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The Role of Risk Forecast and Risk Tolerance in Portfolio Management: A Case Study of the Chinese Financial Sector

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Abstract: Portfolio decisions are affected by the volatility of financial markets and investors' risk tolerance levels. To better allocate portfolios; we introduce risk tolerance into the portfolio management problem by considering the risk contribution of portfolio components. In this paper, portfolio weights are allocated to two stages. In the first stage, the portfolio risks and the risk contribution of each share are forecasted. In the second stage, we put forward three weighting techniques—"aggressive", "moderate" and "conservative", according to three standard levels of risk tolerance. In addition, a new risk measure called "joint extreme risk probability" (*JERP*), with risk tolerance taken into account, is proposed. A case study of the Chinese financial industry is conducted to verify the performance of our methods. The empirical results demonstrate that weighting techniques constrained by risk tolerance lead to higher gains in a normal market and less loss when a market is risky. Compared with risk-tolerance-adjusted strategies, the relationship between the performance of the traditional conditional value at risk (CVaR) minimization method and the market risk level is less obviously demonstrated. Viewed from the results, *JERP* functions as an effective signal that helps investors to deal with potential market risks.

Keywords: risk contribution; one-factor copula with Durante generators; component expected shortfall; conditional value at risk; joint extreme risk probability

MSC: 62; 91

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1. Introduction

The tradeoff between risk and return is a classical issue in portfolio management. Investors look for an optimal portfolio with which to obtain maximum returns. Among all factors that influence their portfolio decisions, risk tolerance, which indicates the degree to which a person is willing to take risks, plays an important role. A longstanding tenet of theories of decision making, dating back to the work of Daniel Bernoulli [1], is that people are risk-averse, at least for decisions with outcomes in the domain of gains and for mixed outcomes that include both gains and losses [2]. Nonetheless, it is undoubted that investors vary greatly in their degree of financial risk tolerance, which indicates the level of discomfort that an individual is willing to accept while risking current wealth for future growth [3]. Some investors would rather buy assets with stable returns than take additional risks for higher returns. For some others, they are willing to take excessive risks in exchange for potential high outcomes. Different levels of risk tolerance lead to different portfolio weighting schemes, but risk tolerance is not the only influence factor. As market risk is constantly changing, no strategy can guarantee profitability all the time. Thus, it is important to forecast risk and adjust weightings in the portfolio to manage market volatility.

Over the years, various investment techniques have been created. Under the assumption that all investors are risk-averse, some scholars focused on the portfolio risk minimization problem. Markowitz [4] proposed the so-called mean-variance framework. His resolution is to maximize returns for a given level of risk and minimize risk when a level of return is given. This method has been criticized by many scholars (e.g., Chopra and Ziemba [5], Konno and Hiroaki [6] as well as Green and Hollifield [7]), and some studies have modified the method to make it more practical (see Leung [8], etc.). Since the variance captures both losses and gains, other methods have emerged to analyze only the downside risk—the tail related to losses, such as the risk measure known as value at risk (VaR). However, there were also several criticisms regarding VaR. Bemporad, Puglia and Gabbriellini [9] argued that VaR cannot capture the magnitude of the loss if it occurs. They also pointed out that the measure does not satisfy the subadditivity condition. In this case, the portfolio's risk may be greater than the sum of each share's risk. Then, Rockafellar and Uryasev [10] presented a risk measure called conditional value at risk (CVaR), a risk measure based on VaR, which indicates the size of the expected loss of the investment. CVaR is a downside risk measure that can be applied to optimize a portfolio within a specified risk range. Such a risk measure satisfies the condition of subadditivity and calculates VaR while minimizing CVaR simultaneously. All the above-mentioned methods have been widely used in the literature, indicating that risk measures are an indispensable part of the portfolio optimization problem.

Instead of portfolio optimization and selection, some studies concentrated on investor behaviors and risk-tolerance levels. Although the concept of financial risk tolerance has been mentioned frequently in the literature, a unique definition does not exist. For example, Grable [11] defined it as "the maximum amount of uncertainty someone is willing to accept when making a financial decision". Gerrans et al. [12] defined it as "the extent to which an investor is willing to risk experiencing a less favorable financial outcome in the pursuit of a more favorable financial outcome". No matter how its definition is expressed, it is shown that financial risk tolerance is non-negligible in the portfolio management problem. Some scholars analyzed the impact of risk tolerance on investment decision making (e.g., Nguyen et al. [13], Yilmazer and Lich [14]). Since factors such as age and income have an influence on investing (Romprasert [15]), many studies have attempted to determine factors related to financial risk tolerance, such as age, gender, cultural background, etc. (e.g., Sung and Hanna [16], Fisher and Yao, [17], Pyles et al. [18] and Hawley and Fujii [19]). Some scholars have attempted to assess investors' risk tolerance levels, such as Ferreira and Dickason-Koekemoer [20], Lawrenson and Dickason-Koekemoer [21] as well as Wahl and Kirchler [22]. Some have even considered the influence of the global financial crisis (GFC) and COVID-19 pandemic on risk tolerance, such as Gerrans et al. [12], Chiang and Xiao [23] as well as Heo et al. [24]. It has been found that financial risk tolerance is difficult to classify accurately, because it is affected by both objective and subjective factors. Beyond all questions, financial risk tolerance is a key factor that influences investors' investment strategies. However, few studies have blended this concept with portfolio optimization models. Zhang et al. [25] proposed a risk tolerance model with transaction costs for adjusting an existing portfolio, using possibilistic moments of a fuzzy number. Gong et al. [26] discussed the portfolio selection problems with the uncertainty of future returns and investors' attitudes towards the stock market (optimistic–pessimistic–neutral) being considered in the model. In this paper, we categorize investors' risk tolerance levels into "conservative", "aggressive" and "moderate", regardless of future returns, and then introduce financial risk tolerance into the portfolio management problem by considering the risk contribution of portfolio components when allocating weights.

Following the literature, this paper assumes that all investors are risk-averse. To satisfy this precondition, Rockafellar and Uryasev's CVaR minimization method is adopted to reduce risk to a given risk capacity level. Note that risk tolerance is only a measure of the risk that a person can take emotionally. The higher the tolerance, the more risk an investor can take. The lower the tolerance, the more conservative an investor is with their portfolio. In a different sense, risk capacity assesses whether an investor can financially take a certain amount of risk with their investments at a particular point in time. In contrast to risk tolerance, which is all about behavior and mental attitude, risk capacity can be measured mathematically. It considers factors such as investing time horizon as well as the ability to withstand volatility and possible losses without affecting investment goals. It is normally calculated during a risk analysis process. Both risk tolerance and risk capacity are essential for determining how much risk should be taken to achieve financial goals. In this paper, we first extract a weight vector for the portfolio by minimizing CVaR at a given risk capacity level. Once the portfolio weights are extracted from the optimization procedure, we further exert the influence of risk tolerance on portfolio weighting techniques. In terms of risk tolerance levels, investors can be generally categorized into "aggressive", "conservative" and "moderate". Thus, three new weighting schemes are proposed accordingly. In the case study of the Chinese financial industry, four strategies, including three with risk tolerance restrictions and one without, are applied to stock data, and their performances are compared under different market risk levels. The results show that strategies adjusted by risk tolerance exhibit a more stable property.

