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Problems of Estimating the Resources of Accompanying Elements: A Case Study from the Cu-Ag Rudna Deposit (Legnica-Głogów Copper District, Poland)

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Abstract: As a result of the exploitation of ore deposits, in addition to the main elements, the accompanying elements are also partially recovered. Some of them increase the profitability of exploitation, while others reduce it because they hinder the recovery of the main elements and thus increase the costs of the recovery process. A comprehensive economic calculation to assess the profitability of ore mining depends on an appropriately accurate estimation of the resources of both the main and associated elements. This issue was analyzed with the example of the Cu-Ag Rudna ore deposit (LGCD, Poland). The subject of the assessment was the resources prediction accuracy of the main element (Cu) and four (4) accompanying elements (Co, Ni, Pb, and V) using geostatistical estimation method, in particular the ordinary kriging after the estimation of the relative variograms for describing the spatial variability structures of elements abundance. It was found that the standard kriging errors (deviations) in accompanying elements resources that are scheduled for exploitation within a one-year period in some parts of deposits are drastically greater (2 to 5 times) than the estimation errors of the main element resources. This is due to the sparse sampling pattern for their determinations and/or the high variability (among others nugget effect) of their abundance. In this situation, without additional sampling and a denser sampling pattern, the possibilities of a reliable assessment of the influence of accompanying elements on the economic consequences of exploitation are very limited.

Keywords: Cu-Ag Rudna deposit; Legnica-Głogów Copper District (LGCD); ore resources; accompanying elements; accuracy; relative variogram; ordinary kriging

1. Introduction

The mining of Cu and Ag ores in the Legnica-Głogów Copper District (LGCD) (SW Poland, Fore-Sudetic Monocline) is carried out by KGHM Polska Miedź S.A. in six deposits (Figure 1). KGHM Polska Miedź S.A. is one of the world's leading producers of silver and copper. In terms of copper and silver production, it ranks eighth and second in the world, respectively (data for 2020) [1]. In 2019, the extraction of ore containing 1.5% Cu and 48.7 g/t Ag amounted to 29.9 million tons. In total, 449 thousand tons of metallic copper and 1455 tons of silver were mined in 2019 [2].

The Cu-Ag ore deposits of the LGCD are rich in accompanying (secondary) elements, including Pb, Zn, Cd, Ni, Co, Fe, Au, Pt, Pd, V, Hg, Sn, Ge, U, Th, Mo, Re (heavy metals), S, Se, Ba, and F (non-metallic elements), and As, Sb, and Bi (elements with intermediate properties) [3]. Some of them increase the value of the deposit, while others are undesirable components that negatively affect the environment and technological processes [4]. In 2019, in addition to the main metals (Cu and Ag), the following were recovered from the LGCD deposits in the technological processing of ores or metallurgical processes: 674 kg of Au, 1.99 thousand tons of nickel sulphate, 28.5 thousand tons of Pb, 75.8 tons of Se, and 8.3 tons of Re [2].

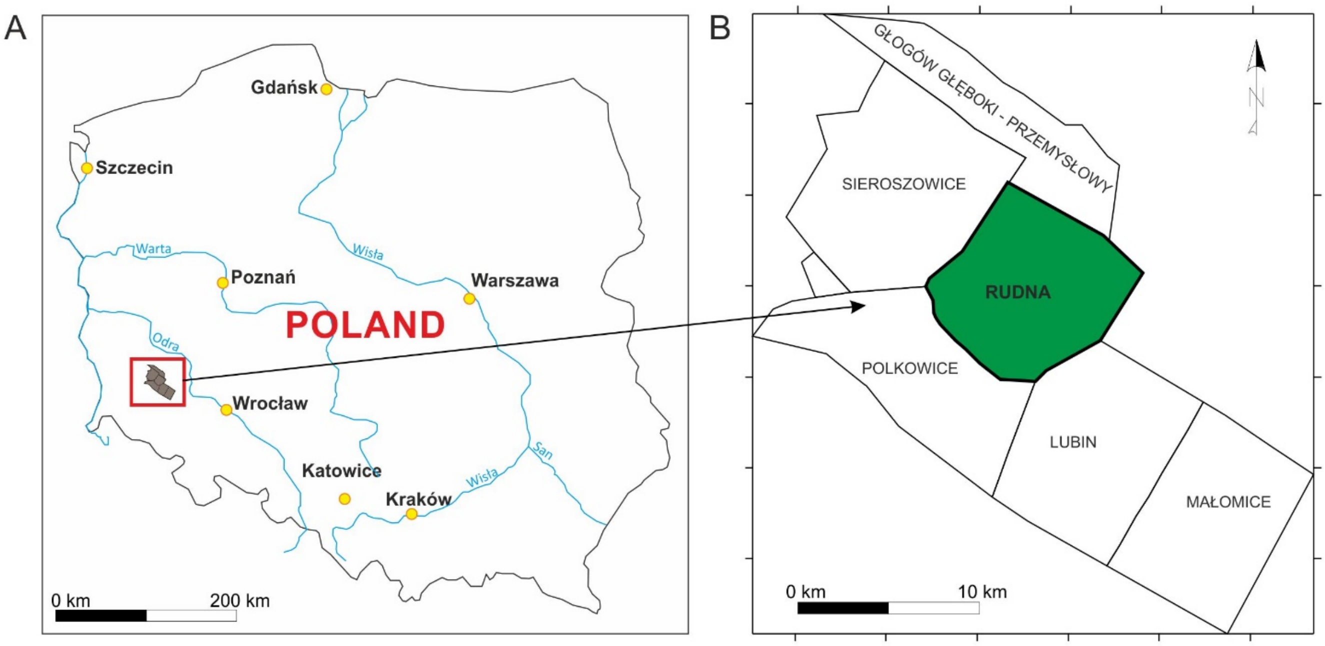

Figure 1. Location of the Cu-Ag deposits of the Legnica-Głogów Copper District against the background of Poland (**A**). Location of the boundaries of the Rudna deposit against the background of developed copper ore deposits in the Fore-Sudetic Monocline (**B**).

Among numerous co-occurring (accompanying) elements in the Cu-Ag ores, only cobalt, nickel, lead, and vanadium resources were reported in the original geological documentations of the Cu-Ag deposits of the LGCD. Their resources in each of the deposits were estimated only on the basis of the results of the drilling exploration using the traditional method (Voronoi's polygons), which does not give the accuracy of these estimates (i.e., the estimation error). The prediction of the resources of these elements in small parts of the deposits is subject to large errors due to the large distance between the boreholes (of the order of 1.5 km). During mining exploration of deposits, the data sets on some accompanying elements are successively enriched with the results of their determinations in samples collected in mining excavations. For the exact prediction of economic effects and profitability of deposit exploitation in short periods of time, it is necessary to accurately estimate not only the resources of the main elements (Cu and Ag) but also the accompanying elements.

The Cu-Ag ore deposits of the LGCD were the subject of numerous geostatistical studies aimed at describing the variability and assessing the accuracy of the estimates of the content and resources of the main element (Cu), less often Ag [5–13]. The subject of this article is to assess the accuracy of the estimation of Co, Ni, Pb, and V resources in parts of the deposit scheduled for exploitation within a settlement period of one year (a normal settlement period at the KGHM mines). The analysis uses only data from the sampling of mining excavations, which is much more numerous than the data from the boreholes used in the original deposit documentation. In Cu-Ag deposits of the LGCD, the issue of the accuracy of the estimation of the resources of the accompanying elements has not been the subject of research so far. The previous studies were focused on the accuracy of estimating the average content of a wider spectrum of accompanying elements in all mined Cu-Ag deposits of the LGCD [14,15].

In general, the accuracy of resource estimation depends on the number and distribution of samples in which the contents of the accompanying elements were determined and their variability. To assess the accuracy of resource estimation, ordinary kriging methods were used, based on the description of the variability structure in the content of elements with the use of relative variograms. The choice of the geostatistical procedure resulted from the occurrence in the variability structure of the abundance of accompanying elements, the more or less marked non-random component of variability. The research was also aimed at answering the question of whether the sampling method currently used in mining excavations for the determination of the considered co-occurring elements is appropriate from the point of view of achieving the assumed accuracy of estimation of their resources.

According to the literature review, the question of estimating the resources of accompanying elements in the world's Cu-Ag deposits has not been of particular interest so far, as evidenced by the lack of publications in this field. However, it cannot be ruled out that it was the subject of internal unpublished mining reports. In this situation, it is impossible to compare the research results presented in the following article with other studies.

2. Geology

2.1. Geological Setting

The Rudna deposit, like other Cu-Ag deposits in the LGCD (Figure 1), is a sedimentary hosted stratiform copper deposit [16–18]. In terms of stratigraphy, the deposit covers the upper part of the Rotliegend sediments and the lower part of the Zechstein sediments (PZ1 cyclothem) (Figure 2). The mineralization occurs in three basic lithological series defined as the carbonate ore (11% of the ore resources), the Kupferschiefer ore (6%) and sandstone ore (83%) [19]. Within all these types of ore, copper sulfides dominate among the following minerals: chalcocite (Cu_2S), digenite (Cu_9S_5), bornite (Cu_5FeS_4), chalcopyrite ($CuFeS_2$), and covelin (CuS). Most of these minerals occur as isolated grains; mineral intergrowths are also found.

In the vertical profile, the deposit series are characterized by a highly diversified thickness and a different intensity of mineralization. The deposit is up to 20 m thick with an average of about 5 m, which is much higher than in the deposits adjacent to the Rudna deposit. The deposit extends NW-SE and dips gently at an angle of about 3 to 6 degrees in the NE direction. The depth of deposition of the Cu-Ag orebody in the Rudna deposit ranges from 844 m up to 1250 m. In the Rudna deposit, the room-and-pillar mining method is generally used, although the detailed procedure depends on deposit thickness and the geotechnical parameters of the orebody and surrounding rocks [20].

2.2. Characteristics of the Studied Accompanying Elements

The conducted research was aimed at geostatistical description of the abundance variability and assessment of the accuracy of the estimation of the resources of 4 co-occurring elements in the Cu-Ag (LGCD) deposits: cobalt (Co), nickel (Ni), lead (Pb) and vanadium (V) against the accuracy of the estimation of resources of the main element (Cu). They appear to be particularly interesting due to their local high content, their importance for the effectiveness of Cu-Ag ore processing, and their impact on the natural environment.

A typical distribution of the highest concentrations of the considered accompanying elements and the main elements (Cu and Ag) in the vertical profile of the deposit against the background of the detailed lithological units is shown in Figure 2.

Cobalt (Co) is on the list of critical raw materials for the EU [21]. Currently, this element is not recovered from the mined ore at KGHM Polska Miedź S.A.; however, previous studies showed that it is possible to recover cobalt with the use of leaching technology [22]. Nickel, despite its low content, close to its Clarke value, is recovered in the form of nickel sulphate ($NiSO_4$). Nickel demand is expected to continue to grow in the coming years, and nickel resources from ore deposits may be depleted in the future (around 2200) [23]. Lead (Pb) negatively affects both the production of copper concentrate and the natural environment, but the Cu-Ag deposits (the LGCD) are currently the main source of lead in Poland due to the depletion of the developed zinc and lead ore deposits in Triassic formations (MVT Cracow-Silesian Zn-Pb ore deposits) and the closure of the last Zn-Pb ore mine.

