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Simplified Neutrosophic Exponential Similarity Measures for the Initial Evaluation/Diagnosis of Benign Prostatic Hyperplasia Symptoms

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Abstract: When a physician carries out the clinical survey of a patient with benign prostatic hyperplasia (BPH) symptoms to reach the initial evaluation/diagnosis of BPH, the existing initial evaluation method of BPH based on the international prostate symptom score (I-PSS) usually uses the objective evaluation/diagnosis method with crisp values without considering fuzzy information. However, this common evaluation/diagnosis method may lead to the loss of a great deal of useful incomplete, uncertain, and inconsistent information in the clinical survey and initial evaluation process of the BPH symptoms for a patient, resulting in an unreasonable evaluation and diagnosis distortion of the BPH symptoms. To overcome this drawback, this paper aims to propose new exponential similarity measures (ESMs) between simplified neutrosophic sets (SNSs), including single-valued neutrosophic ESMs and interval neutrosophic ESMs, and their initial evaluation/diagnosis method of the BPH symptoms with simplified neutrosophic information. Finally, two evaluation/diagnosis examples of the BPH symptoms are provided to demonstrate the effectiveness and rationality of the proposed method.

Keywords: benign prostatic hyperplasia; medical diagnosis; simplified neutrosophic set; single valued neutrosophic set; interval neutrosophic set; exponential similarity measure

1. Introduction

Benign prostatic hyperplasia (BPH) is a common medical problem encountered in aging men, and leads to obstructive and irritative voiding symptoms. The American Urological Association (AUA) uses seven questions as the AUA symptom indices [1,2] for BPH scored on a scale from 0 to 5 points. The international prostate symptom score (I-PSS) [1,2] offers an objective documentation of symptoms: a total score of 0–7 is mildly symptomatic, 8–19 moderately symptomatic, and 20–35 severely symptomatic. However, the objective evaluation is a non-fuzzy evaluation method (a common evaluation method) in I-PSS.

The initial evaluation of the BPH symptoms is obtained by means of clinical survey for a patient to select further examinations (e.g., creatinine, intravenous urography, urethrogram, urodynamics, urethrocystoscopy, etc.) and suitable treatment alternatives (e.g., watchful waiting, medical, surgical, or minimally invasive surgical treatments, etc.). Then, the choice of treatment is reached in a shared decision-making process between the physician and the patient. When the physician carries out the clinical survey of a patient to reach the initial evaluation of the BPH symptoms, the patient gives the responses to the seven questions which may contain a "grey zone" of the uncertainty for the patient about the BPH symptoms. Thus, the clinical data of the BPH symptoms obtained by the physician are incomplete, uncertain, or contradictory. In this case, fuzzy expression is a suitable tool. Zadeh [3] first

introduced the membership/truth degree in 1965 and defined a fuzzy set. Based on a generalization of the fuzzy set, Atanassov [4] introduced the nonmembership/falsity degree in 1986 and defined an intuitionistic fuzzy set (IFS) as a generalization of the fuzzy set. Then, Atanassov and Gargov [5] introduced an interval-valued IFS as a generalization of IFS. Further, Smarandache [6] introduced the degree of indeterminacy/neutrality as an independent component in 1995 and defined a neutrosophic set as the generalization of IFS and the interval-valued IFS. He coined the terms "neutrosophy" and "neutrosophic". From a philosophical point of view, the neutrosophic set can represent uncertain, imprecise, incomplete, and inconsistent information. Its advantage is that the neutrosophic set can express indeterminate and inconsistent information, but IFS and the interval-valued IFS cannot. From a science and engineering point of view, the neutrosophic set will be difficult to apply in real science and engineering fields [7,8] because the truth, indeterminacy, and falsity functions in a neutrosophic set belong to the non-standard interval $]^{-}0, 1^{+}[$. Therefore, Smarandache [6] and Wang et al. [7,8] proposed the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), where the truth, indeterminacy, and falsity functions are constrained in the real standard interval [0, 1] as the subclasses of the neutrosophic set. Further, Ye [9] introduced the concept of a simplified neutrosophic set (SNS), which is a subclass of the neutrosophic set including the concepts of SVNS and INS. SNSs are very suitable for handling medical diagnosis problems, since a symptom may imply a lot of incomplete, uncertain, and inconsistent information for a disease, which characterizes a relation between symptoms and a disease. Recently, SNSs have been applied to medical diagnosis problems. Ye [10] presented the improved cosine similarity measures between SNSs for medical diagnosis. As a generalization of SVNS, Ye et al. [11,12] introduced a single-valued neutrosophic multiset and the Dice similarity measure and distance-based similarity measures of single-valued neutrosophic multisets, and then applied them to medical diagnosis. Broumi and Deli [13] presented a correlation measure of neutrosophic refined sets (neutrosophic multisets) and their application in medical diagnosis. Broumi and Smarandache [14] introduced an extended Hausdorff distance and its similarity measure of refined neutrosophic sets (neutrosophic multisets) and applied the similarity measure to medical diagnosis. Furthermore, Ye and Fu [15] put forward a single-valued neutrosophic similarity measure based on tangent function and the tangent similarity measure-based multi-period medical diagnosis method (a dynamic medical diagnosis method).