Since risk tolerance is the attitude towards risk, portfolio risks are forecasted before implementing risk tolerance restrictions to portfolio management. To forecast risk, the most important element is the choice of a returns forecasting model if probability theory is used to analyze uncertainty. This paper employs a one-factor copula with Durante generators (FDG copula) model to generate simulations, and the rolling window method is adopted to forecast future returns with historical data. As a measure of dependence structures, copulas have been intensively studied by many scholars, such as Wei et al. [27]. The advantage of using copulas is that the dependence between stocks can be taken into account in the simulation process. Besides, an FDG copula model allows for tail asymmetry and is well-suited for high-dimensional modeling. An accurate forecast of future risk can allow investors to adjust their portfolio weightings in a timely manner to reach their investment goals. Hence, we put forward a new method in this paper, called "joint extreme risk probability", to measure risk in a more straightforward way. This method is used to measure the probability that extreme risks occur in multiple assets at the same time. This method works as a signal that helps investors cut losses in time. One interesting thing about this method is that investors can define the range of measurements and the threshold that tells them to cut investments according to their tolerance levels. In this case study, we show that by stopping investing on days with high *JERP*, losses can be well-avoided.

As far as we know, the novelty of this study lies in two aspects. First, we design a new portfolio management approach that values the risk tolerance levels of investors. A fixed portfolio or investment method cannot keep pace with volatility in financial markets and the change in investors' attitudes towards risk. Such strategies assist investors in dealing with the change in risk promptly and efficiently. Investors could consider switching between three techniques by buying or selling assets in response to real-time changes in the risk of the overall market and of individual stocks. Second, we propose a new risk measure, termed "joint extreme risk probability (*JERP*)", that can be applied based on investors' risk tolerances. When the market condition is not optimistic, this method can be put into practice to forecast the probability of extreme risk happening to a portfolio. Once the *JERP* exceeds an investor's tolerance level, assets can be sold in due course to avoid losses.

Portfolio management in this paper is a two-stage process that consists of risk forecasting and weight allocation. In Section 2, we introduce the methods we used to forecast the market risk and the risk contribution of portfolio components. New weighting techniques and risk measures that considers risk tolerance are proposed in Section 3. To assess the performance of the methods, we conducted a case study, and the data and results are presented in Section 4. Finally, Section 5 discusses the main findings and implications. Conclusions are drawn in Section 6. In Section 7, we discuss the limitations and research potential for future studies.

2. Risk Forecasting

To forecast the market risk and the risk contribution of each share in the portfolio, a rolling window approach is adopted. The idea is to forecast risks each day of the out-of-sample period with a fixed window size of consecutive observations. Suppose there are T days in the in-sample period and N stocks in the portfolio. The forecasting process includes modeling the distribution of T observations for each stock, simulating the returns of N stocks for day $t + 1$ and calculating the overall risk and the risk contribution of each stock. Considering the feature of tail dependence in the financial industry, an FDG copula model is adopted to generate simulations with dependence coefficients measured from in-sample data.

2.1. Model for Marginal Distribution

To model the marginal distribution, ARMA (1,1)-Glosten-Jagannathan-Runkle (GJR)-GARCH (1,1) [28] is applied. This model is useful because a leverage effect is included to capture negative shocks. The leverage effect is caused by the fact that negative returns have a more pronounced influence on volatility than positive returns do. Given the non-normality characteristics of the GARCH model residuals, we fitted the in-sample data with three possible distributions—skewed normal, skewed Student's t and skewed generalized error distribution. Parameters in the model are estimated by maximum likelihood, and the best-fit distribution for each stock is selected using Bayesian information criteria (BIC). The model is expressed as

$$r_t = c + \varphi r_{t-1} + \rho \varepsilon_{t-1} + \varepsilon_t \tag{1}$$

with

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2 + \gamma \mathbb{I}_{t-1} \varepsilon_{t-1}{}^2 + \beta \sigma_{t-1}{}^2 \tag{2}$$

where r_t is the stock return at day t, and ε_t is the return residual, which can be split into two parts—a random variable, z_t, and the standard deviation, σ_t. That is,

$$\varepsilon_t = z_t \sigma_t \tag{3}$$

where z_t represents independent and identically distributed variables that follow the best-fit distribution. In Equation (2), the parameter γ captures the negative impact from day $t - 1$ with indicator

$$\mathbb{I}_{t-1} = \begin{cases} 0 \; if \; \varepsilon_{t-1} \geq 0, \\ 1 \; if \; \varepsilon_{t-1} < 0. \end{cases} \tag{4}$$

To model the tail dependence by a copula, the distribution function of the error term is regarded as the marginal distribution. Thus, we standardized the error term by

$$\hat{z_t} = \frac{\hat{\varepsilon_t}}{\hat{\sigma_t}} \tag{5}$$

to ensure that marginal distributions are uniformly distributed in [0, 1]. By applying this method to in-sample data, we obtained all the parameters in ARMA-GJR-GARCH model and the distribution of each stock.

2.2. Simulation

For each day in the out-of-sample period, we first generated 10,000 simulations of return residuals using the FDG copula model. According to Mazo et al. [29], the FDG acronym denotes "One-Factor copula with Durante Generators". It is used to measure the dependence between stocks with in-sample data and simulate data with estimated dependence coefficients. Let $U = (U_1, \ldots, U_d)$ be the margins obtained from the ARMA-GJR-GARCH model with $U_i \sim U(0, 1)$. In this model, the dependence between stocks is based on a latent variable, and $U_1, \ldots, U_d$ are assumed to be conditionally independent given the latent variable, U_0. The FDG copula is expressed as

$$C(u_1, \ldots, u_d) = \int_0^1 C_{1|0}(u_1|u_0) \ldots C_{d|0}(u_\mathrm{d}|u_0) du_0 \tag{6}$$

with

$$C_{i|0}(\cdot|u_0) = \frac{\partial C_{0i}}{\partial u_0} \tag{7}$$

and

$$C_{0i} = min(u_0, u_i) f(max(u_0, u_i)) \tag{8}$$

Two families of the FDG copula—FDG copula with Cuadras–Augé generators (FDG-CA) and FDG copula with Fréchet generators (FDG-F)—were applied to measure dependence coefficients, and the best-fit family selected using mean absolute percentage error (MAPE) was used to generate simulations. These two families are introduced below.

1. FDG copula with Cuadras–Augé generators

In Equation (8), let

$$f_i(t) = t^{1-\theta_i}, \theta_i \in [0,1] \tag{9}$$

The Spearman's rho is given by

$$\rho_{ij} = \frac{3\theta_i\theta_j}{5 - \theta_i - \theta_j} \tag{10}$$

The lower and upper tail dependence coefficients are given by

$$\lambda_{ij}^{(L)} = 0 \; and \; \lambda_{ij}^{(U)} = \theta_i\theta_j \tag{11}$$

The Kendall's tau is given by

$$\tau_{ij} = \begin{cases} \frac{\theta_i\theta_j\left(\theta_i\theta_j + 6 - 2\left(\theta_i + \theta_j\right)\right)}{\left(\theta_i + \theta_j\right)^2 - 8\left(\theta_i + \theta_j\right) + 15} \; if \; \theta_i + \theta_j \neq 1 \\ \frac{\theta(\theta-1)(\theta^2-\theta-4)}{8} \; if \; \theta = \theta_i = 1 - \theta_j. \end{cases} \tag{12}$$

2. FDG copula with Fréchet generators

In Equation (8), let

$$f_i(t) = (1-\theta_i)t + \theta_i, \theta_i \in [0,1] \tag{13}$$

The Spearman's rho, the lower and upper tail dependence coefficients are, respectively, given by

$$\rho_{ij} = \lambda_{ij}^{(L)} = \lambda_{ij}^{(U)} = \theta_i\theta_j \tag{14}$$

The Kendall's tau is given by

$$\tau_{ij} = \frac{\theta_i\theta_j\left(\theta_i\theta_j + 2\right)}{3} \tag{15}$$

The Cuadras–Augé family allows for upper but not lower tail dependence, and the Fréchet family allows for both upper and lower tail dependence. In the Fréchet case, the lower and upper tail dependence coefficients are equal.