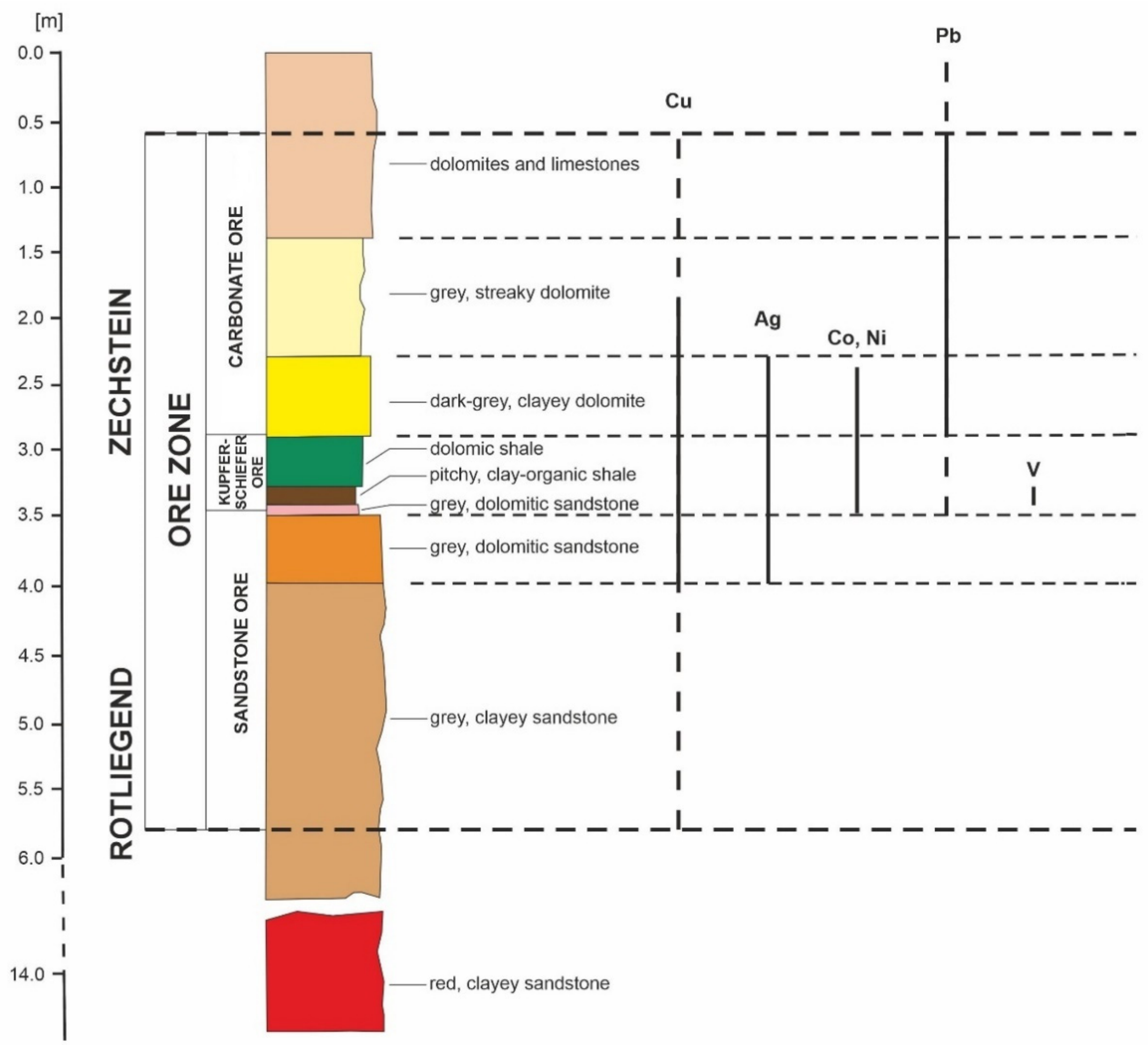

Figure 2. Distribution of the main metals (Cu and Ag) and accompanying elements (Co, Ni, Pb, and V) in the vertical profile of the deposit (based on [3,24]).

Their resources are given as estimates, and for the Rudna deposit, they amount to: Co—16.2 thousand t; Ni—14.2 thousand t; Pb—314.4 thousand t; and V—39.8 thousand t (at 31 December 2019) [2].

The above-mentioned four metals accompanying Cu in ore in the LGCD deposits occur as the form of ore minerals or as isomorphic substitutions in other minerals (Co, Ni, and Pb) and in organometallic compounds (V) [3]. The mineral phases of these metals are summarized in Table 1.

3. Materials and Methods

3.1. Sampling of the Deposit in Mine Workings

The subject of direct research was the abundance of copper and the abundance of four accompanying metals (Co, Ni, Pb, and V) determined within the Cu-Ag Rudna deposit intended for mining (Figure 1). The boundaries of the Cu-Ag deposit in the vertical plane were determined at each sampling station based on the Cu and Ag content in samples taken from sections with an average length of 20 cm, along a vertical line from the roof to the bottom of the mining excavation. The boundaries of the deposit are determined by extreme samples in which the Cu content was not lower than 0.7%, provided that the average equivalent Cu content (considering the Ag content according to the conversion factor of 100 ppm Ag = 1% Cu) in the entire section was not less than 0.7%, while the equivalent abundance of Cu was not less than 50 kg/m^2 (Figure 3).

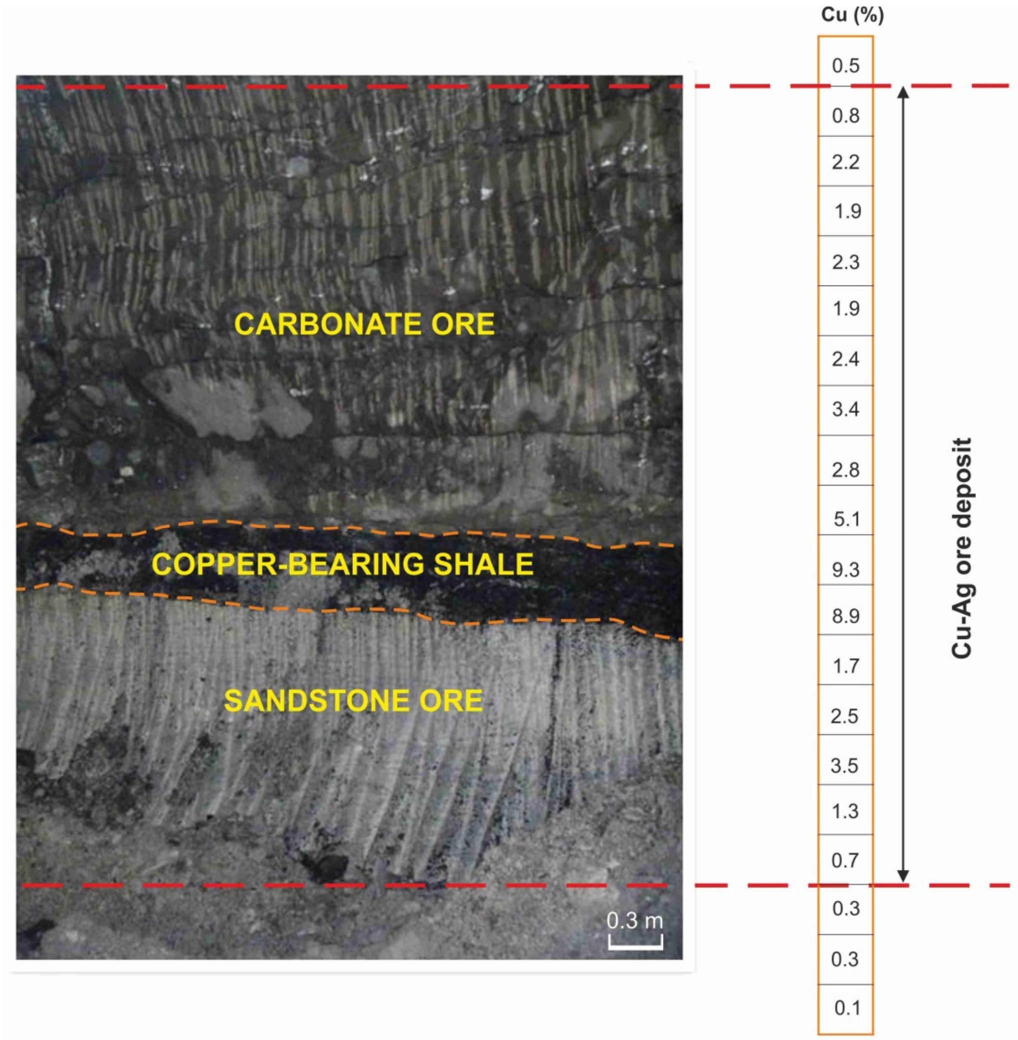

Figure 3. An example of determining the deposit boundary in the vertical profile including three main lithological types of ores: carbonate ore, the Kupferschiefer ore, and sandstone ore with information on the Cu content in point samples.

Table 1. Metal bearing mineral phases in the Cu-Ag deposits (the LGCD).

Metal	Mineral Phases		Recovered [25]
	Main Mineral Phases [26]	Occurrence in the Form of Admixtures [26]	
Cobalt (Co) [27]	cobaltite CoAsS smaltite (Co, Ni)As_3 safflorite $CoAs_2$	pyrite FeS_2 bornite Cu_5FeS_4	no
Nickel (Ni) [25]	nickelite NiAs rammelsbergite $NiAs_2$ gersdorffite NiAsS chloantite $NiAs_3$	pyrite FeS_2 chalcocite Cu_2S	yes

Table 1. *Cont.*

Metal	Mineral Phases		Recovered [25]
	Main Mineral Phases [26]	**Occurrence in the Form of Admixtures [26]**	
Lead (Pb) [28,29]	galena PbS clausthalite (PbSe) cerussite $PbCO_3$	pyrite FeS_2	yes
Vanadium (V) [30]	no occur	in organic matter	no

The abundance of copper and accompanying metals in the entire Cu-Ag deposit was determined at the sampling sites as the sum of their abundance in three main lithological units: sandstone, shale, and carbonates, using the formula:

$$a = \sum_{i=1}^{3} a_i = \sum_{i=1}^{3} M_i \cdot \gamma_{0i} \cdot p_i [\%] \cdot \frac{1}{100\%} \tag{1}$$

where: a—abundance of elements, i—lithological units: 1—sandstone ore, 2—the Kupferschiefer ore, 3—carbonate ore, M_i—thickness of the lithological ore units, γ_{0i} —volumetric density: 1—sandstone ore (2.3 $\frac{t}{m^3}$), 2—the Kupferschiefer ore (2.5 $\frac{t}{m^3}$), 3—carbonate ore (2.6 $\frac{t}{m^3}$), p_i—content of elements in the lithological ore units in % (Cu, Pb) or ppm = g/Mg (Co, Ni, V).

In the vertical profile, the increased content of the accompanying elements of the Cu-Ag deposits of LGCD occurs very often outside their boundaries. However, the results of the study on their abundance presented later in the article refer only to the deposit determined on the basis of the content and the abundance of Cu and Ag.

In mining excavations, the determination of Cu content is made in samples taken with a spacing of 20 to 50 m, determination of Pb content with a spacing of not more than 60 m, and determination of Co, Ni, and V content with a spacing of up to 200 m.

In the Rudna deposit, the average distance between the sampling sites used to determine the basic element (Cu) is approximately 40 m. The sampling pattern for the accompanying elements is much less frequent, with an average sample spacing of approximately 180 m for Pb to approximately 400 m for Co, Ni, and V (Figure 4). For this reason, the size of the data set for Cu was much considerably larger than for the accompanying elements and was 10,243 versus 3255 for Pb, 732 for Co, 722 for Ni, and 676 for V (Table 2).

Table 2. Arithmetic means of the thickness of the deposit (M) and the abundance of elements (a) in the main lithologies and the deposit (the number of data sets is presented in parentheses).

Main Lithologies	M [m]	aCu [kg/m^2]	aCo [g/m^2]	aNi [g/m^2]	aPb [kg/m^2]	aV [g/m^2]
Carbonate ore	1.12	56.0 (6948)	69.3 (718)	116.1 (707)	2.5 (3073)	282.1 (667)
Kupfershiefer	0.27	65.7 (8942)	172.0 (1036)	183.4 (1022)	6.8 (4778)	788.9 (960)
Sandstone ore	4.40	179.5 (10,104)	241.1 (798)	170.9 (787)	2.3 (3387)	203.9 (752)
Deposit	5.34	272.3 (10,243)	426.8 (732)	338.4 (722)	7.75 (3255)	1039.5 (676)

3.2. Methods

The geostatistical ordinary block kriging procedure was used to analyze the accuracy of the estimates of element resources at different parts of the deposit, while the point ordinary kriging and lognormal ordinary kriging methods were applied to verify the accuracy of estimates of the abundance of elements at different points of the deposit. The procedures used were based on the results of describing the variability of the abundance of elements by means of variograms and their theoretical models. All calculations and graphs were made using ISATIS software (2020.12, Geovariances, Avon, France) [31].

