81
11/30/1999	7720.87	7823.06	10,442.80	Root Mean Square Error(RMSE)			99.31

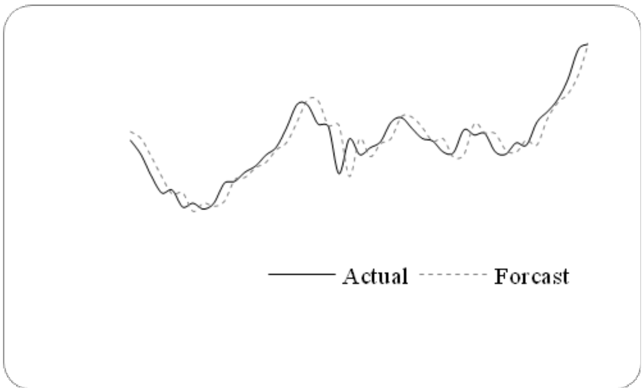

Figure 2. Forecasting results from 1 November1999 to 30 December 1999.

$$MAE = \frac{\sum_{t=1}^{n} |(forecast(t) - actual(t))|}{n} \tag{11}$$

$$MPE = \frac{\sum_{t=1}^{n} |(forecast(t) - actual(t))|/actual(t)}{n} \tag{12}$$

$$S = \frac{\overline{d}}{(Variance(\overline{d}))^{1/2}}, \overline{d} = \frac{\sum_{t=1}^{n} (error \quad of \quad forecast1)_t^2 - \sum_{t=1}^{n} (error \quad of \quad forecast2)_t^2}{n} \tag{13}$$

where n denotes the number of values forecasted, *forecast(t)* and *actual(t)* denote the predicted value and actual valueat time t, respectively. S is a test statistic of the Diebold method which is used to compare predictive accuracy of two forecasts obtained by different methods. *Forecast1* represents the dataset obtained by method 1, and *Forecast2* represents another dataset from method 2. If S>0 and $|S| > Z = 1.64$, at the 0.05 significant level, *Forecast2* has better predictive accuracy than *Forecast1*.

With respect to the proposed method for the sixth-order, the MSE, RMSE, MAE, and MPE are 9862.33, 99.31, 75.22, and 0.01, respectively.

In order to compare the forecasting results with different parameters such as the number n of the nth-order and the element number g of linguistic set used in the fluctuation fuzzifying process, different experiments under different parameters were carried out. Each typeof experiment was repeated 30 times. The forecasting errors of the averages for the experiments are shown in Tables 3 and 4.

Table 3. Comparison of forecasting errors for different nth-orders (g=3).

n	1	2	3	4	5	6	7	8	9	10
RMSE	109.04	105.47	103.04	102.96	101.92	99.12	99.59	99.6	98.75	99

Table 4. Comparison of forecasting errors for different linguistic sets (n=6).

g	3	5	7	None
RMSE	99.12	101.67	105.82	128.97

In Table 4, g = 3 represents that the linguistic set is *{down, equal, up}*, g = 5 means *{greatly down, slightly down, equal, slightly up, greatly up}*, g = 7 means *{very greatly down, greatly down, slightly down, equal, slightly up, greatly up, very greatly up}*, and "*none*" means that the fluctuation values will not be fuzzified at all.

From Tables 3 and 4, we can see that the RMSEs are lower when n is equal to six or more. With respect to the parameter g, obviously, the fuzzified fluctuation trends perform better than none fuzzified ones, and it is proper to let g = 3.

Letting n=6 and g=3, we employ the proposed method to forecast the TAIEX from 1997 to 2005. The forecasting results and errors are shown in Figure 3 and Table 5.