However, it is difficult to adapt the above-mentioned diagnosis methods to deal with the evaluation problems of BPH with simplified neutrosophic information. Generally, the existing initial evaluation method of BPH is commonly based on I-PSS [1,2] and uses the objective evaluation/diagnosis method with crisp values without considering fuzzy information. Hence, this common evaluation/diagnosis method may lose a lot of incomplete, uncertain, and inconsistent information in the clinical survey and initial evaluation process of the BPH symptoms for a patient, resulting in unreasonable evaluation and diagnosis distortion of the BPH symptoms. To overcome this drawback, this paper aims to propose new exponential similarity measures (ESMs) between SNSs, including single-valued neutrosophic ESMs and interval neutrosophic ESMs, and their initial evaluation/diagnosis method of the BPH symptoms with simplified neutrosophic information.

The rest of the article is structured as follows. In Section 2, we briefly introduce some basic concepts of SNSs. Section 3 proposes ESMs between SNSs based on exponential function, including single-valued neutrosophic ESMs and interval neutrosophic ESMs, and investigates their properties. In Section 4, the initial evaluation/diagnosis methods of the BPH symptoms are presented based on the ESMs under a simplified neutrosophic environment, and then two examples of the evaluation of BPH symptoms are given to show the effectiveness and rationality of the proposed evaluation method. Conclusions and further research are given in Section 5.

2. Basic Concepts of SNSs

The SNS introduced by Ye [9] is the generalization of an IFS and an interval-valued IFS, and gives us an additional possibility to represent incomplete, uncertain, and inconsistent information,

which exists in real world. Therefore, it is more suitable for applications in an indeterminate and inconsistent environment. The definition of SNS is introduced as follows.

Definition 1. *In reference [9], let N be an SNS in a universe of discourse X, which is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then, the SNS N can be expressed as $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle | x \in X\}$, where $T_N(x)$, $I_N(x)$, and $F_N(x)$ are singleton subintervals/subsets in the real standard [0, 1], such that $T_N(x)$: $X \to$ [0, 1], $I_N(x)$: $X \to$ [0, 1], and $F_N(x)$: $X \to$ [0, 1].*

As a subclass of the neutrosophic set, the SNS N contains SVNS for $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$, and $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$, and INS for $T_N(x)$, $I_N(x)$, $F_N(x) \subseteq$ [0, 1,] and $0 \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3$ for each point x in X.

Assume that $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle | x \in X\}$ and $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle | x \in X\}$ are two SNSs in X. If $T_M(x)$, $I_M(x)$, $F_M(x) \in [0, 1]$, $0 \leq T_M(x) + I_M(x) + F_M(x) \leq 3$, $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$, and $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$ for each point x in X, then M and N are reduced to two SVNSs. Thus, the inclusion, equation, and complement for SNSs M and N are defined, respectively, as follows [9]:

(1) $N \subseteq M$ if and only if $T_N(x) \leq T_M(x)$, $I_N(x) \geq I_M(x)$, $F_N(x) \geq F_M(x)$ for any x in X;
(2) $N = M$ if and only if $N \subseteq M$ and $M \subseteq N$;
(3) $M^c = \{\langle x, F_M(x), 1 - I_M(x), T_M(x)\rangle | x \in X\}$ and $N^c = \{\langle x, F_N(x), 1 - I_N(x), T_N(x)\rangle | x \in X\}$.

Assume that $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle | x \in X\}$ and $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle | x \in X\}$ are two SNSs in X. If $T_M(x)$, $I_M(x)$, $F_M(x) \subseteq [0, 1]$, $0 \leq \sup T_M(x) + \sup I_M(x) + \sup F_M(x) \leq 3$, $T_N(x)$, $I_N(x)$, $F_N(x) \subseteq [0, 1]$, and $0 \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3$ for each point x in X, then M and N are reduced to two INSs. Thus, the inclusion, equation, and complement for SNSs N and M are defined, respectively, as follows [9]:

(1) $N \subseteq M$ if and only if $\inf T_N(x) \leq \inf T_M(x)$, $\inf I_N(x) \geq \inf I_M(x)$, $\inf F_N(x) \geq \inf F_M(x)$, $\sup T_N(x) \leq \sup T_M(x)$, $\sup I_N(x) \geq \sup I_M(x)$, $\sup F_N(x) \geq \sup F_M(x)$ for any x in X;
(2) $N = M$ if and only if $N \subseteq M$ and $M \subseteq N$;
(3) $M^c = \{\langle x, [\inf F_M(x), \sup F_M(x)], [1 - \sup I_M(x), 1 - \inf I_M(x)], [\inf T_M(x), \sup T_M(x)]\rangle | x \in X\}$ and $N^c = \{\langle x, [\inf F_N(x), \sup F_N(x)], [1 - \sup I_N(x), 1 - \inf I_N(x)], [\inf T_N(x), \sup T_N(x)]\rangle | x \in X\}$.