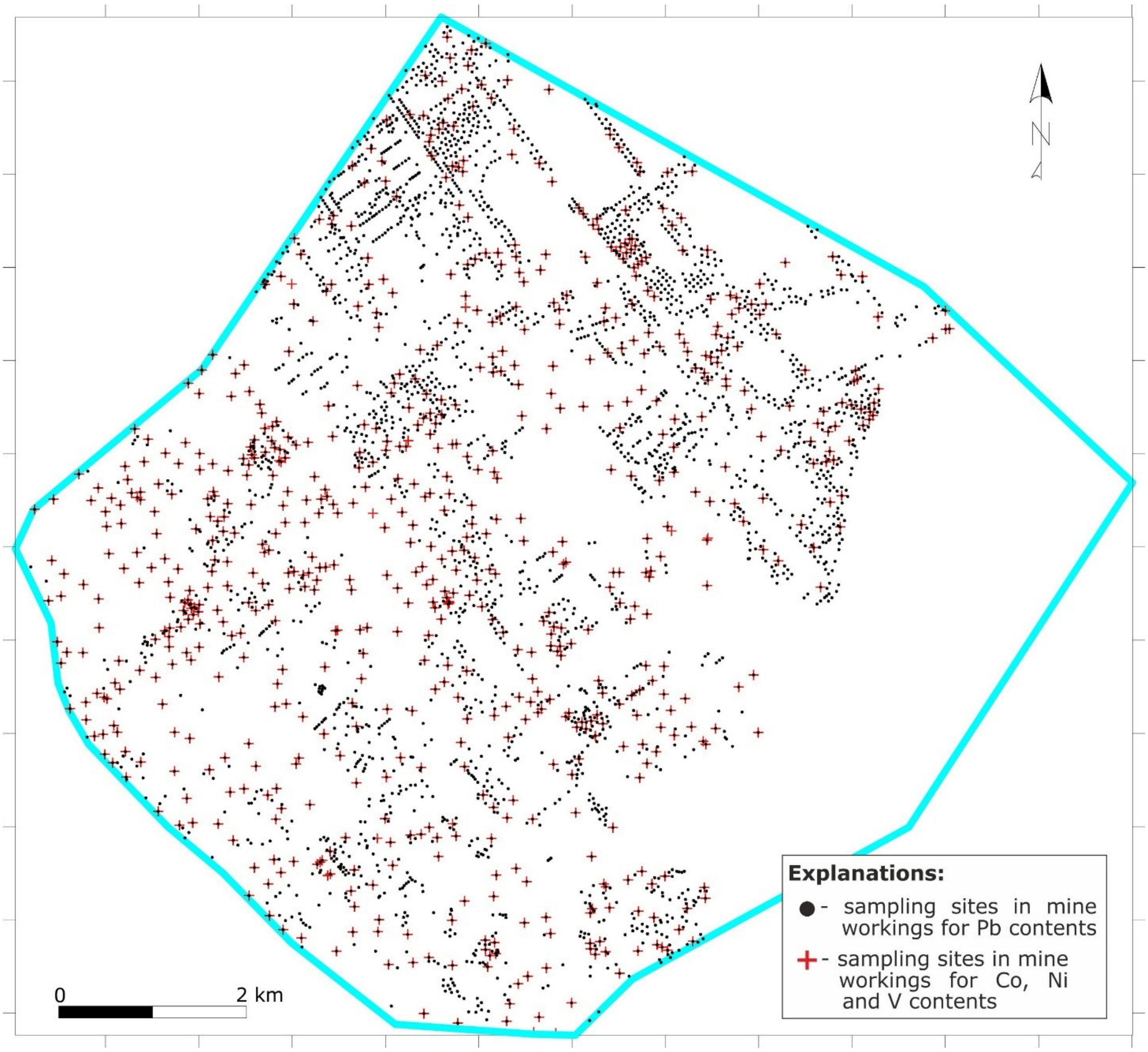

Figure 4. Location of sampling sites in mining excavations for the determination of cobalt, nickel, lead, and vanadium (Rudna deposit).

Geostatistical studies were preceded by a short statistical description covering the calculation of the basic measures of variability and the preparation of histograms and their approximation with the best theoretical models of probability distributions.

The spatial structure of the variability (2D) of copper and the accompanying elements in the Cu-Ag Rudna deposit was described using classical general relative variograms calculated from the formula [32]:

$$\gamma_R(\vec{h}) = \frac{1}{2n_h}\sum_{i=1}^{n_h}\frac{(z_{i+h} - z_i)^2}{(\bar{z}_h)^2} \quad (2)$$

where: $(\gamma_R(h))$—general relative variogram; (n_h)—number of pairs of samples distant by the vector $\vec{h}$; $(z_i,\ z_{i+\vec{h}})$—parameter values (abundance of elements) at sampling sites distant by the vector $\vec{h}$; $(\bar{z}_h)$—the average value of the parameter in all pairs of sampling points distant by the vector $\vec{h}$.

The use of relative variograms filters away the proportional effect and makes it easier to compare the structure of the variability of the abundance of elements with drastically different levels of average values. Relative variograms were approximated with theoretical models used in geostatistics. Directional variogram maps were constructed to investigate the potential anisotropy of the abundance of elements. The quality of fitting theoretical models to empirical relative variograms was verified in a simplified manner by means of linear correlation coefficients between the abundance estimated by ordinary point kriging at sampling sites and the observed abundance at these points (cross-validation procedure) [33,34].

The reliability of the point prediction of the abundance of the selected elements (Ni, Pb) based on variogram models was checked on randomly generated test sets using geostatistical ordinary and ordinary lognormal kriging.

The abundance in points of the test set (z_K^*) was estimated based on the weighted average algorithm:

$$z_K^* = \sum_{i=1}^{n} w_{iK} \cdot z_i \tag{3}$$

where: w_{iK}—weighting factor assigned to the "i" sample; z_i—parameter value at the "i" sampling point; n—number of sampling points used in the estimation (located in the neighborhood).

The weighting factors determined from the kriging equation system [35] take into account the mutual configuration of the sampling points that participate in the estimation, the location of these points in relation to the point of the test set and the previously established model of the relative empirical variogram. The determined weighting factors should guarantee an unbiased estimation (i.e., no systematic error) and minimization of the estimation error.

Lognormal kriging is used to estimate the values of variables characterized by lognormal or close to lognormal distribution, large positive skewness of the distribution and when there are outliers in the data set [36,37]. The variogram is computed and modeled on log-transformed data. Ordinary kriging has been used to estimate log-transformed data values in points, and then the estimates are converted back to original data units using the appropriate back-transformation formula. In practice, the main problems of applying lognormal kriging are related to back-transformation, which is sensitive to deviations from lognormality and may in some cases lead to serious systematic errors [35,38].

The assumptions and mathematical foundations of lognormal kriging were given by Journel and Huijbregts [35]. The difficulties of its practical application and proposals for their solutions were described in many works, including: Journel, Dowd, David, Deutsch and Journel, Sinclair and Blackwell, Cressie and Pavlicova, Webster and Oliver, Yamamoto, Yamamoto, Furuie, and Rossi and Deutsch [37–46].

The last stage of the study was to assess the accuracy of the estimates of the average abundance of elements in square blocks with areas of 0.25 and 1.0 km^2 using ordinary block kriging. As a measure of accuracy, the relative standard error of block kriging was adopted, which depends on the mutual configuration of sampling points in relation to each other and their location in the block, the structure of variability expressed with the use of the empirical variogram model, and the size and shape of the block within which the average abundance of the element is estimated.

4. Results

4.1. Statistical Features of the Abundance of Copper and Accompanying Elements

The abundances of Cu and accompanying elements in the main lithological units are variable. The abundances of Cu and Co dominate in the sandstone ore, while the abundances of Ni, Pb, and V in the Kupfershefer ore (Table 2), despite the small share of this ore in the resources (6%).

Many outliers are observed in the datasets, especially for the abundance of Pb and Co. This results in large values of the coefficients of variation and skewness of empirical distributions of abundance.

The relative variability of the abundance of elements expressed by the coefficient of variation (CV) is highly variable and depends on the studied element (Table 3). The abundance of Cu, Ni, and V (CV from 65–70%) is characterized by similar and high variability. The abundance of cobalt (CV = 96%) is highly variable, while the variability of the abundance of Pb (CV = 340%) is extremely high. The removal of outliers from the basic data sets causes a significant reduction in the value of the coefficient of variation only in the case of the abundance of Pb. Such a high variability of Pb abundance is caused by the tendency to concentrate the minerals of this element in characteristic "islands" or local zones with an irregular course, often exceeding the boundaries of the Cu-Ag deposit. The coefficients of variation of the abundance of elements are positively correlated with the skewness coefficients of the empirical distributions. In all cases, these distributions have positive skewness and the strongest skewness is observed in the case of Pb and Co (Figure 5). An obvious consequence of this is that the arithmetic means are much greater than the median values of the abundance of these elements (Table 3). The coefficients of quartile variation [47,48] are definitely smaller than the classical coefficients of variation, as they are insensitive to outliers. However, they reflect, identical to the classical coefficient of variation, the relations of the degree of variation in the abundance of the studied accompanying elements.

Table 3. Statistics for thickness and Cu abundance (aCu) and abundance of accompanying elements (aCo, aNi, aPb, and aV) in the Cu-Ag Rudna deposit.

Summary Statistics	aCu [kg/m^2]	aCo [g/m^2]	aCo * [g/m^2]	aNi [g/m^2]	aNi * [g/m^2]	aPb [kg/m^2]	aPb * [kg/m^2]	aV [g/m^2]	aV * [g/m^2]
Count	10,243	733	732	723	722	3267	3255	680	676
Arithmetic mean	272	431	427	395	391	8.7	7.7	1066	1039
Median	229	332	332	338	338	2.3	2.3	931	930
Standard deviation	185	429	409	276	253	29.5	12.9	803	724
Coeff. of variation (CV)	68%	99%	96%	70%	65%	340%	166%	75%	70%
Coeff. of quartile variation (CQV) †	39%	49%	49%	40%	40%	77%	77%	49%	49%
Minimum	50	10	10	12	12	0.01	0.01	25	25
Maximum	2066	3949	3143	3312	1764	1417	92	6464	4837
Range	2016	3939	3133	3301	1752	1417	92	6464	4812
Lower quartile (Q_1)	149	180	180	216	216	1.0	1.0	490	490
Upper quartile (Q_3)	337	522	522	505	505	7.7	7.7	1447	1447
Interquartile range	188	345	342	290	290	6.9	6.7	962	958
Skewness	2.27	3.22	2.88	2.82	1.56	34.47	2.81	1.87	1.12
Kurtosis	9.10	14.9	11.30	19.28	3.74	1596.15	8.72	6.99	2.08

Summary Statistics	Thickness			
	Ore Deposit	Carbonate Ore	The Kupferschiefer Ore	Sandstone Ore
Count	10,243	10,243	10,243	10,243
Arithmetic mean	5.34	0.76	0.24	4.34
Median	4.00	0.50	0.20	3.00
Standard deviation	3.54	0.90	0.19	3.55
Coeff. of variation (CV)	66%	118%	79%	82%
Minimum	0.20	0.00	0.00	0.00
Maximum	31.25	11.07	2.17	30.30
Range	31.05	11.07	2.17	30.30
Interquartile range	4.1	1.2	0.30	4.0
Skewness	1.52	1.87	0.78	1.56
Kurtosis	2.64	6.08	1.17	2.73

Explanations: * for data filtered out from outliers and used to estimate the variograms; † coefficient of quartile variation calculated as $CQV = \frac{(Q_3-Q_1)}{(Q_3+Q_1)} \cdot 100\%$ [48].

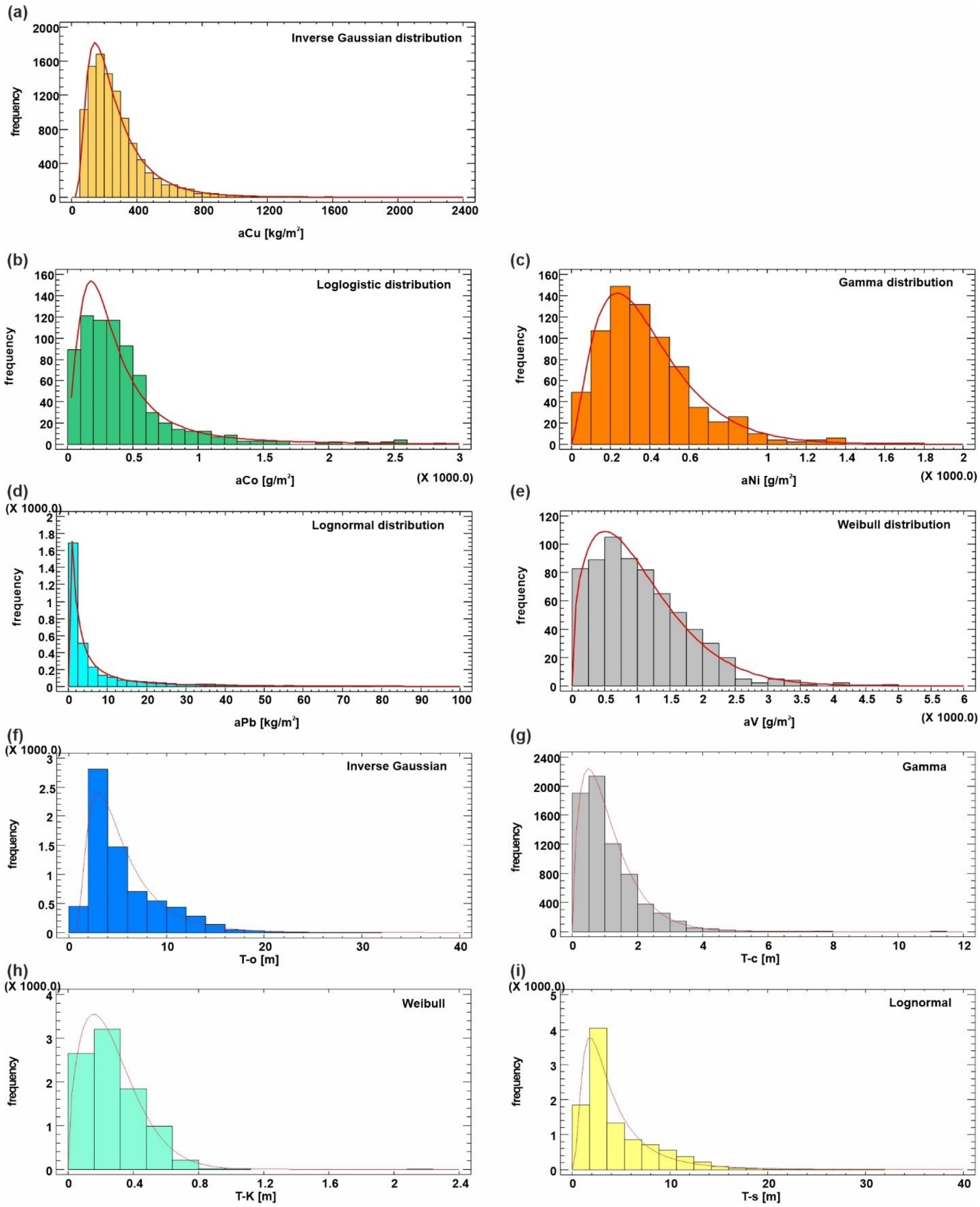

Figure 5. Histograms and optimal theoretical distributions of metal abundance (aCu, aCo, aNi, aPb, and aV (**a**–**e**)) and thickness for fitted to empirical distributions according to the log-likelihood statistic (Explanations: (**f**): (T-o)—thickness of ore deposit, (**g**): (T-c)—the carbonate ore, (**h**): (T-K)—the Kupferschiefer ore, (**i**): (T-s)—sandstone ore); K-S Test—Kolmogorov-Smirnov test of the consistency of the empirical distribution with the assumed theoretical distribution, p-value ≤ 0.05—rejection of the hypothesis on the consistency of the empirical distribution with the assumed theoretical distribution for significance level 0.05, p-value > 0.05—there are no grounds to reject the hypothesis that the empirical distribution is compatible with the assumed theoretical distribution.

