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An Interval Type-2 Fuzzy Logic Approach for Dynamic Parameter Adaptation in a Whale Optimization Algorithm Applied to Mathematical Functions

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Abstract: In this paper, an improved whale optimization algorithm (WOA) based on the utilization of an interval type-2 fuzzy logic system (IT2FLS) is presented. The main idea is to present a proposal for adjusting the values of the $\vec{r}_1$ and $\vec{r}_2$ parameters in the WOA using an IT2FLS to achieve excellent results in the execution of the WOA. The original WOA has already proven itself as an algorithm with excellent results; therefore, a wide variety of improvements have been made to it. Herein, the main purpose is to provide a hybridization of the WOA algorithm employing fuzzy logic to find the appropriate values of the $\vec{\mathrm{r}}_1$ and $\vec{\mathrm{r}}_2$ parameters that can optimize the mathematical functions used in this study, thereby providing an improvement to the original WOA algorithm. The performance of the fuzzy WOA using IT2FLS (FWOA-IT2FLS) shows good results in the case study of the benchmark function optimization. An important comparative with other metaheuristics is also presented. A statistical test and the comparative with other bio-inspired algorithms, namely, the original WOA with type-1 FLS (FWOA-T1FLS) are analyzed. The performance index used is the average of the minimum errors in each proposed method.

Keywords: swarm intelligent; type-1 fuzzy logic system; whale optimization algorithm; mathematical functions; dynamic adjustment

MSC: 03B52; 03E72; 62P30

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1. Introduction

In recent years, metaheuristic algorithms have been utilized and implemented to solve many complex problems. In this paper, a popular and efficient benchmark set of mathematical functions is used, which has been utilized by many authors to analyze and verify their proposed methods. For example, in [1], a tuna search optimization is demonstrated to have excellent results as presented by Xie et al. In [2], an algorithm based on the bird behavior is presented by Meng et al. In [3,4], a mantis search algorithm and exponential distribution optimizer are studied by Abdel-Basset et al. Additionally, in [5], a coronavirus mask protection algorithm is implemented with this problem yielding good results by Yuan et al. In [6,7], an osprey optimization algorithm and green anaconda optimization are presented by Dehghani et al. Moreover, in [8], a comparative study with the bird swarm algorithm and artificial gorilla troops optimizer is outlined with both methods showing excellent results in mathematical functions, as presented by Miramontes et al. Apart from these, several more studies have been conducted.

Nowadays, an increase in the complexity of problems has been visualized; therefore, nature-inspired algorithms have been investigated to improve their performance. A topic of interest that has been shown to yield excellent results is the improvement of the algorithm with innovative and intelligent techniques. Here, it is noteworthy to mention a few cases. In [9], a differential evolution algorithm (DEA) with dynamic adaptation in their main parameters is presented. An improved algorithm for the prediction of heart diseases is presented in [10], and an improved bio-inspired algorithm for the fog computing environment is analyzed in [11]. In [12], an improved algorithm for task mapping in networks-on-chip is presented, and an improved sparrow search algorithm is presented in [13]. A chaotic gaining and sharing knowledge optimization algorithm is presented in [14], and a chaotic dragonfly algorithm is presented in [15]. A marine predator algorithm is improved in [16], and an improved dolphin swarm algorithm is presented in [17]. The primary purpose in the hybridization of these algorithms is to find the optimal values in the main parameters that determine the achievement of an excellent performance in each algorithm; this allows us to analyze some important characteristics such as fast stabilization, high precision, the exact solution, and fast convergence rate.

In this proposal, the first relevant contribution is to explore the execution of the WOA in the case study to observe the behavioral results, and then identify the effect of the two parameters ($\vec{r}_1$ and $\vec{r}_2$) in WOA as well as the effect on exploration and exploitation, as they are crucial for obtaining excellent results.

The remainder of the paper is organized as follows. Several research works that are relevant to this study are reviewed in Section 2. The original WOA and the proposed FWOA-T2FLS are presented in Section 3. The description of the mathematical functions is detailed in Section 4. The results of the experiments are outlined in Section 5. In Section 6, the results are analyzed, statistically compared, and discussed, and finally in Section 7, the conclusions and future works are demonstrated.

2. Related Works

Nowadays, one of the hybridizations in which several authors have shown their interest is with fuzzy systems. Metaheuristic algorithms have provided greater stabilization and better error performances when the fuzzy set method is implemented because problems have grown in complexity. For example, in [18], an adaptive neuro-fuzzy inference system and genetic algorithm (NFGA)-based MPPT controller for the PV system is presented and in [19], a synthesis of the neuro-fuzzy regulator with genetic algorithm is analyzed. Additionally, in [20], a new fuzzy approach to dynamic adaptation applied to the marine predator algorithm is presented, and in [21], an improved chimp optimization algorithm is presented. Moreover, in [22], a hybrid spiral-bacterial foraging algorithm for a fuzzy control design of a flexible manipulator is studied, and in [23], an optimal fuzzy logic controller-based PSO for the photovoltaic system is presented. Furthermore, in [24], a boosted PSO with fuzzy K-nearest neighbor classifier for predicting atopic dermatitis disease is presented.

When the complexity of the problems grows, it is necessary to provide a more precise response. As a result, the IT2FS has the main characteristic of uncertainty in the evaluation with greater potential for providing excellent results, as several authors have demonstrated. Some relevant works have been studied, for example; in [25] an optimization of the T2FL is presented with the hybridization of GSO and FA algorithms, and in [26], a shadowed type-2 FS in the improvement of the cuckoo search and flower pollination algorithms is presented. In [27], a type-2 FL is implemented in the improvement of a metaheuristic algorithm, and in [28], a bat algorithm is implemented in the optimization of an IT2FLS. In [29], a metaheuristic approach for IT2FLS is implemented, and in [30], a type-2 FS is implemented in the dynamic adaptation of harmony search optimization [31] and bee colony optimization [32]. In [33], a T2FL metaheuristic algorithm is implemented in nonlinear dynamic systems, and in [34], a high-speed IT2FS approach for dynamic parameter adaptation is presented.

In recent years, a bio-inspired algorithm called WOA has generated great interest because it has been studied and applied in various fields to solve problems and has been found to yield excellent results in the research. Some relevant works have been applied. In [35], the WOA algorithm is used in image segmentation, in [36], it is implemented to stabilize a control problem, and in [37], it is used for biomedical classification. In [38], WOA is implemented as a planning strategy for mobile robots, in [39], it is used to solve the optimal reactive power dispatch problem, and in [40], it is used as an optimizing deep learning method for software defect prediction. In [41], the WOA algorithm is used in the reconfiguration of distribution networks with simultaneous allocation of distributed generation, and in [42] it is used to classify problems.

Therefore, the WOA algorithm showed significant improvements over various methods. In particular, many authors are interested in the performance of WOA in several problems [43–52]. Some related works where this algorithm has been studied with a similar problem are presented in [53–55].

The proposed improvement with the hybridization using IT2FLS is a new approach in the minimization of benchmark functions.

Once the state-of-the art was analyzed, the WOA algorithm was observed as having the largest number of contributions and applications. As studied by different authors, the WOA algorithm is one of the most popular algorithms due to its rapid convergence in obtaining results. The justification for choosing this algorithm is mainly based on its popularity and this has motivated us to present a new hybridization using type-2 fuzzy logic to improve the original WOA algorithm.

3. Whale Optimization Algorithm

Mirjalili and Lewis [56] created the whale optimization algorithm, drawing inspiration from the behavior of whales in nature. Three critical behaviors have been identified. First, in some parts of their brains, whales have common cells similar to human beings. Second, as the largest mammals in the world, whales are recognized for their intelligence. Third, in the social behavior of whales, they can successfully live alone or in a group. However, they are mostly observed in groups [56].

3.1. Original WOA

A visual representation of the behavior of a whale is illustrated in Figure 1, and in Figure 2, each movement that a whale performs in the process is illustrated. The mechanism of the movement of the whale to find food is based on three interesting techniques: encircling prey (surround prey), spiral bubble-net feeding maneuver (exploitation phase), and search for prey (exploration phase) to find the objective [56]. In this case, during the WOA execution, the best whale with the best solution is updated according to the current location.

