

Article

Geometric Design and Dynamic Analysis of a Compact Cam Reducer

Tsung-Chun Lin ¹, Michael Schabacker ², Yi-Lun Ho ³, Tsu-Chi Kuo ⁴ and Der-Min Tsay ^{1,*}¹ Department of Mechanical and Electro-Mechanical Engineering, National Sun Yat-Sen University, Kaohsiung 80424, Taiwan² Department of Mechanical Engineering, Otto-von-Guericke University Magdeburg, 39106 Magdeburg, Germany³ Compal Electronics Inc., Taoyuan 33371, Taiwan⁴ Innolux Corporation, Tainan 74147, Taiwan

* Correspondence: dermin@mail.nsysu.edu.tw

Abstract: In this paper, the compact reducer which can be used as a rigid drive mechanism with a high-speed reduction ratio is systematically studied for kinematics and dynamics. The speed ratio is determined by the number of cam lobes and engaged rollers. The eccentric rotating conjugate lobe cam profile is synthesized by using the rigid body transformation method. To characterize the input torque based on Newton's second law, the transmission forces of the resisting and driven multi-roller are proportional to the arm length of each actuating roller found by its geometric vector. In consideration of machining undercutting, roller limit load, and contact stresses, favorable designs can be achieved by adjusting the cam size, turret dimension, eccentricity, and roller size. Together with experimental tests, a prototype of the lobe cam reducer is made to verify the feasibility of the proposed design procedure and to investigate its kinematic and dynamic characteristics for speed reduction ratios, torques, and transmission efficiency.



Citation: Lin, T.-C.; Schabacker, M.; Ho, Y.-L.; Kuo, T.-C.; Tsay, D.-M. Geometric Design and Dynamic Analysis of a Compact Cam Reducer. *Machines* **2022**, *10*, 955. <https://doi.org/10.3390/machines10100955>

Academic Editor: Domenico Mundo

Received: 15 September 2022

Accepted: 17 October 2022

Published: 19 October 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

Keywords: lobe cam; compact reducer; rigid body transformation method; kinematic synthesis; dynamic analysis

1. Introduction

The usage of a drive mechanism with a high-speed reduction ratio is ubiquitous in various industries and has to be carefully selected for different engineering applications. For example, the design of a joint drive module used in a robotic system has to ponder its working speed, force/torque load capacity, rigid transmission, space allowance, positioning accuracy, and relatively long service life. To provide a design of reducers with a large reduction ratio in a small space, harmonic reducers [1], cycloidal pinwheel reducers [2,3] (RV reducer), and planetary gear reducers [4] (PGR reducer) having a common structural feature of coaxial input and output shafts are often considered. Harmonic drives with high-speed reduction ratios and relatively small volumes have been employed in machinery for light-load applications and high precision. Because the elastic spline of a harmonic reducer is a thin, flexible part, it may avoid backlash by flexible transmission [5]. However, its expected lifespan may be deteriorated due to high loads.

Elias and Nader [6] propose a compact and high-torque gear mechanism that can be used in joint drive systems for space robots. It can provide a high reduction ratio of 1:2116. Two sets of planetary gears are used to prevent the output gear set from swinging and can effectively improve the output load capacity. This gear mechanism requires high manufacturing accuracy and assembly accuracy because this mechanism is composed of Profile-shifted gears. This article replaces the gears by using a 2D Lobe cam which is easier to manufacture. Due to the high contact ratio of the cam, the attachment capacity of the mechanism is further improved.

A cycloidal reducer can be classified into three types: single-stage cycloidal speed reducer [7,8], two-stage cycloidal speed reducer, and RV reducer. Botsiber and Kingston [9] proposed the basic structure, operation, and synthesis principle of the cycloid reducer and compared it with other types of reducers. Malhotra and Parameswaran [10] calculated the force acting on each element of the cycloid reducer, evaluated the theoretical transmission efficiency, and conducted the optimal design of this type of reducer. Litvin and Feng [11] analyzed the geometry of planar cycloidal gearings and ameliorated the design to avoid geometric singularities. Blagojevic et al. [12] presented a two-stage cycloidal speed reducer and the rollers were assembled on an intermediate disk that had transmission motion from the input shaft to two cycloidal disks. In [13,14], pins were used to transfer movement from the first cycloidal disk to the second cycloidal disk. This design can obtain a more compact configuration by simplifying the intermediate disk from [12]. However, the two cycloidal disks still have a relative speed difference during the transfer of the intermediate disk.

At the same time, there is a sliding contact between the cycloidal discs and the rollers in the two sections. When two sliding contact situations occur during the transmission process, it will be more easily affected by changes in speed and load [15]. For a modified cycloid reducer like a planocentric drive, Jang et al. [16] showed a procedure for producing an epitrochoidal gear contour without using pin-rollers and designed the internal and external gears based on varying tooth thickness ratios as well as the distance from the center of the rolling circle of the epitrochoid. Li et al. [17] studied the effect of ring pin position deviation on the distributed load, contact stress, load transfer error, and instantaneous transmission ratio of mismatched cycloidal pinwheel pairs. However, the action of the lobe cam reducer in this study is rolled by the conjugate cam on the two sets of rollers. The operation of the lobe cam reducer is different from that of the RV reducer [18,19].

The lobe cam reducer proposed in this paper is based on our concept design shown in Taiwan invention patent TW I431209 [20]. The needle roller bearings are used instead of pins used in two-stage cycloidal speed reducers [12] so that this cam reducer can ensure a pure rolling motion between the cam and the roller without sliding contact. Compared to a two-stage cycloidal speed reducer [13], the proposed conjugate cam design is a single unit component with two lobe cams at both ends without any assembly for pins required, as shown in Figure 1.

The main advantage of this design is that, with a low number of components, the design can achieve a rigid transmission with a relatively compact volume. As a result, the design for rigid transmission with a relatively compact volume can be achieved. The lobe cam proposed in this manuscript is not a cycloidal shape. The cam reducer design is based on the relative motion relationship between the output shaft and its input shaft to determine the paths of the rollers. Additionally, since the cam profile is synthesized by the rigid body transformation method, the output rotating turret motion can be accurately defined.

However, the motion synthesis and dynamic analysis of such a reducer driven by a conjugate lobe cam have not yet been reported. Based on the idea above, this study is focused on developing a systematic design procedure for the lobe cam reducer. As shown in Figure 1, one of the obvious advantages of this design is the use of bearing rollers instead of pin rollers to decrease wearing. High reduction ratios can be achieved with a smaller number of rollers. At the same time, a smaller number of assembly elements makes assembly easier.

To provide a systematic design tool, a lobe cam reducer is synthesized, analyzed, made, and tested for the feasibility of the proposed procedure, as shown in the flowchart (Figure 2).