Graphs of theoretical distributions best fitted to empirical distributions according to the log-likelihood statistic are presented in Figure 5. According to this criterion, the best models of empirical distributions of the abundance of each element are different. However, it should be noted that the Kolmogorov-Smirnov test rejects the hypothesis that the empirical distribution is consistent with the assumed theoretical probability distribution for the abundance of Cu and Pb (Figure 5).

The Pearson correlation coefficients and Spearman's rank correlation coefficients between the abundances of the primary variable (Cu) and the secondary variables (Pb, Ni, V, Ag, Co) are shown in Figure 6. In general, the abundance of correlations between the Cu abundance and the abundance of accompanying elements are weak (with correlation coefficients <0.5) and for some pairs of variables, statistically insignificant (for significance level 0.05). Therefore, it can be assumed that the use of more advanced geostatistical methods taking into account auxiliary variables (e.g., collocated cokriging [49], regression kriging [50]) will only contribute to increasing the accuracy of the estimates of the accompanying elements resources to a limited extent.

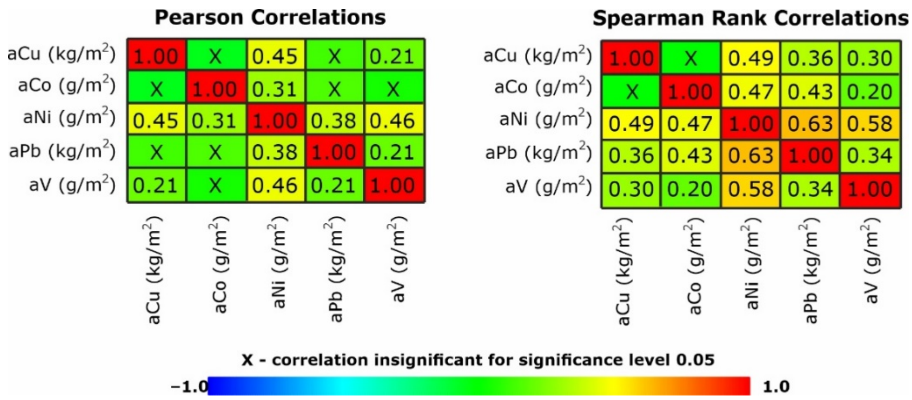

Figure 6. Pearson correlations and Spearman rank correlations between Cu abundance (aCu) and abundance of accompanying elements (aCo, aNi, aPb, and aV) in the Cu-Ag Rudna deposit.

4.2. Variography

Calculations of relative variograms were preceded by the elimination of outliers from data sets, strongly distorting the form of empirical variograms and masking the non-random component of variability. Elimination of outliers was performed on the basis of the variogram cloud in ISATIS software [31]. The significance and purposefulness of using such a procedure was illustrated on the example of the set of Pb abundances distinguished by numerous anomalously high values (Figure 7). Removal of only one outlier of Pb abundance diametrically changes the form of the variogram, which is manifested by a drastic reduction of its amplitude (the maximum level of differentiation) by about 85% and reveals the occurrence of a non-random component, expressed by an increase in the model value over a certain distance range.

The equations of the theoretical models (exponential and spherical) fitted to the relative variograms of the deposit parameters (the thickness and the abundance of elements) are presented in Table 4 and their graphs in Figure 8. In the case of Co, Pb, and V abundance, simple models were fitted, while for Cu and Ni abundance, nested models were applied.

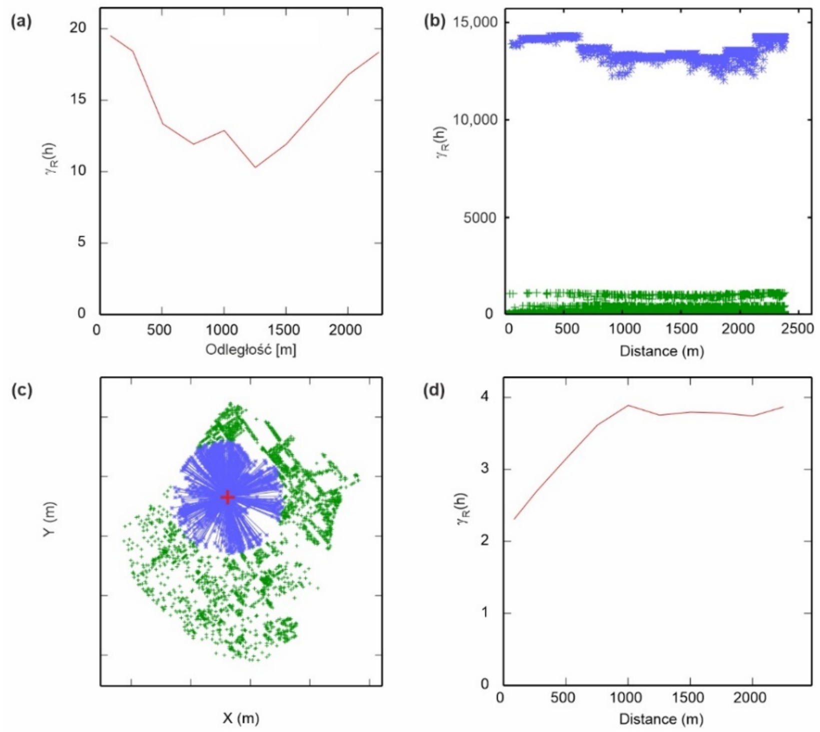

Figure 7. Variogram of Pb abundance before removing the outlier (**a**). Variogram cloud (**b**). Location of sampling points (**c**). Variogram of the Pb abundance after removing the outlier (**d**). Explanations: the values of the variogram related to the outlier value of Pb abundance (**b**) and the pairs of sampling points included in their calculation (**c**) are marked in blue; other values of the variogram (**b**) and the sampling points included in their calculation (**c**) are marked in green; red color—outlier of the Pb abundance (**c**).

Table 4. Equations of geostatistical relative models (omnidirectional) of deposit parameters (copper abundance, abundance of accompanying elements, thickness) in the Cu-Ag Rudna deposit.

Deposit Parameter		Parameters of Geostatistical Models				r	f [%]
		Model	C_0	C_i	a_i [m]		
Abundance	Cu	exponential exponential	0.142	0.180 0.308	765 14,141	0.76	22.5
	Co	spherical	0.36	0.75	6419	0.46	32.4
	Ni	spherical spherical	0.21	0.13 0.07	518 2882	0.38	51.2
	Pb	spherical	1.49	1.19	1002	0.52	55.6
	V	exponential	0.18	0.30	960	0.49	37.5
Thickness	ore deposit	exponential	0.15	0.29	1600	0.76	34.1
	carbonate ore	spherical	0.42	0.16 0.57 0.13	533 4860 1255	0.80	32.8
	the Kupferschiefer ore	spherical	0.20	0.39	1157	0.78	33.9
	sandstone ore	exponential	0.18	0.48	1600	0.78	27.3

Explanations: C_0—nugget variance, C_i—partial sill, a_i—range of variogram model, r—Pearson's correlation coefficient between estimated and raw data at sampling sites, (f) percentage ratio of nugget variance (C_0) in total sill (C_0 + C) calculated using formula $f = \frac{C_0}{C_0+C} \cdot 100\%$.

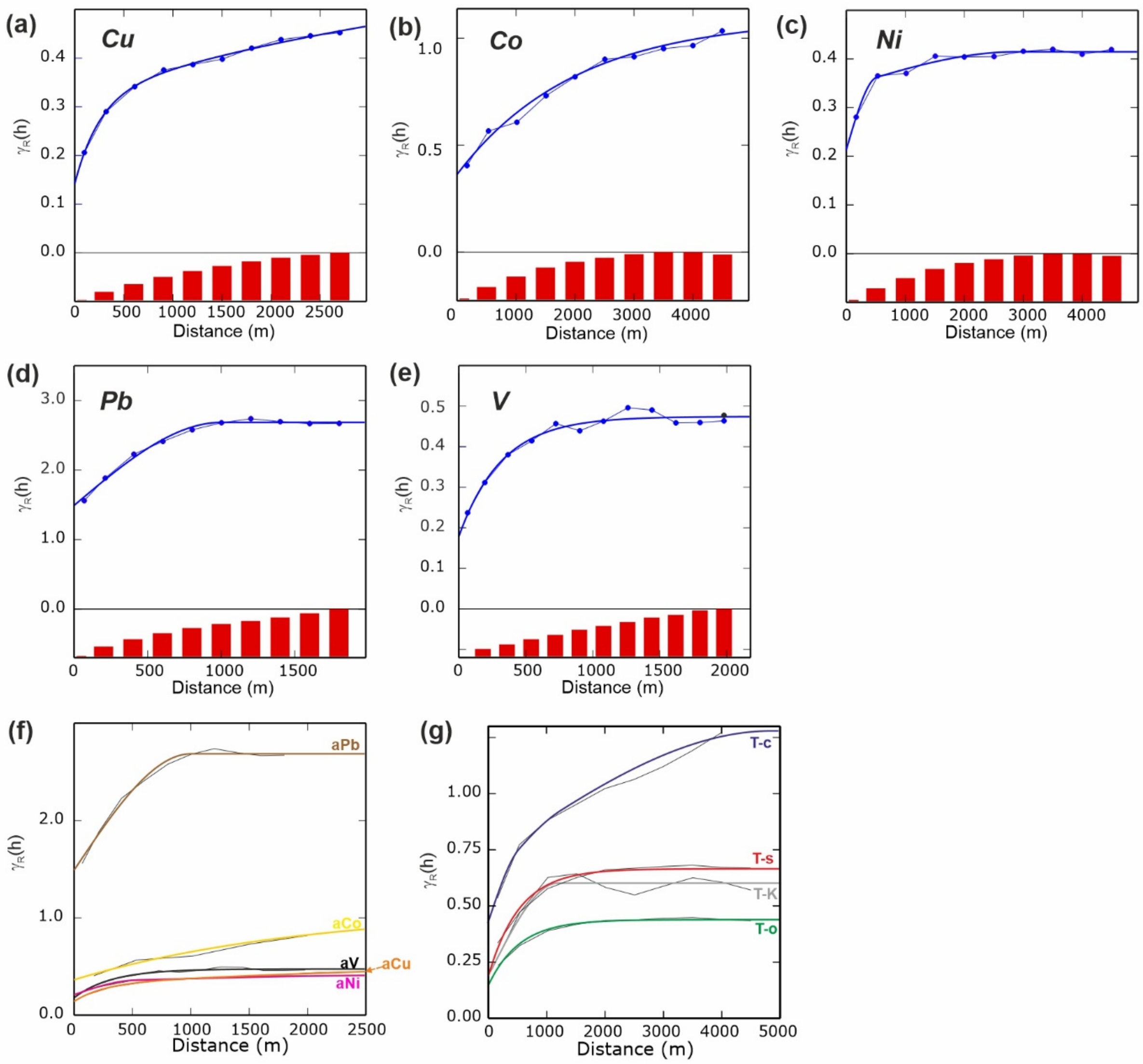

Figure 8. (**a**–**e**) Relative variograms (points) and their theoretical models (solid lines) of the Cu abundance and the abundance of accompanying elements in the Cu-Ag deposit; (**f**) Comparison of relative variograms and their theoretical models in one scale; (**g**) Comparison of relative variograms and their theoretical models of the thickness of ore deposit (T-o) and three basic lithological series (T-c—the carbonate ore, T-K—the Kupferschiefer ore, T-s—sandstone ore).

The percentage ratio of nugget variance (C_0) in total sill (C_0 + C) of the models, calculated on the basis of the theoretical variogram equations (Table 4), is varied for the abundance of the studied elements and increase in the order of Cu, V, Co, Ni, and Pb consistent with the decreasing continuity of changes in the abundance of elements. The linear correlation coefficient calculated in the cross-validation procedure between the estimated and actual abundances at the sampling points is the highest for the Cu abundance (0.76), moderate for the Co, Pb, and V abundance (0.46–0.52), and the lowest for the Ni abundance (0.38). In the case of the deposit thickness and lithological series, the percentage ratio of nugget variance is relatively small (27–34%) while the linear correlation coefficients are as high (0.76–0.8) as in the case of the Cu abundance.