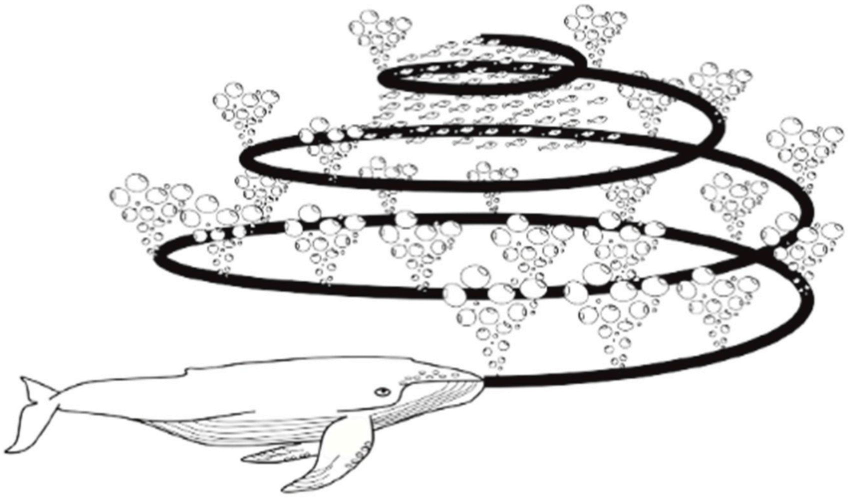


Figure 1. Visual representation of the movement of a whale to find food.

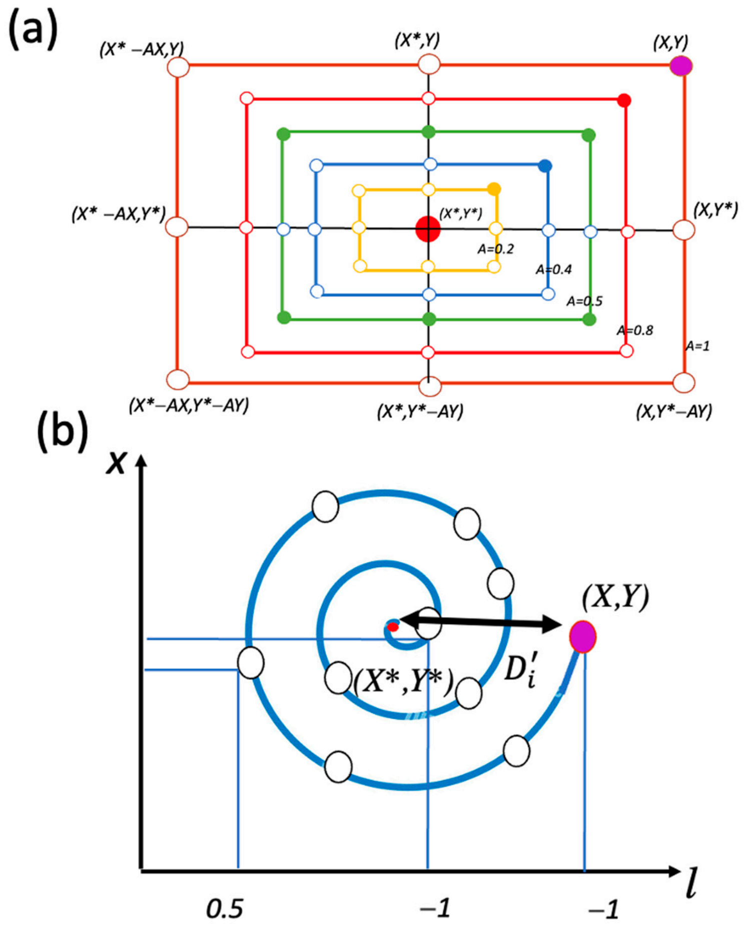


Figure 2. General representation of a bubble-net search mechanism in WOA (X* is the best solution obtained to date): (**a**) shrinking encircling mechanism and (**b**) spiral updating position.

The step-by-step process in WOA is expressed by Algorithm 1.

Algorithm 1: Pseudo-Code of the WOA Algorithm

```
Initialize the whale population Xi (i = 1, 2,. . .. . ., $n$)
Calculate the fitness of each search agent
X* = the best search agent
while (t < maximum number of iterations)
    for each search agent update a, A, C, l, and p
        if1 ($p < 0.5$)
            if2 ($|A| < 1$)
                Update the position of the current agent by Equation (1)
            else if2 ($|A| \geq 1$)
                Select a random search agent ($X_{rand}$)
                Update the position of the current agent by Equation (6)
            end if2
            else if1 ($p \geq 0.5$)
                Update the position of the current search by Equation (5)
            end if1
            end for
            Check if any search agent goes beyond the search space and amend it
            Calculate the fitness of each search agent
            Update X* if there is a better solution
            t = t + 1
end while
return X*
```

3.1.1. Surround Prey

This phase is based on the humpback whale's foam-net feeding method. A relevant movement is that a whale can locate their prey by rotating around it. The optimal solution is defined in the start of this phase and other whales (agents) update their position towards the best individual. This behavior is expressed by Equations (1) and (2) [56]:

$$\vec{V} = \left| \vec{N}.\vec{X}^{*}(m) - \vec{X}(m) \right| \tag{1}$$

$$\vec{X}(m+1) = \vec{X}^{*}(m) - \vec{E} \cdot \vec{V} \tag{2}$$

where the total execution in WOA is expressed by m; the actual position in the optimal solution is expressed by $\vec{X}^{*}$ and is updated in each iteration; $\vec{X}(m)$ is the current position; $\vec{E}$ represent the coefficients of $\vec{X}(m)$ and $\vec{V}$, respectively and are calculated by Equations (3) and (4) [56]:

$$\vec{E} = 2\,\vec{d} \cdot \vec{r}_1 - \vec{d} \tag{3}$$

$$\vec{N} = 2 \cdot \vec{r}_2 \tag{4}$$

In this phase, $\vec{d}$ slowly changes from 2 to 0, and $\vec{r}_1$ and $\vec{r}_2$ are in the range between 0 and 1. (These two parameters are studied in this paper, and the detailed explanation is presented in the following sub-section).

3.1.2. Bubble-Net Attacking Method

In this phase, two movements, namely, shrink and surround are performed by a whale. This is achieved by reducing $\vec{d}$ in Equation (3). Here, $\vec{E}$ is a random value in the interval [$-$d, d], where $\vec{d}$ decreases from 2 in the iterative process to as low as 0. Set d = 1, namely, $\vec{E}$ is a random value of [$-$1, 1]. In WOA, the mechanism to find the new position of a whale is expressed as a spiral equation and is indicated mathematically in Equation [5]:

$$\vec{X}(m+1) = \vec{L} \cdot e^{fj} \cdot \cos(2\pi j) + \vec{X}^{*}(m) \tag{5}$$

where $\vec{L} = \left| \vec{X}^{*}(m) - \vec{X}(m) \right|$; f is the helix constant and a random number in [$-$1, 1] is indicated by j.

3.1.3. Search for Prey

In the following phase, $\vec{E}$ is implemented to find the prey. A random value of $\left|\vec{E}\right|$ which is greater than 1 is established. This process is expressed by Equation (6) [56]:

$$\vec{V} = \left| \vec{N}.\,\vec{X}_{rand} - \vec{X} \right| \tag{6}$$

$$\vec{X}\,(m+1) = \vec{X}_{rand} - \vec{E} \cdot \vec{V}$$

where $\vec{X}_{rand}$ indicates a random position in the current population. In each iteration, a whale chooses which way to update its location according to the value of $\vec{E}$. In this phase, when $|\vec{E}| > 1$, then the selection is represented by a whale (individually), and when $|\vec{E}| < 1$, then the selection is assigned to the best solution of the population.

3.2. Fuzzy WOA

Utilizing the proposed method, the mathematical functions are optimized with the main goal of obtaining minimal values. In this case, the values of $\vec{r}_1$ and $\vec{r}_2$ parameters affect the performance of the WOA algorithm. Based on a previous analysis, the $\vec{r}_1$ parameter (see Equation (3)) has a relationship with the exploration of the algorithm, and $\vec{r}_2$ parameter (see Equation (4)) has a relationship with the exploitation of the algorithm. Both parameters are obtained with the interval type-2 FLS based on fuzzy rules. These rules were designed by analyzing the performance in the original WOA algorithm in previous experiments. To find the optimal values, the IT2FLS has two inputs, namely, iteration and diversity. This section explains this interesting methodology in more detail.

A graphical representation of this paper is depicted in Figure 3.