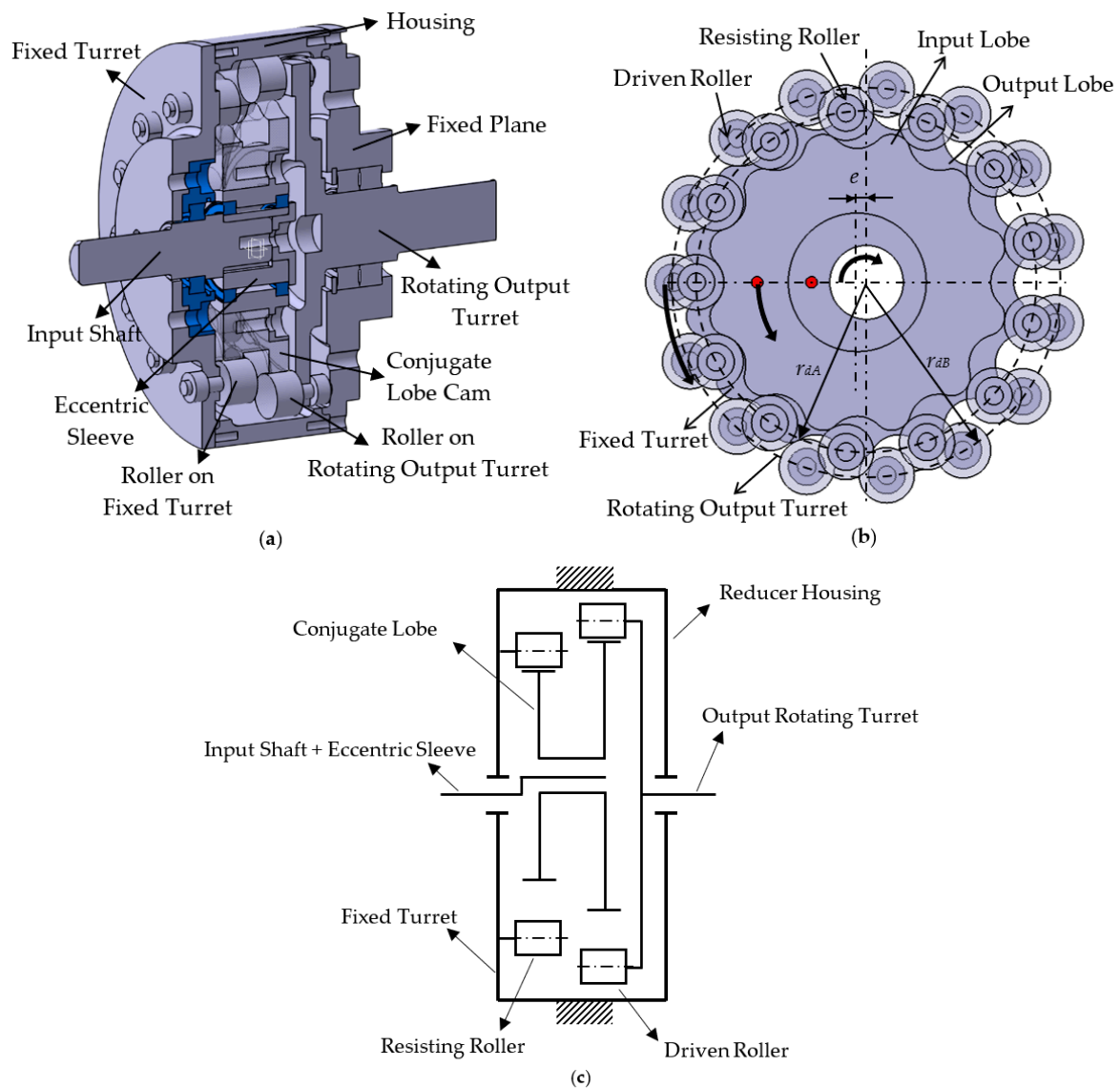


Figure 1. Structure of the conjugate lobe cam reducer: (a) section view; (b) schematic illustration of the conjugate lobe cam and rollers; (c) kinematic scheme.

In the motion synthesis of the reducer, first, the kinematic parameters for a desired speed reduction ratio given by the number of cam lobes and rollers shown in Figure 1b are determined. Then, for determining the conjugate lobe cam profiles, it is described by the relative position between the lobe cam and its eccentric input shaft and the rigid body transformation method [21]. For a given application, the minimum size that a conjugate lobe cam can have must be the minimum size that does not undercutting. To characterize the needed input torque of the reducer, the transmission force derived from Newton's second law is proportional to the actuating arm length of each engaged roller on the fixed and rotating output turrets by its geometric vector. As a result, for a specific application, the lobe cam diameter, roller diameter, eccentricity, number of cam lobes, limiting load of rollers, input/output torques, and contact stresses can be conveniently observed and adjusted during the design process.

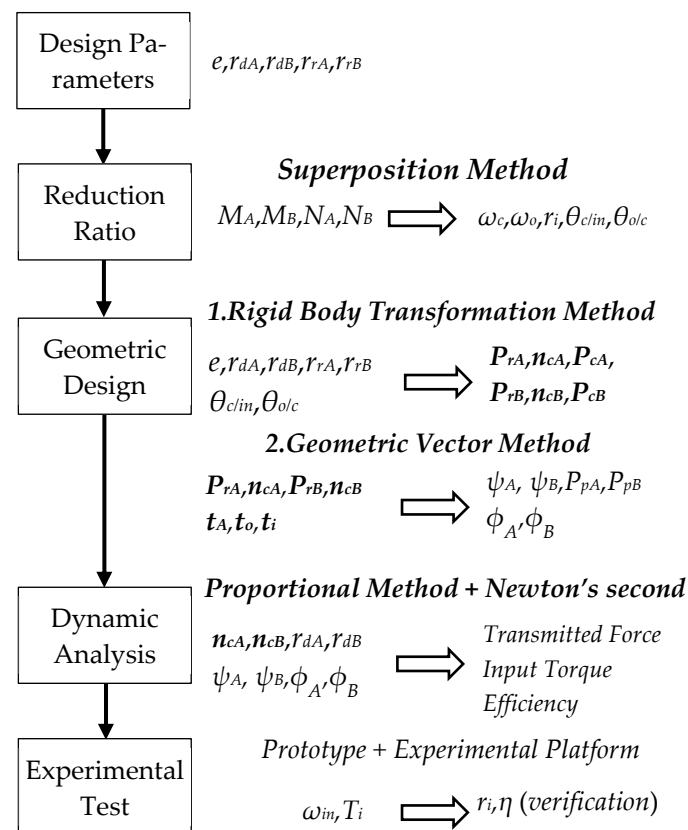


Figure 2. Flowchart for synthesis and analysis of the lobe cam reducer.

2. Kinematics of the Lobe Cam Mechanism

The design of the lobe cam reducer (LCR) can refer to the structure of a schematic diagram of a planetary gear train shown in Figures 8 and 9 of [5] for obtaining a high-speed reduction ratio. By using a similar superposition method, the speed relationship of the lobe cam reducer can be tabulated in Table 1.

Table 1. Speed ratios for a lobe cam reducer with fixed turret R_1 .

Title 1	S	P_1	P_2	R_1	R_2
No. of Teeth	m_A	m_A	m_B	n_A	n_B
Step 1: Rotations with Train Locked	+1	+1	+1	+1	+1
Step 2: Rotations with Planet Carrier Fixed	$+\frac{n_A}{m_A}$	$-\frac{n_A}{m_A}$	$-\frac{n_A}{m_A}$	-1	$-\frac{n_A m_B}{m_A n_B}$
Total Number of Rotations	$1 + \frac{n_A}{m_A}$	$1 - \frac{n_A}{m_A}$	$1 - \frac{n_A}{m_A}$	0	$1 - \frac{n_A m_B}{m_A n_B}$

The rotational speeds of the conjugate rigid lobe cam (ω_c) and output shaft (ω_o) can be expressed as

$$\omega_c = \left(1 - \frac{n_A}{m_A}\right) \omega_{in}, \quad (1)$$

$$\omega_o = \left(1 - \frac{n_A \cdot m_B}{m_A \cdot n_B}\right) \omega_{in}, \quad (2)$$

where ω_{in} is the rotating speed of the eccentric input shaft.