The research on the anisotropy of thickness and abundance was carried out using relative directional variograms and their contour maps (Figures 9 and 10).

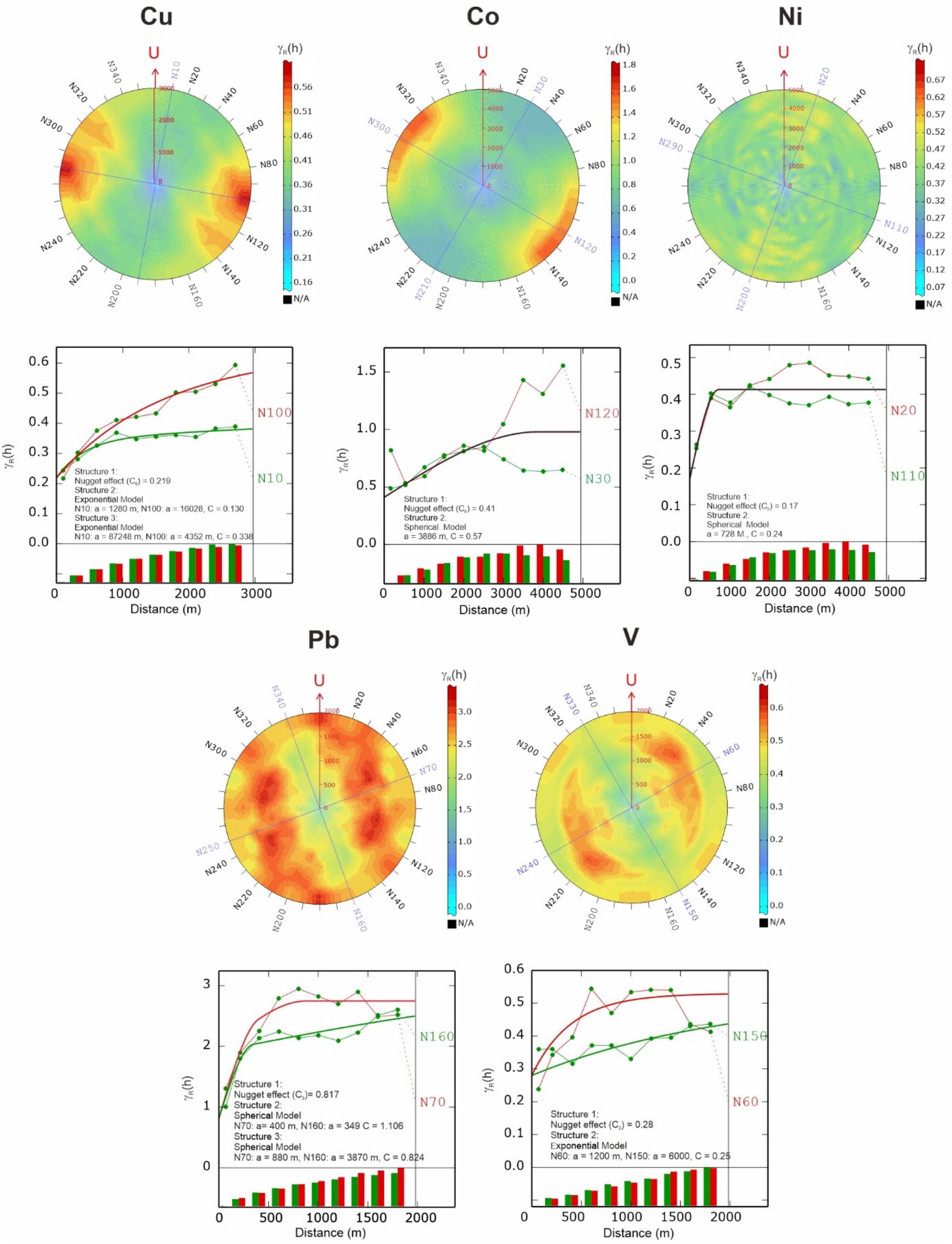

Figure 9. Maps of relative directional variograms of elements abundance and variograms for the directions of minimum (green lines) and maximum (red lines) variability approximated by theoretical models (the anisotropy direction were measured in degrees clock-wise from the North).

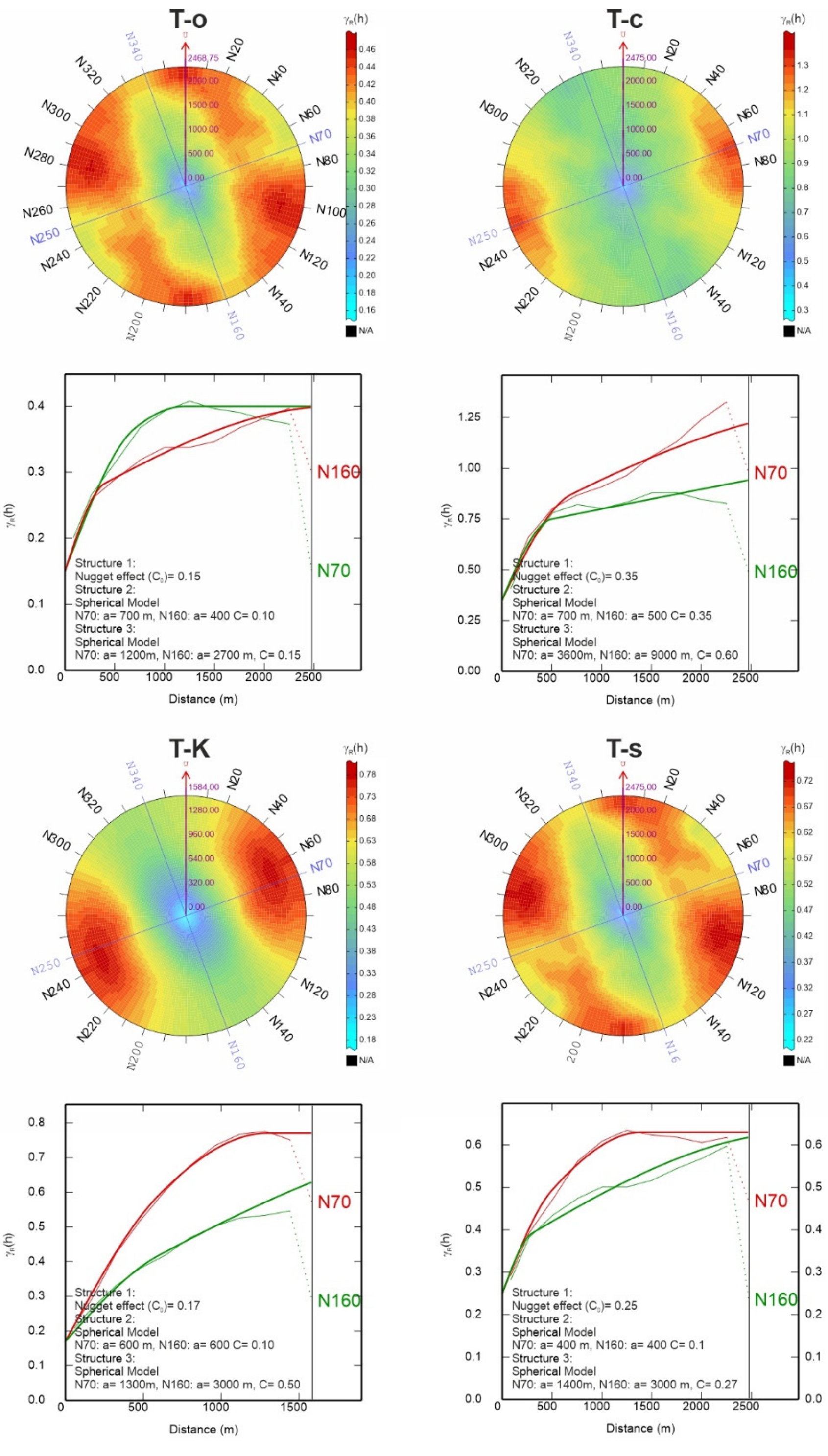

Figure 10. Maps of relative directional variograms of deposit thickness and variograms for the directions of minimum (green lines) and maximum (red lines) variability approximated by theoretical models (the anisotropy direction were measured in degrees clock-wise from the North; T-o—thickness of ore deposit, T-c— the carbonate ore, T-K—the Kupferschiefer ore, T-s—sandstone ore).

In the case of Ni abundance, the practically isotropic nature of the variability is observed, as in the case of Co, for a distance of about 2.5 km. Variability of the abundance of the remaining elements and deposit thickness is usually characterized by moderate or weak anisotropy. It is practically insignificant due to the high local variability represented by the high nugget variance ratio [51]. The cross-validation procedure performed again for the anisotropic models showed only an insignificant increase in the correlation coefficients between the estimated and actual abundances in the sampling sites. For this reason, only isotropic variogram models (omnidirectional) were used in further procedures to estimate the values of the parameters and errors using ordinary kriging. The anisotropy directions of the abundance of elements and deposit thickness can generally be described as NW-SE and NE-SW.

4.3. Accuracy of the Point Prediction of Ni and Pb Abundance

The prediction of Ni and Pb abundance at the test sampling points was performed using ordinary kriging (OK) and lognormal ordinary kriging (LOK). Accuracy assessment was based on a comparison of the predicted abundance with the real abundance in the test set. The reason for choosing Ni and Pb abundance was the extreme difference in their variability as shown by the coefficient of variation (CV): the highest in the case of Pb abundance (CV = 166%) and the lowest in the case of Ni abundance (CV = 65%) (Table 3). The basis of the test procedure consisted of 182 measurements of Ni abundance and 321 measurements of Pb abundance (in separate parts of the deposit, respectively, with an area of 9.4 km^2 and 8 km^2) (Figure 11). The average distance between neighboring test points was about 240 m for Ni and 170 m for Pb.

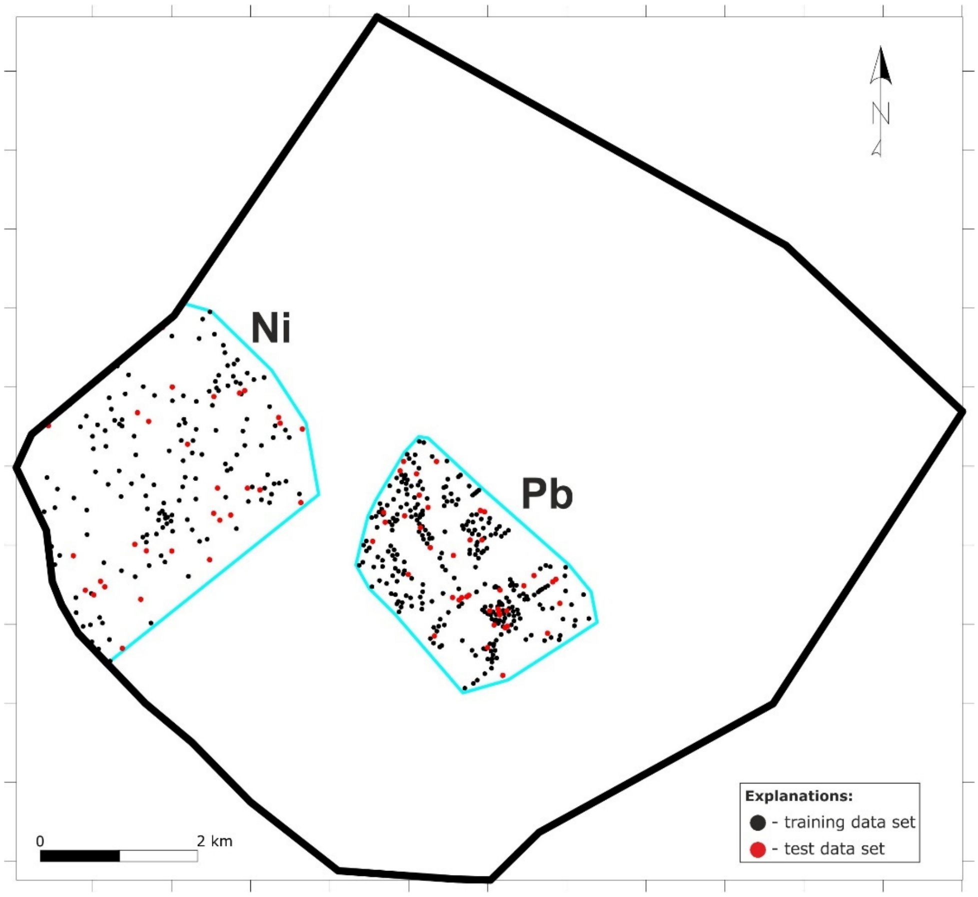

Figure 11. Location of sampling sites selected for the point test of the reliability of Ni and Pb abundance prediction in parts of the Rudna deposit.

From the original data sets, test sets of 30 samples for Ni and 41 samples for Pb were randomly generated for each metal separately. The remaining samples were training sets, which were used to estimate the abundance of elements in the samples of the test sets.