By applying FDG copulas to in-sample data, dependence coefficients—Kendall's tau, Spearman's rho and lower and upper tail dependence—are obtained in each family. The best-fit copula family is selected by the mean absolute percentage errors (*MAPE*s), which are expressed as

$$MAPE_{\tau} = \frac{1}{p}\sum_{i<j}\left|\frac{\hat{\tau}_{i,j} - \tau\left(\hat{\theta}_i, \hat{\theta}_j\right)}{\tau\left(\hat{\theta}_i, \hat{\theta}_j\right)}\right| \quad (16)$$

where $\hat{\tau}_{i,j}$ is the empirical estimator of Kendall's tau calculated in EViews, $\tau\left(\hat{\theta}_i, \hat{\theta}_j\right)$ is the Kendall's tau estimated by FDG copulas and p is the number of variable pairs. Then, the best-fit family is used to generate 10,000 $\widetilde{z}_{t+1}$ for each day in the out-of-sample period. Finally, stock returns at day $t + 1$ are computed by

$$\widetilde{r}_{t+1} = \hat{c} + \hat{\varphi} r_t + \hat{\rho}\hat{\varepsilon}_t + \widetilde{z}_{t+1}\hat{\sigma}_{t+1} \quad (17)$$

where

$$\hat{\sigma}_{t+1}{}^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{\varepsilon}_t{}^2 + \hat{\gamma}\mathbb{I}_t \hat{\varepsilon}_t{}^2 + \hat{\beta}\hat{\sigma}_t{}^2 \quad (18)$$

2.3. Risk Measurement

Following the above procedure, we obtained 10,000 $\widetilde{r}_{t+1}$ of each stock so that risks can be forecasted by computing the negative value of the expected value of the return when the return is less than its 5% quantile. The component expected shortfall (*CES*) method developed by Banulescu and Dumitrescu [30] was used. This model takes into account the weight of financial institutions in the portfolio and decomposes the overall risk of the portfolio into the percentage contribution of each portfolio component. Besides, it only relies on daily market data that can be easily obtained. Compared with other risk measures, such as *VaR*, SRISK and marginal expected shortfall (MES), this method possesses more advantages and satisfies our need of computing portfolio risk and the risk contribution of portfolio components. Let r_{it} be the return of stock i on day t and r_{mt} be the aggregate return, which is thus defined as

$$r_{mt} = \sum_{i=1}^{N} w_{it} r_{it} \quad (19)$$

where w_{it} is the weight of the ith institution in the market on day t. These weights are given by the relative market capitalization of the institutions. That is,

$$w_{it} = \frac{W_{it}}{\sum_{i=1}^{n} W_{it}} \quad (20)$$

where W_{it} is the market capitalization of institution i at day t.

The component expected shortfall on day $t + 1$ given by *CES* is expressed as

$$CES_{it+1} = w_{it}\frac{\partial ES_{m,t}(C)}{\partial w_{it}} = -w_{it}\mathbb{E}_t(\widetilde{r}_{it+1} | \widetilde{r}_{mt+1} < C) \quad (21)$$

where the conditional expected shortfall (with respect to past information) is defined by a threshold, C, which is the

$$ES_{m,t}(C) = -\mathbb{E}_t(\widetilde{r}_{mt+1} | \widetilde{r}_{mt+1} < C) \quad (22)$$

The expected loss of the portfolio at day $t + 1$ equals the sum of all the financial institutions' *CES*, which is given by

$$ES_{m,t}(C) = \sum_{i=1}^{n} CES_{it+1}(C) \quad (23)$$

The risk contribution of each stock is computed without market capitalization, which means with equal weight. The proportion of portfolio risk due to institution i at day $t + 1$ is computed as

$$CES\%_{it+1}(C) = \frac{CES_{it+1}(C)}{ES_{m,t}(C)} \times 100 = \frac{\mathbb{E}_t(\widetilde{r}_{it+1}|\widetilde{r}_{mt+1} < C)}{n\mathbb{E}_t(\widetilde{r}_{it+1}|\widetilde{r}_{mt+1} < C)} \times 100 \tag{24}$$

3. Portfolio Allocation with Risk Tolerance Restrictions

3.1. CVaR Minimization

Before constraining portfolio management with risk tolerance, we minimized CVaR for each day of the out-of-sample period to reduce risks to a given risk capacity level in accordance with the risk-averse nature of investors. Let vector $\boldsymbol{x}$ be the weight in the sense that $x = (x_1, \ldots x_n)$, with $x_i \geq 0$ and $\sum_{i=1}^{N} x_i = 1$. Rockafellar and Uryasev [10] present the $(1 - \beta)$-level CVaR as

$$F_\beta(x, \alpha) = \alpha + (1 - \beta)^{-1} \int_{r\epsilon\mathbb{R}^n} \left[max\left\{-x^T r - \alpha,\ 0\right\}\right] \times p(r)dr \tag{25}$$

where $p(r)$ is the joint probability density function of r.

When Monte Carlo integration is used, it is equivalent to

$$F_\beta(x, \alpha) = \alpha + \frac{1}{q(1 - \beta)} \sum_{k=1}^{q} \left[-x^T r_s - \alpha\right]^+ \tag{26}$$

where q is the number of simulations equaling 10,000; $\beta = 0.05$ in this paper, α is the $(1 - \beta)$-level VaR and r_s is the sth vector of simulated returns.

To minimize CVaR, we minimized Equation (26), subject to linear constraints

$$E\left[x^T r\right] \geq \hat{\mu} \tag{27}$$

$$\sum_{i=1}^{N} x_i = 1\ and\ x_i \geq 0\ for\ all\ i = 1, 2, \ldots, n \tag{28}$$

where $\hat{\mu}$ is the minimum expected portfolio return that the investor can financially tolerate. In this paper, we set $\hat{\mu}$ as the mean value of simulated stock returns. That is,

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{N} \widetilde{r}_{i,t+1} \tag{29}$$

The vector of portfolio weights, x, is extracted from the optimization procedure with a given $\hat{\mu}$, which can be regarded as a proxy of risk capacity in this model. In other words, it represents the lowest return an investor can accept. The first constraint expressed by Equation (27) ensures that CVaR will be minimized within the risk capacity. For each day, there is a vector, x, that generates the portfolio that minimizes CVaR. If risk tolerance is not considered, investors could allocate weights as suggested by x, and the return of the portfolio at day $t + 1$ will be

$$R_{t+1} = \sum_{i=1}^{N} r_{i,t+1} x_{i,t+1} \tag{30}$$

3.2. Weight Allocation with Risk Tolerance Restrictions

On the basis of portfolio optimization by minimizing CVaR, we further introduced the risk tolerance factor into the weight allocation problem. Similar to the three types of risk preferences, risk tolerance level can be broadly categorized into "aggressive", "moderate" and "conservative". Investors who are aggressive tend to invest in assets with greater levels of risk to seek potentially high rewards. They can handle excessive risk in order to maximize returns. Moderate investors can take on some risk and prefer a balanced approach between risky and less volatile assets. For conservative investors, the stability of returns outweighs the potential of a high outcome. They are willing to take little volatility in their investment portfolios. Following this concept, we developed three strategies for three levels of tolerance. To embody the role of risk tolerance in portfolio management, the risk contribution of each stock is taken as the reference for weight allocation.