The speed reduction ratio (r_i), which can be found by the number of cam lobes and rollers, is expressed as

$$r_i = \frac{\omega_{in}}{\omega_o} = \frac{m_A \cdot n_B}{m_A \cdot n_B - m_B \cdot n_A}, \quad (3)$$

where m and n are the numbers of cam lobes and rollers, respectively, and subscripts A and B represent the input and output components.

3. Geometric Design of the Lobe Cam

Generally, a disk cam with a roller follower is driven by its own camshaft, and a variety of processes can be analytically used to find the cam contour [5]. The profile determination of a lobe cam differs significantly from that of a traditional disk cam with a roller follower since the lobe cam is actuated by an eccentric input shaft, and its rotational center becomes variable. Moreover, at an instant time, all the rollers on a turret are simultaneously in contact with their cam contour. To overcome these two difficulties, a convenient approach based on the relative position between the lobe cam and its eccentric input shaft, as well as the rigid body transformation method [22], is proposed to generate the trajectory of the lobe cam's roller and then to find its offset for the conjugate lobe cam profiles. For brevity, the process introduced in [21] for determining a 3D cam contour can be referenced in a similar way for producing the lobe cam profiles.

3.1. Lobe Cam Profile

The lobe cam axis is not fixed and is rotated around the axis of an eccentric input shaft in an opposite direction, as shown in Figure 1b. To generate the lobe cam profiles, the relative position between the lobe cam and its eccentric input shaft must be considered. Since the lobe cam rotates counterclockwise, based on the design parameters shown in Figure 1 and the rigid body transformation method shown in Figure 3a, the trajectory of the roller center of the fixed turret (P_{rA}) in the fixed x_c - y_c coordinate system can be produced by rotating the roller counter clockwise. It can be formulated as

$$\begin{aligned} P_{rA} &= O_c O_{rA} = O_c O_T + O_T O_{rA} = (x_{rAt} \ y_{rAt}), \\ &= [e \ 0] \begin{bmatrix} \cos \theta_{c/in} & -\sin \theta_{c/in} \\ \sin \theta_{c/in} & \cos \theta_{c/in} \end{bmatrix} + [-r_{dA} \ 0] \begin{bmatrix} \cos \theta_c & -\sin \theta_c \\ \sin \theta_c & \cos \theta_c \end{bmatrix} \\ &= (e \cdot \cos \theta_{c/in} - r_{dA} \cdot \cos \theta_c \quad -e \cdot \sin \theta_{c/in} + r_{dA} \cdot \sin \theta_c) \end{aligned} \quad (4)$$

where r_{dA} is the radius of the pitch circle, and O_{rA} , as well as O_T , are the positions of the roller center on the fixed turret and the input shaft center, respectively.

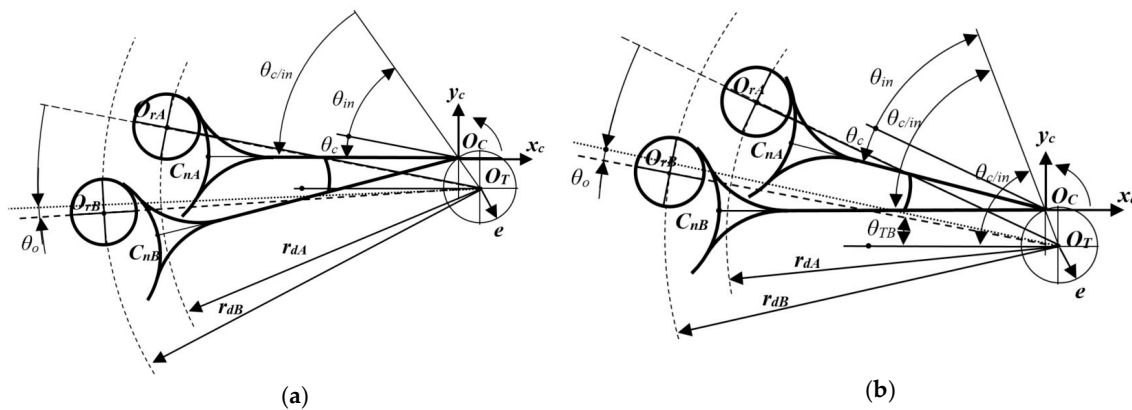


Figure 3. Schematics of the conjugate lobe cam for synthesis: (a) the input lobe cam; (b) the output lobe cam.

To obtain the offset of P_{rA} , the unit normal vector with the input cam axial direction vector (k) can be written as

$$n_{cA} = \frac{P'_{rA} \times k}{|P'_{rA} \times k|} = (n_{xcA} \ n_{ycA} \ 0), \quad (5)$$

where the components are

$$n_{xcA} = (-e \cdot \theta'_{c/in} \cdot \cos \theta_{c/in} + r_{dA} \cdot \theta'_c \cdot \cos \theta_c) / c_1,$$

$$n_{ycA} = -(-e \cdot \theta'_{c/in} \cdot \sin\theta_{c/in} + r_{dA} \cdot \theta'_c \cdot \sin\theta_c) / c_1,$$

$$c_1 = \sqrt{(e \cdot \theta'_{c/in})^2 - 2e \cdot r_{dA} \cdot \theta'_c \cdot \theta'_{c/in} \cdot \cos(\theta_c - \theta_{c/in}) + (r_{dA} \cdot \theta'_c)^2},$$

Then, the input lobe cam profile can be found as an offset of the roller center trajectory with a distance equal to the roller radius (r_{rA}) along the positive direction of the unit normal vector. Therefore, it can be derived as

$$P_{cA} = P_{rA} + r_{rA} \cdot n_{cA} = (x_{cA} \ y_{cA}) = (x_{rAt} + r_{rA} \cdot n_{xcA} \ y_{rAt} + r_{rA} \cdot n_{ycA}), \quad (6)$$

Similarly, the output lobe cam profile is

$$P_{cB} = P_{rB} + r_{rB} \cdot n_{cB} = (x_{cB} \ y_{cB}) = (x_{rBt} + r_{rB} \cdot n_{xcB} \ y_{rBt} + r_{rB} \cdot n_{ycB}), \quad (7)$$

where r_{dB} is the radius of the pitch circle and O_{rB} is the position of the roller center on the rotating output turret. The trajectory of the roller center of the rotating output turret (P_{rB}) in the fixed x_c - y_c coordinate system is shown in Figure 3b and can be described as

$$P_{rB} = (x_{rBt} \ y_{rBt}) = (e \cdot \cos\theta_{c/in} - r_{dB} \cdot \cos\theta_{o/c} \ -e \cdot \sin\theta_{c/in} + r_{dB} \cdot \sin\theta_{o/c}), \quad (8)$$