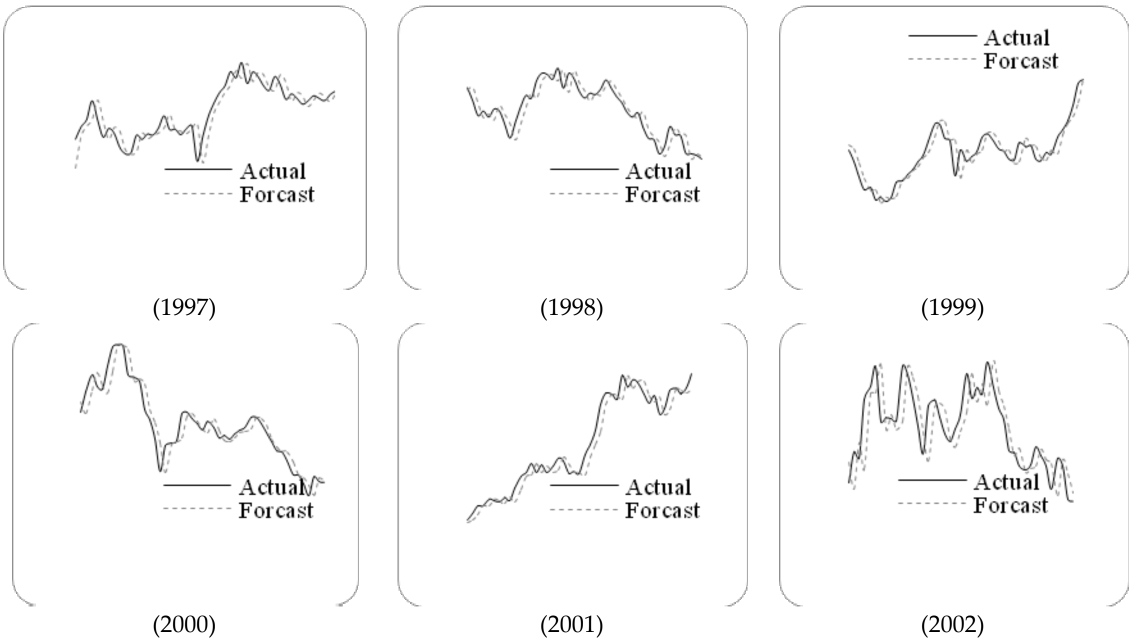

Figure 3. *Cont.*

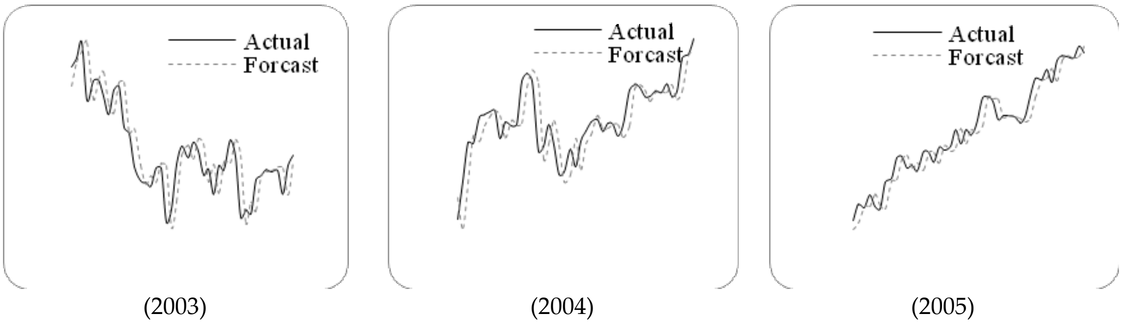

Figure 3. The stock marketfluctuation for TAIEX test dataset (1997–2005).

Table 5. RMSEs of forecast errors for TAIEX 1997 to 2005.

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
RMSE	143.60	115.34	99.12	125.70	115.91	70.43	54.26	57.24	54.68

Table 6 shows a comparison of RMSEs for different methods for forecasting the TAIEX1999. From this table, we can see that the performance of the proposed method is acceptable. The greatest advantage of the proposed method is that it puts forward a method relying completely on the machine learning mechanism. Though RMSEs of some of the other methods outperform the proposed method, they oftenneed to determine complex discretization partitioning rulesor use adaptive expectation models to justify the final forecasting results. The method proposed in this paper is simpler and easily realized by a computer program.