To assess the accuracy of the prediction of the abundance of metals in the test sets, two types of errors were calculated: mean error (ME) and mean absolute error (MAE):

$$\mathrm{ME} = \frac{1}{n}\sum_{i=1}^{n}(z_i^* - z_i) \quad (4)$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}|z_i^* - z_i| \quad (5)$$

where: z_i^* i z_i—respectively: estimated and actual (measured) metal abundance at the "i" sampling point of the test set, n—number of data in the test set.

The mean error (ME) can be treated as a measure of the systematic prediction error, while the mean absolute error (MAE) can be treated as a measure of the random prediction error.

The Kolmogorov-Smirnov test used to assess the compliance of empirical distributions with theoretical lognormal (Ln) and lognormal three-parameter (Ln-3P) distributions (Figure 12) gave the following P-value: for Ni abundance 0.10 and 0.45, respectively, and for Pb abundance, respectively, 0.38 and 0.21. At the significance level of $\propto$ = 0.05, the calculated P-value greater than 0.05 does not provide grounds for rejecting the hypothesis of a lognormal or lognormal three-parameter distribution of the abundance of both elements in the general population.

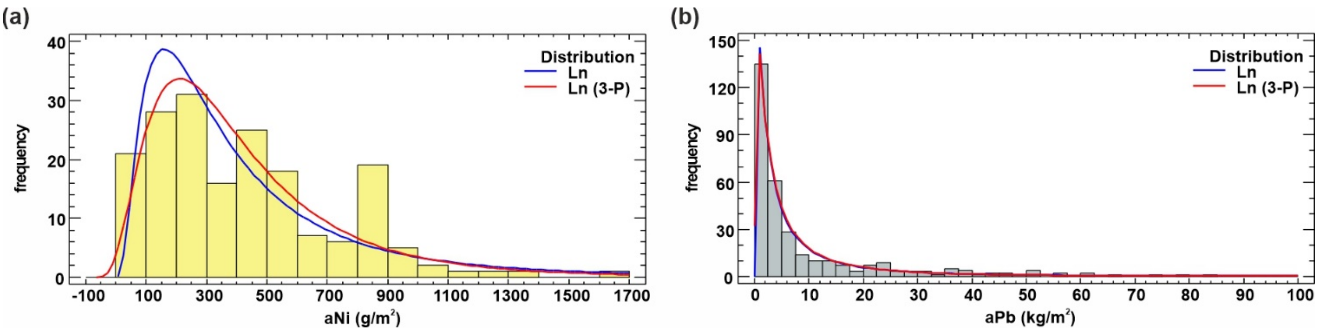

Figure 12. Histograms and theoretical distributions (lognormal (Ln) and lognormal three-parameter (Ln 3-P)) of the abundances of Ni ((**a**): aNi) and Pb ((**b**): aPb) for sets used to validate the prediction quality (combined training and testing datasets) (Figure 11).

The results of the assessment of the accuracy of the prediction of metal abundance, presented in Table 5, indicate that the random error of the prediction of Ni abundance (MAE: 43–51%) is clearly lower than the analogous error of the prediction of Pb abundance (MAE: 66–69%).

Table 5. The results of the assessment of the accuracy of the Ni and Pb abundance estimates in the test sets based on 2 geostatistical methods (the errors related to the average abundance in the test set, expressed as a percentage, are shown in brackets).

Method	**Ni [g/m^2] The Size of the Test Set: 30**		**Pb [kg/m^2] The Size of the Test Set: 41**	
	Mean error (ME) [g/m^2]	Mean absolute error (MAE) [g/m^2]	Mean error (ME) [kg/m^2]	Mean absolute error (MAE) [kg/m^2]
Ordinary kriging (OK)	37.2 (8.6%)	218.5 (50.5%)	−0.41 (−4.3%)	6.43 (67.3%)
Lognormal kriging (LOK)	−2.4 (−0.6%)	189.7 (43.9%)	2.22 (23.3%)	6.35 (66.5%)

However, for the abundance of both metals, the error is very high. The use of lognormal kriging (LOK) gives, compared to ordinary kriging (OK), a slight (several percent) reduction in the magnitude of the random error of the prediction of Ni abundance, but from a practical point of view, it is of little importance. The sizes of the random errors in the prediction of the abundance of Pb are practically identical for both methods. It can be assumed that the random errors of prediction for the remaining elements (Co and V) will be within the limits of random errors for the prediction of Ni and Pb abundance, i.e., roughly in the range of 45–65%. Systematic errors (ME) in the prediction of abundance are generally relatively small (from −4.3% to 8.6% of the mean abundance in the test set), except for the abundance of Pb predicted using lognormal kriging (LOK) for which it is significant and amounts to 23% of the mean abundance of this metal in the test set (Table 5). This unfavorable result may be caused by a too large disagreement of the empirical distribution of Pb abundance from the ideal lognormal distribution, despite the fact that the Kolmogorov-Smirnov test does not provide grounds for rejecting such a hypothesis. According to Sinclaire and Blackwell [42], in general, lognormal kriging should be avoided and an equivalent alternative procedure, such as ordinary kriging with a relative variogram, should be used.

Generally, the low accuracy of the prediction of the abundance of both metals is related to the large spacing of the samples and additionally, in the case of Pb abundance, to its extremely high variability and the presence of numerous outliers. For this reason, the estimation of the abundance of elements accompanying the Cu-Ag deposits of the LGCD may be rationally justified only within larger parts of the deposit corresponding to the area of, for example, the annual exploitation of the deposit.

4.4. Accuracy of Estimation of Accompanying Elements in Deposit Blocks

For the initial assessment and comparison of the accuracy of the estimates of the average abundance and thus the Cu resources and resources of the four accompanying elements in the deposit blocks, ordinary block kriging, based on the models of relative variograms, was used. The procedure was carried out for two types of square blocks with highly differentiated areas: 0.25 km^2 (0.5 km × 0.5 km) and 1 km^2 (1 km × 1 km), covering the area of the Rudna deposit recognized by mining excavations. The total area of annual exploitation of deposits in LGCD mines is within the given limits. The theoretical relative standard kriging error was adopted as a measure of the accuracy of the estimates. The average abundance was estimated based on all data contained in the block, provided that their number was at least 2 (otherwise, the estimation was abandoned).

The calculation results for all elementary blocks, including the medians of the kriging errors and their lower and upper quartiles, are summarized in Figure 13 and Table 6. Relative errors were ranked in ascending order, which allowed us to present the ranking of the accuracy of the estimated abundance of the analyzed elements (Figure 13).

Table 6. Quartiles of relative standard kriging errors of estimates (σ_{KR} [%]) of the average Cu abundance and resources and the accompanying elements in blocks with areas 0.25 and 1.0 km^2.

Statistics	**Blocks 1000 m × 1000 m**					**Blocks 500 m × 500 m**				
	aCu [kg/m^2]	**aNi [g/m^2]**	**aV [g/m^2]**	**aCo [g/m^2]**	**aPb [kg/m^2]**	**aCu [kg/m^2]**	**aNi [g/m^2]**	**aV [g/m^2]**	**aCo [g/m^2]**	**aPb [kg/m^2]**
	σ_{KR} [%]									
Number of blocks	72	70	69	70	77	279	173	162	173	254
Lower quartile	10	19	21	24	32	11	29	30	35	42
Median	10	22	24	27	39	12	33	35	41	51
Upper quartile	12.5	27	30	33	47	15	39	42	48	71

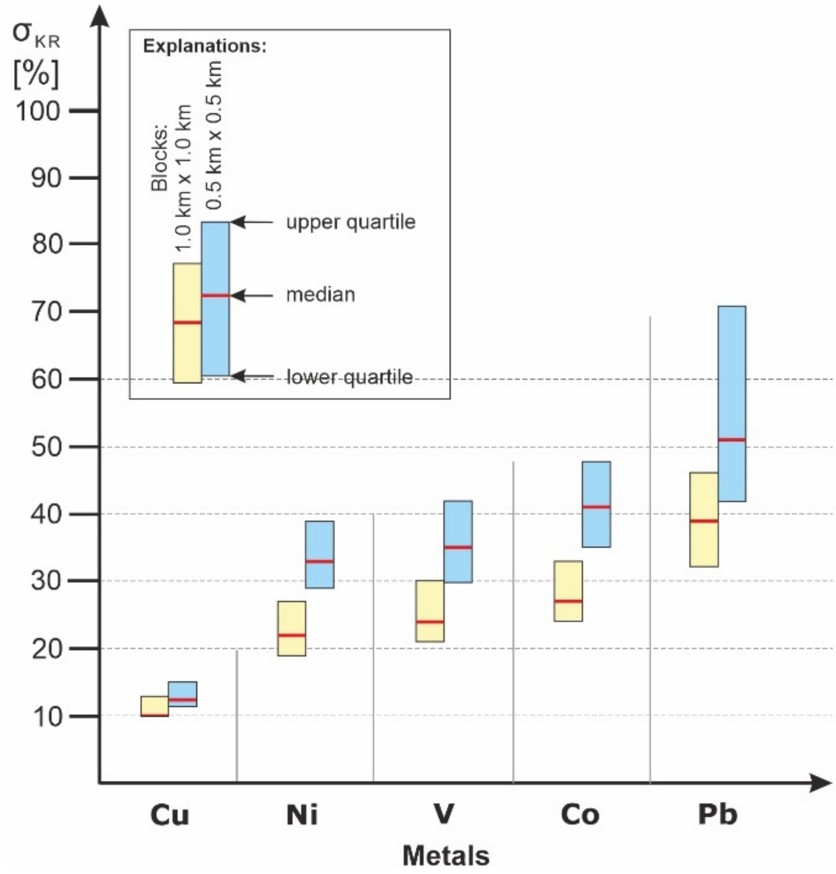

Figure 13. The ranking of the accuracy of the estimation of the average abundance of Cu and accompanying elements in blocks with an area of 0.25 and 1.0 km^2 in the Rudna deposit, expressed with the use of quartiles of standard block kriging relative error (σ_{KR}).

The average abundance of the accompanying elements is estimated, in comparison to the abundance of the basic Cu element, with much less accuracy. The medians of errors of the Ni, V, and Co abundance estimation are roughly 2–3 times and in the case of $\times$the abundance of Pb 4–5 times greater than the medians of the estimates of the abundance of Cu, which are of the order of 10–12% (Table 6). It is also seen in Figure 13 that the estimation kriging error (σ_{KR}) decreases as the estimation surface area increases, a well-known result established by Krige [52,53] and demonstrated by Matheron [54,55] using the dispersion variance expressed in function of the variogram.

The effect of the number of samples within square blocks with a side of 1 km on the value of the kriging error of the estimation of the average abundance of accompanying elements is shown in Figure 14. In the case of the abundance of all accompanying elements, except for lead, a significant decrease in the estimation error was observed when considering at least 5 samples in the calculations (Figure 15). Assuming the maximum relative kriging error of the estimation of the average abundance of accompanying elements of 25%, the approximate minimum number of samples located within the 1 km^2 calculation block should be: 13 for Co, 7 for Ni, 8 for V, and as much as 417 for Pb.