The components of the unit normal vector are

$$n_{xcB} = (-e \cdot \theta'_{c/in} \cdot \sin\theta_{c/in} + r_{dB} \cdot \theta'_{o/c} \cdot \sin\theta_{o/c}) / c_2,$$

$$n_{ycB} = -(-e \cdot \theta'_{c/in} \cdot \cos\theta_{c/in} + r_{dB} \cdot \theta'_{o/c} \cdot \cos\theta_{o/c}) / c_2,$$

$$c_2 = \sqrt{(e \cdot \theta'_{c/in})^2 - 2e \cdot r_{dB} \cdot \theta'_{c/in} \cdot \theta'_{o/c} \cdot \cos(\theta_{c/in} - \theta_{o/c}) + (r_{dB} \cdot \theta'_{o/c})^2},$$

The angle between the transmitted force (the common normal at the cam-roller contact point) and the tangential direction of rotation of the turret is defined as the pressure angle [5]. The transmitted force direction (the unit normal vector n_{cA} of the input lobe cam) of a resisting roller of the fixed turret for the input lobe cam is from the roller center (O_{rA}) to the contact point (P_{pAi}). The pressure angle (ψ_A) between each roller of the fixed turret and the input lobe cam is shown in Figure 4a. The pressure angle (ψ_B) between each roller of the rotating output turret and the output lobe cam is illustrated in Figure 4b,c and can be seen similarly with the pressure angle (ψ_A).

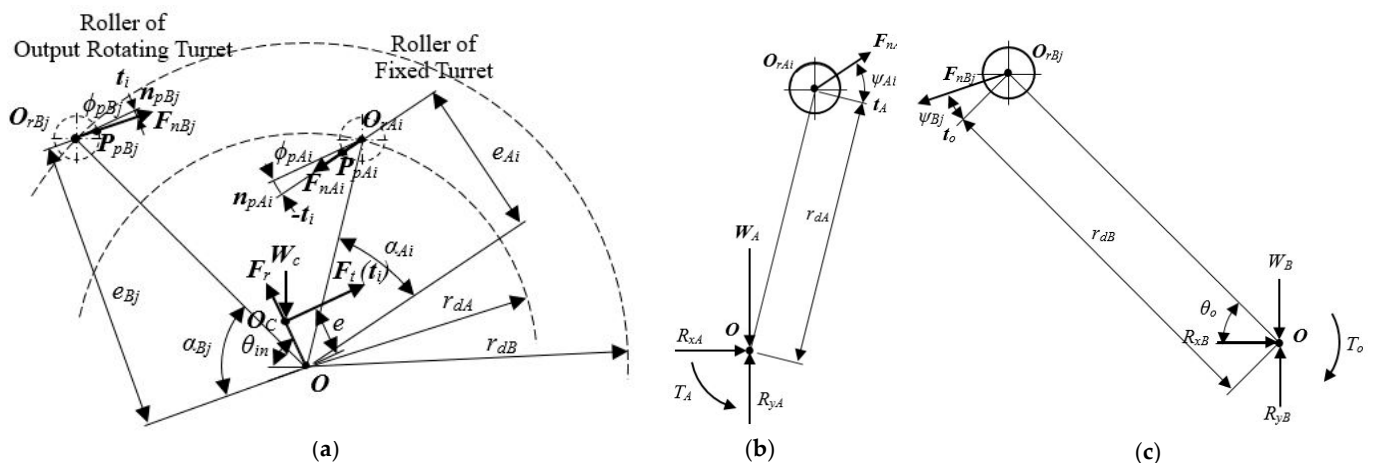


Figure 4. Analysis of forces and torques of turrets: (a) the conjugate rigid lobe cam; (b) the fixed turret; (c) the rotating output turret.

For machining the lobe cam profiles with a tool diameter (d_t) not greater than the diameter of roller (d_r), the tool path (P_t) can be defined by Equations (6) and (7) with the roller center trajectory (P_r) and its unit normal vector (n_c) as

$$P_t = P_r + n_c (d_r - d_t)/2, \quad (9)$$

3.2. Contact Point

As described earlier, the conjugate rigid lobe cam is rotated around the eccentric input shaft and all of the rollers of the turrets are simultaneously in contact with the lobe cams during operation. For locating the contact points needed for dynamic analysis between the lobe cams and their turret rollers, a computation method is developed to determine the distance between the profile position of the lobe cam and its roller center. When this distance is equal to the radius of the roller, the corresponding point of the lobe cam is then the contact point.

To find the distance for locating contact points, the view of the lobe cam profile from the input rotating center (O) shown in Figure 3a, Equation (6) for P_{cA} must be transformed from the lobe cam center to the input rotating center and rotated clockwise about the axis of the input shaft by an angle θ_c of the cam rotation. Then, it can be denoted as

$$\begin{aligned} P_{cTA} &= (x_{cTA} \quad y_{cTA}) = [P_{cA} + OO_c][T(\theta_c)], \\ &= [x_{cA} - e \cdot \cos\theta_{in} \quad y_{cA} + e \cdot \sin\theta_{in}] \begin{bmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{bmatrix} \end{aligned} \quad (10)$$

Hence, the roller center of the fixed turret is $O_{rAi} = r_{dA} \cdot [-\cos\phi_{Ai} \quad \sin\phi_{Ai}]$ and the angular position of the i -th roller on the fixed turret is $\phi_{Ai} = (i - 1) \cdot 360^\circ / n_A$, $i = 1 \sim n_A$. As a result, the distance between the roller center (O_{rA}) and the transformed input lobe cam profile (P_{cTA}) is

$$L_A = \sqrt{(x_{cTA} - x_{rA})^2 + (y_{cTA} - y_{rA})^2}, \quad (11)$$

When the distance (L_A) equals the radius of the roller, the point of the cam profile (P_{cTA}) is the contact point (P_{pAi}) corresponding to i -th roller of the fixed turret.

In a similar way, the output lobe cam profile, P_{cB} shown in Equation (7), can be transformed from the lobe cam center (O_c) to the input rotating center (O) shown in Figure 3b and rotated clockwise about the axis of the input shaft by an angle θ_c of the cam rotation. Then, it can be represented as

$$\begin{aligned} P_{cTB} &= (x_{cTB} \quad y_{cTB}) = [P_{cB} + OO_c][T(\theta_c)], \\ &= [x_{cB} - e \cdot \cos\theta_{in} \quad y_{cB} + e \cdot \sin\theta_{in}] \begin{bmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{bmatrix} \end{aligned} \quad (12)$$

Since the j -th roller center position of the rotating output turret can be described as $O_{rBj} = (x_{rB} \quad y_{rB}) = r_{dB} \cdot [-\cos\phi_{Bj} \quad \sin\phi_{Bj}]$ and $\phi_{Bj} = (j - 1) \cdot 360^\circ / n_B$, $j = 1 \sim n_B$, its rotated center position can be expressed as $O_{rTB} = (x_{rTB} \quad y_{rTB}) = [x_{rB} \quad y_{rB}] \begin{bmatrix} \cos\theta_o & \sin\theta_o \\ -\sin\theta_o & \cos\theta_o \end{bmatrix}$, when the output turret is rotated at an angle of θ_o . As a result, the distance between the roller center (O_{rTB}) and the transformed output lobe cam profile (P_{cTB}) is

$$L_B = \sqrt{(x_{cTB} - x_{rTB})^2 + (y_{cTB} - y_{rTB})^2}, \quad (13)$$

When the distance (L_B) is the same as the radius of the roller, the point of the cam profile (P_{cTB}) is then the contact point (P_{pBj}) corresponding to j -th roller of the rotating output turret.