Especially when the upper and lower ends of the interval numbers $T_M(x)$, $I_M(x)$, $F_M(x)$ in M and $T_N(x)$, $I_N(x)$, $F_N(x)$ in N are equal, the INSs M and N are reduced to the SVNSs M and N. Therefore, SVNSs are the special cases of INSs, and also SVNSs and INSs are also the special cases of SNSs.

3. ESMs of SNSs

Based on an exponential function, this section proposes ESMs between SNSs, including single-valued neutrosophic ESMs and interval neutrosophic ESMs, and investigates their properties.

Definition 2. *Let $M = \{\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j)\rangle \mid x_j \in X\}$ and $N = \{\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j)\rangle \mid x_j \in X\}$ be any two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$. Thus, we can define an ESM between N and M as follows:*

$$E_1(M, N) = \frac{1}{n}\sum_{j=1}^{n} \frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)\right) - \exp(-1)}{1 - \exp(-1)}. \quad (1)$$

Obviously, ESM has the following proposition.

Proposition 1. *For two SVNSs M and N in $X = \{x_1, x_2, \ldots, x_n\}$, the ESM $E_1(M, N)$ should satisfy the following properties (1)–(4):*

(1) $0 \le E_1(M, N) \le 1$;

(2) $E_1(M, N) = 1$ *if and only if* $M = N$, *i.e.*, $T_M(x_j) = T_N(x_j)$, $I_M(x_j) = I_N(x_j)$, *and* $F_M(x_j) = F_N(x_j)$ *for* $x_j \in X$ $(j = 1, 2, \ldots, n)$;

(3) $E_1(M, N) = E_1(N, M)$;

(4) *If* P *is an SVNS in* X *and* $M \subseteq N \subseteq P$, *then* $E_1(M, P) \le E_1(M, N)$ *and* $E_1(M, P) \le E_1(N, P)$.

Proof. (1) Since there are $T_M(x_j)\, I_M(x_j)\, F_M(x_j) \in [0, 1]$ and $T_N(x_j)\, I_N(x_j)\, F_N(x_j) \in [0, 1]$ in the two SVNSs M and N, the distance $\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)/3$ lies between 0 and 1. By applying Equation (1), ESM also lies between 0 and 1. Hence, there is $0 \le E_1(M, N) \le 1$.

(2) For the two SVNSs M and N, if $M = N$, this implies $T_M(x_j) = T_N(x_j)$, $I_M(x_j) = I_N(x_j)$, $F_M(x_j) = F_N(x_j)$ for $x_j \in X$ and $j = 1, 2, \ldots, n$. Hence, there are $\left|T_M(x_j) - T_N(x_j)\right| = 0$, $\left|I_M(x_j) - I_N(x_j)\right| = 0$, and $\left|F_M(x_j) - F_N(x_j)\right| = 0$. Thus, we can obtain the following result:

$$\begin{aligned} E_1(M, N) &= \frac{1}{n}\sum_{j=1}^{n} \frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j)-T_N(x_j)\right|+\left|I_M(x_j)-I_N(x_j)\right|+\left|F_M(x_j)-F_N(x_j)\right|\right)\right)-\exp(-1)}{1-\exp(-1)} \\ &= \frac{n(1-\exp(-1))/(1-\exp(-1))}{n} = 1. \end{aligned}$$

If $E_1(M, N) = 1$, we have the following equation:

$$E_1(M, N) = \frac{1}{n}\sum_{j=1}^{n} \frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j)-T_N(x_j)\right|+\left|I_M(x_j)-I_N(x_j)\right|+\left|F_M(x_j)-F_N(x_j)\right|\right)\right)-\exp(-1)}{1-\exp(-1)} = 1.$$

Then, there exists the following result:

$$\frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)\right) - \exp(-1)}{(1 - \exp(-1)} = 1.$$

This implies $(1 - \exp(-1))/(1 - \exp(-1)) = 1$, and then there are $\left|T_M(x_j) - T_N(x_j)\right| = 0$, $\left|I_M(x_j) - I_N(x_j)\right| = 0$, and $\left|F_M(x_j) - F_N(x_j)\right| = 0$. Thus, these equalities indicate that $T_M(x_j) = T_N(x_j)$, $I_M(x_j) = I_N(x_j)$, and $F_M(x_j) = F_N(x_j)$ for $x_j \in X$ and $j = 1, 2, \ldots, n$. Hence $M = N$.

(3) Proof is straightforward.