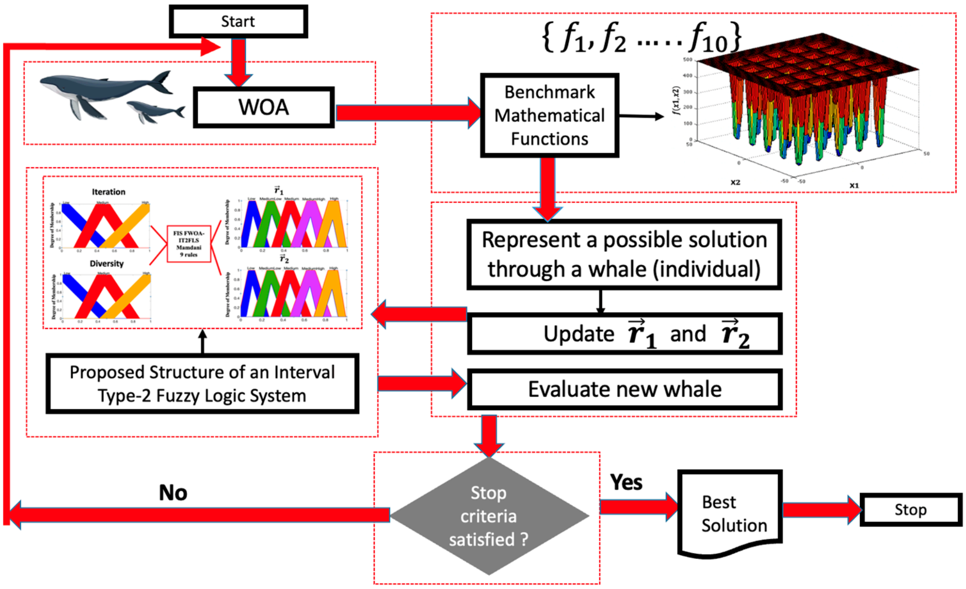

Start
WOA
$\{f_1, f_2 \ldots\ldots f_{10}\}$
Benchmark Mathematical Functions
Iteration
Diversity
FIS FWOA-IT2FLS Mamdani 9 rules
$\vec{r}_1$
$\vec{r}_2$
Represent a possible solution through a whale (individual)
Update $\vec{r}_1$ and $\vec{r}_2$
Proposed Structure of an Interval Type-2 Fuzzy Logic System
Evaluate new whale
No
Yes
Stop criteria satisfied ?
Best Solution
Stop

Figure 3. Proposed idea in the fuzzy WOA.

3.2.1. Type-1 Fuzzy Logic System

Zadeh created a way to analyze complex problems with uncertainty or manage the way humans make deductions, which is called a fuzzy logic system (FLS). The main components of type-1 FLS are depicted in Figure 4 [57].

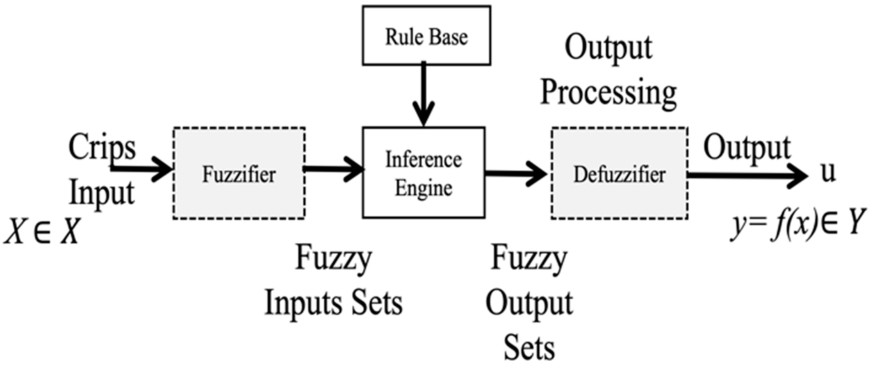


Figure 4. Structure of a type-1 FLS.

Equation (7) indicates the representation of a fuzzy set (FS) in universe X [58]:

$$A = \{(x, \mu_A(x)) \mid x \in X\} \tag{7}$$

where $\mu_A : X \to [0,1]$. In Equation (7), $\mu_A(x)$ indicates the membership degree of element x ∈ X to set A. The main processes that compose a T1FLS are a fuzzifier which maps a crisp input into a fuzzy set, an inference engine which processes the rule base, and a defuzzifier which is used to find the output of the T1FLS as a final crisp value.

3.2.2. Interval Type-2 Fuzzy System

Based on Zadeh's ideas, Mendel et al. presented the mathematical definition of a type-2 fuzzy set, as follows [59].

An interval type-2 fuzzy set $\tilde{A}$, denoted by $\underline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$, is represented by the lower and upper membership functions of $\mu_{A^{\vee}}(x)$, where $x \in X$. In this case, Equation (8) shows the definition of an IT2FS [60]:

$$\tilde{A} = \{((x,u),1) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{8}$$

where X is the primary domain and Jx is the secondary domain. All secondary degrees $\left(\mu_{\tilde{A}}(x,u)\right)$ are equal to 1. Figure 5 shows the representation of an interval type-2 fuzzy system.

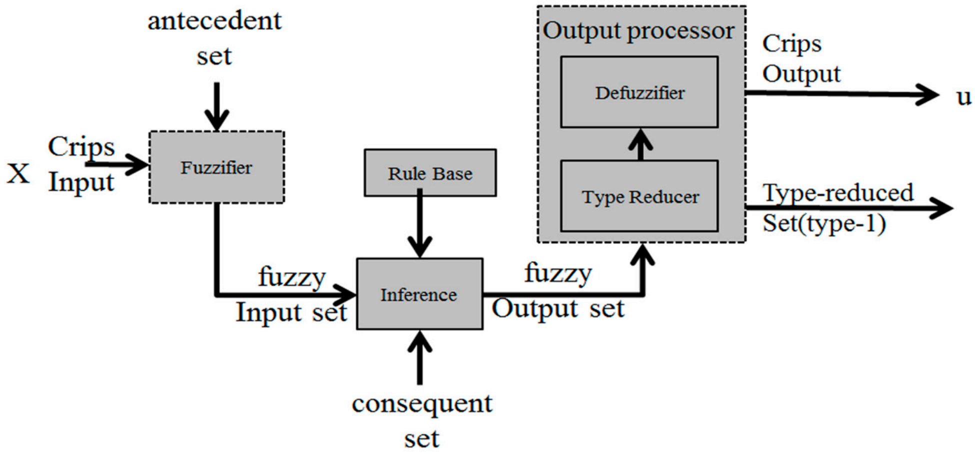


Figure 5. Structure of an interval type-2 fuzzy system.

The output processor includes a type reducer and defuzzifier that generates a type-1 fuzzy set output (from the type reducer) or a crisp number (from the defuzzifier) [60]. An interval type-2 FLS is also characterized by IF-THEN rules, but their fuzzy sets are now of the interval type-2 form. The type-2 fuzzy set can be used when circumstances are very uncertain to determine the exact membership degrees, as is the case with the membership functions (MFs) in a fuzzy controller that can take different values. Additionally, we want to find the distribution of MFs that shows better results in the stability of the fuzzy controller.

The structure of the proposal with IT2FLS to find the optimal values of the $\vec{r}_1$ and $\vec{r}_2$ parameters of the WOA is shown below. Figure 6 illustrates the inputs and outputs with the triangular membership functions (MFs), and Figure 7 illustrates the proposal with T1FLS.

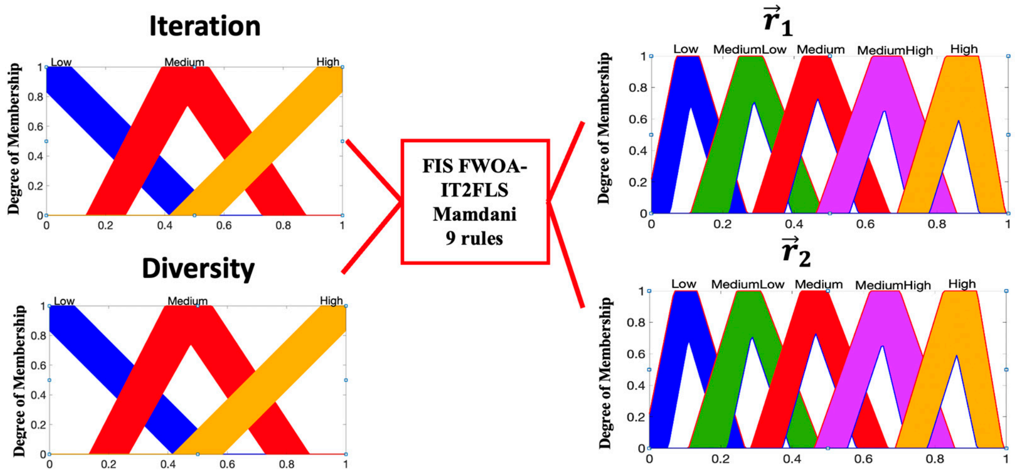


Figure 6. Design of the inputs and outputs of the proposal with IT2FLS.