3.3. Actuating Angle

In this lobe cam reducer, the engagement resistance and drive of the cam and roller are not accomplished by the engagement of a single roller, but by the simultaneous engagement action of multiple rollers. Before the kinematic analysis, the number of actuating rollers on the turrets should be determined. In this study, the geometric-vector method is used to evaluate the number of actuating rollers. As illustrated in Figure 4a, the angle between the reverse direction of the tangential force of the eccentric input shaft ($-t_i$) and the direction of the resisted cam transmission force (n_{cAi}) is the actuating angle of the input lobe cam, and it can be written as

$$\cos\phi_{pAi} = -t_i \cdot n_{cAi} = \frac{(x_{pAi} - x_{rAi}) \cdot (-\sin\theta_{in}) + (y_{pAi} - y_{rAi}) \cdot (-\cos\theta_{in})}{\sqrt{(x_{pAi} - x_{rAi})^2 + (y_{pAi} - y_{rAi})^2}}, \quad (14)$$

where the direction of the tangential force of the eccentric input shaft is represented as $t_i = (\sin\theta_{in}, \cos\theta_{in})$.

Similarly, the angle between the direction of the tangential force of the input shaft (t_i) and the direction of the driving cam transmission force (n_{cBj}) is the actuating angle of the output lobe cam, which can be expressed as

$$\cos\phi_{pBj} = t_i \cdot n_{cBj} = \frac{(x_{pBj} - x_{rBT}) \cdot \sin\theta_{in} + (y_{pBj} - y_{rBT}) \cdot \cos\theta_{in}}{\sqrt{(x_{pBj} - x_{rBT})^2 + (y_{pBj} - y_{rBT})^2}} \quad (15)$$

If the actuating angle is acute, the contact between the roller and its lobe cam will exert a force to rotate the conjugate lobe cam in a reverse direction to the eccentric input shaft. When the actuating angle becomes obtuse, no force will be transmitted between the roller and its lobe cam. As a result, the number of turret rollers that will exert transmission forces can be counted.

3.4. Actuating Arm Length

The transmission force is proportional to the actuating arm length which is the normal distance between the direction of the transmission force and the center of the input eccentric shaft. The actuating arm length of the i -th actuating roller is

$$e_{Ai} = r_{dA} \cdot \sin\alpha_{Ai}, \quad (16)$$

where α_{Ai} is the actuating arm angle that is equal to the angle between the direction of the transmitted force and the line connecting the centers of the input shaft and the roller center. As a result, the actuating arm angle can be found as

$$\cos\alpha_{Ai} = \frac{OO_{rAi}}{|OO_{rAi}|} \cdot n_{pAi} = (n_{xpAi} \cdot x_{rAi} + n_{ypAi} \cdot y_{rAi}) / r_{dA}, \quad (17)$$

The actuating arm length of the j -th actuating roller is

$$e_{Bj} = r_{dB} \cdot \sin\alpha_{Bj}, \quad (18)$$

where α_{Bj} is the actuating angle that is equal to the angle between the direction of the transmitted force and the line connecting the centers of the input shaft and the roller center. Again, the actuating arm angle can be computed as

$$\cos\alpha_{Bj} = \frac{OO_{rBj}}{|OO_{rBj}|} \cdot n_{pBj} = (n_{xpBj} \cdot x_{rBj} + n_{ypBj} \cdot y_{rBj}) / r_{dB}, \quad (19)$$

4. Dynamic Analysis of the Lobe Cam Mechanism

As shown in Figure 1b, it can be seen that the conjugate lobe cam has a pure rolling motion contact with its bearing rollers of the fixed and rotating turrets. During operation, though all the rollers are simultaneously in contact with the lobes of the conjugate cam, a partial number of turret rollers will not bear or exert forces on the conjugate lobe cam. Therefore, for identifying the dynamic characteristic of the reducer here, the number of actuating turret rollers should be determined.

4.1. Transmission Forces

The torque needs to be distributed according to the actuating roller with its arm. The transmission force of each actuating roller is proportional to the length of the actuating arm, as shown in Figure 4a. The transmission force of the i -th actuating roller with the input lobe cam can be written as

$$F_{nAi} = (C_A \cdot e_{Ai}) n_{cAi}, \quad (20)$$

where C_A is the constant of proportionality shown on page 371 in [23]. As described in Figure 4a, the transmission force is proportional to the actuating arm length which is the normal distance (e_{Ai}) between the direction of the transmission force (n_{cAi}) and the center of the eccentric input shaft.

The transmission force of the j -th actuating roller of the output lobe cam is

$$F_{nBj} = (C_B \cdot e_{Bj}) n_{cBj}, \quad (21)$$

where C_B is the constant of proportionality. The transmission force is proportional to the actuating arm length which is the normal distance (e_{Bj}) between the direction of the transmission force (n_{cBj}) and the center of the input eccentric shaft.

In a similar process, the transmission force of the j -th actuating roller with the output lobe cam shown in Equation (21) can be evaluated.

4.2. Torques and Reducer Efficiency

According to Newton's second law of motion in dynamic analysis, the reducer mechanism is divided into four elements, the eccentric input shaft, conjugate rigid lobe cam, fixed turret, and rotating output turret. In this study, the acceleration and deceleration of the reducer were not considered.

As shown in Figure 4, the force balance of the conjugate rigid lobe cam can be divided into a tangential component and a radial component expressed as

$$F_t - \sum_{i=1}^{k_A} F_{nAi} \cdot \cos\phi_{pAi} + \sum_{j=1}^{k_B} F_{nBj} \cdot \cos\phi_{pBj} - W_c \cdot \cos\theta_{in} = 0, \quad (22)$$

$$F_r - \sum_{i=1}^{k_A} F_{nAi} \cdot \sin\phi_{pAi} + \sum_{j=1}^{k_B} F_{nBj} \cdot \sin\phi_{pBj} - W_c \cdot \sin\theta_{in} = 0, \quad (23)$$

where F_t and F_r are the tangential and radial forces of the eccentric input shaft, respectively. W_c is the weight of the conjugate rigid lobe cam, and k_A and k_B are the numbers of resisting and driven rollers, respectively. The torque balance at the center of the lobe cam can also be represented by the tangential force of the transmitted force.

$$F_t \cdot e - \sum_{i=1}^{k_A} (F_{nAi} \cdot \cos\phi_{pAi}) \cdot e_{Ai} + \sum_{j=1}^{k_B} (F_{nBj} \cdot \cos\phi_{pBj}) \cdot e_{Bj} - W_c \cdot (e \cdot \cos\theta_{in}) = 0, \quad (24)$$

As described in Figure 4b, the torque balance at the center of the fixed turret can be obtained as

$$\sum_{i=1}^{k_A} (F_{nAi} \cdot \cos\psi_{Ai}) \cdot r_{dA} - T_A = 0, \quad (25)$$

where T_A and $\cos\psi_{Ai}$ denote the resisting torque and the pressure angle of i -th roller of the fixed turret at the reducer support, respectively.