For people who prefer outcomes with low uncertainty as opposed to those outcomes with high uncertainty, they could adopt a "conservative" strategy by allocating a larger weight to the stock that contributes least to the portfolio risk. For people who are willing to accept greater uncertainty in exchange for the potential of higher returns, they could employ an "aggressive" scheme by investing in the riskiest stock with the highest weight. For investors that are moderate, they could allocate equal weight to stocks. To implement three strategies, there are four steps to follow.

Step one. According to the results of CVaR minimization, eliminate stocks with zero weight in vector x. For example, if $x_3 = 0$, the third stock should be removed from the portfolio.

Step two. Sort non-zero values in x from the smallest to the largest; a new weight vector, $\vec{x}$, is formed, with $0 < \vec{x}_1 \leq \vec{x}_2 \leq \ldots \leq \vec{x}_g < 1$, where G is the number of the remaining stocks.

Step three. Rank the remaining companies according to the risk contribution rate to ensure that the first company in the array has the largest contribution and the last has the smallest, which means $CES\%_1 > CES\%_2 > \ldots > CES\%_k$. Let $r = (r_1 \ldots r_g)$ be the real returns of the remaining companies, where r_i is the return of the ith company in the ranked array.

Step four. Allocate weights in $\vec{x}$ to the remaining companies according to the three levels of risk tolerance.

1. Conservative: Allocate the smallest weight to the stock with the highest risk contribution. The portfolio return will be

$$R_{con} = r_1 \vec{x}_1 + r_2 \vec{x}_2 + \ldots + r_g \vec{x}_g \tag{31}$$

2. Aggressive: Allocate the largest weight to the stock with the highest risk contribution. The portfolio return will be

$$R_{agg} = r_1 \vec{x}_g + r_2 \vec{x}_{g-1} + \ldots + r_g \vec{x}_1 \tag{32}$$

3. Moderate: Allocate equal weight to the remaining companies. The portfolio return will be the average return of the remaining stocks.

$$R_{mod} = \frac{1}{G} \sum_{i=1}^{G} r_i \tag{33}$$

Note that all steps are based on the forecasted results, except for the returns, $r = (r_1 \ldots r_k)$, that we used to compute the real returns of the portfolio. Compared to portfolio optimization by minimizing CVaR, the risk-tolerance-adjusted weighting techniques are more individually targeted and flexible. An investor's risk tolerance level is not invariable. Investors can easily switch among the three strategies according to the change in their tolerance levels. It is worth mentioning that, apart from the three strategies mentioned above, investors can customize their strategies by combining two or three strategies together. For example, based on the forecasted risk contribution and weight vector, x, an investor can adopt an "aggressive" strategy for half of the companies in the portfolio and a "moderate" one for the other half. Yet, a demonstration and the performance of other possible methods of investing following from these three will not be discussed in this paper.

3.3. Joint Extreme Risk Probability

In addition to portfolio allocation, risk tolerance can also be introduced to risk measurement. In this paper, we propose a new method called "joint extreme risk probability" to measure risk in a more specific manner. *JERP* measures the joint probability of extreme risk occurring in multiple financial assets simultaneously. This method helps to assess the probability that an investor suffers a great loss from more than a certain proportion of their portfolio in the same day. In contrast to traditional risk measures, *JERP* can be computed according to investors' risk tolerance levels. If *JERP* exceed the bounds they decide on some days, they can cut or stop their investing on these days to avoid risk. As with *CES*, *JERP* is measured by the rolling window method. Daily *JERP* is computed using the following steps.

Step one. Use the rolling window method to forecast stock returns for each day in the out-of-sample period.

Step two. Calculate *VaR* for each stock in the portfolio.

Step three. Define a threshold, C, for *VaR* and an indicator, such that

$$J_i = I(VaR_i > C) \tag{34}$$

In this model, the threshold, C, represents the highest risk level an investor can accept.

Equation (34) means that if the risk of the ith stock is greater than the threshold, $J_i = 1$. Otherwise, $J_i = 0$.

Step four. Repeat steps one–three M times and obtain a $M \times N$ matrix, which is

$$J = \begin{bmatrix} J_1^1 & J_2^1 & \cdots & J_N^1 \\ J_1^2 & J_2^2 & & J_N^2 \\ \vdots & & \ddots & \vdots \\ J_1^M & J_2^M & \cdots & J_N^M \end{bmatrix} \tag{35}$$

Step five. Compute *JERP* by

$$JERP = \frac{1}{M}\sum_{m=1}^{M} I_m\left(\frac{1}{N}\sum_{n=1}^{N} J_i^m > \frac{K}{N}\right) \tag{36}$$

where J_i^m is the J_i obtained from the mth computation and K is the number of stocks defined by investors according to their tolerance level. If an investor is concerned about the probability of risk increasing in 50% of the stocks in the portfolio, K/N should be set as $1/2$. To obtain the value of *JERP*, we first sum up the J_i in each row of the matrix, J, and take the average. If the average is larger than the threshold K/N, $I_m\left(\frac{1}{N}\sum\limits_{i=1}^{N} J_i^m > \frac{K}{N}\right) = 1$. Otherwise, $I_m\left(\frac{1}{N}\sum\limits_{i=1}^{N} J_i^m > \frac{kK}{N}\right) = 0$. When the indicator function, I, equals one, it means that extreme risk occurs in at least K/N percent of the assets. Thus, an $M \times 1$ matrix is given as

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{bmatrix} \tag{37}$$

Finally, by taking the average values of I_m, we obtain *JERP*, which implies how many times extreme risk occurred in K/N proportion of assets in M calculations. Investors can set a critical value for themselves. When *JERP* exceeds this value, they should immediately cut or even stop their investing. For two investors with the same threshold of *JERP*, the higher the K/N the higher the tolerance level. For instance, an investor that sets K/N = 90% will stop investing when *JERP* reaches the threshold. This means that the investor chooses to reduce risk when extreme risk is likely to occur in 90% of stocks. For the other investor that sets K/N = 50%, when investing is stopped, due to the same level of *JERP*, extreme risk is likely to occur to 50% of stocks in the portfolio. Obviously, the first investor has a higher level of risk tolerance.

In the case study of the Chinese financial industry, this method is applied after risk forecasting. The computational process is detailed below.