Table 6. A comparison of RMSEs for different methods for forecasting the TAIEX1999.

Methods	RMSE	S
Yu's Method(2005) [36]	145	1.62 **
Hsieh et al.'s Method(2011) [51]	94	−0.32
Chang et al.'s Method(2011) [48]	100	0.11
Cheng et al.'s Method(2013) [50]	103	0.34
Chen et al.'s Method(2013) [49]	102.11	0.21
Chen and Chen's Method(2015) [11]	103.9	0.36
Chen and Chen's Method(2015) [10]	92	−0.42
Zhao et al.'s Method(2016) [27]	110.85	1.08
The Proposed Method	99.12	-

** The proposed method has better predictive accuracy than the method at the 5% significance level.

5.2. Forecasting DAX30

The German DAX30 index is an important stock index in Germany. The RMSEs of different models forecastingyear 2015 of DAX30 are shown in Table 7.

Table 7. RMSEs of the forecast errors for year 2015 of DAX30.

Year	Yu (2005) [36]	Cheng et al. (2008) [37]	Wang et al. (2013) [25]	Rubio et al. (2017) [13]	Proposed Model
RMSE	172.69	170.56	376.80	153.15	159.22
S	1.31	1.23	3.68 **	−0.26	-

** The proposed method has better predictive accuracy than the method at the 5% significancelevel.

From Table 7, we can see that the proposed method can successfully predict the DAX30 index.

5.3. Forecasting SHSECI

The SHSECI (Shanghai Stock Exchange Composite Index) is the most famous stock market index in China. In the following, we apply the proposed method to forecast the SHSECI from 2007 to 2015. For each year, the authentic datasets of historical daily SHSECI closing prices from January to October are used as the training data, and the datasets from November to December are used as the testing data. The RMSEs of forecast errors are shown in Table 8.

Table 8. RMSEs of forecast errors for SHSECI from 2007 to 2015.

Year	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**
RMSE	113.11	55.28	49.59	45.73	28.45	25.05	19.86	41.44	59.5

From Table 8, we can see that the proposed method can successfully predict the SHSECI stock market.

6. Conclusions

In this paper, a novel forecasting model is proposed based on high-order fuzzy-fluctuation logical trends and the PSO machine learning method. The proposed method is based on the fluctuations of the time series. The PSO method is employed to look for the best parameters to minimize the RMSE of a historical training dataset. Experiments show that the parameters generated from the training dataset can be successfully used for future datasets as well. In order to compare the performance with that of other methods, we take the TAIEX1999 as an example. We also forecasted TAIEX1997–2005, DAX30 2015 and SHSECI 2007–2015 to verify its effectiveness and universality. In the future, we will consider other factors which might affect the fluctuation of the stock market, such as the trade volume, the beginning value, the end value, etc. We will also consider the influence of other stock markets, such as the Dow Jones, the NASDAQ, the M1b, and so on.

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Appendix A

The Pseudo-code of PSO-based machine learning algorithm for seeking for global best position of each constant parameter in the forecasting model is shown in Table A1.

Table A1. Pseudo-code of the PSO-based machine learning algorithm.

PSO-Based Machine Learning Algorithm for the Training Process	
INPUT:	X: training time series, containing T cases, denoted as $X[1], X[2], ..., X[i] ..., X[T]$. S: a fuzzy-fluctuation time series of training data, containing $T-1$ cases, denoted as $S[2], S[3], ..., S[i] ..., S[T]$. n: the number of nth-order. *itern*: the number of iterations. x_{min}, x_{max}: lower and upper bounds of space. w, c_1, c_1: parameters described in Equations (3) and (4).