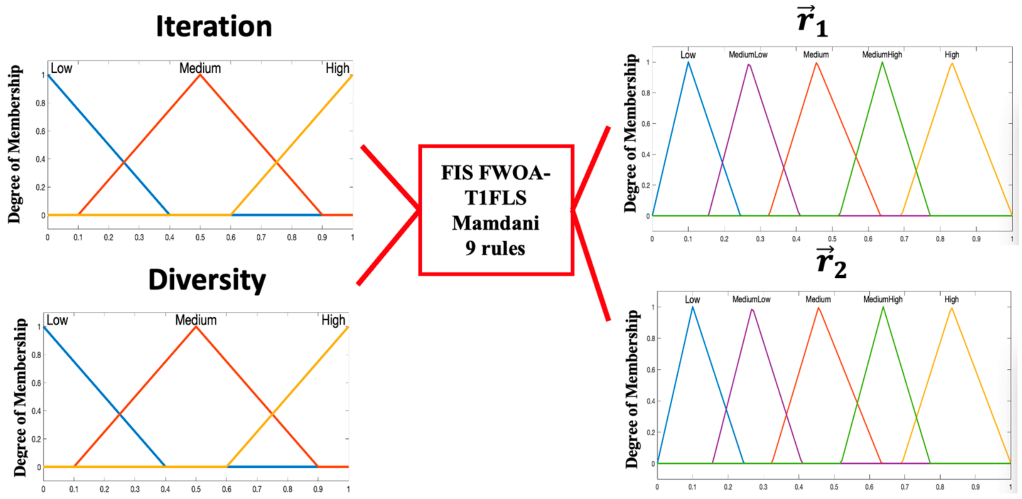


Figure 7. Design of the inputs and outputs of the proposal with T1FLS.

The FLS is detailed in the following paragraphs. Triangular membership functions (MFs) are used because this type of MF has been shown to give stability in the results in previous works. A total of two inputs and two outputs with three triangular MFs in each variable are used, each MF requires six values in the solution for the IT2FLS; therefore, the IT2FLS comprises 96 values and T1FLS contains 48 values.

The idea comprises finding the values of the $\vec{r}_1$ and $\vec{r}_2$ parameters and is based on the methodology outlined in [31,61]. Based on the preliminary experiments, the value of $\vec{r}_1$ is observed, which is an important parameter in the process of exploring good solutions in the algorithm. On the other hand, when the value of $\vec{r}_2$ is close to 1, this indicates that a better solution is close to the best solution and the process is exploited in the WOA.

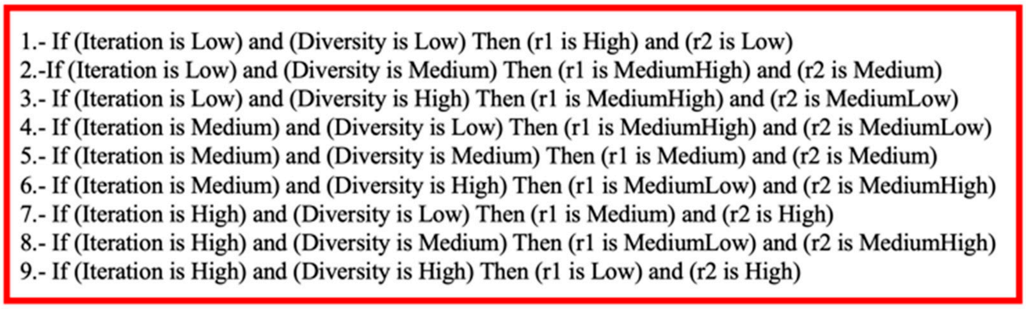
To verify the effect of the iterations in the WOA, the percentage of the iterations is used as a variable, then in the start of WOA, the iterations will be considered "low"; otherwise, at the end, all iterations will be considered "high" or close to 100%. The representation of the idea is expressed by Equation (9) [31,61]:

$$Iteration = \frac{Current\ Iteration}{Maximum\ of\ Iterations} \tag{9}$$

The second input in the T1FLS is called the diversity measure as defined by Equation (10). It measures the degree of dispersion of the whales, i.e., when the whales are closer together, then there is less diversity, and when whales are separated, then the diversity is higher. This evaluation is considered as the average of the Euclidean distance between each whale and the best whale. The main idea in the use of diversity is to prevent the algorithm from running into a local minimum. This mechanism is analyzed with the fuzzy rules that integrate the T1FLS, as shown in more detail in Figure 8 [31,61]:

$$Diversity(S(t)) = \frac{1}{n_s}\sum_{i=1}^{n_x}\sqrt{X_{ij}(t) - \overline{X}_j(t)^2} \tag{10}$$

where t represents the current iteration, n_s is used with the value of the population, i indicates the whale, the number of possible solutions is expressed by n_x (in this case, it indicates the number of dimensions) and indicates the number of solutions, j is expressed as the next solution, X_{ij} indicates solution j of the whale i, and finally, $\overline{X}_j$ represents solution j of the best whale in the space search.



1.- If (Iteration is Low) and (Diversity is Low) Then (r1 is High) and (r2 is Low)
2.-If (Iteration is Low) and (Diversity is Medium) Then (r1 is MediumHigh) and (r2 is Medium)
3.- If (Iteration is Low) and (Diversity is High) Then (r1 is MediumHigh) and (r2 is MediumLow)
4.- If (Iteration is Medium) and (Diversity is Low) Then (r1 is MediumHigh) and (r2 is MediumLow)
5.- If (Iteration is Medium) and (Diversity is Medium) Then (r1 is Medium) and (r2 is Medium)
6.- If (Iteration is Medium) and (Diversity is High) Then (r1 is MediumLow) and (r2 is MediumHigh)
7.- If (Iteration is High) and (Diversity is Low) Then (r1 is Medium) and (r2 is High)
8.- If (Iteration is High) and (Diversity is Medium) Then (r1 is MediumLow) and (r2 is MediumHigh)
9.- If (Iteration is High) and (Diversity is High) Then (r1 is Low) and (r2 is High)

Figure 8. Fuzzy rules of the IT2FLS.

Based on the natural characteristics of a bio-inspired algorithm that present the intelligent behavior of living beings, which in this case is a whale, it is observed that when the food is closer to the current solution, then an exploitation mechanism occurs in the area. For this, the value of the $\vec{r}_2$ parameter that represents the exploitation in the proposal is considered low or close to 0, which is part of the analysis when designing the fuzzy rules (see Figure 8). This is why the fuzzy rules start with a low exploitation when the iterations are in beginning of the algorithm. On the other hand, when the iterations are at the end of the execution, the value of the $\vec{r}_2$ parameter is considered high or close to 1. Figure 8 shows the nine fuzzy rules are based on previous works [61].

4. Set of Benchmark Functions

A case study in the optimization of benchmark functions is herein demonstrated. A total of ten classical benchmark functions are used with the idea of validating FWOA-IT2FLS such as Sphere, Griewangk, Rastringin, Shewefel, Sum of Different Powers, Zakharov, Dixon and Price, Levy, Sum of Squares, and Rotated Hyper Ellipsoid in the simulation of results. Each function was evaluated with 10, 20, 30, 50, and 100 dimensions (artificial whale). The mathematical representation for each function is shown in Table 1, and the plot for each mathematical function is illustrated in Figure 9.