(4) If $M \subseteq N \subseteq P$, then this implies $T_M(x_j) \le T_N(x_j) \le T_P(x_j)$, $I_M(x_j) \ge I_N(x_j) \ge I_P(x_j)$, $F_M(x_j) \ge F_N(x_j) \ge F_P(x_j)$ for $x_j \in X$ and $j = 1, 2, \ldots, n$. Then, we have

$$\left|T_M(x_j) - T_N(x_j)\right| \le \left|T_M(x_j) - T_P(x_j)\right|, \left|T_N(x_j) - T_P(x_j)\right| \le \left|T_M(x_j) - T_P(x_j)\right|,$$

$$\left|I_M(x_j) - I_N(x_j)\right| \le \left|I_M(x_j) - I_P(x_j)\right|, \left|I_N(x_j) - I_P(x_j)\right| \le \left|I_M(x_j) - I_P(x_j)\right|,$$

$$\left|F_M(x_j) - F_N(x_j)\right| \le \left|F_M(x_j) - F_P(x_j)\right|, \left|F_N(x_j) - F_P(x_j)\right| \le \left|F_M(x_j) - F_P(x_j)\right|.$$

Hence, $E_1(M, P) \le E_1(M, N)$ and $E_1(M, P) \le E_1(N, P)$ since the exponential function $\exp\left(-\frac{1}{3}\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)\right)$ is a decreasing function.

Therefore, the proofs of these properties are completed. □

Generally, one takes the weight of each element x_j for $x_j \in X$ into account and assumes that the weight of an element x_j is w_j $(j = 1, 2, \ldots, n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Hence, we can introduce the following weighted ESM between M and N:

$$W_1(M, N) = \sum_{j=1}^{n} w_j \frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j)-T_N(x_j)\right|+\left|I_M(x_j)-I_N(x_j)\right|+\left|F_M(x_j)-F_N(x_j)\right|\right)\right)-\exp(-1)}{1-\exp(-1)}. \quad (2)$$

Clearly, the ESM $W_1(M, N)$ should satisfy the properties (1)–(4) in Proposition 1. Especially when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Equation (2) reduces to Equation (1).

Similarly, we can extend the ESMs of SVNSs to propose ESMs between INSs.

Let $M = \{\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j)\rangle \mid x_j \in X\}$ and $N = \{\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j)\rangle \mid x_j \in X\}$ be any two INSs in $X = \{x_1, x_2, \ldots, x_n\}$, where $T_M(x_j) = [\inf T_M(x_j), \sup T_M(x_j)] \subseteq [0, 1]$, $I_M(x_j) = [\inf I_M(x_j), \sup I_M(x_j)] \subseteq [0, 1]$, and $F_M(x_j) = [\inf F_M(x_j), \sup F_M(x_j)] \subseteq [0, 1]$ in M for any $x_j \in X$ are denoted by $T_M(x_i) = [T_M^L(x_i), T_M^U(x_i)]$, $I_M(x_i) = [I_M^L(x_i), I_M^U(x_i)]$, and $F_M(x_i) = [F_M^L(x_i), F_M^U(x_i)]$, respectively, and $T_N(x_j) = [\inf T_N(x_j), \sup T_N(x_j)] \subseteq [0, 1]$, $I_N(x_j) = [\inf I_N(x_j), \sup I_N(x_j)] \subseteq [0, 1]$, and $F_N(x_j) = [\inf F_N(x_j), \sup F_N(x_j)] \subseteq [0, 1]$ in N for any $x_j \in X$ are denoted by $T_N(x_i) = [T_N^L(x_i), T_N^U(x_i)]$, $I_N(x_i) = [I_N^L(x_i), I_N^U(x_i)]$, and $F_N(x_i) = [F_N^L(x_i), F_N^U(x_i)]$, respectively, for convenience. Then, based on the extension of the above similarity measures Equations (1) and (2), we can introduce the following two ESMs between M and N:

$$E_2(M,N) = \frac{1}{n}\sum_{j=1}^{n} \frac{\exp\left(-\frac{1}{6}\left(\begin{array}{c}|T_M^L(x_j) - T_N^L(x_j)| + |I_M^L(x_j) - I_N^L(x_j)| + |F_M^L(x_j) - F_N^L(x_j)| \\ +|T_M^U(x_j) - T_N^U(x_j)| + |I_M^U(x_j) - I_N^U(x_j)| + |F_M^U(x_j) - F_N^U(x_j)|\end{array}\right)\right) - \exp(-1)}{1 - \exp(-1)}, \quad (3)$$

$$W_2(M,N) = \sum_{j=1}^{n} w_j \frac{\exp\left(-\frac{1}{6}\left(\begin{array}{c}|T_M^L(x_j) - T_N^L(x_j)| + |I_M^L(x_j) - I_N^L(x_j)| + |F_M^L(x_j) - F_N^L(x_j)| \\ +|T_M^U(x_j) - T_N^U(x_j)| + |I_M^U(x_j) - I_N^U(x_j)| + |F_M^U(x_j) - F_N^U(x_j)|\end{array}\right)\right) - \exp(-1)}{1 - \exp(-1)}, \quad (4)$$

