Evaluating Investment Risks of Metallic Mines Using an Extended TOPSIS Method with Linguistic Neutrosophic Numbers

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Abstract: The investment in and development of mineral resources play an important role in the national economy. A good mining project investment can improve economic efficiency and increase social wealth. Faced with the complexity and uncertainty of a mine's circumstances, there is great significance in evaluating investment risk scientifically. In order to solve practical engineering problems, this paper presents an extended TOPSIS method combined with linguistic neutrosophic numbers (LNNs). Firstly, considering that there are several qualitative risk factors of mining investment projects, the paper describes evaluation information by means of LNNs. The advantage of LNNs is that major original information is reserved with linguistic truth, indeterminacy, and false membership degrees. After that, a number of distance measures are defined. Furthermore, a common status is that the decision makers can't determine the importance degrees of every risk factor directly for a few reasons. With respect to this situation, the paper offers a weight model based on maximizing deviation to obtain the criteria weight vector objectively. Subsequently, a decision-making approach through improving classical TOPSIS with LNNs comes into being. Next, a case study of the proposed method applied in metallic mining projects investment is given. Some comparison analysis is also submitted. At last, the discussions and conclusions are finished.

Keywords: metallic mine project; investment risks evaluation; linguistic neutrosophic numbers; maximum deviation; extended TOPSIS

1. Introduction

The assessment of investment risk has always attracted the attention of many researchers in different fields [1]. For example, Wu et al. [2] proposed an improved Analytical Hierarchy Process (AHP) approach to select an optimal financial investment strategy. An extended TOPSIS method was provided by Hatami-Marbini and Kangi [3], and applied in the Tehran stock exchange. Yazdani-Chamzini et al. [4] constructed a model on the basis of AHP, decision-making trial and evaluation, and TOPSIS to evaluate investment risk in the private sector of Iran. A VIKOR-DANP method was presented by Shen et al. [5] and a case study of Taiwan's semiconductor industry was also given to demonstrate the effectiveness of the approach. Dincer and Hacioglu [6] discussed the relationships of financial stress and conflict risk in emerging capital markets with a fuzzy AHP-TOPSIS and VIKOR method. In high-tech fields, such as nanotechnology, Hashemkhani, Zolfani, and Bahrami [7] provided a SWARA-COPRAS decision-making method. Unlike other general industries, investment in the mining industry usually has a long cycle and large uncertainty [8]. There are a lot of risk factors in the process of mining investment. Consequently, identifying and assessing the investment venture of a mine accurately and properly is vital for any project.

The widely-used risk evaluation methods of mining investment can be divided into two main categories [9]. Traditional methods include fault tree analysis, Monte Carlo Simulation, breakeven analysis, the decision tree method, and so on. Another kind contains Analytic Hierarchy process (AHP), fuzzy comprehensive evaluation, and so on. Many researchers have paid particular attention to the latter method, which is based on fuzzy mathematics. Chen et al. [10] sorted and summarized the risk elements of metallic mines, and then presented a method based on a fuzzy set and a neural network. Wang et al. [11] constructed the fuzzy comprehensive appraisal model through creating a risk estimation indicator system. San et al. [12] focused on Tongxin mine, and established an investment risk assessment model with a fuzzy analytic hierarchy process. These methods take the ambiguity of the assessment process into consideration.

However, the fuzzy numbers, such as interval numbers [13], triangular fuzzy numbers [14,15], and trapezoidal fuzzy numbers [16], used in most approaches have some limitations. On the one hand, they only described limited consistent information, while the hesitant and inconsistent values are not indicated. Furthermore, qualitative information is also not expressed. Smarandache [17] firstly put forward the concept of neutrosophic sets (NSs) to deal with consistent, hesitant, and inconsistent information simultaneously. After that, many extensions based on NSs have been presented [18–20]. Related decision-making methods include TOPSIS [21], VIKOR [22], TODIM [23], COPRAS [24,25], WASPAS [26], MULTIMOORA [27], ELECTRE [28,29], QUALIFLEX [30], and other approaches [31,32]. Among them, TOPSIS is widely used. The basic idea of this method is that the distance of the optimal alternative with the positive ideal solution is nearest, and the negative-position ideal solution is farthest [21]. It is easy to understand and operate for decision makers.

In order to qualitatively evaluate risk, like social environment risk and management risk in a mining project, linguistic variables may be a good description [33,34]. Much literature has focused on risk assessment with linguistic information. A venture analysis method on the basis of Dempster–Shafer theory under linguistic environment was presented in the literature [35]. Liu et al. [36] established a risk linguistic decision matrix and discussed the situation when weight informationis unknown. An unbalanced linguistic weighted geometric average operator was proposed to deal with fuzzy risk evaluation problems in [37]. Peiris et al. [38] built three linguistic models to assess alien plants' invasion risks.

For the sake of keeping as much linguistic evaluation information as possible, multiple extensions about language were suggested. For example, the notion of 2-dimensional uncertain linguistic variables occurred some researchers [39–41]. The idea of single-valued neutrosophic linguistic numbers occurred to Ye [42]. Other extensive forms are intuitionistic linguistic sets [43], hesitant fuzzy linguistic term sets [44,45], probabilistic linguistic term sets [46,47], and so on [48,49]. It is worth noting that Chen et al. [50] proposed a group decision-making method in the light of linguistic intuitionistic fuzzy numbers (LIFNs). They connected linguistic values with intuitionistic fuzzy numbers [51]. Then, the linguistic intuitionistic Frank Heronian average operator [52] and some improved linguistic intuitionistic fuzzy operators [53] were proposed.

However, there are only linguistic membership degrees and linguistic non-membership degrees reflected in LIFNs. To overcome this shortcoming, Fang and Ye [54] came up with the concept of linguistic neutrosophic numbers (LNNs). They are based on linguistic terms and simplified neutrosophic numbers [55]. The truth-membership, indeterminacy-membership, and false-membership in a linguistic neutrosophic number (LNN) are found using linguistic information. The difference of LNNs with neutrosophic linguistic numbers (NLNs) [56] is that there is only a linguistic value in NLNs, and the truth-membership, indeterminacy-membership, and false-membership are crisp numbers. For instance, (s_1, s_2, s_3) is a LNN, while $(s_1, < 0.1, 0.2, 0.3 >)$ is a neutrosophic linguistic number (NLN). Of course, they are independent of each other as well. In addition, Fang and Ye [54] defined the operations and comparison rules of LNNs, and then decision-making methods based on of several weighted mean operators were raised.

From this we can see, considering the complicacy of mine environment and the ambiguity of the human mind, assessing the ventures of mining projects on the basis of LNNs may be feasible and advisable. As a result, this paper considers metallic mine investment risk under a linguistic neutrosophic situation with incomplete weight information. A new and reasonable way to evaluate risk degrees by means of LNNs is proposed. In summary, the fundamental innovations of this article are conveyed as follows:

(1) Present a number of distance measures between two LNNs, such as the Hamming distance, the Euclidean distance, and the Hausdorff distance. Equally important, prove relevant properties of these formulas;
(2) Use the thought of maximum deviation for our reference, build a model with respect to linguistic neutrosophic environment to obtain the values of mine risk evaluation criteria weight;
(3) Come up with the extended TOPSIS model with LNNs. Importantly, utilize this method to cope with investment decision-making matter of metallic mine projects;
(4) Compare with other methods, in order to demonstrate the significance and superiority.

We methodize the rest of this article as follows. In Section 2, basic background and knowledge related to risk factors, linguistic information, and LNNs are presented. The extended TOPSIS method with LNNs is depicted after defining the distance measures of LNNs and constructing the weight model in Section 3. Section 4 studies a case of metallic mining investment, and the proposed approach is applied in it. In Section 5, we make comparison with several current literatures. And then, conclusions are made in the last section.

2. Background

In this section, some preliminaries about mining investment risk factors, linguistic term sets, linguistic scale functions, and LNNs are presented.

2.1. Risk Factors of Mining Project Investment

The economic factors of mines and the risk influence factors of metallic mines are introduced in this subsection.

According to the situation of mining investment in China and the research results of the World Bank's investment preference, Pan [57] divided the economic venture of mining investment into five types. They are financial risk, production risk, market risk, personnel risk, and environmental risk, respectively. More details can be seen in Table 1.

Table 1. The economic risk factors of mines.