1. For return forecasting, we follow the same procedure as the one we use in the calculation of *CES*.
2. The 1%-*VaR* for each stock is calculated, which is denoted by $VaR_{i,1\%}$.
3. A threshold, $TMVaR_{i,1\%}$, is defined for $VaR_{i,1\%}$, which is given by

$$TMVaR_{1\%} = \frac{1}{N}\sum_{i=1}^{N} MVaR_{i,1\%} \tag{38}$$

with

$$MVaR_{1\%} = \frac{1}{D}\sum_{t=1}^{D} VaR_{i,1\%} \tag{39}$$

N is the number of stocks in the portfolio and D is the number of days in the out-of-sample period. Additionally, $MVaR_{i,1\%}$ is the mean value of *VaR* in the whole out-of-sample period for each stock, while $TMVaR_{1\%}$ is the total mean value of *VaR* of all stocks. The indicator is thus expressed as

$$J^{m}_{i,t+1} = I\left(VaR^{m}_{i,t+1} > TMVaR_{1\%}\right) \tag{40}$$

4. Steps one–three are repeated 100 times (M = 100), and K/N is set to be 60%. *JERP* is then given by

$$JERP_{t+1} = \frac{1}{100}\sum_{m=1}^{100} I_m\left(\frac{1}{N}\sum_{i=1}^{N} J^{m}_{i,t+1} > 0.6\right) \tag{41}$$

4. Case Study

4.1. The Data

This work studied portfolio management in the following four periods: October 2006 to September 2007 (pre-GFC), October 2007 to December 2009 (GFC), the year 2019 (post-GFC) and January 2020 to June 2021 (COVID-19). The 2008 global financial crisis and the 2020 COVID-19 pandemic are the most representative black swan events since the 21st century, and have had huge impacts on the global stock market. Thus, data are divided into different periods to validate the forecasting ability of our methods and compare the performance of weighting techniques under different market risk levels. The out-of-sample period starts from October 2006 to contain as many financial institutions as possible, considering their time to market. October 2007 is taken as a demarcation between the pre-GFC and GFC periods because the Shanghai Composite Index peaked and then declined sharply in this month. By the end of 2009, the turbulence in the market ended and the Shanghai Composite Index began to rise. Ten years after the GFC, its negative impact has long been mitigated, and the market is in a normal state. However, a pandemic occurred, beginning in January 2020. Thus, the year 2009 can be viewed as a post-event period for the GFC as well as a pre-event period for the COVID-19 pandemic.

For the pre-GFC and GFC periods, we selected 17 and 14 stocks, respectively. There are three reasons to select these financial institutions. First, to improve the accuracy of forecast, we require historical data three years ahead of the out-of-sample period, so we must ensure that these selected companies were public at that time. Second, due to the financial crisis, all stocks had been suspended, including long or short. To avoid computational errors, we only selected companies that had been suspended for a relatively shorter period. Third, the selected companies possess about 70% of the market share during these two periods, so that the portfolio risk can, to some extent, represent the market risk. For the post-GFC and COVID-19 periods, we selected 30 institutions that take more than 90% of the market value. Similarly, these companies had been listed three years before the forecast period and had fewer suspensions. Daily stock returns and market capitalization data are collected from Choice data. Stock returns were transformed using $r_t = lnR_t - lnR_{t-1}$, where R_t is the closing price at day t. Table 1 lists 47 financial institutions in the Chinese financial industry with their trading codes, full names and the period containing them.

Table 1. Sample description.

Code	Name	Period
000627	Hubei Biocause Pharmaceutical Co., Ltd., Jingmen, China	1
000666	Jingwei Textile Machinery Company Limited, Beijing, China	1
000958	Spic Dongfang New Energy Corporation, Shijiazhuang, China	1
600643	Shanghai AJ Group Co., Ltd., Shanghai, China	1
600783	Luxin Venture Capital Group Co., Ltd., Jinan, China	1
600061	SDIC Capital Co., Ltd., Beijing, China	2
600830	Sunny Loan Top Co., Ltd., Ningbo, China	2
600120	Zhejiang Orient Financial Holdings Group Co., Ltd., Hangzhou, China	1,2
000046	Oceanwide Holdings Co., Ltd., Beijing, China	1,2
000416	Minsheng Holdings Co., Ltd., Beijing, China	1,2
000567	Hainan Haide Capital Management Co., Ltd., Beijing, China	1,2
600390	Minmetals Capital Company Limited, Changsha, China	1,2
000890	Jiangsu Fasten Company Limited, Jiangyin, China	1,2
000987	Guangzhou Yuexiu Financial Holdings Group Co., Ltd., Guangzhou, China	1,2
600621	Shanghai Chinafortune Co., Ltd., Shanghai, China	1,2
601788	Everbright Securities Company Limited, Shanghai, China	3
601901	Founder Securities Co., Ltd., Changsha, China	3
000617	Cnpc Capital Company Limited, Beijing, China	4
600015	Hua Xia Bank Co., Ltd., Beijing, China	4
601009	Bank Of Nanjing Co., Ltd., Nanjing, China	3,4
601166	Industrial Bank Co., Ltd., Fuzhou, China	3,4
601169	Bank of Beijing Co., Ltd., Beijing, China	3,4
601288	Agricultural Bank Of China Limited, Beijing, China	3,4
601328	Bank Of Communications Co., Ltd., Shanghai, China	3,4
601398	Industrial Additionally, Commercial Bank Of China Limited, Beijing, China	3,4
601818	China Everbright Bank Co., Ltd., Beijing, China	3,4
601939	China Construction Bank Corporation, Beijing, China	3,4
601988	Bank Of China Limited, Beijing, China	3,4
601998	China Citic Bank Corporation Limited, Beijing, China	3,4
000776	Gf Securities Co., Ltd., Guangzhou, China	3,4
002736	Guosen Securities Co., Ltd., Shenzhen, China	3,4
600837	Haitong Securities Co., Ltd., Shanghai, China	3,4
600958	Orient Securities Company Limited, Shanghai, China	3,4
600999	China Merchants Securities Co., Ltd., Shenzhen, China	3,4
601211	Guotai Junan Securities Co., Ltd., Shanghai, China	3,4
601318	Ping An Insurance (Group) Company Of China, Ltd., Shenzhen, China	3,4
601336	New China Life Insurance Company Ltd., Beijing, China	3,4
601601	China Pacific Insurance(group) Co., Ltd., Shanghai, China	3,4
601628	China Life Insurance Company Limited, Beijing, China	3,4
601688	Huatai Securities Co., Ltd., Nanjing, China	3,4
300059	East Money Information Co., Ltd., Shanghai, China	3,4
002142	Bank of Ningbo Co., Ltd., Ningbo, China	3,4
000001	Ping An Bank Co., Ltd., Shenzhen, China	1,3,4
600000	Shanghai Pudong Development Bank Co., Ltd., Shanghai, China	2,3,4
600016	China Minsheng Banking Corp., Ltd., Beijing, China	1,2,3,4
600036	China Merchants Bank Co., Ltd., Shenzhen, China	1,2,3,4
600030	CITTI Securities Company Limited, Shenzhen, China	1,2,3,4

Notes: In the column of "Period", "1" = pre-GFC, "2" = "GFC", "3" = "post-GFC" and "4" = "COVID-19". For example, Hubei Biocause Pharmaceutical Co., Ltd. is only included in the pre-GFC period, and Citic Securities Company Limited is included in all four periods.

4.2. *Empirical Results*

4.2.1. Market Risk

The daily expected shortfall (ES) is computed to represent the risk level. For a clearer display, weekly average results are reported in Figure 1. Larger magnitudes of ES indicate higher risk. Since the selected stocks in our portfolio represent a large proportion of the market value of the Chinese financial sector, the forecasted portfolio risk can largely represent the market risk level. The risk before and during the GFC is higher than that in the post-GFC and COVID-19 periods. From January to July 2020, when the epidemic situation was severe, expected loss of the portfolio increases and the volatility is larger compared to the post-GFC period. The risk reached its highest level in October 2008 and gradually declined after a series of policies were promulgated. The result is in line with the facts, which shows that our forecasting method is practical and accurate.