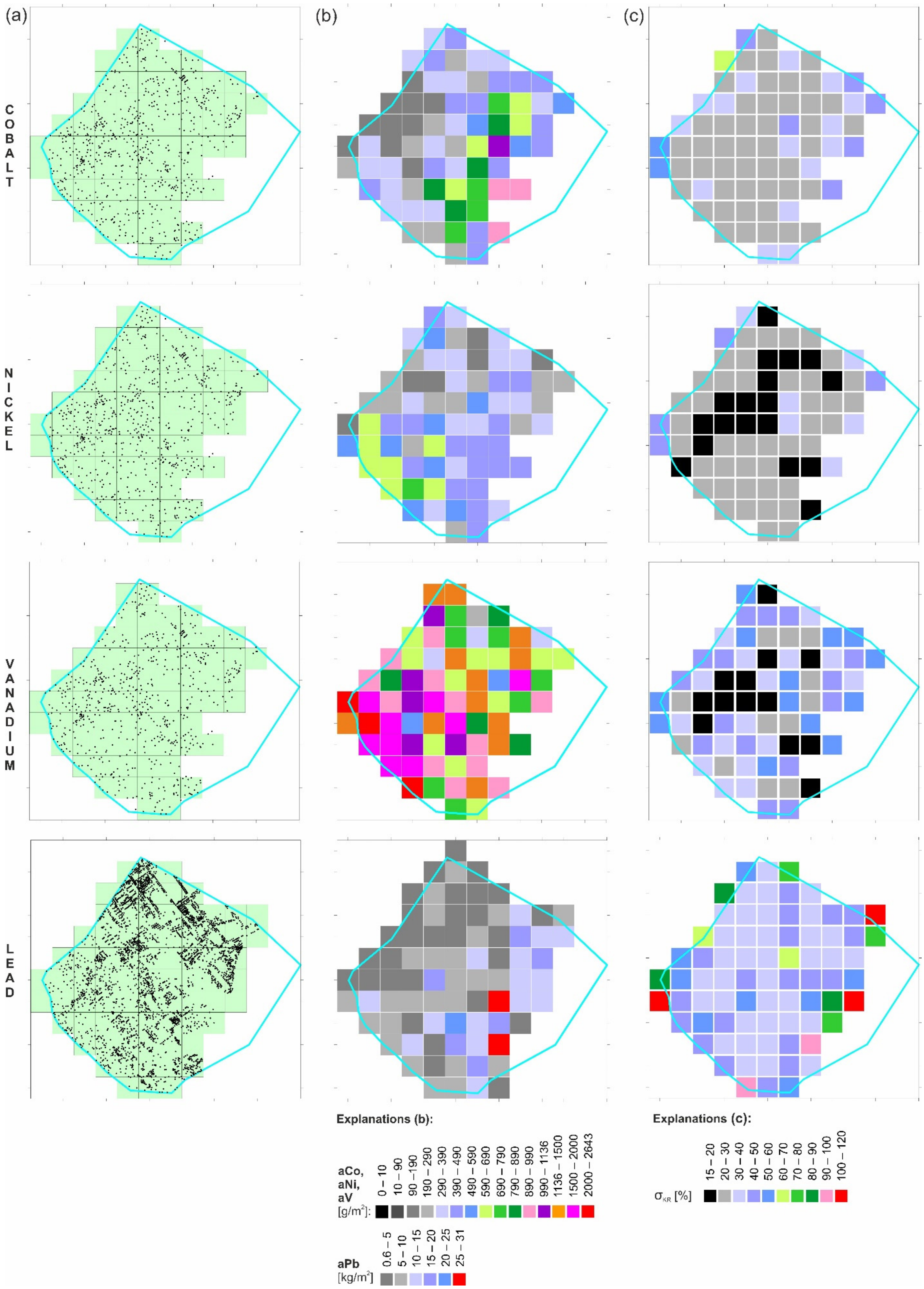

Figure 14. Location of the Co, Ni, V, and Pb sampling points in the Rudna deposit and division into calculation blocks (1 km × 1 km), where the estimates were made (**a**). The summary of the estimated mean abundance of Co, Ni, V, and Pb (aCo, aNi, aV, and aPb) (**b**) and relative standard kriging errors (σ_{KR}) (**c**).

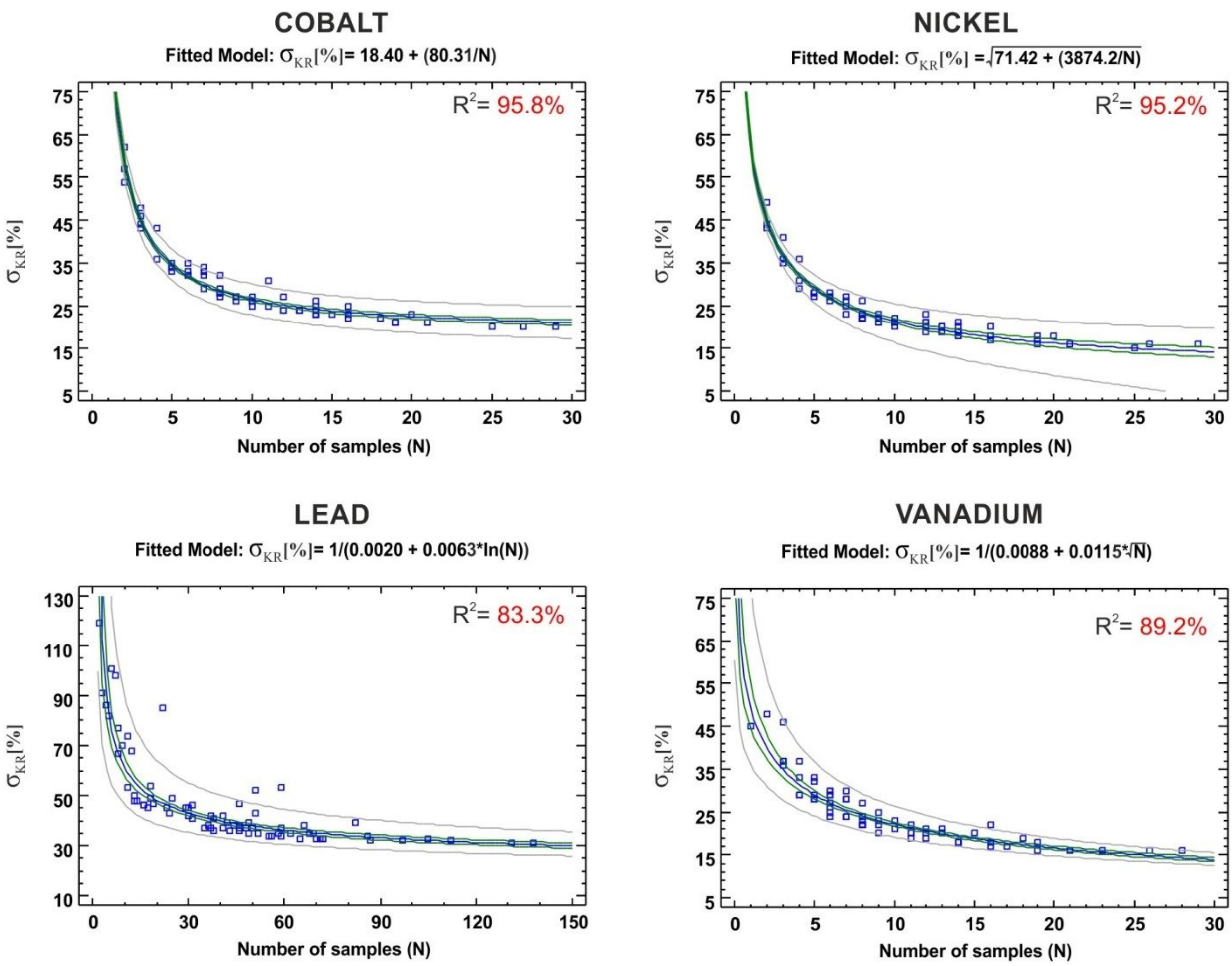

Figure 15. Plots of dependence between the relative standard kriging errors (σ_{KR}) of the estimated mean abundance of elements and the number of samples in 1 km × 1 km blocks (Explanation: R^2—coefficient of determination).

5. Summary and Discussion

A common feature of the empirical distributions of the abundance of the studied elements (Co, Ni, Pb, and V) of the Cu-Ag Rudna ore deposit is their positive skewness and the presence of outliers, exceptionally numerous in the case of Pb abundance. The theoretical models that best approximate the empirical distributions are different for each element. The statistical and geostatistical features of the abundance of Ni and V are similar to the features of the abundance of copper (main metal). This is expressed by a similar intensity of variability with variability coefficients in a narrow range of 65–70% and a similar geostatistical structure of variability represented by the values of relative variograms. Extremely different statistical and geostatistical properties are revealed by the abundance of Pb, which is distinguished by a large variability (with a coefficient of variation of 340%) and high values of the relative variograms despite the removal of anomalous values from the data set, completely masking the nonrandom structure of variability. This can be explained by the specificity of the distribution of Pb minerals (mainly galena) in the deposit zone, occurring small "island-like" patches, often exceeding the boundaries of the Cu-Ag deposit in the vertical profile. The abundance of cobalt, due to high relative variability (with a coefficient of variation of 96%) and the high values of the relative variograms, occupies an intermediate position between the abundance of Pb and the abundance of Cu, Ni, and V.

Copper abundance is characterized by the highest horizontal continuity with the lowest percentage ratio of nugget variance amounting to 23%. In the case of the abundance of other elements, it is much lower, as evidenced by a higher percentage ratio of nugget variances, ranging from 32% (Co) to 56% (Pb).

The mentioned unfavorable statistical characteristics related to the abundance of accompanying elements, combined with increased intervals of sample collection, resulted in significantly lower accuracy of estimation of the average abundance and, at the same time due to the established block boundaries, lower accuracy of estimation of the resources of these elements.

The ordinary kriging procedure used for blocks with an area of 0.25 km^2 and 1 km^2 showed that the medians of the relative standard kriging errors of the copper resources estimates of 12% and 10%, respectively, can be considered suitable for geological and mining applications. In comparison, the median errors of the Ni, V, and Co resource estimates are about 2–3 times greater and about 4–5 times greater for the Pb resource estimates. The drastically low accuracy of the estimation of the Pb resource is mainly due to the extremely high variability of the abundance of Pb because the average sampling interval for determinations of this element (~180 m) is more than two times smaller than for Co, Ni, and V (~400 m). Assuming square blocks with an area equal to 1 km^2 and the permissible (maximum) standard error of 25% for resource calculations, it is theoretically necessary to collect at least 7–13 samples of Ni, V, and Co and as many as 417 samples of Pb. Despite the adoption of lower requirements with regard to the accuracy of the estimates of the abundance of accompanying metals, the achievement of the assumed accuracy of the Pb resource estimates requires a significant increase in the number of collected samples (in relation to the currently collected ones) by appropriately reducing the sampling interval.

6. Conclusions

So far, the accuracy of the estimates of the resources of accompanying elements in Cu-Ag ore deposits has not been thoroughly studied. This is probably due to the structure of the sales value of recovered metals, where the share of accompanying elements is generally low. This also explains the lower interest of users or deposit owners in obtaining high accuracy of the estimates of their resources and the use of larger sampling intervals for determining their content than in the case of the main metals. For this reason, it seems understandable to apply lower requirements regarding the accuracy of the estimates of the accompanying elements compared to the corresponding requirements for the main elements. Under the conditions of the Cu-Ag Rudna deposit, the assumption of the maximum size of the standard kriging error equal to 25% when estimating the resources of accompanying elements in the parts of the deposit to be exploited within 1 year can be considered rational. This criterion is easily obtained for the estimation of Co, Ni, and V resources when there are roughly a dozen samples available. Additionally, considering that Co and V are not currently recovered from the mined ore, even an approximate estimate of their resources seems to be fully sufficient.

In the case of Pb, the accuracy of estimating the resources of this metal is insufficient given the current sampling pattern of the deposit in mine workings. Considering that the Cu-Ag deposits of LGCD will soon become the only source of Pb production from Polish deposits, it seems advisable to consider actions aimed at increasing the accuracy of the prediction of the size of this metal resource. The use of more advanced geostatistical methods for this purpose would not bring the expected result due to the extremely uneven distribution of the minerals of this element in the Cu-Ag deposit. The simplest solution resulting in an increase in the accuracy of the estimates would be to reduce the horizontal sampling interval for Pb and match it with the Cu sampling interval. To limit the number of additional Pb samples, it is sufficient to take a single sample from each lithological unit through its entire thickness within the boundaries of the deposit, rather than collecting samples from vertical 20 cm long sections as is the case with samples collected to determine the Cu content.

Undoubtedly, the proposed procedure will contribute to a certain increase in sampling costs, but it can be considered justified and does not impose an excessive financial and organizational burden.

Obligatory control sampling may be recommended to verify the correctness of primary determinations in places where outlier contents of accompanying elements were found. When the outliers form compact zones, it is advisable to contour them, e.g., by means of indicator kriging with the assumed probability level, and to estimate the resources of the accompanying elements in these zones separately in relation to other parts of the deposit with no anomalous values. If outliers occur locally throughout the study area and do not form clusters, contouring them is unnecessary. In this case, consideration should be given to excluding outliers from the data set, e.g., based on a variogram cloud, in order to avoid overestimating resources and the magnitude of predicted errors.

Increasing the accuracy of estimating the resources of accompanying elements in parts of the deposit scheduled for exploitation in the following years is necessary to achieve a precise and reliable assessment of the profitability of exploitation, which is difficult to achieve with the current sampling pattern.