Risk Factors	Explanations
Financial risk	Caused by the unexpected changes in the mine's balance of payments. It largely consists of financial balance, exchange rate, interest rate, and other factors.
Production risk	Caused by accident, which makes it impossible to produce the production plan according to the predetermined cost. Mainly including production cost, technical conditions, selection scheme, and so on.
Market risk	Caused by the unexpected changes in the market, which makes the mine unable to sell its products according to the original plan. It chiefly contains demand forecasting, substitution products, peer competition, and other factors.
Personnel risk	Caused by accident or change of the important personnel in the mine, which causes a significant impact on the production and operation of the mine. The main factors include accidental casualties, confidential leaks, and personnel changes.
Environmental risk	Caused by the changes of the external environment of the mining industry, which primarily comprises the national policies, geological conditions, and pollution control.

In 2011, Chen et al. [10] summarized and classified the influence factors of the metallic mining investment process based on strategic angle of investment implementation. The presented risk system (see Table 2.) includes two levels of indicators, which are five primary indicators and sixty secondary indicators. The secondary index corresponds to the attributes of the primary one.

Table 2. Investment risk evaluation system of metallic mining.

Assessment Indicators	
Primary indicators	Secondary indicators
Production risk	Mining type, production equipment level, and mining technology
Geological risk	Geological grade, mine reserves, hydrogeology, and surrounding rock conditions
Social environment	Marco economy, national industrial policy, and international environment
Market risk	Marketing ability, product market price, and potential competition
Management risk	Rationality of enterprise organization, scientific decision, and management personnel

2.2. Linguistic Term Sets and Linguistic Scale Function

Xu [58] first put forward the concept of linguistic term sets. For a certain linguistic term set, there are a group of linguistic values s_i ($i = 0, 1, \ldots, 2g$). Consequently, the linguistic term set can be denoted as $\overline{S} = \{s_i | i = 0, 1, \ldots, 2g\}$.

While the linguistic values in the above-mentioned linguistic term set are discrete, they may not work on aggregated linguistic information. Accordingly, Xu [58] redefined the linguistic term set with $S = \{s_i | i \in [0, 2u]\}$ ($u > g$), where the elements are continuous. Moreover, we can compare arbitrary linguistic terms in accordance with their subscripts. Namely, when $i > j$, $s_i > s_j$ is established. The operational rules of any two linguistic values $s_i, s_j \in S$ are indicated: (1) the addition operator $s_i \oplus s_j = s_{i+j}$; (2) the scalar multiplication $\tau s_i = s_{\tau i}$, $0 \le \tau \le 1$; (3) the negation operator $ne(s_i) = s_{-i}$.

Definition 1. *[59] The linguistic scale function is regarded as a mapping from linguistic values s_i ($i = 0, 1, \cdots, 2g$) to a corresponding crisp number $cn_i \in [0, 1]$. Furthermore, it should meet the requirement of monotonically increasing, that is to say, $0 \le cn_0 < cn_1 < \cdots < cn_{2g} \le 1$.*

As the continuous linguistic term sets are defined, we use $f(s_i) = cn_i = \frac{i}{2u}$ ($i \in [0, 2u]$) as the linguistic scale function in this essay. The inverse function can be described as $f^{-1}(cn_i) = 2u \cdot cn_i$ ($i \in [0, 2u]$).

2.3. Linguistic Neutrosophic Numbers

Definition 2. *[54] Given the linguistic term set $S = \{s_i | i \in [0, 2u]\}$, if $s_T, s_I, s_F \in S$, then $\eta = (s_T, s_I, s_F)$ can be regarded as a LNN, where s_T, s_I, and s_F are independent, and describe the linguistic truth-membership degree, the linguistic indeterminacy-membership degree, and the linguistic falsity-membership degree in turn.*

Definition 3. *[54] Assume $\eta_1 = (s_{T_1}, s_{I_1}, s_{F_1})$ and $\eta_2 = (s_{T_2}, s_{I_2}, s_{F_2})$ are two LNNs, then the operations of them are represented as follows:*

(1) $\eta_1 \oplus \eta_2 = (s_{T_1}, s_{I_1}, s_{F_1}) \oplus (s_{T_2}, s_{I_2}, s_{F_2}) = (s_{T_1 + T_2 - \frac{T_1 T_2}{2u}}, s_{\frac{I_1 I_2}{2u}}, s_{\frac{F_1 F_2}{2u}})$;

(2) $\eta_1 \otimes \eta_2 = (s_{T_1}, s_{I_1}, s_{F_1}) \oplus (s_{T_2}, s_{I_2}, s_{F_2}) = (s_{\frac{T_1 T_2}{2u}}, s_{I_1 + I_2 - \frac{I_1 I_2}{2u}}, s_{F_1 + F_2 - \frac{F_1 F_2}{2u}})$;

(3) $q\eta_1 = q(s_{T_1}, s_{I_1}, s_{F_1}) = (s_{2u - 2u(1 - \frac{T_1}{2u})^q}, s_{2u(\frac{I_1}{2u})^q}, s_{2u(\frac{F_1}{2u})^q})$, $q > 0$;

(4) $\eta_1^q = (s_{T_1}, s_{I_1}, s_{F_1})^q = (s_{2u(\frac{T_1}{2u})^q}, s_{2u - 2u(1 - \frac{I_1}{2u})^q}, s_{2u - 2u(1 - \frac{F_1}{2u})^q})$, $q > 0$.

Definition 4. *[54] Suppose $\eta = (s_T, s_I, s_F)$ is an optional LLN, the following are the score function and the accuracy function, respectively:*

$$SC(\eta) = (4u + T - I - F)/(6u), \tag{1}$$

$$AC(\eta) = (T - F)/(2u). \tag{2}$$

Definition 5. *[54] If $\eta_1 = (s_{T_1}, s_{I_1}, s_{F_1})$ and $\eta_2 = (s_{T_2}, s_{I_2}, s_{F_2})$ are two LNNs, then the comparison rule is:*

(1) $\eta_1 > \eta_2$ if $SC(\eta_1) > SC(\eta_2)$;
(2) $\eta_1 > \eta_2$ if $SC(\eta_1) = SC(\eta_2)$ and $AC(\eta_1) > AC(\eta_2)$;
(3) $\eta_1 = \eta_2$ if $SC(\eta_1) = SC(\eta_2)$ and $AC(\eta_1) = AC(\eta_2)$.

Definition 6. *[54] Assume there are a group of LNNs $\eta_i = (s_{T_i}, s_{I_i}, s_{F_i})$ $(i = 1, 2, \ldots, n)$, the linguistic neutrosophic weight arithmetic mean (LNWAM) operator is:*

$$LNWAM(\eta_1, \eta_2, \ldots, \eta_n) = \sum_{i=1}^{n} \gamma_i \eta_i = (s_{2u - 2u \prod_{i=1}^{n} (1 - \frac{T_i}{2u})^{\gamma_i}}, s_{2u \prod_{i=1}^{n} (\frac{I_i}{2u})^{\gamma_i}}, s_{2u \prod_{i=1}^{n} (\frac{F_i}{2u})^{\gamma_i}}), \tag{3}$$

where γ_i is the corresponding weight value of η_i, $0 \leq \gamma_i \leq 1$ and $\sum_{i=1}^{n} \gamma_i = 1$.

Definition 7. *[54] Assume $\eta_i = (s_{T_i}, s_{I_i}, s_{F_i})$ $(i = 1, 2, \ldots, n)$ are a set of LNNs, the linguistic neutrosophic weight geometric mean (LNWGM) operator is:*

$$LNWGM(\eta_1, \eta_2, \ldots, \eta_n) = \sum_{i=1}^{n} \eta_i^{\gamma_i} = (s_{2u \prod_{i=1}^{n} (\frac{T_i}{2u})^{\gamma_i}}, s_{2u - 2u \prod_{i=1}^{n} (1 - \frac{I_i}{2u})^{\gamma_i}}, s_{2u - 2u \prod_{i=1}^{n} (1 - \frac{F_i}{2u})^{\gamma_i}}), \tag{4}$$

where γ_i is the related weight value of η_i, $0 \leq \gamma_i \leq 1$ and $\sum_{i=1}^{n} \gamma_i = 1$.

3. Extended TOPSIS Method with Incomplete Weight Information

In this section, we present the idea of an extended TOPSIS method with LNNs, and discuss the situation in which weight information is completely unknown.