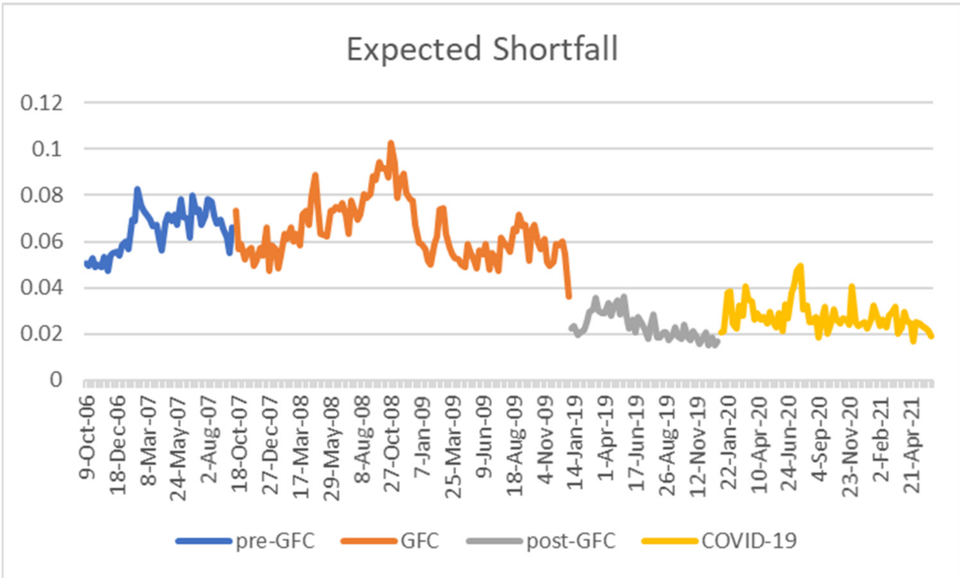

Figure 1. Expected shortfall of portfolios in four periods.

4.2.2. Portfolio Management

Suppose an investor invests RMB one million at the beginning of each period and leaves the money in the stock market until the end of the period. In Figures 2–5, we display the investor's cumulative wealth earned from four strategies in four periods.

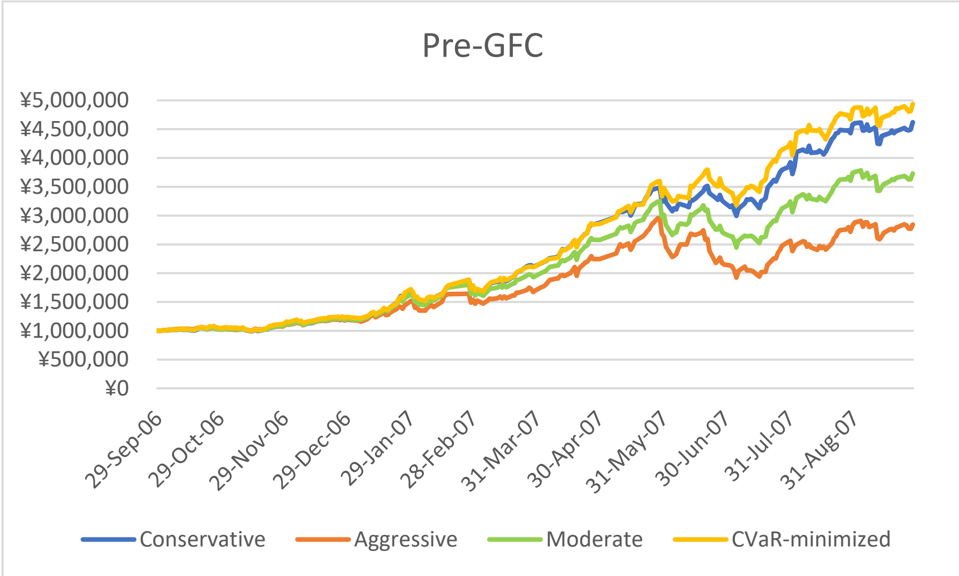

Figure 2. Cumulative gains from four investment schemes in the pre-GFC period.

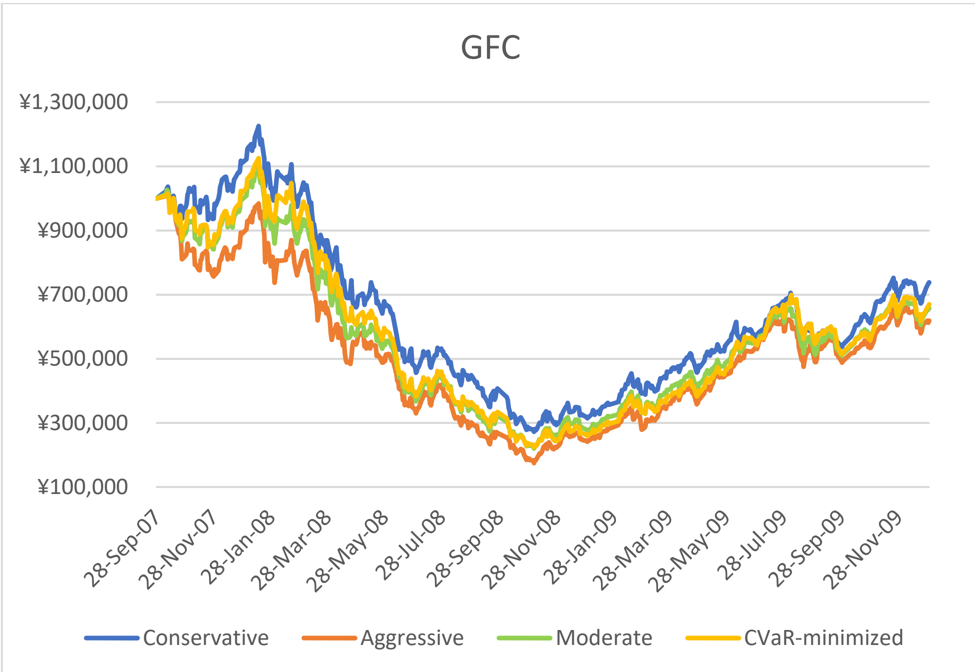

Figure 3. Cumulative gains from four investment schemes in the GFC period.

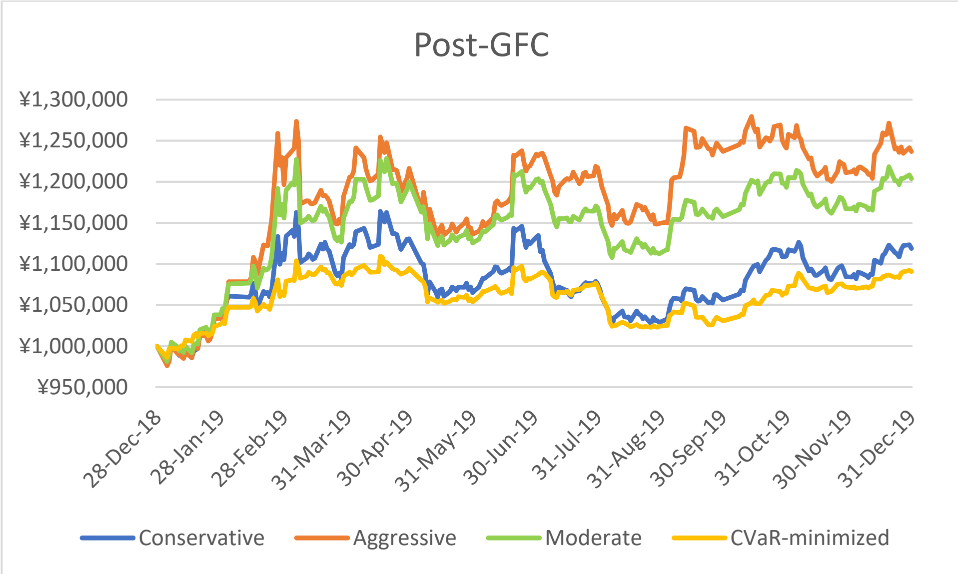

Figure 4. Cumulative gains from four investment strategies in the post-GFC period.