Table 1. Set of benchmark functions.

f_x	Name	Search Domain	f Min	Mathematical Representation
F1	Sphere	$[-5.12,\ 5.12]^n$	0	$\mathrm{f(x)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{x_i^2}$
F2	Griewangk	$[-600,\ 600]^n$	0	$\mathrm{f(x)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \frac{\mathrm{x_i^2}}{400} - \prod_{\mathrm{i=1}}^{\mathrm{n}} \cos\left(\frac{\mathrm{x_i}}{\sqrt{\mathrm{i}}}\right) + 1$

Table 1. *Cont.*

f_x	Name	Search Domain	f Min	Mathematical Representation
F3	Rastringin	$[-5.12,\ 5.12]^n$	0	$f(x) = 10n + \sum_{i=1}^{n}\left[x_i^2 - 10\cos(2x_i)\right]$
F4	Shewefel	$[-500,\ 500]^n$	−837.9658	$f(x) = 418.9829n - \sum_{i=1}^{dn} x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)$
F5	Sum of Different Powers	$[-1,\ 1]^n$	0	$f(x) = \sum_{i=1}^{n} \lvert x_i\rvert^{i+1}$
F6	Zakharov	$[-5,\ 10]^n$	0	$f(x) = \sum_{i=1}^{n} x_i^2 + (\sum_{i+1}^{n} 0.5ix_i)^2 + (\sum_{i=1}^{n} 0.5ix_i)^4$
F7	Dixon and Price	$[-10,\ 10]^n$	0	$f(x) = (x_1 - 1)^2 + \sum_{i=2}^{n} i\left(2x_i^2 - x_{i-1}\right)^2$
F8	Levy	$[-10,\ 10]^n$	0	$f(x) = \sin^2(\pi\omega_1) + \sum_{i=1}^{n-1}(\omega_1 - 1)^2[1 + 10\sin^2(\pi\omega_1 + 1)] + (\omega_n - 1)^2[1 + \sin^2(2\pi\omega_n)]$, where $\omega_i = 1 + \frac{x_i - 1}{4}$, for all i = 1,. . . . .,n
F9	Sum of Squares	$[-10,\ 10]^n$	0	$f(x) = \sum_{i=1}^{n} ix_i^2$
F10	Rotated Hyper Ellipsoid	$[-65.536,\ 65536]^n$	0	$f(x) = \sum_{i=1}^{n}\sum_{j=1}^{i} x_j^2$

(**a**)

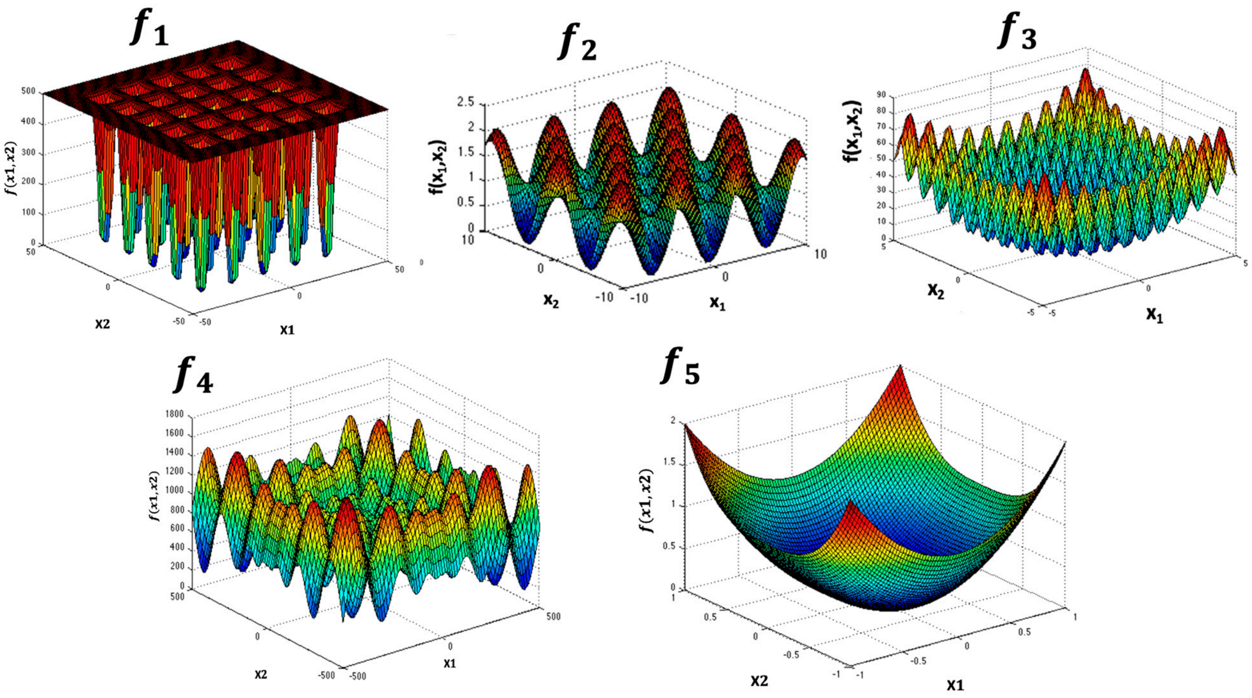


Figure 9. *Cont.*

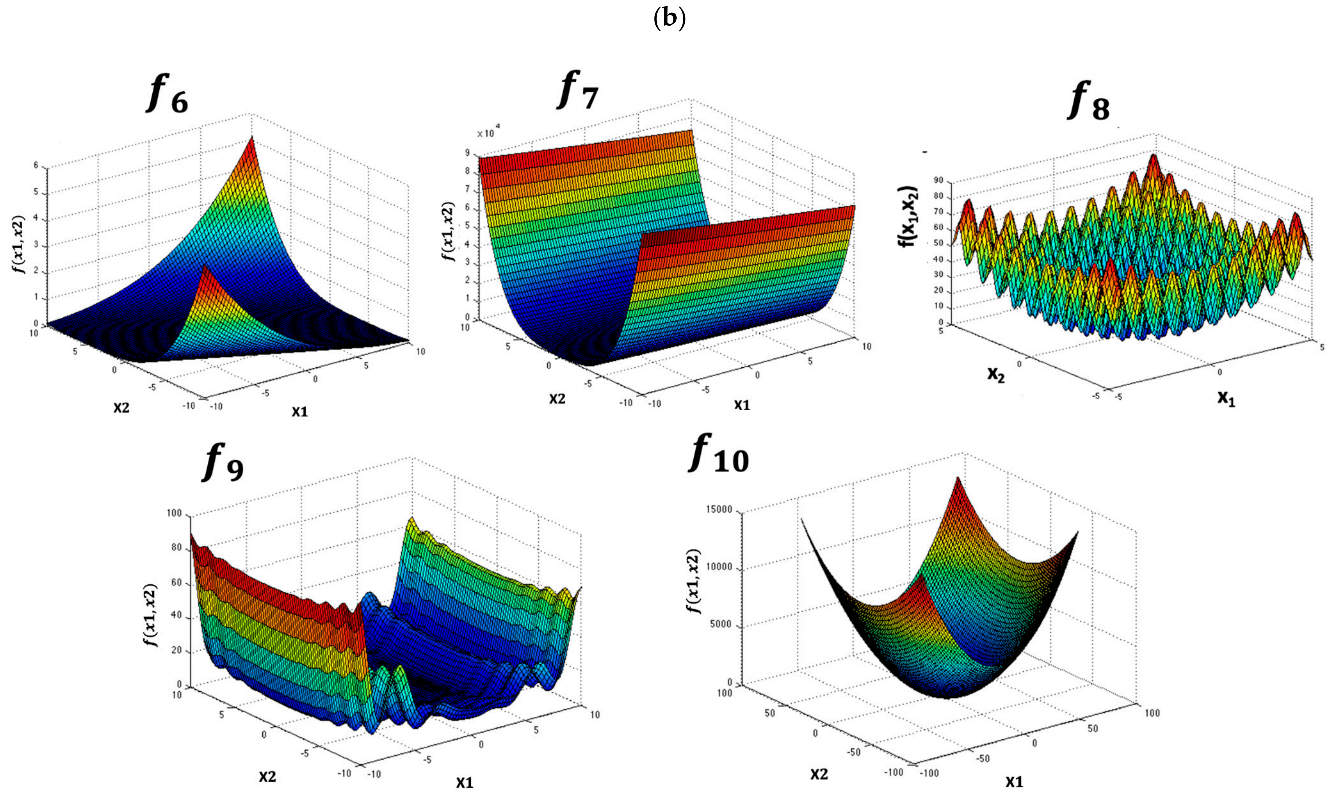


Figure 9. Plot of the mathematical functions. (**a**) Sphere, Griewangk, Rastringin, Shewefel, and Sum of Different Powers. (**b**) Zakharov, Dixon and Price, Levy, Sum of Squares, and Rotated Hyper Ellipsoid.