3.1. Descriptions

With respect to the multi-criteria decision-making problems under linguistic neutrosophic situations, k decision makers evaluate a set of options $X = \{x_1, x_2, \ldots, x_n\}$ under some attributes $A = \{a_1, a_2, \ldots, a_m\}$. ω_j is the corresponding weight of a_j, which is completely unknown, but satisfies $\omega_j \in [0, 1]$ and $\sum_{i=1}^{m} \omega_j = 1$. There are k decision makers $\{b_1, b_2, \ldots, b_k\}$ with the related weight $\{\gamma_1, \gamma_2, \ldots, \gamma_k\}$, $0 \leq \gamma_l \leq 1$ $(l = 1, 2, \ldots, k)$ and $\sum_{l=1}^{k} \gamma_l = 1$. $S = \{s_i | i \in [0, 2u]\}$ is the predefined linguistic term set. In order to rank the objects or pick out the optimal one(s), each decision-maker $(b_l(l = 1, 2, \ldots, k))$ makes evaluations and then constructs the corresponding decision-making matrix, that is:

$$N^{(l)} = (\eta_{ij}^{(l)})_{n \times m} = \begin{bmatrix} \eta_{11}^{(l)} & \cdots & \eta_{1m}^{(l)} \\ \eta_{21}^{(l)} & \cdots & \eta_{2m}^{(l)} \\ \vdots & \cdots & \vdots \\ \eta_{n1}^{(l)} & \cdots & \eta_{nm}^{(l)} \end{bmatrix} = \begin{bmatrix} (s_{T_{11}}^{(l)}, s_{I_{11}}^{(l)}, s_{F_{11}}^{(l)}) & \cdots & (s_{T_{1m}}^{(l)}, s_{I_{1m}}^{(l)}, s_{F_{1m}}^{(l)}) \\ (s_{T_{21}}^{(l)}, s_{I_{21}}^{(l)}, s_{F_{21}}^{(l)}) & \cdots & (s_{T_{2m}}^{(l)}, s_{I_{2m}}^{(l)}, s_{F_{2m}}^{(l)}) \\ \vdots & \cdots & \vdots \\ (s_{T_{n1}}^{(l)}, s_{I_{n1}}^{(l)}, s_{F_{n1}}^{(l)}) & \cdots & (s_{T_{nm}}^{(l)}, s_{I_{nm}}^{(l)}, s_{F_{nm}}^{(l)}) \end{bmatrix}, (l = 1, 2, \ldots, k).$$

The basic elements of the matrix $N^{(l)}$ are by means of LNNs, where $\eta_{ij}^{(l)} = (s_{T_{ij}}^{(l)}, s_{T_{ij}}^{(l)}, s_{T_{ij}}^{(l)})(s_{T_{ij}}^{(l)}, s_{T_{ij}}^{(l)}, s_{T_{ij}}^{(l)} \in S)$ means the assessment information of b_l about x_i related to criteria a_j.

3.2. Distance Measures of LNNs

In this subsection, we intend to introduce several distance formulas of LNNs, so that the discussion behind these can be smoothly advanced.

Definition 8. *Let $\eta_1 = (s_{T_1}, s_{I_1}, s_{F_1})$ and $\eta_2 = (s_{T_2}, s_{I_2}, s_{F_2})$ be two haphazard LNNs. $S = \{s_i | i \in [0, 2u]\}$ is the linguistic term set, and $f(s_i) = \frac{i}{2u}$ is the linguistic scale function. Then, the distance between η_1 and η_2 are denoted as follows:*

$$d(\eta_1, \eta_2) = (\tfrac{1}{3}(|f(s_{T_1}) - f(s_{T_2})|^\lambda + |f(s_{2u-I_1}) - f(s_{2u-I_2})|^\lambda + |f(s_{2u-F_1}) - f(s_{2u-F_2})|^\lambda))^{\frac{1}{\lambda}}, \lambda > 0. \quad (5)$$

Remarkably:

(1) when $\lambda = 1$, the Hamming distance

$$d_{Hm}(\eta_1, \eta_2) = \frac{1}{3}(|f(s_{T_1}) - f(s_{T_2})| + |f(s_{2u-I_1}) - f(s_{2u-I_2})| + |f(s_{2u-F_1}) - f(s_{2u-F_2})|); \quad (6)$$

(2) when $\lambda = 2$, the Euclidean distance

$$d_{Ed}(\eta_1, \eta_2) = \sqrt{\frac{1}{3}(|f(s_{T_1}) - f(s_{T_2})|^2 + |f(s_{2t-I_1}) - f(s_{2t-I_2})|^2 + |f(s_{2t-F_1}) - f(s_{2t-F_2})|^2)}; \quad (7)$$

(3) the Hausdorff distance

$$d_{Hd}(\eta_1, \eta_2) = \max\{|f(s_{T_1}) - f(s_{T_2})|, |f(s_{2t-I_1}) - f(s_{2t-I_2})|, |f(s_{2t-F_1}) - f(s_{2t-F_2})|\}. \quad (8)$$

Property 1. *Given three arbitrary LNNs $\eta_1 = (s_{T_1}, s_{I_1}, s_{F_1})$, $\eta_2 = (s_{T_2}, s_{I_2}, s_{F_2})$ and $\eta_3 = (s_{T_3}, s_{I_3}, s_{F_3})$. The linguistic term set is $S = \{s_i | i \in [0, 2u]\}$, and the universal set of LNNs is Ω. For any $\eta_1, \eta_2, \eta_3 \in \Omega$, the following properties are met:*

(1) $0 \le d(\eta_1, \eta_2) \le 1$;
(2) $d(\eta_1, \eta_2) = d(\eta_2, \eta_1)$;
(3) $d(\eta_1, \eta_2) = 0$ if $\eta_1 = \eta_2$;
(4) $d(\eta_1, \eta_3) \le d(\eta_1, \eta_2) + d(\eta_2, \eta_3)$.

Proof.

(1) Because $f(s_i) = \frac{i}{2u} \in [0, 1] \Rightarrow |f(s_{T_1}) - f(s_{T_2})| \in [0, 1]$, $|f(s_{2u-I_1}) - f(s_{2u-I_2})|$ and $|f(s_{2u-F_1}) - f(s_{2u-F_2})|$, as $\lambda > 0$, then $0 \le d(\eta_1, \eta_2) \le 1$.
(2) This proof is obvious.

(3) Since $\eta_1 = \eta_2$, then $SC(\eta_1) = SC(\eta_2)$ and $AC(\eta_1) = AC(\eta_2)$

$\Rightarrow (4u + T_1 - I_1 - F_1)/(6u) = (4u + T_2 - I_2 - F_2)/(6u)$ and $(T_1 - F_1)/(2u) = (T_2 - F_2)/(2u)$

$\Rightarrow T_1 - I_1 - F_1 = T_2 - I_2 - F_2$ and $T_1 - F_1 = T_2 - F_2 \Rightarrow I_1 = I_2$ and $T_1 - F_1 = T_2 - F_2$.

Thus, $d(\eta_1, \eta_2)$

$= (\frac{1}{3}(|f(s_{T_1}) - f(s_{T_2})|^\lambda + |f(s_{2u-I_1}) - f(s_{2u-I_2})|^\lambda + |f(s_{2u-F_1}) - f(s_{2u-F_2})|^\lambda))^{\frac{1}{\lambda}}$

$= (\frac{1}{3}(|\frac{T_1-T_2}{2u}|^\lambda + |\frac{I_2-I_1}{2u}|^\lambda + |\frac{F_2-F_1}{2u}|^\lambda))^{\frac{1}{\lambda}}$

$= (\frac{1}{3}(|\frac{T_1-F_1+F_1-T_2}{2u}|^\lambda + |\frac{I_2-I_1}{2u}|^\lambda + |\frac{F_2-T_2+T_2-F_1}{2u}|^\lambda))^{\frac{1}{\lambda}}$

$= (\frac{1}{3}(|\frac{T_2-F_2+F_1-T_2}{2u}|^\lambda + |\frac{I_2-I_1}{2u}|^\lambda + |\frac{F_1-T_1+T_2-F_1}{2u}|^\lambda))^{\frac{1}{\lambda}}$

$= (\frac{1}{3}(|\frac{F_1-F_2}{2u}|^\lambda + |\frac{I_1-I_2}{2u}|^\lambda + |\frac{T_2-T_1}{2u}|^\lambda))^{\frac{1}{\lambda}}$

$= (\frac{1}{3}(|f(s_{T_2}) - f(s_{T_1})|^\lambda + |f(s_{2u-I_2}) - f(s_{2u-I_1})|^\lambda + |f(s_{2u-F_2}) - f(s_{2u-F_1})|^\lambda))^{\frac{1}{\lambda}}$

$= d(\eta_2, \eta_1)$

(4) As $|f(s_{T_1}) - f(s_{T_2})| = |f(s_{T_1}) - f(s_{T_2}) + f(s_{T_2}) - f(s_{T_3})|$

$\leq |f(s_{T_1}) - f(s_{T_2})| + |f(s_{T_2}) - f(s_{T_3})|$,

$|f(s_{2u-I_1}) - f(s_{2u-I_2})| = |f(s_{2u-I_1}) - f(s_{2u-I_2}) + f(s_{2u-I_2}) - f(s_{2u-I_3})|$

$\leq |f(s_{2u-I_1}) - f(s_{2u-I_2})| + |f(s_{2u-I_2}) - f(s_{2u-I_3})|$,

and $|f(s_{2u-F_1}) - f(s_{2u-F_2})| = |f(s_{2u-F_1}) - f(s_{2u-F_2}) + f(s_{2u-F_2}) - f(s_{2u-F_3})|$

$\leq |f(s_{2u-F_1}) - f(s_{2u-F_2})| + |f(s_{2u-F_2}) - f(s_{2u-F_3})|$,

hence, $d(\eta_1, \eta_3) \leq d(\eta_1, \eta_2) + d(\eta_2, \eta_3)$.