Similarly, as illustrated in Figure 4c, the torque balance at the center of the rotating output turret can be denoted as

$$\sum_{j=1}^{k_B} (F_{nBj} \cdot \cos\psi_{Bj}) \cdot r_{dB} - T_o = 0, \quad (26)$$

where T_o and $\cos\psi_{Bj}$ represent the output loading torque and the pressure angle of the j -th roller of the rotating output turret at the reducer support, respectively.

By substituting Equations (14) and (15) into Equations (20)–(25), each constant of proportionality [23] for the input and output lobe cams can be separately obtained as

$$C_B = \frac{T_o}{r_{dB} \cdot \sum_{j=1}^{k_B} (e_{Bj} \cdot \cos\psi_{Bj})}, \quad (27)$$

$$C_A = C_B \frac{e \cdot \sum_{j=1}^{k_B} (e_{Bj} \cdot \cos\phi_{Bj}) - \sum_{j=1}^{k_B} (e_{Bj}^2 \cdot \cos\phi_{Bj})}{e \cdot \sum_{i=1}^{k_A} (e_{Ai} \cdot \cos\phi_{pAi}) - \sum_{i=1}^{k_A} (e_{Ai}^2 \cdot \cos\phi_{pAi})}, \quad (28)$$

The tangential and radial forces of the eccentric input shaft are then denoted as

$$F_t = C_A \cdot \sum_{i=1}^{k_A} (e_{Ai} \cdot \cos\phi_{pAi}) - C_B \cdot \sum_{j=1}^{k_B} (e_{Bj} \cdot \cos\phi_{Bj}) + W_c \cdot \cos\theta_{in}, \quad (29)$$

$$F_r = C_A \cdot \sum_{i=1}^{k_A} (e_{Ai} \cdot \sin\phi_{pAi}) - C_B \cdot \sum_{j=1}^{k_B} (e_{Bj} \cdot \sin\phi_{Bj}) + W_c \cdot \sin\theta_{in}, \quad (30)$$

The contact stresses between the conjugate lobe cam and its rollers can be calculated by using the Hertz theory [23]. Based on the contact force, the radius of curvature [5] of the lobe cam, Young's modulus, and Poisson's ratio for the selected material, the contact stress can be found with a reasonable effort. Because the contact force and radius of curvature vary with the curved surface of the lobe cam and its roller in operation, the contact stress also changes.

The resisting torque of the fixed turret at the reducer support is then written as

$$T_A = c_A \cdot r_{dA} \cdot \sum_{i=1}^{k_A} (e_{Ai} \cdot \cos\psi_{Ai}), \quad (31)$$

The needed input torque, T_i , can be determined by multiplying the sum of the input tangential and frictional forces with the eccentricity. The friction force is equal to the multiplication of the radial force and the friction coefficient. Then, T_i can be found as

$$T_i = [F_t + \mu \cdot F_r] \cdot e, \quad (32)$$

As for the reducer efficiency (η), it can be defined by using the relationship as

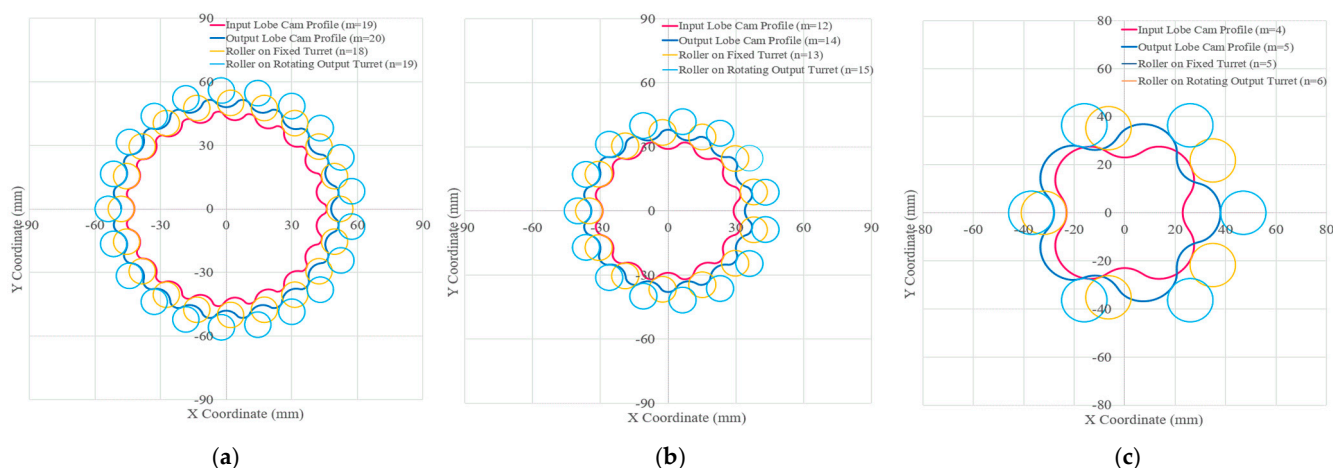
$$\eta = \frac{T_o \cdot \omega_o}{T_i \cdot \omega_i} \times 100\% = \frac{T_o}{T_i \cdot r_i} \times 100\%, \quad (33)$$

5. Application Examples and Experimental Tests

In this section, three LCRs I, II, and III separately corresponding to three different speed reduction ratios of -399 , -90 , and -24 are presented. By adjusting the radii of the turrets, roller diameter, and eccentricity, the maximum outer diameter of the LCR was 92 mm. The fundamental design parameters for these three lobe cam reducers are listed in Table 2. As shown in Figure 5, the cam profile of the three LCRs can be easily synthesized by the aforementioned method and analyzed as not undercutting. Under suitable design parameters, the LCR can have the characteristics of a large reduction ratio with a small volume.

Table 2. Design parameters of three lobe cam reducers.

	LCR I ($r_i = -399$)		LCR II ($r_i = -90$)		LCR III ($r_i = -24$)	
	Input	Output	Input	Output	Input	Output
Lobe Number	19	20	12	14	4	5
Roller Number	20	21	13	15	5	6
Radii of Turret (mm)	39	44	37	42	36	41
Diameter of Roller (mm)	6	6	12	12	18	18
Eccentricity (mm)	1	1	2	2	4	4
Length of Roller (mm)	5	5	10	10	16	16
Thickness of Lobe Cam (mm)	4	4	8	8	14	14

**Figure 5.** Lobe cam profiles for LCRs I, II, and III: (a) LCR I ($r_i = -399$); (b) LCR II ($r_i = -90$); (c) LCR III ($r_i = -24$).

Based on the same output torque of 80 Nm, the dynamic analysis results are shown in Table 3. If LCR has more numbers of actuating rollers, the needed force of a single roller will be less. LCR III has only three actuating rollers, so each roller needs to load a considerable force. Even if the radius of curvature is small, the contact stress will be too large due to the excessive force of each roller. Although the LCR can be designed with a small volume mechanism with a large reduction ratio, an excessively small volume will reduce the efficiency, just like the LCR I. After considering the allowable analysis results, the design of LCR II can be applied in practice. In addition, a prototype based on the design parameters of LCR II together with a test bed is manufactured for experimental tests to verify the feasibility of the developed design procedure and to investigate the efficiency of LCR II.

Table 3. Characteristic analyses of three lobe cam reducers ($T_o = 80$ Nm).