where w_j is the weight of an element x_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Obviously, Equations (1) and (2) are the special cases of Equations (3) and (4) when the upper and lower ends of the interval numbers $T_M(x_j)$, $I_M(x_j)$, $F_M(x_j)$ in M and $T_N(x_j)$, $I_N(x_j)$, $F_N(x_j)$ in N are equal. Therefore, the above ESMs of INSs also satisfy properties (1)–(4) in Proposition 1. The proof is similar to that of Proposition 1, and thus it is not repeated here.

4. Initial Evaluation/Diagnosis Method of BPH Using the ESMs

According to the seven questions in the AUA symptom indexes [1,2] for BPH, we can consider a set of the seven questions $Q = \{Q_1$ (Over the past month, how often have you had a sensation of not emptying your bladder completely after you finished urinating?), Q_2 (Over the past month, how often have you had to urinate again less than two hours after you finished urinating?), Q_3 (Over the past month, how often have you found you stopped and started again several times when you urinated?), Q_4 (Over the past month, how often have you found it difficult to postpone urination?), Q_5 (Over the past month, how often have you had a weak urinary stream?), Q_6 (Over the past month, how often have you had to push or strain to begin urination?), Q_7 (Over the past month, how many times did you most typically get up to urinate from the time you went to bed at night until the time you got up in the morning?)$\}$ for a physician to survey the patients' BPH symptoms. The clinical survey of the number of BPH symptoms in the 5 times for a patient P_k ($k = 1, 2, \ldots, t$) can be constructed by Table 1, where T, I, and F denote truth, indeterminacy, and falsity, respectively.

Based on I-PSS [1,2], BPH can be divided into the four types of symptoms, which are represented by a set of the four types of symptoms $S = \{S_1$ (Normal symptom), S_2 (Mild symptom), S_3 (Moderate symptom), S_4 (Severe symptom)$\}$ as the symptom knowledge for the initial evaluation of BPH patients, as shown in Table 2.

Table 1. The number of benign prostatic hyperplasia (BPH) symptoms in the 5 times for a patient P_k. T: true; I: indeterminate; F: false.

Question	*T*	*I*	*F*
Q_1: Over the past month, how often have you had a sensation of not emptying your bladder completely after you finished urinating?			
Q_2: Over the past month, how often have you had to urinating again less than two hours after you finished urinating?			
Q_3: Over the past month, how often have you found you stopped and started again several times when you urinated?			
Q_4: Over the past month, how often have you found it difficult to postpone urination?			
Q_5: Over the past month, how often have you had a week urinary stream?			
Q_6: Over the past month, how often have you had to push or strain to begin urination?			
Q_7: Over the past month, how many times did you most typically get up to urinate from the time you went to bed at night until the time you got up in the morning?			

Table 2. Four types of BPH symptoms with simplified neutrosophic information.

S_i (Symptom Type)	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
S_1 (Normal symptom)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>
S_2 (Mild symptom)	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>
S_3 (Moderate symptom)	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>
S_4 (Severe symptom)	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>

From Table 2, the BPH symptom types of patients with respect to all the questions can be represented by the following SNS information:

$S_1 = \{\langle Q_1, 0, 0, 1\rangle, \langle Q_2, 0, 0, 1\rangle, \langle Q_3, 0, 0, 1\rangle, \langle Q_4, 0, 0, 1\rangle, \langle Q_5, 0, 0, 1\rangle, \langle Q_6, 0, 0, 1\rangle, \langle Q_7, 0, 0, 1\rangle\}$,

$S_2 = \{\langle Q_1, 0, 0.2, 0.8\rangle, \langle Q_2, 0, 0.2, 0.8\rangle, \langle Q_3, 0, 0.2, 0.8\rangle, \langle Q_4, 0, 0.2, 0.8\rangle, \langle Q_5, 0, 0.2, 0.8\rangle, \langle Q_6, 0, 0.2, 0.8\rangle, \langle Q_7, 0, 0.2, 0.8\rangle\}$,

$S_3 = \{\langle Q_1, 0.2, 0.4, 0.4\rangle, \langle Q_2, 0.2, 0.4, 0.4\rangle, \langle Q_3, 0.2, 0.4, 0.4\rangle, \langle Q_4, 0.2, 0.4, 0.4\rangle, \langle Q_5, 0.2, 0.4, 0.4\rangle, \langle Q_6, 0.2, 0.4, 0.4\rangle, \langle Q_7, 0.2, 0.4, 0.4\rangle\}$.