Example 1. *If* $u = 4$, *two LNNs* $\eta_1 = (s_1, s_2, s_4)$ *and* $\eta_2 = (s_5, s_3, s_6)$, *the Hamming distance is* $d_{Hm}(\eta_1, \eta_2) \approx 0.292$, *the Euclidean distance is* $d_{Ed}(\eta_1, \eta_2) \approx 0.331$, *and the Hausdorff distance is* $d_{Hd}(\eta_1, \eta_2) = 0.500$.

3.3. Weight Model Based on Maximum Deviation

Because the weight information is completely unknown, we use the maximum deviation approach to determine the weight vector of criteria in this subsection.

The basic idea of the maximum deviation method is that [60]:

(1) If there is a tiny difference of evaluation values η_{ij} among all objects under criteria $a_j(j = 1, 2, \ldots, m)$, it indicates that the criteria a_j has little effect on the sorting results. Accordingly, it is appropriate to allocate a small value of the related weight ω_j.

(2) Conversely, if there is a significant variance of assessment information η_{ij} among all alternatives under criteria $a_j(j = 1, 2, \ldots, m)$, then the criteria a_j may be very important to the ranking orders. In this case, giving a large weight value ω_j is reasonable.

(3) Notably, if η_{ij} are the same values among all options under criteria a_j $((j = 1, 2, \ldots, m))$, it means that the criteria a_j doesn't affect the ranking results. Therefore, we can make the corresponding weight $\omega_j = 0$.

For the sake of obtaining the difference values, we define the deviation degree of a certain object $x_i(i = 1, 2, \ldots, n)$ to all objects for a certain criteria $a_j(j = 1, 2, \ldots, m)$ as follows:

$$D_{ij}(\omega_j) = \sum_{e=1}^{n} d(\eta_{ij}, \eta_{ej})\omega_j, \tag{9}$$

where $d(\eta_{ij}, \eta_{ej})$ is the distance measure between η_{ij} and η_{ej}.

Subsequently, the deviation degrees of all options under the criteria $a_j(j = 1, 2, \ldots, m)$ can be denoted as:

$$D_j(\omega_j) = \sum_{i=1}^{n} D_{ij}(\omega_j) = \sum_{i=1}^{n} \sum_{e=1}^{n} d(\eta_{ij}, \eta_{ej})\omega_j. \tag{10}$$

Thus, the total deviation of all alternatives with all criteria is proposed in the following:

$$D(\omega) = \sum_{j=1}^{m} D_j(\omega_j) = \sum_{j=1}^{m} \sum_{i=1}^{n} D_{ij}(\omega_j) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{e=1}^{n} d(\eta_{ij}, \eta_{ej})\omega_j. \tag{11}$$

As a result, we can build the weight model based on maximum deviation as follows:

$$\max D(\omega) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{e=1}^{n} d(\eta_{ij}, \eta_{ej})\omega_j$$
$$= \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{e=1}^{n} \left(\frac{1}{3}\left(|f(s_{T_{ij}}) - f(s_{T_{ej}})|^\lambda + |f(s_{2u-I_{ij}}) - f(s_{2u-I_{ej}})|^\lambda + |f(s_{2u-F_{ij}}) - f(s_{2u-F_{ej}})|^\lambda\right)\right)^{\frac{1}{\lambda}} \omega_j$$

$$s.t \quad \begin{cases} \sum_{j=1}^{m} \omega_j^2 = 1 \\ 0 \leq \omega_j \leq 1, \ j = 1, 2, \ldots, m \end{cases} \tag{12}$$

In order to get the solution, we can construct the Lagrange function as that:

$$L(\omega, p) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{e=1}^{n} \left(\frac{1}{3}\left(|f(s_{T_{ij}}) - f(s_{T_{ej}})|^\lambda + |f(s_{2u-I_{ij}}) - f(s_{2u-I_{ej}})|^\lambda + \right.\right.$$
$$\left.\left. |f(s_{2u-F_{ij}}) - f(s_{2u-F_{ej}})|^\lambda\right)\right)^{\frac{1}{\lambda}} \omega_j + \frac{p}{2}\left(\sum_{j=1}^{m} \omega_j^2 - 1\right) \tag{13}$$

Taking the partial deviation of this function, we have:

$$\begin{cases} \frac{\partial L(\omega,p)}{\partial \omega} = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{e=1}^{n} \left(\frac{1}{3}\left(|f(s_{T_{ij}}) - f(s_{T_{ej}})|^\lambda + |f(s_{2u-I_{ij}}) - f(s_{2u-I_{ej}})|^\lambda + |f(s_{2u-F_{ij}}) - f(s_{2u-F_{ej}})|^\lambda\right)\right)^{\frac{1}{\lambda}} + p\omega_j = 0 \\ \frac{\partial L(\omega,p)}{\partial p} = \sum_{j=1}^{m} \omega_j^2 - 1 = 0 \end{cases}$$
$$\Rightarrow p = \sqrt{\sum_{j=1}^{m} \left(\sum_{i=1}^{n} \sum_{e=1}^{n} \left(\frac{1}{3}\left(|f(s_{T_{ij}}) - f(s_{T_{ej}})|^\lambda + |f(s_{2u-I_{ij}}) - f(s_{2u-I_{ej}})|^\lambda + |(s_{2u-F_{ij}}) - f(s_{2u-F_{ej}})|^\lambda\right)\right)^{\frac{1}{\lambda}}\right)^2} \text{ and} \tag{14}$$
$$\omega_j = \frac{\sum_{i=1}^{n} \sum_{e=1}^{n} \left(\frac{1}{3}\left(|f(s_{T_{ij}})-f(s_{T_{ej}})|^\lambda + |f(s_{2u-I_{ij}})-f(s_{2u-I_{ej}})|^\lambda + |(s_{2u-F_{ij}})-f(s_{2u-F_{ej}})|^\lambda\right)\right)^{\frac{1}{\lambda}}}{\sqrt{\sum_{j=1}^{m} \left(\sum_{i=1}^{n} \sum_{e=1}^{n} \left(\frac{1}{3}\left(|f(s_{T_{ij}})-f(s_{T_{ej}})|^\lambda + |f(s_{2u-I_{ij}})-f(s_{2u-I_{ej}})|^\lambda + |(s_{2u-F_{ij}})-f(s_{2u-F_{ej}})|^\lambda\right)\right)^{\frac{1}{\lambda}}\right)^2}}.$$

In the end, we can use the following formula to normalize the criteria weights:

$$\omega_j^* = \frac{\omega_j}{\sum\limits_{j=1}^{m} \omega_j}, j = 1, 2, \ldots, m. \tag{15}$$

3.4. The Extended TOPSIS Method with LNNs

In this subsection, an extended TOPSIS approach under a linguistic neutrosophic environment is proposed.