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References

1. Newman, P.; Meader, N.; Klapwijk, P.; Liang, J.; Chou, E.; Gao, Y.; Barot, H.; Furuno, A.; Rey, F.; Yau, S.; et al. *Metals Focus. World Silver Survey*; The Silver Institute and Metals Focus: Washington, DC, USA, 2021.
2. Malon, A.; Tymiński, M.; Mikulski, M.Z.; Oszczepalski, S. Surowce metaliczne. In *Bilans Zasobów Złóż Kopalin w Polsce*; Szuflicki, M., Malon, A., Tymiński, M., Eds.; Państwowy Instytut Geologiczny—Państwowy Instytut Badawczy: Warszawa, Poland, 2020.
3. Banaś, M.; Kijewski, P.; Salamon, W.; Pieczonka, J.; Piestrzyński, A. *Monografia KGHM Polska Miedź S.A.*; Pierwiastki Towarzyszące w złożu rud miedzi; KGHM Cuprum Sp. z.o.o. CBR: Wrocław, Poland, 2007; pp. 214–228.
4. Chmielewski, T. Development of a Hydrometallurgical Technology for Production of Metals from Kghm Polska Miedz S.A. Concentrates. *Physicochem. Probl. Miner. Process.* **2015**, *51*, 335–350. [CrossRef]
5. Namysłowska-Wilczyńska, B. Application of Kriging to the Determination of Homogeneous Blocks of Cu Ore Deposits. *Sci. Terre Inf. Geol.* **1988**, *7*, 279–290.
6. Namysłowska-Wilczyńska, B. Geostatistical Estimation of Cu Ore Deposit. *Sci. Terre Inf. Geol.* **1990**, *29*, 63–74.
7. Namysłowska-Wilczyńska, B. Support Effect in Light of Results of Geostatistical Analysis of Copper Ore Deposits Variability. *Sci. Terre Inf. Geol.* **1995**, *32*, 279–300.
8. Namysłowska-Wilczyńska, B. Geostatistical Methods Used to Estimate Sieroszowice Copper Ore Deposit Parameters. *Z. Geol. Wiss.* **2012**, *40*, 329–361.
9. Namysłowska-Wilczyńska, B. Application of Turning Bands Technique to Simulate Values of Copper Ore Deposit Parameters in Rudna Mine (Lubin-Sieroszowice Region in SW Part of Poland). *Georisk Assess. Manag. Risk Eng. Syst. Geohazards* **2015**, *9*, 224–241. [CrossRef]
10. Namyslowska-Wilczynska, B. Application of Geostatistical Techniques for the Determining of an Anomalous Zone of Copper Ore Deposit in the Area of Polkowice Mine (Region of Lubin-Sieroszowice, SW Part of Poland). *Geoinf. Geostat. Overv.* **2019**, *7*, 1–22. [CrossRef]
11. Wasilewska-Błaszczyk, M.; Mucha, J. Geochemical Modeling of the Cu-Ag Deposits from the Lubin-Głogów Copper District (Poland) Supported by Lithological Modeling. *Gospod. Surow. Miner.—Miner. Resour. Manag.* **2017**, *33*, 63–78. [CrossRef]
12. Wasilewska-Błaszczyk, M.; Mucha, J. The Credibility of the Extrapolation of Resource Parameters of the Cu-Ag Deposit on the Basis of a 3D Geochemical Model (the Lubin-Głogów Copper District, Poland). In *International Multidisciplinary Scientific GeoConference-SGEM*; 3. Exploration and Mining; Proceedings.com: Albena, Bulgaria, 2018; Volume 18, pp. 925–932. [CrossRef]
13. Wasilewska-Błaszczyk, M.; Dudek, M.; Mucha, J. The Methodology of Defining Geochemical Domains in Cu-Ag Deposit (the Lubin-Głogów Copper District, Poland). In *International Multidisciplinary Scientific GeoConference-SGEM*; 3. Exploration and Mining; Proceedings.com: Albena, Bulgaria, 2018; Volume 18, pp. 1005–1012. [CrossRef]

14. Mucha, J.; Nieć, M.; Sermet, E.; Auguścik, J. Evaluation Accuracy of Accompanying Elements Contents in the Cu−Ag Sulphide Ore Deposit of the Lubin-Głogów Copper District (Fore-Sudetic Monocline, Poland). In *Geology, Hydrogeology, Engineering Geology and Geotechnics, Proceedings of the International Multidisciplinary Scientific GeoConference-SGEM 2016, Albena, Bulgaria, 28 June–6 July 2016*; STEF92 Technology Ltd.: Albena, Bulgaria, 2016; Volume 1, pp. 27–34.
15. Pieczonka, J.; Piestrzyński, A.; Mucha, J.; Głuszek, A.; Kotarba, M.J.; Więcław, D. The Red-Bed-Type Precious Metal Deposit in the Sieroszowice-Polkowice Copper Mining District, SW Poland. *Ann. Soc. Geol. Pol.* **2008**, *78*, 151–280.
16. Cox, D.P. Descriptive Model of Sediment-Hosted Cu. In *Mineral Deposit Models*; Cox, D.P., Singer, D.A., Eds.; U.S. Geological Survey Bulletin: Reston, VA, USA, 1986; Volume 1693, p. 205.
17. Nieć, M.; Piestrzyński, A. Forma i budowa złoża. In *Monografia KGHM Polska Miedź S.A.*; KGHM CUPRUM Sp. z.o.o. CBR: Wrocław, Poland, 2007; pp. 157–163.
18. Borg, G.; Piestrzyński, A.; Bachmann, G.H.; Püttmann, W.; Walther, S.; Fiedler, M. An Overview of the European Kupferschiefer Deposits. In *Geology and Genesis of Major Copper Deposits and Districts of the World: A Tribute to Richard H. Sillitoe*; Hedenquist, J.W., Harris, M., Camus, F., Eds.; Special Publications of the Society of Economic Geologists; Society of Economic Geologists: Littleton, CO, USA, 2012; Volume 16, pp. 455–486. [CrossRef]
19. Report on the Mining Assets of KGHM Polska Miedź S.A. located within the Legnica-Głogów Copper Belt Area. 50/2012; KGHM Polska Miedź S.A.; 2012; p 45. Available online: https://kghm.com/pl/aktywa-gornicze-kghm-polska-miedz-sa-w-rejonie-legnicko-glogowskiego-okregu-miedziowego (accessed on 16 December 2021).
20. Bartlett, S.C.; Burgess, H.; Damjanowić, B.; Gowans, R.M.; Lattanzi, C.R. Technical Report on the Copper-Silver Production Operations of KGHM Polska Miedź S.A. In *The Legnica-Głogów Copper Belt Area of Southwestern Poland*; 5/2013; Micon International Limited: Toronto, ON, Canada, 2013; p. 159.
21. Blengini, G.A.; Latunussa, C.E.; Eynard, U.; Torres de Matos, C.; Wittmer, D.; Georgitzikis, K.; Pavel, C.; Carrara, S.; Mancini, L.; Unguru, M.; et al. *Study on the EU's List of Critical Raw Materials—Final Report*; European Commission: Luxemburg, 2020.
22. Pazik, P.M.; Chmielewski, T.; Glass, H.J.; Kowalczuk, P.B. World Production and Possible Recovery of Cobalt from the Kupferschiefer Stratiform Copper Ore. *E3S Web Conf.* **2016**, *8*, 01063. [CrossRef]
23. Olafsdottir, A.H.; Sverdrup, H.U. Modelling Global Nickel Mining, Supply, Recycling, Stocks-in-Use and Price under Different Resources and Demand Assumptions for 1850–2200. *Min. Metall. Explor.* **2021**, *38*, 819–840. [CrossRef]
24. Stala-Szlugaj, K. Outline of the Lower Silesian Copper Deposits Geology. In *The World of KGHM Minerals*; Fundacja Dla Akademii Górniczo-Hutniczej im. Stanisława Staszica w Krakowie: Kraków, Poland, 2019; pp. 15–26.
25. Sobierajski, S.; Kubacz, N.; Tora, B. Problemy kompleksowego wykorzystania rud miedzi w procesach wzbogacania. In *Monografia KGHM Polska Miedź S.A.*; KGHM CUPRUM Sp. z.o.o. CBR: Wrocław, Poland, 2007; pp. 542–554.
26. Piestrzyński, A. Minerals from the Copper Ore Deposit of the Fore-Sudetic Monocline. In *The World of KGHM Minerals*; Fundacja dla Akademii Górniczo-Hutniczej im. Stanisława Staszica w Krakowie: Kraków, Poland, 2016; pp. 27–40.
27. Horn, S.; Gunn, A.G.; Petavratzi, E.; Shaw, R.A.; Eilu, P.; Törmänen, T.; Bjerkgård, T.; Sandstad, J.S.; Jonsson, E.; Kountourelis, S.; et al. Cobalt Resources in Europe and the Potential for New Discoveries. *Ore Geol. Rev.* **2021**, *130*, 103915. [CrossRef]
28. Wedepohl, K.H.; Delevaux, M.H.; Doe, B.R. The Potential Source of Lead in the Permian Kupferschiefer Bed of Europe and Some Selected Paleozoic Mineral Deposits in the Federal Republic of Germany. *Contrib. Mineral. Petrol.* **1978**, *65*, 273–281. [CrossRef]
29. Mayer, W.; Piestrzyński, A. Ore Minerals from Lower Zechstein Sediments at Rudna Mine, Fore-Sudetic Monocline, SW Poland. *Prace Mineral.* **1985**, *75*, 1–72.
30. Kucha, H.; Mayer, W.; Piestrzyński, A. Vanadium in the Copper Ore Deposit on the Fore-Sudetic Monocline (Poland). *Mineral. Pol.* **1983**, *14*, 35–43.
31. Bourassi, A.; Foucher, B.; Geffroy, F.; Marin, J.Y.; Martin, B.; Meric, Y.M.; Perseval, S.; Renard, D.; Robinot, L.; Touffait, Y. *ISATIS Software. User's Guide*; Ecole des Mines de Paris & Geovariances: Fontainebleau, France, 2018.
32. Cressie, N. When Are Relative Variograms Useful in Geostatistics? *Math. Geol.* **1985**, *17*, 693–702. [CrossRef]
33. Clark, I. *Proceedings of 19th Application of Computers and Operations Research in the Mineral Industry*; The Art of Cross Validation in Geostatistical Applications; Society of Mining Engineers, Inc.: Littleton, CO, USA, 1986; pp. 211–220.
34. Dubrule, O. Cross Validation of Kriging in a Unique Neighborhood. *Math. Geol.* **1983**, *15*, 687–699. [CrossRef]
35. Journel, A.G.; Huijbregts, C.J. *Mining Geostatistics*; Academic Press: London, UK, 1979.
36. Kishné, A.S.; Bringmark, E.; Bringmark, L.; Alriksson, A. Comparison of Ordinary and Lognormal Kriging on Skewed Data of Total Cadmium in Forest Soils of Sweden. *Environ. Monit. Assess.* **2003**, *84*, 243–263. [CrossRef]
37. Yamamoto, J.K. On Unbiased Backtransform of Lognormal Kriging Estimates. *Comput. Geosci.* **2007**, *11*, 219–234. [CrossRef]
38. David, M. *Handbook of Applied Advanced Geostatistical Ore Reserve Estimation*; Elsevier: Amsterdam, The Netherlands, 1988.
39. Journel, A.G. The Lognormal Approach to Predicting Local Distributions of Selective Mining Unit Grades. *Math. Geol.* **1980**, *12*, 285–303. [CrossRef]
40. Dowd, P.A. Lognormal Kriging—The General Case. *Math. Geol.* **1982**, *14*, 475–499. [CrossRef]
41. Deutsch, C.V.; Journel, A.G. *GSLIB Geostatistical Software Library and User's Guide*; Oxford University Press: New York, NY, USA, 1992.
42. Sinclair, A.J.; Blackwell, G.H. *Applied Mineral Inventory Estimation*; Cambridge University Press: Cambridge, UK, 2002. [CrossRef]
43. Cressie, N.; Pavlicová, M. Lognormal Kriging: Bias Adjustment and Kriging Variances. In *Geostatistics Banff 2004*; Leuangthong, O., Deutsch, C.V., Eds.; Springer: Dordrecht, The Netherlands, 2005; pp. 1027–1036. [CrossRef]

44. Webster, R.; Oliver, M.A. *Geostatistics for Environmental Scientists*, 2nd ed.; Wiley: Hoboken, NJ, USA, 2007.
45. Yamamoto, J.K.; de Aguiar Furuie, R. Survey into Estimation of Lognormal Data. *Geosciences* **2010**, *29*, 5–19.
46. Rossi, M.E.; Deutsch, C.V. *Mineral Resource Estimation*, 1st ed.; Springer: Dordrecht, The Netherlands, 2014.
47. Bonett, D.G. Confidence Interval for a Coefficient of Quartile Variation. *Comput. Stat. Data Anal.* **2006**, *50*, 2953–2957. [CrossRef]
48. Arachchige, C.N.P.G.; Prendergast, L.A.; Staudte, R.G. Robust Analogs to the Coefficient of Variation. *J. Appl. Stat.* **2020**, 1–23. [CrossRef]
49. Samson, M.; Deutsch, C. Collocated Cokriging. Available online: https://geostatisticslessons.com/lessons/collocatedcokriging (accessed on 4 December 2021).
50. Hengl, T.; Heuvelink, G.B.M.; Rossiter, D.G. About Regression-Kriging: From Equations to Case Studies. *Comput. Geosci.* **2007**, *33*, 1301–1315. [CrossRef]
51. Mucha, J.; Wasilewska-Błaszczyk, M. Variability Anisotropy of Mineral Deposits Parameters and Its Impact on Resources Estimation—A Geostatistical Approach. *Gospod. Surow. Miner.—Miner. Resour. Manag.* **2012**, *28*, 113–135. [CrossRef]
52. Krige, D.G. A Statistical Approach to Some Mine Valuation and Allied Problems on the Witwatersrand. Master's Thesis, University of the Witwatersrand, Johannesburg, South Africa, 1951.
53. Krige, D.G.; Magri, E.J. Geostatistical Case Studies of the Advantages of Lognormal-de Wijsian Kriging with Mean for a Base Metal Mine and a Gold Mine. *Math. Geol.* **1982**, *14*, 547–555. [CrossRef]
54. Matheron, G. *La Theorie Des Variables Regionalisees et Ses Applications*; Masson: Paris, France, 1986.
55. Matheron, G. *Traité de Géostatistique Appliquée*; Mémoires du Bureau de Recherches Géologiques et Minières; Editions Technip: Paris, France, 1962.