$S_4 = \{\langle Q_1, 0.6, 0.4, 0\rangle, \langle Q_2, 0.6, 0.4, 0\rangle, \langle Q_3, 0.6, 0.4, 0\rangle, \langle Q_4, 0.6, 0.4, 0\rangle, \langle Q_5, 0.6, 0.4, 0\rangle, \langle Q_6, 0.6, 0.4, 0\rangle, \langle Q_7, 0.6, 0.4, 0\rangle\}$.

Assume that we give the clinical survey for t BPH patients by using Table 1 to obtain the t patients' responses of the BPH symptom which are represented by the form of truth, indeterminacy, and falsity values. For a patient P_k ($k = 1, 2, \ldots, t$) with SNS information, we can give the following evaluation/diagnosis method.

To give a proper evaluation/diagnosis for a patient P_k with BPH symptoms, we can calculate the similarity measure $W_q(P_k, S_i)$ for $q = 1$ or 2, $i = 1, 2, 3, 4$ and $k = 1, 2, \ldots, t$. The proper BPH symptom evaluation S_{i^*} for patient P_k is derived by $i^* = \arg\max_{1 \le i \le 4} \{W_q(P_k, S_i)\}$.

To illustrate the evaluation/diagnosis process of the BPH symptoms, we provide two evaluation/diagnosis examples of the BPH symptoms to demonstrate the applications and effectiveness of the proposed evaluation/diagnosis method under simplified neutrosophic (single-valued neutrosophic and interval neutrosophic) environments.

4.1. Initial Evaluation of the BPH Symptoms Under a Single-Valued Neutrosophic Environment

In some cases, we can obtain that data collected from the clinical survey of patients are single values rather than interval values for T, I, and F. In this case, ESM of SVNSs is a better tool to give a proper initial evaluation of a patient's BPH symptoms.

Example 1. *Assume that we give a clinical survey for three BPH patients by using Table 1, and then we can obtain the three patients' responses of the BPH symptoms which are represented by the form of truth, indeterminacy, and falsity values, as shown in Table 3.*

Table 3. The number of BPH symptoms (single values) in the 5 times for three patients.

Question	P_1			P_2			P_3		
	T	I	F	T	I	F	T	I	F
Q_1	2/5	1/5	2/5	1/5	1/5	3/5	3/5	0/5	2/5
Q_2	2/5	2/5	1/5	2/5	1/5	2/5	3/5	1/5	1/5
Q_3	2/5	1/5	2/5	1/5	0/5	4/5	3/5	1/5	1/5
Q_4	2/5	1/5	2/5	2/5	1/5	2/5	4/5	1/5	0/5
Q_5	3/5	2/5	0/5	1/5	2/5	2/5	3/5	1/5	1/5
Q_6	2/5	0/5	3/5	2/5	0/5	3/5	4/5	1/5	0/5
Q_7	3/5	0/5	2/5	1/5	1/5	3/5	2/5	2/5	1/5

From Table 3, the BPH symptom responses of the patient P_k (k = 1, 2, 3) with respect to all the questions can be represented by the following SVNS information:

P_1 = {⟨Q_1, 0.4, 0.2, 0.4⟩, ⟨Q_2, 0.4, 0.4, 0.2⟩, ⟨Q_3, 0.4, 0.2, 0.4⟩, ⟨Q_4, 0.4, 0.2, 0.4⟩, ⟨Q_5, 0.6, 0.4, 0.0⟩, ⟨Q_6, 0.4, 0.0, 0.6⟩, ⟨Q_7, 0.6, 0.0, 0.4⟩},

P_2 = {⟨Q_1, 0.2, 0.2, 0.6⟩, ⟨Q_2, 0.4, 0.2, 0.4⟩, ⟨Q_3, 0.2, 0.0, 0.8⟩, ⟨Q_4, 0.4, 0.2, 0.4⟩, ⟨Q_5, 0.2, 0.4, 0.4⟩, ⟨Q_6, 0.4, 0.0, 0.6⟩, ⟨Q_7, 0.2, 0.2, 0.6⟩},

P_3 = {⟨Q_1, 0.6, 0.0, 0.4⟩, ⟨Q_2, 0.6, 0.2, 0.2⟩, ⟨Q_3, 0.6, 0.2, 0.2⟩, ⟨Q_4, 0.8, 0.2, 0.0⟩, ⟨Q_5, 0.6, 0.2, 0.2⟩, ⟨Q_6, 0.8, 0.2, 0.0⟩, ⟨Q_7, 0.4, 0.4, 0.2⟩}.

Assume that the weight of each element Q_j is $w_j = 1/7$ for $j = 1, 2, \ldots, 7$. By applying Equation (2), we can obtain the results of the similarity measure between the patient P_k (k = 1, 2, 3) and the considered symptom S_i (i = 1, 2, 3, 4), as shown in Table 4.

Table 4. Similarity measure values between P_k and S_i with single-valued neutrosophic sets (SVNSs).