Table A1. *Cont.*

PSO-Based Machine Learning Algorithm for the Training Process	
OUPUT:	*Φ[k]* and ε: parameters for the forecasting model, $k = 1,2, \ldots ,n$.
1.	Initialize the position and velocity for each particle *i*: *pn* = *T*−1−*n;* /* the number of particles. */ *For i = 1 to pn* *For j = 1 to n* *x[i,j] = rand*(x_{min}, x_{max})*;* *v[i,j] = rand*(x_{min}, x_{max})*;*
2.	Calculate the fitness value for each particle *i* according to Equation (6): Set *x[pbest]* to current *x[i]* for each particle. Locate the global best fitness value *x[gbest]* and set *Φ[k]* and ε to the corresponding *x[gbest]*.
3.	for *m=1* to *itern* loop For each particle *i* Calculate particle velocity according to Equation (3). Update particle position according to Equations (4) and (5) If the fitness value is better than the best fitness value *x[pbest]* of particle *i* in history: Set current value as the new *x[pbest]* for particle *i* Locate the current global best fitness value, if it is better than the *x[gbest]* in history: Set current global best fitness value as the new *x[gbest]*, and set *Φ[k]* and ε to *x[gbest]*.
4.	Output Φ*[k]* and ε

Appendix B

The historical training dataset can be represented by a fuzzified fluctuation dataset as shown in Table A2.

Table A2. Historical training data and fuzzified fluctuation data of TAIEX1999.

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
1/5/1999	6152.43	-	-	4/17/1999	7581.5	114.68	3	7/26/1999	7595.71	−128.81	1
1/6/1999	6199.91	47.48	3	4/19/1999	7623.18	41.68	2	7/27/1999	7367.97	−227.74	1
1/7/1999	6404.31	204.4	3	4/20/1999	7627.74	4.56	2	7/28/1999	7484.5	116.53	3
1/8/1999	6421.75	17.44	2	4/21/1999	7474.16	−153.58	1	7/29/1999	7359.37	−125.13	1
1/11/1999	6406.99	−14.76	2	4/22/1999	7494.6	20.44	2	7/30/1999	7413.11	53.74	3
1/12/1999	6363.89	−43.1	1	4/23/1999	7612.8	118.2	3	7/31/1999	7326.75	−86.36	1
1/13/1999	6319.34	−44.55	1	4/26/1999	7629.09	16.29	2	8/2/1999	7195.94	−130.81	1
1/14/1999	6241.32	−78.02	1	4/27/1999	7550.13	−78.96	1	8/3/1999	7175.19	−20.75	2
1/15/1999	6454.6	213.28	3	4/28/1999	7496.61	−53.52	1	8/4/1999	7110.8	−64.39	1
1/16/1999	6483.3	28.7	2	4/29/1999	7289.62	−206.99	1	8/5/1999	6959.73	−151.07	1