	LCR I ($r_i = -399$)		LCR II ($r_i = -90$)		LCR III ($r_i = -24$)	
	Input	Output	Input	Output	Input	Output
Actuating Roller Number	11	10	7	7	3	3
Max Roller Force (N)	810	731	960	720	3657	2656
Contact Stress (MPa)	122	115	186	161	243	198
Input Torque (Nm)	1.62		1.34		6.54	
Efficiency (%)	54.6		66.6		13.7	

Regarding the analyses of actuating angles and transmission forces of LCR II with an output load of 80 Nm, they are illustrated in Figure 6 separately for the fixed turret rollers and the rotating output turret rollers. As shown in Figure 6b, the computed maximum

transmission force is 960 N, which is less than the bearing limit load of 1050 N [24]. At the output lobe cam in Figure 6d, the maximum transmission force is 720 N, which is lower than the maximum transmission force of the input lobe cam. Referring to Figure 6a,c, we can see that each turret has seven actuating rollers ($k_A = k_B = 7$) with various actuating angles to transmit different magnitudes of contact forces along with the rotating input angles.

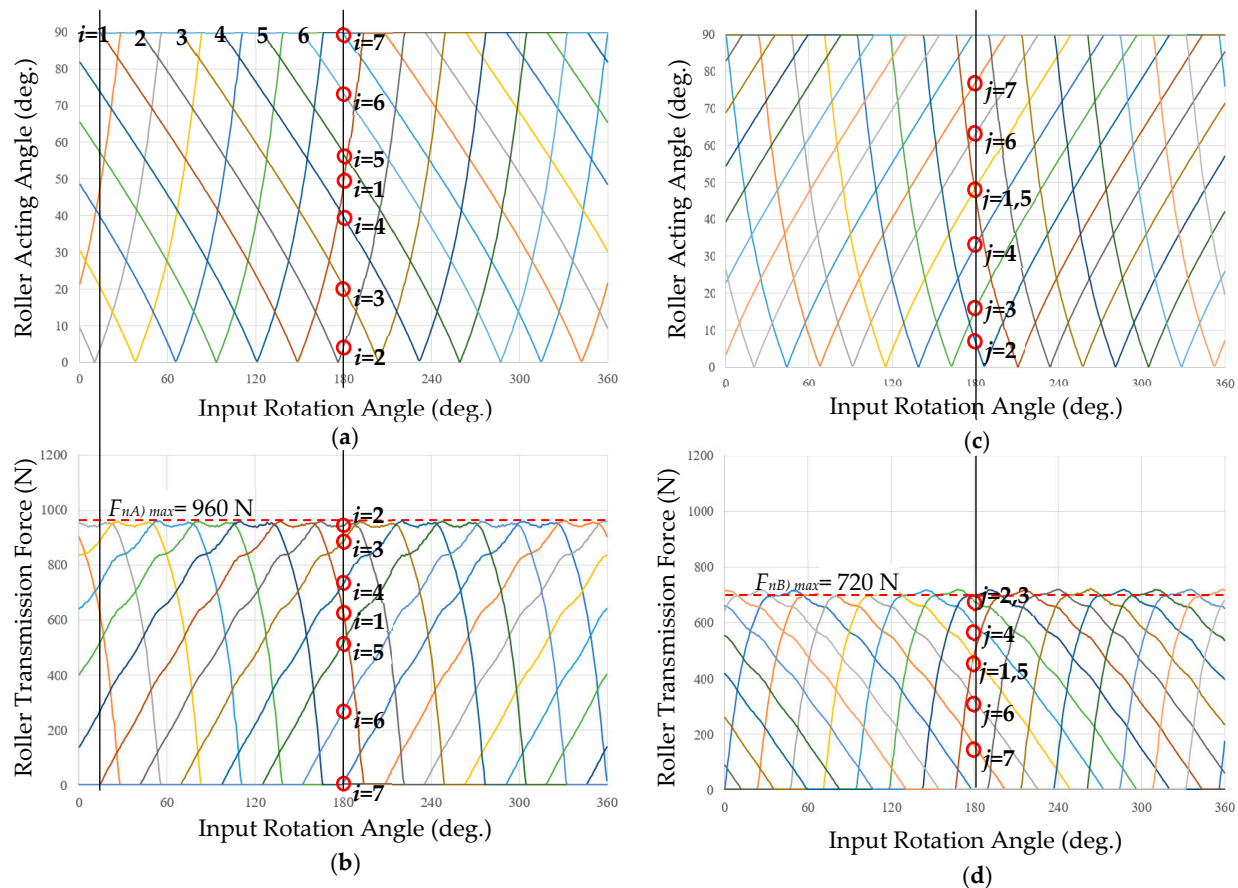


Figure 6. Analysis of the resisting rollers for LCR II: (a) the acting angles; (b) the transmission forces. Analysis of the actuating rollers for LCR II: (c) the acting angles; (d) the transmission forces.

The contact stress analysis of LCR II for its two lobe cams and their turret rollers is shown in Figure 7. The calculated maximum contact stress plotted in Figure 7a is 185 MPa, which is less than the allowable compression stress ($\sigma_a = 480$ MPa) of the medium carbon steel (S45C). As illustrated in Figure 7b, the evaluated maximum contact stress of the output lobe cam is 161 MPa, which is smaller than that of the input lobe cam. Based on Equations (32) and (33) for the needed input torque with a friction coefficient of 0.05 for the input shaft and efficiency of the LCR II, the computed efficiency varies in the range from 60% to 70% with an average efficiency of about 66.6%.

To further investigate the reducer efficiency, a prototype of LCR II (Figure 8a) without lubricant inside was made for experimental tests. In addition, for conducting the experiments, a test bed photographed in Figure 8b,c was also built. The operational speed was maintained at 900 rpm and the output load torques were regulated at 5.71, 16.66, and 24.71 Nm. The test results presented in Table 4 show that the average reduction ratio is 89.31, and the reducer efficiency is in the 49%–52.6% range. Compared with the theoretical values of input torques and efficiency, the major deviation may be reasonably attributed to the extra loads of installed equipment and the absence of lubricant inside the housing of the conjugate lobe cam reducer.

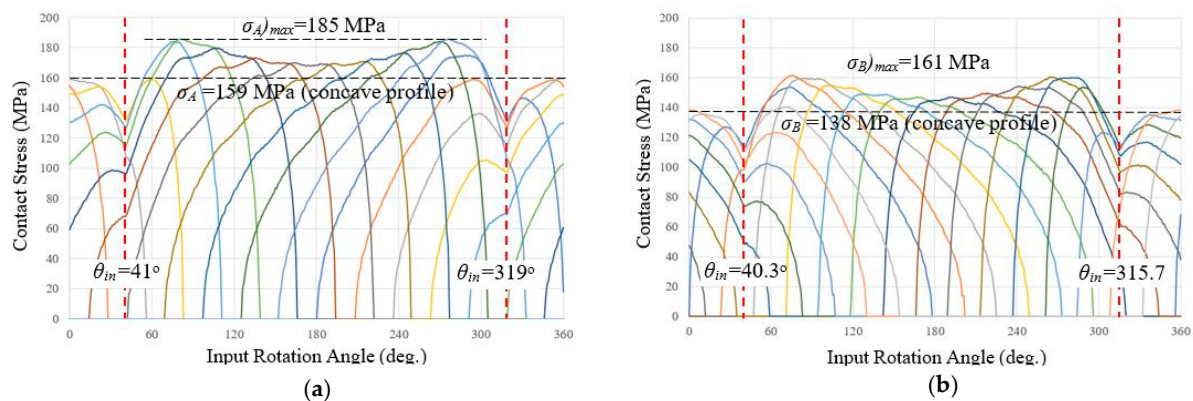


Figure 7. Contact stress analysis for conjugate lobe cam reducer I: (a) the input lobe cam and (b) the output lobe cam.