	S_1	S_2	S_3	S_4
$W_1(P_1, S_i)$	0.4457	0.5460	0.7285	0.6857
$W_1(P_2, S_i)$	0.5896	0.7038	0.7814	0.5244
$W_1(P_3, S_i)$	0.3319	0.4406	0.6112	0.7778

In Table 4, the largest similarity measure indicates the proper evaluation/diagnosis. Therefore, in initial clinical evaluations for the three patients, Patients P_1 and P_2 have moderate symptoms, and Patient P_3 has severe symptoms.

4.2. Initial Evaluation of the BPH Symptoms Under an Interval Neutrosophic Environment

In some cases, we can obtain that data collected from the clinical survey of patients are interval values rather than single values for T, I, and F, since patients easily express real situations by using the interval values. In this case, ESM of INSs is a better tool to give a proper initial evaluation of the BPH symptoms.

Example 2. *Assume that we give the clinical survey for three BPH patients by using Table 1, and then we can obtain the three patients' responses of the BPH symptoms, which are represented by the interval values of T, I, and F, as shown in Table 5.*

Table 5. The number of BPH symptoms (interval values) in the 5 times for three patients.

Question	P_1			P_2			P_3		
	T	*I*	*F*	*T*	*I*	*F*	*T*	*I*	*F*
Q_1	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[1/5, 2/5]	[0/5, 1/5]	[2/5, 3/5]	[3/5, 4/5]	[0/5, 0/5]	[1/5, 2/5]
Q_2	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]	[2/5, 2/5]	[0/5, 1/5]	[1/5, 2/5]	[3/5, 4/5]	[1/5, 1/5]	[0/5, 1/5]
Q_3	[2/5, 3/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[0/5, 1/5]	[3/5, 4/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_4	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[2/5, 3/5]	[1/5, 2/5]	[0/5, 2/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_5	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]	[1/5, 2/5]	[2/5, 3/5]	[0/5, 1/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 1/5]
Q_6	[2/5, 3/5]	[0/5, 1/5]	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]	[2/5, 3/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_7	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]

From Table 5, the BPH symptom responses of patient P_k (k = 1, 2, 3) with respect to all the questions can be represented by the following INS information:

P_1 = {⟨Q_1, [0.4, 0.6], [0, 0.2], [0.2, 0.4]⟩, ⟨Q_2, [0.2, 0.4], [0.4, 0.6], [0, 0.2]⟩, ⟨Q_3, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]⟩, ⟨Q_4, [0.4, 0.6], [0, 0.2], [0.2, 0.4]⟩, ⟨Q_5, [0.6, 0.8], [0.2, 0.4], [0, 0]⟩, ⟨Q_6, [0.4, 0.6], [0, 0.2], [0.4, 0.6]⟩, ⟨Q_7, [0.4, 0.6], [0, 0.2], [0.2, 0.4]⟩},

P_2 = {⟨Q_1, [0.2, 0.4], [0, 0.2], [0.4, 0.6]⟩, ⟨Q_2, [0.4, 0.4], [0, 0.2], [0.2, 0.4]⟩, ⟨Q_3, [0.2, 0.4], [0, 0.2], [0.6, 0.8]⟩, ⟨Q_4, [0.4, 0.6], [0.2, 0.4], [0, 0.4]⟩, ⟨Q_5, [0.2, 0.4], [0.4, 0.6], [0, 0.2]⟩, ⟨Q_6, [0.4, 0.6], [0, 0.2], [0.4, 0.6]⟩, ⟨Q_7, [0.2, 0.4], [0.2, 0.4], [0.2, 0.4]⟩},

P_3 = {⟨Q_1, [0.6, 0.8], [0, 0], [0.2, 0.4]⟩, ⟨Q_2, [0.6, 0.8], [0.2, 0.2], [0, 0.2]⟩, ⟨Q_3, [0.6, 0.8], [0.2, 0.4], [0, 0]⟩, ⟨Q_4, [0.6, 0.8], [0.2, 0.4], [0, 0]⟩, ⟨Q_5, [0.6, 0.8], [0.2, 0.4], [0, 0.2]⟩, ⟨Q_6, [0.6, 0.8,] [0.2, 0.4], [0, 0]⟩, ⟨Q_7, [0.4, 0.6], [0.4, 0.6], [0, 0.2]⟩}.

Assume that the weight of each element Q_j is $w_j = 1/7$ for $j = 1, 2, \ldots, 7$. By applying Equation (4), we can obtain the results of the similarity measure between Patient P_k (k = 1, 2, 3) and the considered symptom S_i (i = 1, 2, 3, 4), as shown in Table 6.

Table 6. Similarity measure values of between P_k and S_i with interval neutrosophic sets (INSs).