1/18/1999	6377.25	−106.05	1	4/30/1999	7371.17	81.55	3	8/6/1999	6823.52	−136.21	1
1/19/1999	6343.36	−33.89	2	5/3/1999	7383.26	12.09	2	8/7/1999	7049.74	226.22	3
1/20/1999	6310.71	−32.65	2	5/4/1999	7588.04	204.78	3	8/9/1999	7028.01	−21.73	2
1/21/1999	6332.2	21.49	2	5/5/1999	7572.16	−15.88	2	8/10/1999	7269.6	241.59	3
1/22/1999	6228.95	−103.25	1	5/6/1999	7560.05	−12.11	2	8/11/1999	7228.68	−40.92	2
1/25/1999	6033.21	−195.74	1	5/7/1999	7469.33	−90.72	1	8/12/1999	7330.24	101.56	3
1/26/1999	6115.64	82.43	3	5/10/1999	7484.37	15.04	2	8/13/1999	7626.05	295.81	3
1/27/1999	6138.87	23.23	2	5/11/1999	7474.45	−9.92	2	8/16/1999	8018.47	392.42	3
1/28/1999	6063.41	−75.46	1	5/12/1999	7448.41	−26.04	2	8/17/1999	8083.43	64.96	3
1/29/1999	5984	−79.41	1	5/13/1999	7416.2	−32.21	2	8/18/1999	7993.71	−89.72	1
1/30/1999	5998.32	14.32	2	5/14/1999	7592.53	176.33	3	8/19/1999	7964.67	−29.04	2
2/1/1999	5862.79	−135.53	1	5/15/1999	7576.64	−15.89	2	8/20/1999	8117.42	152.75	3
2/2/1999	5749.64	−113.15	1	5/17/1999	7599.76	23.12	2	8/21/1999	8153.57	36.15	2
2/3/1999	5743.86	−5.78	2	5/18/1999	7585.51	−14.25	2	8/23/1999	8119.98	−33.59	2
2/4/1999	5514.89	−228.97	1	5/19/1999	7614.6	29.09	2	8/24/1999	7984.39	−135.59	1
2/5/1999	5474.79	−40.1	2	5/20/1999	7608.88	−5.72	2	8/25/1999	8127.09	142.7	3
2/6/1999	5710.18	235.39	3	5/21/1999	7606.69	−2.19	2	8/26/1999	8097.57	−29.52	2
2/8/1999	5822.98	112.8	3	5/24/1999	7588.23	−18.46	2	8/27/1999	8053.97	−43.6	1
2/9/1999	5723.73	−99.25	1	5/25/1999	7417.03	−171.2	1	8/30/1999	8071.36	17.39	2
2/10/1999	5798	74.27	3	5/26/1999	7426.63	9.6	2	8/31/1999	8157.73	86.37	3
2/20/1999	6072.33	274.33	3	5/27/1999	7469.01	42.38	2	9/1/1999	8273.33	115.6	3
2/22/1999	6313.63	241.3	3	5/28/1999	7387.37	−81.64	1	9/2/1999	8226.15	−47.18	1
2/23/1999	6180.94	−132.69	1	5/29/1999	7419.7	32.33	2	9/3/1999	8073.97	−152.18	1
2/24/1999	6238.87	57.93	3	5/31/1999	7316.57	−103.13	1	9/4/1999	8065.11	−8.86	2
2/25/1999	6275.53	36.66	2	6/1/1999	7397.62	81.05	3	9/6/1999	8130.28	65.17	3
2/26/1999	6318.52	42.99	3	6/2/1999	7488.03	90.41	3	9/7/1999	7945.76	−184.52	1
3/1/1999	6312.25	−6.27	2	6/3/1999	7572.91	84.88	3	9/8/1999	7973.3	27.54	2