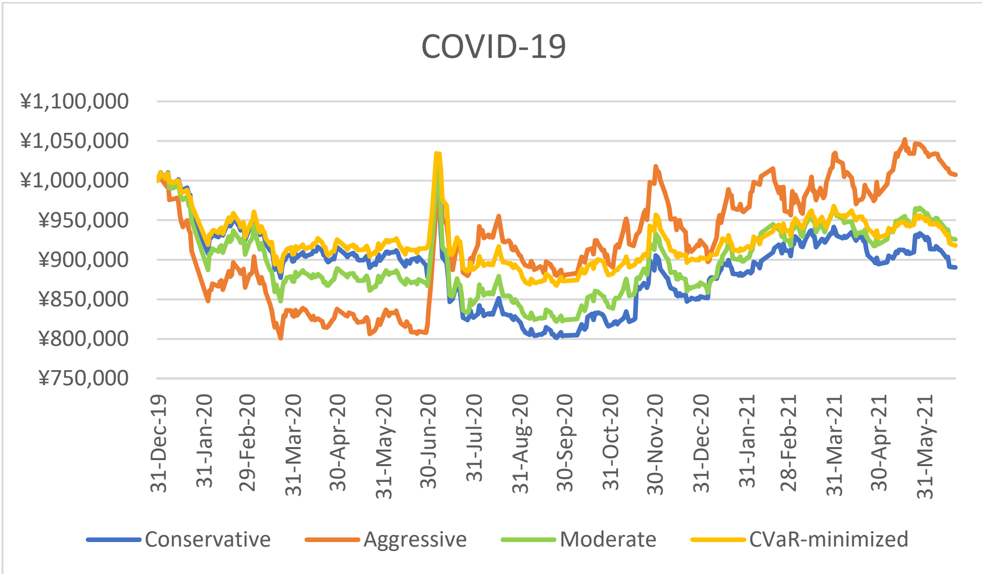

Figure 5. Cumulative gains from four investment schemes in the COVID-19 period.

In Figures 2 and 3, the "aggressive" strategy performs the worst. This shows that when the market is volatile, investing in risky stocks is not recommended. In Figure 4, it brings the highest returns, far more than other strategies do. In Figure 5, when the market risk is high, this strategy results in the lowest return. However, it becomes the best as the market risk reduces. On the contrary, the "conservative" method of investing performs better before and during the GFC, but worse in the post-GFC period. This suggests that seeking low uncertainty is not always the best strategy. In a normal market, taking some risks is beneficial, but it is wise to be cautious in an extremely risky market. The "conservative" strategy causes less loss during the GFC. The comparison between these two strategies indicates that when the market is relatively volatile, allocating more money to risky stocks will cause greater loss. If the market is in a normal state, it is likely to be a case of "high risk, high reward". The pursuit of stable income may not be as good as we expect.

Compared to other strategies, the "moderate" type is not sensitive to market risk. This strategy assigns equal weight to stocks selected by minimizing CVaR. We found that, in the four figures, the green line is always in-between the red line and the blue line. Compared to the “aggressive” and “conservative” strategies, this strategy never performs the best nor the worst, no matter what the market risk is.

Compared to weighting schemes constrained by risk tolerance, “CVaR-minimized” shows more interesting traits. Although it can bring returns larger than the lowest return an investor can accept, the relationship between its performance and market risk level is not evidently shown in the results. In Figure 4, it can be seen that it generates much lower returns than other schemes. However, in Figure 5, when the market risk level is close to that in the post-GFC period, it can be seen to be more advantageous than the “moderate” and “conservative” ones. In the later stage, the “moderate” and “CVaR-minimized” strategies are evenly matched. If this strategy is adopted, the minimum requirement can be reached, but its performance is unpredictable. An interesting discovery is that this strategy is the best in the pre-GFC period and the early stage of the COVID-19 period (from January to July 2020), in which the fluctuation range of the market risk is about 0.03, as seen in Figure 1.

It is worth mentioning that during the GFC, there exists small difference in the returns of the four strategies. This shows that when the market is extremely risky, the most important thing is not to adjust the strategy. The wise thing to do is to cut losses in time because the weighting strategy no longer matters. During the COVID-19 period, the “aggressive” strategy is the only one that brings benefits instead of losses at the end of the period, indicating that when the market conditions become better, adjusting the strategy in time helps to cut losses and gain more profits.

Table 2 shows the financial institutions that have the largest weights in the best strategy of each period. China Merchants Bank Co., Ltd. is the only one that appears twice in the table. In 242 days of the pre-GFC period, the top three institutions were invested in for at least 218 days. The largest average weight was 15.1%. During the GFC, three companies were invested in for more than half of the time, and the largest average weight was 15.7%. Institutions in the other two periods have fewer investment days, but the average weights are much larger. For example, we invested in East Money Information Co., Ltd. for only 11 days, but its average weight was 44%. The results imply that strategies in the pre-GFC and GFC periods are more dispersive with smaller variation. In the other two periods, the distribution of weights is more centralized and standard deviations imply greater fluctuations in weight.

4.3. Joint Extreme Risk Probability

Based on the results in Section 4.2.1, we selected two days that had the highest market risks in each period and computed the probability of extreme risk occurring in 60% of the stocks for them. The results are shown in Table 3. For example, during the pre-GFC period, the forecasted market risk on 7 August 2007 was the highest, followed by a value of 0.090 on 29 June 2007. There were 17 stocks in the portfolio, and we measured the joint extreme risk probability of 10 ($17 \times 0.6 \approx 10$) stocks for these two days. The results show that seven out of eight days had a *JERP* of 100%. Given the threshold we set for “extreme risk”, the results suggest that, on these eight days, the probability of receiving a return lower than the forecasted mean return of the out-of-sample period is high, for example, 100% on 7 August 2007. We can see that when the forecasted risk is high, the *JERP* is also high, but not necessarily 100%. Given the same dataset, the value of *JERP*, greatly influenced by the number of K and the selection of the threshold may not be the same. If we take a larger number of K or a threshold higher than the forecasted mean return, the *JERP* in Table 3 may become lower.

Table 2. Financial institutions with the largest weights in the best strategies.