	S_1	S_2	S_3	S_4
$W_2(P_1, S_i)$	0.3956	0.4910	0.6784	0.7164
$W_2(P_2, S_i)$	0.4799	0.5831	0.7331	0.6297
$W_2(P_3, S_i)$	0.2718	0.3734	0.5741	0.8322

In Table 6, the largest similarity measure indicates the proper evaluation/diagnosis. Therefore, in initial clinical evaluations for the three patients, Patients P_1 and P_3 have severe symptoms, and Patient P_2 has moderate symptoms.

4.3. Comparison and Analysis

For convenient comparison with the common evaluation method [1,2], based on Tables 3 and 5 we only give the BPH symptom responses with the values of T (without the values of I and F in I-PSS) of the patient P_k (k = 1, 2, 3) with respect to all the questions of Examples 1 and 2, which are shown in Tables 7 and 8, respectively. According to the common evaluation method of I-PSS [1,2], where one time means one score in I-PSS [1,2], we can give the clinical initial evaluation results of three patients (P_1, P_2, P_3), which are also shown in Tables 7 and 8, respectively.

In Table 7, all the initial evaluation/diagnosis results of Example 1 are the same as the ones of the new evaluation method proposed in this paper. Then, all the initial evaluation/diagnosis results of Example 2 in Table 8 are almost the same as the ones of the new evaluation method, but the evaluation/diagnosis of P_1 is difficult to determine the moderate and/or severe symptoms in the common evaluation/diagnosis method based on I-PSS [1,2]. However, P_1 has severe symptoms based on the comprehensive evaluation/diagnosis results in this study. Therefore, the diagnosis results of the two examples demonstrate the effectiveness of the proposed diagnosis method under simplified neutrosophic environments.

Table 7. The number of BPH symptoms (single values of T) in the 5 times for three patients, where one time means one score in the international prostate symptom score (I-PSS).

Question	P_1			P_2			P_3		
	T (time)	I	F	T (time)	I	F	T (time)	I	F
Q_1	2	/	/	1	/	/	3	/	/
Q_2	2	/	/	2	/	/	3	/	/
Q_3	2	/	/	1	/	/	3	/	/
Q_4	2	/	/	2	/	/	4	/	/
Q_5	3	/	/	1	/	/	3	/	/
Q_6	2	/	/	2	/	/	4	/	/
Q_7	3	/	/	1	/	/	2	/	/
Total score	16			10			22		
BPH symptom	Moderate			Moderate			Severe		

Table 8. The number of BPH symptoms (interval values of T) in the 5 times for three patients.

Question	P_1			P_2			P_3		
	T (time)	I	F	T (time)	I	F	T (time)	I	F
Q_1	[2, 3]	/	/	[1,2]	/	/	[3, 4]	/	/
Q_2	[2, 3]	/	/	[2, 2]	/	/	[3, 4]	/	/
Q_3	[2, 3]	/	/	[1,2]	/	/	[3, 4]	/	/
Q_4	[2, 3]	/	/	[2, 3]	/	/	[3, 4]	/	/
Q_5	[3, 4]	/	/	[1,2]	/	/	[3, 4]	/	/
Q_6	[2, 3]	/	/	[2, 3]	/	/	[3, 4]	/	/
Q_7	[2, 3]	/	/	[1,2]	/	/	[2, 3]	/	/
Total score	[15, 22]			[10, 16]			[20, 27]		
BPH symptom	Moderate and/or severe			Moderate			Severe		

Compared with the existing initial evaluation method based on I-PSS [1,2], the proposed evaluation method demonstrates their effectiveness and rationality because the developed initial evaluation method with simplified neutrosophic information contains much more evaluation information (truth, indeterminacy, and falsity information) than the existing initial evaluation method based on I-PSS (crisp values) without indeterminacy and falsity information [1,2]. Obviously, the common initial evaluation method based on I-PSS may lose much useful information (indeterminacy and falsity information), resulting in the unreasonable/difficult evaluation and diagnosis distortion for some patients. Therefore, the developed diagnosis method is more suitable and more practical in the initial evaluation of BPH symptoms, is superior to the existing initial evaluation method [1,2], and also shows an advantage in terms of the effectiveness and rationality of the proposed diagnosis method.

5. Conclusions

Based on an exponential function, this paper proposed ESMs of SNSs, including single-valued neutrosophic ESMs and interval neutrosophic ESMs. Then, an initial evaluation/diagnosis method for BPH symptoms was established based on ESMs under a simplified neutrosophic environment. Finally, two illustrative examples of the initial evaluations of BPH symptoms are provided to demonstrate the application and effectiveness of the proposed evaluation method in a simplified neutrosophic setting. The advantage of the evaluation method developed in this paper is that it can deal with medical diagnosis problems with incomplete, uncertain, and inconsistent information, while the existing initial evaluation method [1,2] cannot handle them and may lose much useful information in evaluation process.

In further work, it is necessary to apply ESMs of SNSs to other medical problems, such as medical image processing and medical clustering analysis.

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