The detailed steps are described as follows:

Step 1: Obtain the normalized decision-making matrix $N^{\bullet(l)} = \left(\eta_{ij}^{\bullet(l)}\right)_{n\times m} = \left(s_{T_{ij}}^{\bullet(l)}, s_{I_{ij}}^{\bullet(l)}, s_{F_{ij}}^{\bullet(l)}\right)_{n\times m}$. If the criteria belong to cost type, let $s_{T_{ij}}^{\bullet(l)} = s_{2u-T_{ij}}^{(l)}$, $s_{I_{ij}}^{\bullet(l)} = s_{2u-I_{ij}}^{(l)}$ and $s_{F_{ij}}^{\bullet(l)} = s_{2u-F_{ij}}^{(l)}$. If the criteria belong to benefit type, then the matrix remains, that is to say $s_{T_{ij}}^{\bullet(l)} = s_{T_{ij}}^{(l)}$, $s_{I_{ij}}^{\bullet(l)} = s_{I_{ij}}^{(l)}$ and $s_{F_{ij}}^{\bullet(l)} = s_{F_{ij}}^{(l)}$.

Step 2: Get the comprehensive decision-making matrix $N^{\bullet} = \left(\eta_{ij}^{\bullet}\right)_{n\times m} = \left(s_{T_{ij}}^{\bullet}, s_{I_{ij}}^{\bullet}, s_{F_{ij}}^{\bullet}\right)_{n\times m}$ using the LNWAM operator or LNWGM operator on the basis of Formula (3) or Formula (4).

Step 3: Use the weight model to calculate the weight values ω_j $(j = 1, 2, \ldots, m)$ based on Formula (9), and then normalize the weight information in line with Formula (10), denoted as $\omega_j^{\bullet}$ $(j = 1, 2, \ldots, m)$.

Step 4: Establish the weight standardized decision-making matrix $N^* = (\eta_{ij}^*)_{n \times m} = (s_{T_{ij}}^*, s_{I_{ij}}^*, s_{F_{ij}}^*)_{n \times m}$ through multiplying the normalized matrix with weight vector, where $s_{T_{ij}}^* = \omega_j s_{T_{ij}}^{\bullet}$, $s_{I_{ij}}^* = \omega_j s_{I_{ij}}^{\bullet}$ and $s_{F_{ij}}^* = \omega_j s_{F_{ij}}^{\bullet}$.

Step 5: Distinguish the positive ideal solution η^+ and the negative ideal solution η^-, respectively, then:

$$\eta^+ = (\eta_1^+, \eta_1^+, \ldots, \eta_m^+), \eta_j^+ = \max_i(\eta_{ij}^*), \ (j = 1, 2, \ldots, m) \tag{16}$$

And

$$\eta^- = (\eta_1^-, \eta_1^-, \ldots, \eta_m^-), \eta_j^- = \min_i(\eta_{ij}^*), \ (j = 1, 2, \ldots, m) \tag{17}$$

Step 6: Based on Formula (5), calculate the distance measures of the positive ideal solution to all options, and the distance measures of the negative ideal solution to all options in proper sequence. The computation formulas are:

$$d^+ = (d_1^+, d_1^+, \ldots, d_n^+), d_i^+ = \sum_{j=1}^{m} d(\eta_j^+, \eta_{ij}^*), \ (i = 1, 2, \ldots, n) \tag{18}$$

And

$$d^- = (d_1^-, d_1^-, \ldots, d_n^-), d_i^- = \sum_{j=1}^{m} d(\eta_j^-, \eta_{ij}^*), \ (i = 1, 2, \ldots, n). \tag{19}$$

Step 7: For each option x_i $(i = 1, 2, \ldots, n)$, compute the values of correlation coefficient D_i with the following equation:

$$D_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{20}$$

Step 8: Achieve the ranking orders according to the values of D_i $(i = 1, 2, \ldots, n)$. The bigger the value of D_i, the better the alternative x_i is.

4. Case Study

In this section, we study a case of evaluating investment risks of a gold mine using the proposed approach.

Recently, a construction investment company in Hunan province, called JK MINING Co., Ltd., had a plan for investing in a domestic metal mine. After an initial investigation and screening, four famous metal mines, described as $\{x_1, x_2, x_3, x_4\}$, have been under consideration. The enterprise establishes a team of three experts to conduct field explorations and surveys in depth, so that the optimal mine can be selected. The specialists need to evaluate the investment risk in line with their findings, professional knowledge, and experience. Assume the importance of each professional is equal, that is to say $\gamma_1 = \gamma_2 = \gamma_3 = \frac{1}{3}$. After heated discussions, five attributions are recognized as the evaluation criteria. They are geological risk (a_1), production risk (a_2), market risk (a_3), management risk (a_4), and social environment risk (a_5), separately. Then, the experts defined the linguistic term set, $S = \{s_i | i \in [0, 8]\}$, where $s = \{s_0 = exceedingly\ low, \ s_1 = pretty\ low, \ s_2 = low, \ s_3 = slightly\ low, \ s_4 = medium, \ s_5 = slightly\ high, \ s_2 = high, \ s_3 = pretty\ high, \ s_4 = exceedingly\ high\}$. Afterwards, they can give scores (or score ranges) or linguistic information directly of options under each attribute. The corresponding relationships between grade and linguistic term can been seen in Table 3.

Table 3. Reference of investment risk evaluation.

Grade	0~19	20~29	30~39	40~49	50~59	60~69	70~79	80~89	90~100
Evaluation	exceedingly low	pretty low	low	slightly low	medium	slightly high	high	pretty high	exceedingly high
Linguistic term	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8

In order to describe the ambiguity and uncertainty of risks, their evaluation information is represented by LNNs. Subsequently, these assessment matrices are formed as Tables 4–6:

Table 4. Decision-making matrix $N^{(1)}$.

$N^{(1)}$	a_1	a_2	a_3	a_4	a_5
x_1	(s_1, s_2, s_1)	(s_2, s_3, s_2)	(s_4, s_4, s_3)	(s_1, s_5, s_1)	(s_3, s_3, s_2)
x_2	(s_2, s_6, s_2)	(s_3, s_8, s_2)	(s_2, s_4, s_1)	(s_3, s_1, s_2)	(s_1, s_2, s_1)
x_3	(s_2, s_3, s_1)	(s_3, s_2, s_3)	(s_1, s_4, s_1)	(s_3, s_5, s_1)	(s_5, s_2, s_4)
x_4	(s_3, s_1, s_2)	(s_1, s_7, s_1)	(s_4, s_6, s_3)	(s_2, s_5, s_1)	(s_4, s_6, s_4)

Table 5. Decision-making matrix $N^{(2)}$.

$N^{(2)}$	a_1	a_2	a_3	a_4	a_5
x_1	(s_1, s_6, s_1)	(s_4, s_3, s_4)	(s_2, s_6, s_2)	(s_3, s_5, s_2)	(s_5, s_2, s_4)
x_2	(s_1, s_4, s_1)	(s_3, s_2, s_1)	(s_2, s_3, s_4)	(s_4, s_0, s_5)	(s_2, s_6, s_4)
x_3	(s_3, s_5, s_2)	(s_2, s_4, s_3)	(s_1, s_6, s_5)	(s_3, s_5, s_3)	(s_2, s_6, s_1)
x_4	(s_2, s_7, s_2)	(s_4, s_6, s_1)	(s_3, s_7, s_2)	(s_4, s_4, s_2)	(s_3, s_8, s_4)

Table 6. Decision-making matrix $N^{(3)}$.

$N^{(3)}$	a_1	a_2	a_3	a_4	a_5
x_1	(s_2, s_4, s_1)	(s_3, s_5, s_2)	(s_5, s_1, s_4)	(s_2, s_6, s_1)	(s_3, s_3, s_2)
x_2	(s_1, s_2, s_1)	(s_2, s_4, s_2)	(s_1, s_5, s_3)	(s_4, s_2, s_0)	(s_0, s_5, s_6)
x_3	(s_2, s_3, s_3)	(s_1, s_5, s_2)	(s_2, s_4, s_5)	(s_0, s_4, s_6)	(s_3, s_2, s_4)
x_4	(s_2, s_3, s_2)	(s_4, s_2, s_1)	(s_1, s_4, s_3)	(s_3, s_4, s_5)	(s_0, s_4, s_5)

Next, the extended TOPSIS approach presented in Section 3.4 is employed to identify the optimal metal mine. A concrete calculation process is delivered as follows:

Step 1: Obtain the normalized decision matrix. As all the criteria are risk element, regarded as a part of cost, then normalizing evaluation values with function $s_{T_{ij}}^{\bullet(l)} = s_{2u-T_{ij}}^{(l)}$, $s_{I_{ij}}^{\bullet(l)} = s_{2u-I_{ij}}^{(l)}$ and $s_{F_{ij}}^{\bullet(l)} = s_{2u-F_{ij}}^{(l)}$. The followings (Tables 7–9) are the normalized decision-making matrix of each expert.