5. Experimental Results

This section outlines the results of the benchmark functions implemented in this paper. The objective of this experiment is to determine the behavior of the FWOA with IT2FLS to find the optimal values of $\vec{r}_1$ and $\vec{r}_2$ parameters in the WOA. Each experiment was executed with 1000 iterations in the WOA and FWOA. In the original WOA, the values of $\vec{r}_1$ and $\vec{r}_2$ were obtained with a random number in the range of [0, 1] and a fixed value of 0.5 for each parameter. Tables 2–6 demonstrate a comparative using the original WOA with fixed and random parameters, with 10 dimensions in Table 2, 20 dimensions in Table 3, 30 in Table 4, 50 in Table 5, and 100 in Table 6. The average for each execution is shown.

Table 2. Benchmark functions of varying $\vec{r}_1$ and $\vec{r}_2$ values with 10 dimensions.

f_x	**Methods**			
	Original WOA $\vec{r}_1$ **and** $\vec{r}_2$ **Fixed**	**Original WOA** $\vec{r}_1$ **and** $\vec{r}_2$ **Random**	**FWOA-T1FLS**	**FWOA-IT2FLS**
F1	6.92×10^{-51}	2.49×10^{-1}	3.41×10^{-1}	$\mathbf{6.29 \times 10^{-53}}$
F2	$\mathbf{1.61 \times 10^{-9}}$	$6.99 \times 10^{+1}$	$7.72 \times 10^{+6}$	2.91×10^{-9}
F3	8.79×10^{-10}	$2.15 \times 10^{+1}$	$1.71 \times 10^{+13}$	$\mathbf{3.74 \times 10^{-10}}$
F4	$\mathbf{2.91 \times 10^{-2}}$	$2.15 \times 10^{+1}$	$5.00 \times 10^{+2}$	$9.88 \times 10^{+1}$
F5	$\mathbf{6.35 \times 10^{-35}}$	8.21×10^{-2}	-1.69×10^{-15}	1.86×10^{-28}
F6	4.54×10^{-1}	3.06×10^{-1}	$8.96 \times 10^{+13}$	$\mathbf{3.97 \times 10^{-1}}$
F7	$\mathbf{1.20 \times 10^{-2}}$	2.90×10^{-2}	2.90×10^{-2}	1.26×10^{-2}
F8	$\mathbf{3.97 \times 10^{-1}}$	1.16×10^{0}	$3.84 \times 10^{+14}$	6.20×10^{-1}
F9	$\mathbf{4.94 \times 10^{-54}}$	1.16×10^{-1}	3.93×10^{-32}	1.22×10^{-48}
F10	$\mathbf{2.21 \times 10^{-52}}$	$1.01 \times 10^{+1}$	$-6.55 \times 10^{+13}$	7.48×10^{-44}

Table 3. Benchmark functions of varying $\vec{r}_1$ and $\vec{r}_2$ values with 20 dimensions.

f_x	Methods			
	Original WOA **$\vec{r}_1$ and $\vec{r}_2$ Random**	**Original WOA** **$\vec{r}_1$ and $\vec{r}_2$ Fixed**	**FWOA-T1FLS**	**FWOA-IT2FLS**
F1	**1.18×10^{-70}**	5.26×10^{-1}	1.06×10^{-41}	7.57×10^{-60}
F2	2.57×10^{-9}	8.75×10^{-1}	$3.33 \times 10^{+6}$	**1.75×10^{-10}**
F3	9.47×10^{-10}	6.70×10^{-1}	$1.17 \times 10^{+13}$	**8.22×10^{-12}**
F4	$4.06 \times 10^{+2}$	$1.33 \times 10^{+1}$	$1.40 \times 10^{+13}$	**$9.20 \times 10^{+1}$**
F5	**4.15×10^{-49}**	2.43×10^{-2}	2.25×10^{-36}	2.97×10^{-42}
F6	**2.17×10^{-1}**	3.29×10^{-1}	$1.22 \times 10^{+13}$	3.88×10^{-1}
F7	**1.13×10^{-2}**	1.06×10^{0}	1.06×10^{0}	1.40×10^{-2}
F8	8.45×10^{-1}	1.16×10^{0}	6.27×10^{-1}	**5.67×10^{-1}**
F9	**3.17×10^{-72}**	4.03×10^{-1}	1.21×10^{-52}	5.25×10^{-55}
F10	**1.16×10^{-71}**	4.19×10^{0}	4.29×10^{-53}	1.25×10^{-57}

Table 4. Benchmark functions of varying $\vec{r}_1$ and $\vec{r}_2$ values with 30 dimensions.

f_x	Methods			
	Original WOA **$\vec{r}_1$ and $\vec{r}_2$ Random**	**Original WOA** **$\vec{r}_1$ and $\vec{r}_2$ Fixed**	**FWOA-T1FLS**	**FWOA-IT2FLS**
F1	**9.58×10^{-84}**	2.39×10^{-1}	1.20×10^{-51}	2.84×10^{-59}
F2	**2.57×10^{-9}**	$6.35 \times 10^{+1}$	$5.60 \times 10^{+5}$	7.22×10^{-9}
F3	1.36×10^{-10}	$4.36 \times 10^{+2}$	$1.17 \times 10^{+13}$	**1.39×10^{-9}**
F4	$3.76 \times 10^{+2}$	$4.39 \times 10^{+2}$	$1.40 \times 10^{+13}$	**$2.23 \times 10^{+2}$**
F5	**3.75×10^{-54}**	1.03×10^{-1}	4.47×10^{-27}	1.56×10^{-42}
F6	5.65×10^{-1}	3.73×10^{-1}	5.65×10^{-1}	**1.24×10^{-1}**
F7	1.39×10^{-2}	4.50×10^{-1}	$8.01 \times 10^{+5}$	**1.31×10^{-2}**
F8	9.39×10^{-1}	9.35×10^{-1}	$4.77 \times 10^{+14}$	**4.32×10^{-1}**
F9	**6.38×10^{-81}**	$6.65 \times 10^{+1}$	-1.28×10^{-62}	1.19×10^{-61}
F10	**2.14×10^{-78}**	4.60×10^{0}	-4.37×10^{0}	7.51×10^{-66}

Table 5. Benchmark functions of varying $\vec{r}_1$ and $\vec{r}_2$ values with 50 dimensions.

f_x	Methods			
	Original WOA **$\vec{r}_1$ and $\vec{r}_2$ Random**	**Original WOA** **$\vec{r}_1$ and $\vec{r}_2$ Fixed**	**FWOA-T1FLS**	**FWOA-IT2FLS**
F1	**1.69×10^{-93}**	-3.57×10^{-1}	1.00×10^{-56}	2.35×10^{-60}
F2	5.28×10^{-9}	-6.03×10^{0}	1.42×10^{6}	**2.15×10^{-9}**
F3	8.64×10^{-10}	7.55×10^{-2}	2.16×10^{5}	**1.96×10^{-10}**
F4	4.21×10^{2}	**1.27×10^{2}**	$5.00 \times 10^{+2}$	$3.03 \times 10^{+2}$
F5	**1.55×10^{-64}**	1.00×10^{-1}	3.91×10^{-38}	3.50×10^{-42}
F6	5.65×10^{-1}	1.71×10^{0}	3.75×10^{13}	**4.35×10^{-1}**
F7	**1.12×10^{-2}**	5.63×10^{-1}	2.28×10^{14}	1.30×10^{-2}
F8	9.93×10^{-1}	7.21×10^{-1}	3.97×10^{14}	**5.81×10^{-1}**
F9	**1.14×10^{-91}**	6.25×10^{-1}	-1.33×10^{0}	4.88×10^{-59}
F10	**1.64×10^{-89}**	-2.57×10^{0}	-4.37×10^{0}	2.80×10^{-59}

Table 2 shows that the FWOA-IT2FLS method with functions F1, F3, and F6 provides the best solution, while in Table 3, the FWOA-IT2FLS method with functions F2, F3, F4, and F8 is better. When the dimensions are 30, the proposed method shows an improvement in functions F3, F4, F6, and F7 (see Table 4). When the dimensions are 50, the complexity increases and functions F3, F3, F6, and F8 continue with minimum average errors (see Table 5). When the dimensions are 100, the proposed FWOA-IT2FLS method with functions F2, F6, and F8 is maintained with better results (see Table 6).