Period	Company	Days Invested	Average Return	Average Weight (%)
CVaR-minimized (pre-GFC)	Ping An Bank Co., Ltd., Shenzhen, China	218/242	0.007 (0.032)	15.1 (0.105)
	Shanghai AJ Group Co., Ltd., Shanghai, China	228/242	0.005 (0.031)	14.5 (0.074)
	China Merchants Bank Co., Ltd., Shenzhen, China	232/242	0.006 (0.031)	11.6 (0.059)
Conservative (GFC)	China Minsheng Banking Corp., Ltd., Beijing, China	392/500	0.001 (0.030)	15.7 (0.083)
	Shanghai Pudong Development Bank Co., Ltd., Shanghai, China	454/550	−0.002 (0.042)	14.3 (0.104)
	China Merchants Bank Co., Ltd., Shenzhen, China	305/550	−0.001 (0.035)	14.0 (0.080)
Aggressive (post-GFC)	Everbright Securities Company Limited, Shanghai, China	12/244	0.009 (0.027)	25.4 (0.130)
	East Money Information Co., Ltd., Shanghai, China	27/244	0.016 (0.028)	25.4 (0.129)
	Guosen Securities Co., Ltd., Shenzhen, China	63/244	0.004 (0.021)	23.6 (0.143)
Aggressive (COVID-19)	East Money Information Co., Ltd., Shanghai, China	11/354	0.014 (0.030)	44.0 (0.160)
	CITTI Securities Company Limited, Shenzhen, China	8/354	0.0002 (0.021)	37.7 (0.100)
	Industrial Bank CO.,LTD., Fuzhou, China	109/354	0.002 (0.024)	34.8 (0.193)

Note: In the bracket is the standard deviation.

Table 3. Joint extreme risk probability.

Period (K/N)	Date	Forecasted Market Risk	Mean Return of the Portfolio	Max. Return in the Portfolio	Min. Return in the Portfolio	*JERP*
Pre-GFC (10/17)	7 August 2007	0.094	−0.005	0.040	−0.075	100%
	29 June 2007	0.090	−0.035	0.095	−0.105	90%
GFC (9/14)	3 November 2008	0.110	−0.011	0.018	−0.065	100%
	23 September 2008	0.102	−0.060	0.033	−0.105	100%
Post-GFC (18/30)	23 May 2019	0.053	−0.006	0.029	−0.029	100%
	26 March 2019	0.052	−0.014	0.012	−0.042	100%
COVID−19 (18/30)	27 July 2020	0.069	−0.005	0.009	−0.045	100%
	21 July 2020	0.062	−0.011	0.033	−0.044	100%

[1] K/N means that there are totally N stocks in the portfolio, and we measured the *JERP* of K stocks.[2] Mean, Max. and Min. returns are real returns, not the forecasted returns.

It is worth mentioning that the threshold which represents the risk capacity is not taken randomly. As a key determinant of *JERP*, it should be decided by investors according to the actual situation. For example, if *JERP* is used to forecast risk, the threshold should be extracted from the data before the day that needs to be forecasted, and hence provide references for portfolio management. Investors could adjust weights in their portfolios or stop investing if the forecasted *JERP* exceeds their tolerance levels. Suppose the "investor" in Section 4.2 stops investing in the eight days shown in Table 3 and continues to invest the next day; we calculated the losses they can stop and display the results in Table 4. It is found that, no matter which strategy the investor adopts, losses can be prevented. Among the four weighting techniques, the "conservative" way of investing can prevent the largest amount of loss, except on 29 June 2007, when the *JERP* is not 100%. From the perspective of cutting losses, the "conservative" strategy is the best.

Table 4. Amount of loss stopped by *JERP* (unit: yuan).

Date	Conservative	Aggressive	Moderate	CVaR-Minimized
7 August 2007	64,835	31,815	56,639	52,522
29 June 2007	31,362	22,401	27,382	42,766
3 November 2008	19,288	10,716	13,219	15,982
23 September 2008	25,396	15,911	17,542	20,912
23 May 2019	3562	1029	526	2415
26 March 2019	3629	1043	535	2424
27 July 2020	10,283	9388	9084	7959
21 July 2020	7524	5628	6342	4342

5. Discussion

The portfolio optimization problem is essentially the asset selection and weight allocation problem. Investors hope to maximize returns within their bearable risk range or minimize risks above their acceptable return level. Although CVaR-based portfolio optimization methods have been widely used, factors such as investors' personal risk tolerance and future risk levels that affect investment decision making are not considered. The empirical results in this study indicate that the traditional method, adjusted by risk tolerance and risk forecasts, has better performance under some market risk levels. The new risk measure, "JERP", if applied properly, functions well in helping investors cut losses.

We find, from the empirical results, that strategies considering risk tolerance do not always outperform the conventional CVaR optimization method. The results confirm that the best method of investment is to be conservative in an extremely risky market and be aggressive in a normal market. This is consistent with an individual's general behavior. As was discovered by Browne et al. [31] and Hoffmann et al. [32], the financial crisis decreased individuals' willingness to take risks, yet their risk tolerance recovered after the crisis. This study speculates that when the market risk or its fluctuation is within a certain range, the conventional method can effectively manage risks and provide optimal investment suggestions. However, once beyond the range, higher or lower, weighting techniques with risk tolerance constraints are better. In other words, the application of the CVaR optimization method has requirements for market risk level. More empirical work must be performed to prove or overturn this speculation.

This study provides investors with a complete and feasible method, including risk forecasting and portfolio optimization. As reported by Corter and Chen [2], "risk-taking behavior is a situation-specific behavior, not a general personality trait". The manner in which we name the strategies (conservative, aggressive and moderate) does not mean that a certain type of investor will always adopt the corresponding strategy. A normally conservative investor may also choose an aggressive scheme when the market risk is expected to be very low, as long as this scheme can bring them higher returns. The method we propose helps investors to prepare for future risks and supports them in selecting the strategy most likely to be profitable instead of the self-defined most suitable strategy. Moreover, the GFC and COVID-19 pandemic are two special cases that occasionally occur. The normal state of the market is actually similar to that in 2019, in which all three strategies with constraints of risk tolerance outperform the original one. Thus, the results provide support for investment advisors who pay attention to measuring clients' investment risk attitudes and taking these preferences into account in designing investment portfolios for these clients.

6. Conclusions

This paper studied the portfolio management problem, in which weights of portfolio components are allocated according to forecasted risk and the risk tolerance levels of investors. Based on the risk contribution forecasted by an FDG copula and *CES* method, weights computed using a CVaR minimization procedure are reallocated to each share in accordance with three risk tolerance levels. To verify the performance of risk-tolerance-adjusted weighting strategies, we conducted empirical research and divided the data into four periods, representing different market risk levels. The results show that weighting strategies adjusted by risk tolerance have better performance when the market is in a normal state. In addition, we put forward a new risk measure, "*JERP*", for which the threshold can be customized according to an individual's risk tolerance level. We found, in the case study, that stopping investing on days with high *JERP* helps to cut losses.

This study contributes to the literature by providing one possibility for the integration of risk tolerance into the portfolio optimization problem. Meanwhile, the case study indicates that our methods are applicable in practice, providing investors with alternative portfolio management strategies. The *JERP* method, which integrates risk capacity and risk tolerance into the computation process, satisfies the needs of practical application.

7. Limitations of the Study and Future Directions of Research

Despite the contribution of this paper, there are also some limitations that should be acknowledged. First, among all the types of copula models, only two families were adopted. Second, the empirical data only covered portfolios composed of several stocks. Stocks in other sectors and other asset classes such as funds, deposits and bonds were not included in the sample. Third, risk tolerance was simply categorized into three levels, which would be much more complicated in real life.

Since the idea of taking the influence of risk tolerance into account is relatively new, future studies could examine the performance of the methods we proposed by expanding the scope of data. The effectiveness and applicability of the new risk measure, *JERP*, could also be investigated and improved. In addition, a better way of introducing risk tolerance into the portfolio optimization problem could be explored in future research.

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