Table A2. *Cont.*

Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified	Date (MM/DD/YYYY)	TAIEX	Fluctuation	Fuzzified
3/2/1999	6263.54	−48.71	1	6/4/1999	7590.44	17.53	2	9/9/1999	8025.02	51.72	3
3/3/1999	6403.14	139.6	3	6/5/1999	7639.3	48.86	3	9/10/1999	8161.46	136.44	3
3/4/1999	6393.74	−9.4	2	6/7/1999	7802.69	163.39	3	9/13/1999	8178.69	17.23	2
3/5/1999	6383.09	−10.65	2	6/8/1999	7892.13	89.44	3	9/14/1999	8092.02	−86.67	1
3/6/1999	6421.73	38.64	2	6/9/1999	7957.71	65.58	3	9/15/1999	7971.04	−120.98	1
3/8/1999	6431.96	10.23	2	6/10/1999	7996.76	39.05	2	9/16/1999	7968.9	−2.14	2
3/9/1999	6493.43	61.47	3	6/11/1999	7979.4	−17.36	2	9/17/1999	7916.92	−51.98	1
3/10/1999	6486.61	−6.82	2	6/14/1999	7973.58	−5.82	2	9/18/1999	8016.93	100.01	3
3/11/1999	6436.8	−49.81	1	6/15/1999	7960	−13.58	2	9/20/1999	7972.14	−44.79	1
3/12/1999	6462.73	25.93	2	6/16/1999	8059.02	99.02	3	9/27/1999	7759.93	−212.21	1
3/15/1999	6598.32	135.59	3	6/17/1999	8274.36	215.34	3	9/28/1999	7577.85	−182.08	1
3/16/1999	6672.23	73.91	3	6/21/1999	8413.48	139.12	3	9/29/1999	7615.45	37.6	2
3/17/1999	6757.07	84.84	3	6/22/1999	8608.91	195.43	3	9/30/1999	7598.79	−16.66	2
3/18/1999	6895.01	137.94	3	6/23/1999	8492.32	−116.59	1	10/1/1999	7694.99	96.2	3
3/19/1999	6997.29	102.28	3	6/24/1999	8589.31	96.99	3	10/2/1999	7659.55	−35.44	2
3/20/1999	6993.38	−3.91	2	6/25/1999	8265.96	−323.35	1	10/4/1999	7685.48	25.93	2
3/22/1999	7043.23	49.85	3	6/28/1999	8281.45	15.49	2	10/5/1999	7557.01	−128.47	1
3/23/1999	6945.48	−97.75	1	6/29/1999	8514.27	232.82	3	10/6/1999	7501.63	−55.38	1
3/24/1999	6889.42	−56.06	1	6/30/1999	8467.37	−46.9	1	10/7/1999	7612	110.37	3
3/25/1999	6941.38	51.96	3	7/2/1999	8572.09	104.72	3	10/8/1999	7552.98	−59.02	1
3/26/1999	7033.25	91.87	3	7/3/1999	8563.55	−8.54	2	10/11/1999	7607.11	54.13	3
3/29/1999	6901.68	−131.57	1	7/5/1999	8593.35	29.8	2	10/12/1999	7835.37	228.26	3
3/30/1999	6898.66	−3.02	2	7/6/1999	8454.49	−138.86	1	10/13/1999	7836.94	1.57	2
3/31/1999	6881.72	−16.94	2	7/7/1999	8470.07	15.58	2	10/14/1999	7879.91	42.97	3
4/1/1999	7018.68	136.96	3	7/8/1999	8592.43	122.36	3	10/15/1999	7819.09	−60.82	1
4/2/1999	7232.51	213.83	3	7/9/1999	8550.27	−42.16	2	10/16/1999	7829.39	10.3	2
4/3/1999	7182.2	−50.31	1	7/12/1999	8463.9	−86.37	1	10/18/1999	7745.26	−84.13	1
4/6/1999	7163.99	−18.21	2	7/13/1999	8204.5	−259.4	1	10/19/1999	7692.96	−52.3	1
4/7/1999	7135.89	−28.1	2	7/14/1999	7888.66	−315.84	1	10/20/1999	7666.64	−26.32	2
4/8/1999	7273.41	137.52	3	7/15/1999	7918.04	29.38	2	10/21/1999	7654.9	−11.74	2
4/9/1999	7265.7	−7.71	2	7/16/1999	7411.58	−506.46	1	10/22/1999	7559.63	−95.27	1
4/12/1999	7242.4	−23.3	2	7/17/1999	7366.23	−45.35	1	10/25/1999	7680.87	121.24	3
4/13/1999	7337.85	95.45	3	7/19/1999	7386.89	20.66	2	10/26/1999	7700.29	19.42	2
4/14/1999	7398.65	60.8	3	7/20/1999	7806.85	419.96	3	10/27/1999	7701.22	0.93	2
4/15/1999	7498.17	99.52	3	7/21/1999	7786.65	−20.2	2	10/28/1999	7681.85	−19.37	2
4/16/1999	7466.82	−31.35	2	7/22/1999	7678.67	−107.98	1	10/29/1999	7706.67	24.82	2
4/17/1999	7581.5	114.68	3	7/23/1999	7724.52	45.85	3	10/30/1999	7854.85	148.18	3

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