Table 6. Benchmark functions of varying $\vec{r}_1$ and $\vec{r}_2$ values with 100 dimensions.

f_x	Methods			
	Original WOA $\vec{r}_1$ and $\vec{r}_2$ Random	**Original WOA $\vec{r}_1$ and $\vec{r}_2$ Fixed**	**FWOA-T1FLS**	**FWOA-IT2FLS**
F1	$\mathbf{2.34 \times 10^{-101}}$	2.68×10^{-1}	5.17×10^{-63}	2.63×10^{-59}
F2	4.00×10^{-9}	$7.57 \times 10^{+1}$	$3.44 \times 10^{+5}$	$\mathbf{4.59 \times 10^{-10}}$
F3	$\mathbf{8.19 \times 10^{-10}}$	4.53×10^{-1}	$1.17 \times 10^{+13}$	8.94×10^{-10}
F4	$\mathbf{4.21 \times 10^{2}}$	5.00×10^{2}	$1.40 \times 10^{+13}$	$4.87 \times 10^{+2}$
F5	2.40×10^{-64}	7.40×10^{-2}	$\mathbf{7.65 \times 10^{-79}}$	2.67×10^{-43}
F6	5.00×10^{0}	1.61×10^{0}	$1.62 \times 10^{+14}$	$\mathbf{1.29 \times 10^{0}}$
F7	$\mathbf{1.12 \times 10^{-2}}$	1.33×10^{0}	1.33×10^{0}	1.20×10^{-2}
F8	1.00×10^{0}	6.78×10^{-1}	$7.05 \times 10^{+14}$	$\mathbf{4.38 \times 10^{-1}}$
F9	$\mathbf{2.52 \times 10^{-102}}$	2.91×10^{0}	3.33×10^{-1}	7.47×10^{-59}
F10	$\mathbf{1.03 \times 10^{-98}}$	-2.17×10^{0}	2.53×10^{-80}	2.01×10^{-60}

6. Analysis of Results

To verify the performance of the results obtained with the proposed FWOA-IT2FLS method, a statistical test and a comparative with the original WOA and fuzzy bee colony optimization algorithm (FBCO) [61] are presented.

6.1. Statistical Test

The parameters of the statistical z-test include the 95% confidence level, alpha of 5%, $H_a = \mu_1 < \mu_2$, $H_0 = \mu_1 \geq \mu_2$, and critical value of -1.645. The description in each hypothesis is as follows. H_o is the proposed FWOA-IT2FLS method which is greater or equal to the original WOA with random values and H_a denotes that the results of the proposed FWOA-IT2FLS method are smaller (better) than the original WOA. According to the values used in Tables 3–7, a sample of 30 experiments was randomly chosen, which shows a rejection zone for values lower than -1.64. Equation (11) expresses the mathematical function of the z-test:

$$Z = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - (\mu_1 - \mu_2)}{\sigma_{\overline{X}_1 - \overline{X}_2}} \tag{11}$$

Table 7. Statistical test of original WOA and FWOA-IT2FLS.

f_x	Methods			
	FWOA-IT2FLS	**Original WOA $\vec{r}_1$ and $\vec{r}_2$ Random**	**Z-Value**	**Evidence**
F1	-2.63×10^{-59}	2.34×10^{-101}	-1.265	NS
F2	4.59×10^{-10}	4.00×10^{-9}	-1.004	NS
F3	8.94×10^{-10}	8.19×10^{-10}	-3.531	S
F4	$4.87 \times 10^{+2}$	$4.21 \times 10^{+2}$	-97.494	S
F5	2.67×10^{-43}	2.40×10^{-64}	1.281	NS
F6	1.29×10^{0}	5.00×10^{0}	-0.016	NS
F7	1.20×10^{-2}	1.12×10^{-2}	0.0543	NS
F8	4.38×10^{-1}	1.00×10^{0}	77.992	NS
F9	7.47×10^{-59}	-2.52×10^{-102}	-0.852	NS
F10	2.01×10^{-60}	-1.03×10^{-98}	-1.0054	NS

The results of the statistical test show that when the z-value is smaller than -1.645, then the evidence is "Significant" to reject the null hypothesis. As a result, the alternative hypothesis is accepted. In this case, in the comparison between FWOA-IT2FLS and original WOA, the average (avg) with 100 dimensions is presented in Table 7. A statistical test compares FWOA-T1FLS and original WOA with the results of 100 dimensions in Table 8, and the statistical test of FWOA-IT2FLS and FWOA-T1FLS is presented in Table 9.

In Table 8, the results show that the proposed method has a significant evidence to accept the null hypothesis with functions F3 and F4.

In Table 9, the results show that the FWOA-T1FLS method has a significant evidence to accept the null hypothesis with functions F3, F5, and F8.

In Table 9, the results show that the FWOA-IT2FLS method has a significant evidence to accept the null hypothesis with functions F2 and F8.

Table 8. Statistical test of original WOA and FWOA-T1FLS.

f_x	Methods			
	FWOA-T1FLS	**Original WOA** $\vec{r}_1$ **and** $\vec{r}_2$ **Random**	**Z-Value**	**Evidence**
F1	5.71×10^{-63}	2.34×10^{-101}	0.560	NS
F2	$3.44 \times 10^{+5}$	4.00×10^{-9}	3.085	NS
F3	1.17×10^{-13}	8.19×10^{-10}	−2.902	S
F4	$1.40 \times 10^{+13}$	$4.21 \times 10^{+2}$	591.500	NS
F5	7.65×10^{-79}	2.40×10^{-64}	−1.600	S
F6	$1.62 \times 10^{+14}$	5.00×10^{0}	−0.183	NS
F7	1.33×10^{0}	1.12×10^{-2}	−1.002	NS
F8	$7.05 \times 10^{+14}$	1.00×10^{0}	−1.686	S
F9	3.33×10^{-1}	-2.52×10^{-102}	−1.000	NS
F10	2.53×10^{-80}	-1.03×10^{-98}	3.122	NS

Table 9. Statistical test of FWOA-IT2FLS and FWOA-T1FLS.

f_x	Methods			
	FWOA-IT2FLS	**FWOA-T1FLS**	**Z-Value**	**Evidence**
F1	2.63×10^{-59}	9.04×10^{-62}	−1.265	NS
F2	-4.59×10^{-10}	$3.44 \times 10^{+5}$	−3.09	S
F3	-8.94×10^{-10}	$-2.02 \times 10^{+5}$	2.902	NS
F4	$-4.87 \times 10^{+2}$	$-5.02 \times 10^{+2}$	1.4388	NS
F5	2.67×10^{-43}	7.65×10^{-79}	1.281	NS
F6	1.29×10^{0}	5.00×10^{0}	0.166	NS
F7	1.20×10^{-2}	1.22×10^{-2}	1.807	NS
F8	4.38×10^{-1}	9.69×10^{-1}	−8.242	S
F9	7.47×10^{-59}	3.33×10^{-1}	1.000	NS
F10	2.01×10^{-60}	2.53×10^{-80}	−1.005	NS

The results of the statistical tests demonstrate that our proposed method is better on average than the original WOA; therefore, it is possible to mention that FWOA-IT2FLS is statistically better than the original WOA compared to the averages. The results of the proposed FWOA-IT2FLS method are very close to the original WOA, for example, in F1, F2, and F10, the z-value is closer to the acceptance of H_a.

The comparative results with the fuzzy bee colony optimization (FBCO-IT2FLS) [61] with 50 dimensions are presented in Table 10.

The results of the statistical tests demonstrate that our proposed method is better on average than the original WOA; therefore, it is possible to state that FWOA is statistically better than the original WOA compared to the averages. The results of the proposed FWOA-IT2FLS method are very close to the original WOA.

Table 11 shows the performance index of standard deviation to FWOA-IT2FLS and original WOA and comparison with FBCO-IT2FLS algorithm.