Table 7. Normalized decision-making matrix $N^{\bullet(1)}$.

$N^{\bullet(1)}$	a_1	a_2	a_3	a_4	a_5
x_1	(s_7, s_6, s_7)	(s_6, s_5, s_6)	(s_4, s_4, s_5)	(s_7, s_3, s_7)	(s_5, s_5, s_6)
x_2	(s_6, s_2, s_6)	(s_5, s_0, s_6)	(s_6, s_4, s_7)	(s_6, s_7, s_6)	(s_7, s_6, s_7)
x_3	(s_6, s_5, s_7)	(s_5, s_6, s_5)	(s_7, s_4, s_7)	(s_5, s_3, s_7)	(s_3, s_6, s_4)
x_4	(s_5, s_7, s_6)	(s_7, s_1, s_7)	(s_4, s_2, s_5)	(s_6, s_3, s_7)	(s_4, s_2, s_4)

Table 8. Normalized decision-making matrix $N^{\bullet(2)}$.

$N^{\bullet(2)}$	a_1	a_2	a_3	a_4	a_5
x_1	(s_7, s_2, s_7)	(s_4, s_5, s_4)	(s_6, s_2, s_6)	(s_5, s_3, s_6)	(s_3, s_6, s_4)
x_2	(s_7, s_4, s_7)	(s_5, s_6, s_7)	(s_6, s_5, s_4)	(s_4, s_8, s_3)	(s_6, s_2, s_4)
x_3	(s_5, s_3, s_6)	(s_6, s_4, s_5)	(s_7, s_2, s_3)	(s_5, s_3, s_5)	(s_6, s_2, s_7)
x_4	(s_6, s_1, s_6)	(s_4, s_3, s_7)	(s_5, s_1, s_6)	(s_4, s_4, s_6)	(s_5, s_0, s_4)

Table 9. Normalized decision-making matrix $N^{\bullet(3)}$.

$N^{\bullet(3)}$	a_1	a_2	a_3	a_4	a_5
x_1	(s_6, s_4, s_7)	(s_5, s_3, s_6)	(s_3, s_7, s_4)	(s_6, s_2, s_7)	(s_5, s_5, s_6)
x_2	(s_7, s_6, s_7)	(s_6, s_4, s_6)	(s_7, s_3, s_5)	(s_4, s_6, s_8)	(s_8, s_3, s_2)
x_3	(s_6, s_5, s_5)	(s_7, s_3, s_6)	(s_6, s_4, s_3)	(s_8, s_4, s_2)	(s_5, s_6, s_4)
x_4	(s_6, s_5, s_6)	(s_4, s_6, s_7)	(s_7, s_4, s_5)	(s_5, s_4, s_3)	(s_8, s_4, s_3)

Step 2: Using the LNWAM operator in line with Formula (3) to get the comprehensive decision matrix as Table 10:

Table 10. Comprehensive decision-making matrix $N^{\bullet}$.

$N^{\bullet}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_{6.74}, s_{3.63}, s_7)$	$(s_{5.12}, s_{4.22}, s_{5.24})$	$(s_{4.58}, s_{3.83}, s_{4.93})$	$(s_{6.18}, s_{2.62}, s_{6.65})$	$(s_{4.44}, s_{5.31}, s_{5.24})$
x_2	$(s_{6.74}, s_{3.63}, s_{6.65})$	$(s_{5.38}, s_0, s_{6.32})$	$(s_{6.18}, s_{3.91}, s_{5.19})$	$(s_{4.83}, s_{6.95}, s_{5.24})$	$(s_8, s_{3.3}, s_{3.83})$
x_3	$(s_{5.71}, s_{4.22}, s_{5.94})$	$(s_{6.18}, s_{4.16}, s_{5.31})$	$(s_{6.74}, s_{3.17}, s_{3.98})$	$(s_8, s_{3.3}, s_{4.12})$	$(s_{4.89}, s_{4.16}, s_{4.82})$
x_4	$(s_{5.71}, s_{3.27}, s_6)$	$(s_{5.48}, s_{2.62}, s_7)$	$(s_{5.71}, s_2, s_{5.31})$	$(s_{5.11}, s_{3.63}, s_{5.01})$	$(s_8, s_0, s_{3.63})$

Step 3: Calculate the values of the criteria weight ω_j (suppose $\lambda = 1$) on the basis of Formula (9) as follows: $\omega_1 \approx 0.17$, $\omega_2 \approx 0.42$, $\omega_3 \approx 0.31$, $\omega_4 \approx 0.55$ and $\omega_1 \approx 0.63$. Normalize them based on Formula (10): $\omega_1^{\bullet} = \frac{\omega_1}{\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5} \approx 0.08$, $\omega_2^{\bullet} \approx 0.20$, $\omega_3^{\bullet} \approx 0.15$, $\omega_4^{\bullet} \approx 0.27$ and $\omega_5^{\bullet} \approx 0.30$.

Step 4: Establish the weight standardized decision-making matrix as Table 11.

Table 11. Weight standardized decision-making matrix N^w.

N^w	a_1	a_2	a_3	a_4	a_5
x_1	$(s_{1.1}, s_{7.51}, s_{7.91})$	$(s_{1.48}, s_{7.04}, s_{7.35})$	$(s_{0.96}, s_{7.16}, s_{7.44})$	$(s_{2.64}, s_{5.92}, s_{7.61})$	$(s_{1.73}, s_{7.07}, s_{7.05})$
x_2	$(s_{1.1}, s_{7.51}, s_{7.88})$	$(s_{1.6}, s_0, s_{7.63})$	$(s_{1.59}, s_{7.19}, s_{7.5})$	$(s_{1.77}, s_{7.7}, s_{7.14})$	$(s_8, s_{6.13}, s_{6.41})$
x_3	$(s_{0.76}, s_{7.6}, s_{7.81})$	$(s_{2.05}, s_{7.02}, s_{7.37})$	$(s_{1.94}, s_{6.96}, s_{7.2})$	$(s_8, s_{6.3}, s_{6.69})$	$(s_{1.97}, s_{6.57}, s_{6.87})$
x_4	$(s_{0.76}, s_{7.45}, s_{7.82})$	$(s_{1.65}, s_{6.4}, s_{7.79})$	$(s_{1.37}, s_{6.5}, s_{7.53})$	$(s_{1.92}, s_{6.46}, s_{7.05})$	$(s_8, s_0, s_{6.31})$

Step 5: Identify the positive ideal solution and the negative ideal solution, respectively. See Table 12.

Table 12. Positive ideal solution and negative ideal solution.

η_1^+	η_2^+	η_3^+	η_4^+	η_5^+
$(s_{1.1}, s_{7.51}, s_{7.88})$	$(s_{1.6}, s_0, s_{7.63})$	$(s_{1.94}, s_{6.96}, s_{7.2})$	$(s_8, s_{6.3}, s_{6.69})$	$(s_8, s_0, s_{6.31})$

η_1^-	η_2^-	η_3^-	η_4^-	η_5^-
$(s_{0.76}, s_{7.6}, s_{7.81})$	$(s_{1.48}, s_{7.04}, s_{7.35})$	$(s_{0.96}, s_{7.16}, s_{7.44})$	$(s_{1.77}, s_{7.7}, s_{7.14})$	$(s_{1.73}, s_{7.07}, s_{7.05})$

Step 6: In line with Formula (5), the distances are measured as follows (assume $\lambda = 1$): $d_1^+ \approx 9.88$, $d_2^+ \approx 5.06$, $d_3^+ = 7.13$, $d_4^+ = 5.01$, $d_1^- \approx 1.22$, $d_2^- \approx 5.50$, $d_3^- \approx 3.68$ and $d_4^- \approx 6.04$.

Step 7: Compute the values of correlation coefficient: $D_1 \approx 0.11$, $D_2 \approx 0.52$, $D_3 \approx 0.34$ and $D_4 \approx 0.55$.

Step 8: Since $D_4 > D_2 > D_3 > D_1$, then the ranking order is $x_4 > x_2 > x_3 > x_1$, and the best metal mine is x_4.

5. Comparison Analysis

In this section, several related studies are compared through solving the same problem of gold mine venture assessment.

The comparison results can be seen in Table 13, and the particularized discussions and analysis are depicted in the following:

Table 13. Ranking orders using different approaches.