Table 10. Comparative results (best and worst) with FBCO, original WOA, and the proposed method.

f_x	Methods							
	Original WOA $\vec{r}_1$ and $\vec{r}_2$ Random		**Fuzzy WOA-T1FLS**		**Fuzzy WOA-IT2FLS**		**FBCO-IT2FLS [61]**	
	Best	**Worst**	**Best**	**Worst**	**Best**	**Worst**	**Best**	**Worst**
F1	**1.60×10^{-97}**	1.98×10^{-92}	$5.\ 43 \times 10^{-55}$	$2.\ 23 \times 10^{-55}$	1.74×10^{-59}	2.94×10^{-60}	7.74×10^{-8}	6.94×10^{-7}
F2	3.57×10^{-8}	1.32×10^{-8}	**1.04×10^{-8}**	2.34×10^{7}	3.73×10^{-8}	4.02×10^{-8}	8.40×10^{-5}	3.26×10^{-2}
F3	4.81×10^{-9}	9.58×10^{-9}	$4.67 \times 10^{+4}$	$8.69 \times 10^{+5}$	**4.78×10^{-9}**	8.90×10^{-9}	7.52×10^{0}	$1.51 \times 10^{+1}$
F4	4.20×10^{2}	$4.22 \times 10^{+2}$	5.00×10^{2}	$5.00 \times 10^{+2}$	**$3.03 \times 10^{+2}$**	$3.03 \times 10^{+2}$	1.01×10^{3}	$2.47 \times 10^{+3}$
F5	3.92×10^{-63}	3.24×10^{-94}	5.00×10^{2}	1.35×10^{-36}	**1.77×10^{-86}**	7.93×10^{-41}	1.87×10^{-10}	1.08×10^{-6}
F6	**4.92×10^{0}**	9.24×10^{0}	$8.52 \times 10^{+14}$	$7.69 \times 10^{+14}$	4.99×10^{0}	8.53×10^{0}	5.46×10^{2}	$7.20 \times 10^{+2}$
F7	**2.18×10^{-6}**	3.33×10^{-1}	$1.13 \times 10^{+8}$	$1.00 \times 10^{+1}$	4.14×10^{-4}	3.18×10^{-1}	1.76×10^{0}	9.06×10^{0}
F8	7.72×10^{-1}	1.03×10^{0}	$1.01 \times 10^{+14}$	$9.98 \times 10^{+14}$	**9.63×10^{-2}**	1.22×10^{0}	6.37×10^{-7}	1.04×10^{-5}
F9	**2.96×10^{-90}**	2.56×10^{-90}	$1.00 \times 10^{+1}$	1.63×10^{-7}	1.70×10^{-57}	7.68×10^{-58}	5.10×10^{6}	1.29×10^{-4}
F10	**2.56×10^{-89}**	2.88×10^{-88}	$6.55 \times 10^{+1}$	5.68×10^{-7}	6.29×10^{-59}	5.96×10^{-58}	1.35×10^{-4}	2.44×10^{-3}

Table 11. Standard deviation results of FBCO-IT2FLS, original WOA, and the proposed method.

f_x	Methods		
	Original WOA $\vec{r}_1$ and $\vec{r}_2$ Random	**FWOA-IT2FLS**	**FBCO-IT2FLS [61]**
F1	**2.34×10^{-101}**	2.63×10^{-59}	1.74×10^{-7}
F2	4.00×10^{-9}	**1.36×10^{-8}**	6.12×10^{-3}
F3	8.19×10^{-10}	**8.14×10^{-10}**	1.98×10^{0}
F4	4.21×10^{2}	**3.37×10^{-12}**	$3.59 \times 10^{+2}$
F5	**2.40×10^{-64}**	2.67×10^{-43}	2.46×10^{-7}
F6	5.00×10^{0}	**1.29×10^{0}**	$3.89 \times 10^{+1}$
F7	**1.12×10^{-2}**	1.20×10^{-2}	1.76×10^{0}
F8	1.00×10^{0}	4.38×10^{-1}	**1.91×10^{-6}**
F9	**2.52×10^{-102}**	7.47×10^{-59}	2.50×10^{-5}
F10	**1.03×10^{-98}**	2.01×10^{-60}	6.83×10^{-4}

6.2. Discussion of the Results

This paper addresses some important points in the discussion. First, the use of IT2FLS as a tool for enhancing the performance of the WOA algorithm in the optimization of mathematical functions was proposed. The results in Tables 2–6 show that the optimal values in the heuristic parameters improve the performance of the original WOA algorithm in the problem of benchmark functions.

Second, based on the experiments presented in Section 5, we can summarize an analysis of the results. The original WOA algorithm is an excellent technique in the optimization of 10 sets of mathematical functions. The results with 10, 20, 30, 50, and 100 dimensions show that a better performance can be achieved when the FWOA-IT2FLS algorithm is used. Another important aspect to mention with respect to the IT2FLS method is the quick convergence when the number of dimensions of the WOA algorithm are increasing.

Regarding the computational complexity of the interval type-2 fuzzy logic system, IT2FLS requires more calculations; however, we have found that WOA requires a smaller number of iterations to find a good solution. In addition, better stability is shown in the results in Figure 10 with respect to the original WOA with (random and fixed values), while FWOA-T1FLS and FWOA-IT2FLS are expressed with two mathematical functions of Zakharov (F6) with 100 dimensions.

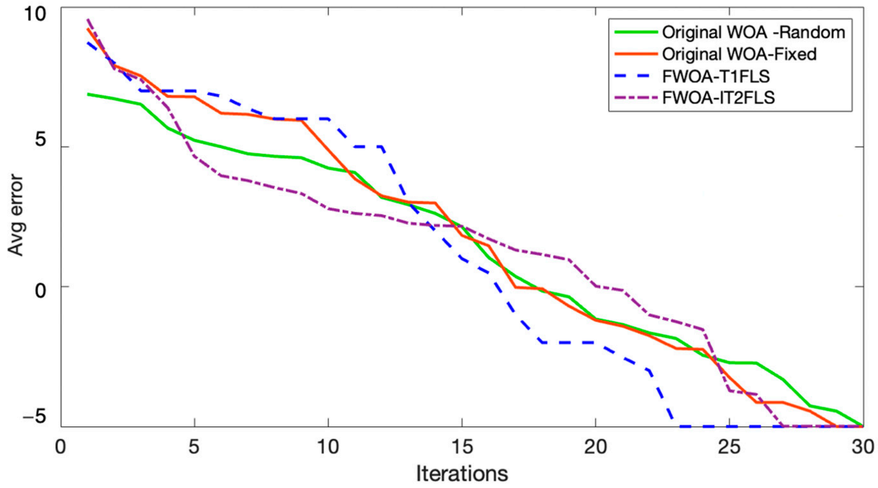


Figure 10. Behavioral results and comparative of Zakharov function with 100 dimensions.

Figure 10 shows a fast convergence when the errors are found with FWOA-IT2FLS in the experiments (see the purple line).

7. Conclusions

The research presented in this paper allows us to demonstrate that using a hybridization based on IT2FLS and T1FLS to find the optimal values of the $\vec{r}_1$ and $\vec{r}_2$ parameters shows good results when compared with the original WOA. This is mainly based on the fuzzy rules, as the design of the rules was obtained with the results previously found and the ability to analyze the effects of each parameter on the exploration and exploitation of the WOA. The proposed method is called FWOA-IT2FLS. The statistical test shows that the FWOA-IT2FLS algorithm has a statistically significant improvement when compared with the original WOA.

For the F1 function, the best results of the $\vec{r}_1$ and $\vec{r}_2$ values with 100 dimensions are in the range of [0.47, 0.85] and [0.11, 0.41]. For the F2 function, the best results are in the range of [0.47, 0.69] and [0.11, 0.24], for F3, in the range of [0.56, 0.82] and [0.17, 0.38], for F4, in the range of [0.51, 0.70] and [0.14, 0.25], for F5, in the range of [0.47, 0.47] and [0.11, 0.11], for F6, in the range of [0.47, 0.85] and [0.11, 0.44], for F7, in the range of [0.61, 0.75] and [0.20, 0.33], for F8, in the range of [0.47, 0.63] and [0.11, 0.22], for F9, in the range of [0.47, 0.75] and [0.11, 0.32], for F10, in the range of [0.47, 0.58] and [0.11, 0.19], respectively. Based on the results, it can be concluded that the optimal value of $\vec{r}_1$ is 0.47 and $\vec{r}_2$ is 0.11.

In future work, the FWOA-IT2FLS method will be applied to real control problems, for example, in the trajectory stabilization of an autonomous mobile robot, as using this proposed algorithm in fuzzy controllers is an excellent application. Another interesting aspect is to find the values of $\vec{r}_1$ and $\vec{r}_2$ parameters in the WOA through a better uncertainty analysis based on an interval type-3 FLS.

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