Approaches	Ranking Orders	Optimal Alternatives	Worst Alternatives
Approach with the LNWAM operator [54]	$x_4 > x_2 > x_3 > x_1$	x_4	x_1
Approach with the LNWGM operator [54]	$x_4 > x_3 > x_2 > x_1$	x_4	x_1
Approach with $u_{ij} = \frac{1}{3}s_{T_{ij}}$ [50]	$x_4 > x_3 > x_2 > x_1$	x_4	x_1
Approach with $u_{ij} = \frac{1}{3}s_{T_{ij}} + \frac{1}{6}s_{I_{ij}}$ [50]	$x_1 > x_3 > x_2 > x_4$	x_1	x_4
Approach with $u_{ij} = \frac{1}{3}s_{T_{ij}} + \frac{1}{3}s_{F_{ij}}$ [50]	$x_2 > x_1 > x_3 > x_4$	x_2	x_4
Approach with SVNLN-TOPSIS [42]	$x_4 > x_2 > x_3 > x_1$	x_4	x_1
The presented approach	$x_4 > x_2 > x_3 > x_1$	x_4	x_1

(1) The information in Reference [54] is LNNs. The multi-criteria group decision-making methods based on the LNWM operator or LNGM operator are presented. If we use the LNWAM operator to deal with the same problem in this paper, we have the comprehensive evaluations of each alternative as follows: $c_1 = (s_{5.4}, s_{3.87}, s_{5.67})$, $c_2 = (s_8, s_0, s_{5.04})$ $c_3 = (s_8, s_{3.76}, s_{4.65})$ $c_4 = (s_8, s_0, s_{4.98})$. Since the score function $SC(c_1) \approx 0.494$, $SC(c_2) = 0.790$, $SC(c_3) \approx 0.650$, $SC(c_4) \approx 0.793$, then $SC(c_4) > SC(c_2) > SC(c_3) > SC(c_1) \Rightarrow x_4 > x_2 > x_3 > x_1$. If the LNWGM operator is used, then $c_1^{\#} = (s_{5.19}, s_{4.15}, s_{5.87})$, $c_2^{\#} = (s_{6.12}, s_{4.6}, s_{5.32})$, $c_3^{\#} = (s_{6.22}, s_{3.81}, s_{4.75})$, $c_4^{\#} = (s_{6.08}, s_{2.24}, s_{5.43})$. As $SC(c_1^{\#}) \approx 0.465$, $SC(c_2^{\#}) \approx 0.508$, $SC(c_3^{\#}) \approx 0.569$, $SC(c_4^{\#}) \approx 0.600$, we have $SC(c_4^{\#}) > SC(c_3^{\#}) > SC(c_2^{\#}) > SC(c_1^{\#}) \Rightarrow x_4 > x_3 > x_2 > x_1$.

(2) The information in Reference [50] is linguistic intuitionistic fuzzy numbers (LIFNs). In the first place, it is necessary to translate LNNs into LIFNs. However, there is no existing universal conversion method. In this case, we have three ideas. The first idea is that all the linguistic indeterminacy-membership degrees in LNNs are allocated to linguistic non-membership degrees in LIFNs. In other words, $u_{ij} = \frac{1}{3}s_{T_{ij}}$ and $v_{ij} = \frac{1}{3}s_{I_{ij}} + \frac{1}{3}s_{F_{ij}}$. For example, a LNN (s_3, s_6, s_6) can be changed into a linguistic intuitionistic fuzzy number (LIFN) (s_1, s_4). The second opinion is that linguistic indeterminacy-membership degrees in LNNs are assigned to linguistic membership degrees and linguistic non-membership degrees in LIFNs on average. That is to say, $u_{ij} = \frac{1}{3}s_{T_{ij}} + \frac{1}{6}s_{I_{ij}}$ and $v_{ij} = \frac{1}{6}s_{I_{ij}} + \frac{1}{3}s_{F_{ij}}$. For instance, the LIFN (s_2, s_3) may take the place of a LNN (s_3, s_6, s_6). On the contrary, the last attitude is that all the linguistic indeterminacy-membership degrees in LNNs are allotted to linguistic membership degrees in LIFNs. So to speak, $u_{ij} = \frac{1}{3}s_{T_{ij}} + \frac{1}{3}s_{F_{ij}}$ and $v_{ij} = \frac{1}{3}s_{F_{ij}}$. As an example, a LNN (s_3, s_6, s_6) may be replaced by a LIFN (s_3, s_2).

Owing to the limited space, we take the first idea as an example in the following. The converted decision-making matrices of each expert are shown as Tables 14–16:

Table 14. Converted decision-making matrix $N^{co(1)}$.

$N^{co(1)}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_{7/3}, s_{13/3})$	$(s_2, s_{11/3})$	$(s_{4/3}, s_3)$	$(s_{7/3}, s_{10/3})$	$(s_{5/3}, s_{11/3})$
x_2	$(s_2, s_{8/3})$	$(s_{5/3}, s_2)$	$(s_2, s_{11/3})$	$(s_2, s_{13/3})$	$(s_{7/3}, s_{13/3})$
x_3	(s_2, s_4)	$(s_{5/3}, s_{11/3})$	$(s_{7/3}, s_{11/3})$	$(s_{5/3}, s_{10/3})$	$(s_1, s_{10/3})$
x_4	$(s_{5/3}, s_{13/3})$	$(s_{7/3}, s_{8/3})$	$(s_{4/3}, s_{7/3})$	$(s_2, s_{10/3})$	$(s_{4/3}, s_2)$

Table 15. Converted decision-making matrix $N^{co(2)}$.

$N^{co(2)}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_{7/3}, s_3)$	$(s_{4/3}, s_3)$	$(s_2, s_{8/3})$	$(s_{5/3}, s_3)$	$(s_1, s_{10/3})$
x_2	$(s_{7/3}, s_{11/3})$	$(s_{5/3}, s_{13/3})$	(s_2, s_3)	$(s_{4/3}, s_{11/3})$	(s_2, s_2)
x_3	$(s_{5/3}, s_3)$	(s_2, s_3)	$(s_{7/3}, s_{5/3})$	$(s_{5/3}, s_{8/3})$	(s_2, s_3)
x_4	$(s_2, s_{7/3})$	$(s_{4/3}, s_{10/3})$	$(s_{5/3}, s_{7/3})$	$(s_{4/3}, s_{10/3})$	$(s_{5/3}, s_{4/3})$

Table 16. Converted decision-making matrix $N^{co(3)}$.

$N^{co(3)}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_2, s_{11/3})$	$(s_{5/3}, s_3)$	$(s_1, s_{11/3})$	(s_2, s_3)	$(s_{5/3}, s_{11/3})$
x_2	$(s_{7/3}, s_{13/3})$	$(s_2, s_{10/3})$	$(s_{7/3}, s_{8/3})$	$(s_{4/3}, s_{14/3})$	$(s_{8/3}, s_{5/3})$
x_3	$(s_2, s_{10/3})$	$(s_{7/3}, s_3)$	$(s_2, s_{7/3})$	$(s_{8/3}, s_2)$	$(s_{5/3}, s_{10/3})$
x_4	$(s_2, s_{11/3})$	$(s_{4/3}, s_{13/3})$	$(s_{7/3}, s_3)$	$(s_{5/3}, s_{7/3})$	$(s_{8/3}, s_{7/3})$

Then, using the method in Reference [50], the collective evaluations of each option are $x_1 = (s_{83/48}, s_{173/52})$, $x_2 = (s_{103/52}, s_{68/21})$, $x_3 = (s_{133/73}, s_{257/84})$ and $x_4 = (s_{173/96}, s_{29/11})$ (let the position weight $w = (0.2, 0.3, 0.5)^T$). Then the score functions are $L(x_1) \approx -1.60$, $L(x_2) \approx -1.26$, $L(x_3) \approx -1.24$ and $L(x_4) \approx -0.83$. Because $L(x_4) > L(x_3) > L(x_2) > L(x_1)$, the ranking result is $x_4 > x_3 > x_2 > x_1$.

Likewise, we use the approach in Reference [50] with the second and third thought to deal with the same problem, successively. Afterwards, we get the corresponding ranking orders are $x_1 > x_3 > x_2 > x_4$ and $x_2 > x_1 > x_3 > x_4$, respectively (suppose the position weight is constant and that $w = (0.2, 0.3, 0.5)^T$).

(3) The information in Reference [42] consists of single valued neutrosophic linguistic numbers (SVNLNs). The first step is to change the LNNs into SVNLNs. For a certain LNN $\eta = (s_T, s_I, s_F)$, if $g = \max(T, I, F)$, we can make the linguistic value in a single valued neutrosophic linguistic number (SVNLN) equal to s_g, then the truth-membership, indeterminacy-membership, and false-membership degrees in a SVNLN are described as T/g, I/g and F/g in proper order. So to say, a LNN $\eta = (s_T, s_I, s_F)$ may be converted into a SVNLN $(s_g, < T/g, I/g, F/g >)$. For example, a LNN (s_3, s_3, s_6) and a SVNLN $(s_6, < 0.5, 0.5, 1 >)$ are equivalent in manner.

The transformed decision-making matrices of each specialist are listed as Tables 17–19:

Table 17. Transformed decision-making matrix $N^{tr(1)}$.

$N^{tr(1)}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_7, < 1, 6/7, 1 >)$	$(s_6, < 1, 5/6, 1 >)$	$(s_5, < 4/5, 4/5, 1 >)$	$(s_7, < 1, 3/7, 1 >)$	$(s_6, < 5/6, 5/6, 1 >)$
x_2	$(s_6, < 1, 1/3, 1 >)$	$(s_6, < 5/6, 0, 1 >)$	$(s_7, < 6/7, 4/7, 1 >)$	$(s_7, < 6/7, 1, 6/7 >)$	$(s_7, < 1, 6/7, 1 >)$
x_3	$(s_7, < 6/7, 5/7, 1 >)$	$(s_6, < 5/6, 1, 5/6 >)$	$(s_7, < 1, 4/7, 1 >)$	$(s_7, < 5/7, 3/7, 1 >)$	$(s_6, < 1/2, 1, 2/3 >)$
x_4	$(s_7, < 5/7, 1, 6/7 >)$	$(s_7, < 1, 1/7, 1 >)$	$(s_5, < 4/5, 2/5, 1 >)$	$(s_7, < 6/7, 3/7, 1 >)$	$(s_4, < 1, 1/2, 1 >)$

Table 18. Transformed decision-making matrix $N^{tr(2)}$.

$N^{tr(2)}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_7, <1, 2/7, 1>)$	$(s_5, <4/5, 1, 4/5>)$	$(s_6, <1, 1/3, 1>)$	$(s_6, <5/6, 1/2, 1>)$	$(s_6, <1/2, 1, 2/3>)$
x_2	$(s_7, <1, 4/7, 1>)$	$(s_7, <5/7, 6/7, 1>)$	$(s_6, <1, 5/6, 2/3>)$	$(s_8, <1/2, 1, 3/8>)$	$(s_6, <1, 1/3, 2/3>)$
x_3	$(s_6, <5/6, 1/2, 1>)$	$(s_6, <1, 2/3, 5/6>)$	$(s_7, <1, 2/7, 3/7>)$	$(s_5, <1, 3/5, 1>)$	$(s_7, <6/7, 2/7, 1>)$
x_4	$(s_6, <1, 1/6, 1>)$	$(s_7, <4/7, 3/7, 1>)$	$(s_6, <5/6, 1/6, 1>)$	$(s_6, <2/3, 2/3, 1>)$	$(s_5, <1, 0, 4/5>)$

Table 19. Transformed decision-making matrix $N^{tr(3)}$.

$N^{tr(3)}$	a_1	a_2	a_3	a_4	a_5
x_1	$(s_7, <6/7, 4/7, 1>)$	$(s_6, <5/6, 1/2, 1>)$	$(s_7, <3/7, 1, 4/7>)$	$(s_7, <6/7, 2/7, 1>)$	$(s_6, <5/6, 5/6, 1>)$
x_2	$(s_7, <1, 6/7, 1>)$	$(s_6, <1, 2/3, 1>)$	$(s_7, <1, 3/7, 5/7>)$	$(s_8, <1/2, 3/4, 1>)$	$(s_8, <1, 3/8, 1/4>)$
x_3	$(s_6, <1, 5/6, 5/6>$	$(s_7, <1, 3/7, 6/7>)$	$(s_6, <1, 2/3, 1/2>)$	$(s_8, <1, 1/2, 1/4>)$	$(s_6, <5/6, 1, 2/3>)$
x_4	$(s_6, <1, 5/6, 1>)$	$(s_7, <4/7, 6/7, 1>)$	$(s_7, <1, 4/7, 5/7>)$	$(s_5, <1, 4/5, 3/5>)$	$(s_8, <1, 1/2, 3/8>)$

After that, the extended SVNLN-TOPSIS approach in literature [42] is employed to assess the metal mine's investment venture. The relative closeness coefficients of each mine are calculated as follows: $rc_1 = 21/25$, $rc_2 = 54/67$, $rc_3 = 5/6$ and $rc_4 = 29/36$. Because $rc_4 < rc_2 < rc_3 < rc_1$, we have $x_4 > x_2 > x_3 > x_1$.

From Table 13, we can see that there are diverse ranking results with distinct methods. In order to attain the ideal ranking order, we can assign grades for alternatives in these seven rankings successively. The better the option is, the higher the score is. That is to say, the optimal alternative in a ranking may be distributed with 4, the second is 3, the third is 2, and the worst is 1. As an illustration, according to the ranking $x_4 > x_2 > x_3 > x_1$ in literature [54] with the LNWAM operator, we have $G_1(x_4) = 4$, $G_1(x_2) = 3$, $G_1(x_3) = 2$ and $G_1(x_1) = 1$. Similarly, grades in other ranking methods can be determined. In the end, the overall grades of all alternatives can be earned through summation as follows: $G(x_1) = 12$, $G(x_2) = 19$, $G(x_3) = 17$ and $G(x_4) = 22$. Because $G(x_4) > G(x_2) > G(x_3) > G(x_1)$, the ideal ranking result may be regarded as $x_4 > x_2 > x_3 > x_1$. It is obvious that the result is the same with the proposed method in this paper. The feasibility and availability of the presented approach are indicated.

Besides, the best and worst objects are identical in the literature [42,54] and our approach. The reasons for the differences between literature [54] with our method may be the decision-making thought. Our measure is based on distance, while the literature [54] is based on aggregation operators. Some initial information may be missing in the process of aggregating. Moreover, diverse conclusions may occur with different aggregation operators, which has been demonstrated in the second and third line in Table 13. Both the method in Reference [42] and ours are in line with TOPSIS, and the same orders are received. However, there may be some limitations in [42]. Because the attribute weight vector is given directly, the positive and negative ideal solutions are absolute. In addition, the rankings in literature [50] are all different from the presented method. The reason for the distinction may be that the indeterminacy-membership information in LNNs is unavoidably distorted in LIFNs to some extent.

From the analysis above, the advantages of the proposed method can be summarized as follows:

(1) Evaluating the risk degree of mining projects under qualitative criteria by means of LNNs is a good choice. As all the consistent, hesitant, and inconsistent linguistic information are taken into account.

(2) The flexibility has increased because various distance measures, aggregation operators, and linguistic scale functions can be chosen according to the savants' experience or reality.

(3) A common situation, in which the criteria weight information is unknown, is under consideration. There are many complex risk factors in the process of metallic mining investment. Thus, it is difficult or unrealistic for decision makers to give the weight vector directly. The weight

model based on the thought of maximum deviation may be a simple and suitable way to resolve this problem.

(4) Instead of using absolute ideal points, the extended TOPSIS method defined the relative ideal solutions. The strength of it is that different ideal solutions are calculated corresponding with the different original information of different mining projects. This may be more in line with reality.

6. Discussion and Conclusions

To evaluate risk is the beginning of a metallic mining project investment. Proper risk assessments have great significance on the success of investments. Owing to the uncertainty and complexity in mine surroundings, this paper advised an extended TOPSIS method with LNNs to rise to this challenge. LNNs were suggested to manifest the indeterminate and inconsistent linguistic values, so that the evaluation information can be retained as much as possible. Then, generalized distance formulas were presented to calculate the difference degrees of two LNNs. As it is not easy for the mining investment decision makers to directly determine criteria weight values, a weight model based on maximum deviation was recommended. Afterwards, the method of ranking mines was shown by a case study. Furthermore, the effectiveness and highlights of the presented approach can be reflected in the comparison analysis.

Even though the extended TOPSIS with LNNs method is a good solution, there are still some limitations. For example, the determination of the criteria weight values does not take the subjective elements into consideration. Hence, a more reasonable weight determination method should be further proposed. Besides, the sub-attribute risk factors may be considered in the future. The presented method with LNNs for evaluating the investment risks may be extended to interval linguistic neutrosophic numbers.

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