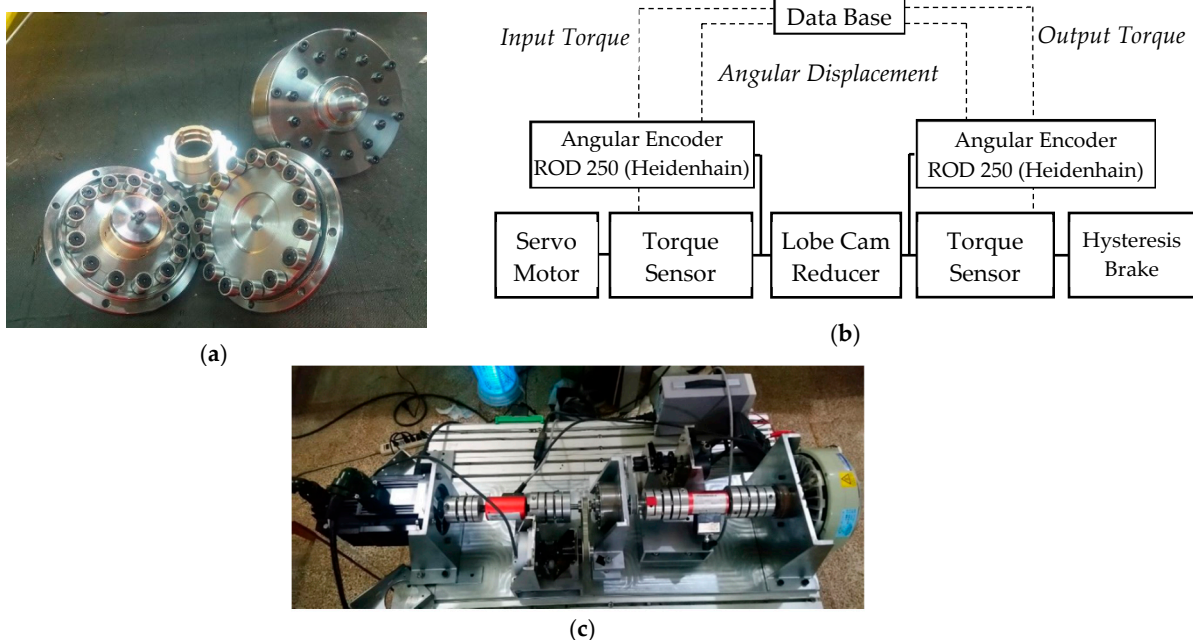


Figure 8. Experimental platform for LCR II: (a) prototype of LCR II; (b) the framework of the experimental platform; (c) the image of the experimental platform.

Table 4. Experimental results of LCR II.

Output Torque (Nm)	Speed Ratio	Measured Value							Theoretical Value			
		Input Torque			Output Torque			Eff. (%)	Input Torque			Eff. (%)
		Ave.	SD.	CV.	Ave.	SD.	CV.		Ave.	SD.	CV.	
5.71	88.88	0.131	0.005	4.03%	−5.701	0.413	7.25%	48.96	0.095	0.008	8.60%	66.86
16.66	89.67	0.359	0.008	2.34%	−16.66	0.440	2.64%	51.75	0.278	0.018	6.47%	66.66
24.71	89.39	0.526	0.009	1.77%	−24.71	0.387	1.57%	52.55	0.413	0.026	6.27%	66.65

Ave. = The average value of three experiments for each output torque. SD. = The standard deviation of three experiments for each output torque. CV. = The coefficient of variation of three experiments for each output torque.

6. Conclusions

As described above, this study mainly focuses on the systematic methodology of kinematic synthesis, geometric design, and dynamic analysis for conjugate lobe cam reducers. Three application examples have been displayed to illustrate the synthesis of reducer motions. Moreover, a case with its real prototype has been provided to show its

kinematic and dynamic characteristics together with experimental tests. The feasibility and effectiveness of the developed approach have been demonstrated through computational and experimental results.

The conjugate lobe cam profile can be easily generated by using the rigid body transformation method of the eccentric rotating shaft, and the conjugate lobe cam can be manufactured with a reasonable effort. Furthermore, the lobe cam reducer is compact, and its cam roller is in rolling contact. The engagement actions between both the lobe cams and their turret rollers have been identified through the analysis of multiple actuating rollers. Based on Newton's second law in dynamic analysis, the resisting and driven rollers of the turrets of the reducer have been analyzed for correct torque transmissions. Employing the developed procedure can significantly reduce the time needed for a specific application design of the conjugate lobe cam reducer.

Author Contributions: Conceptualization, D.-M.T. and T.-C.K.; methodology, T.-C.L.; software, T.-C.K.; validation, T.-C.L. and T.-C.K.; formal analysis, Y.-L.H.; investigation, Y.-L.H. and M.S.; resources, D.-M.T.; data curation, M.S.; writing—original draft preparation, T.-C.L.; writing—review and editing, D.-M.T. and M.S.; supervision, D.-M.T.; project administration, M.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research has been supported by grants 110-2927-I-110-504, 110-2221-E-110-031, 111-2927-I-110-504, and 111-2218-E-110-004 from the Ministry of Science and Technology, Taiwan and DAAD project ID: 57602200 from the Deutscher Akademischer Austauschdienst, Germany.

Data Availability Statement: The authors attest that all data for this study are included in the paper.

Acknowledgments: The authors wish to acknowledge that Ministry of Science and Technology, Taiwan supported this research. We also thank the anonymous reviewers for providing critical comments and suggestions that improved the manuscript.

Conflicts of Interest: There are no conflict of interest.

References

1. Musser, C.W. Strain Wave Gearing. U.S. Patent No. 2,906,143, 29 September 1959.
2. Braren, R. Cycloidal Gears. U.S. Patent No. 4,050,331, 27 September 1977.
3. Nabtesco Corporation. *Precision Reduction Gear RV N Series*. CAT.180410; Nabtesco Corporation: Tokyo, Japan, 2018.
4. Fujimoto, K. Eccentric Oscillating-Type Planetary Gear Device. U.S. Patent No. 7,476,174 B2, 13 January 2009.
5. Wilson, C.E.; Sadler, J.P. *Kinematics and Dynamics of Machinery*, 3rd ed.; Pearson Education: London, UK, 2006.
6. Brassitos, E.; Jalili, N. Design and Development of a Compact High-Torque Robotic Actuator for Space Mechanisms. *J. Mech. Robot.* **2017**, *9*, 061002. [\[CrossRef\]](#)
7. Shin, J.H.; Kwon, S.M. On the Lobe Profile Design in a Cycloid Reducer Using Instant Velocity Center. *Mech. Mach. Theory* **2006**, *41*, 596–616. [\[CrossRef\]](#)
8. Vasic, M.; Blagojevic, M.; Matejic, M.; Cofaru, I.I. The Influence of the Cycloid Disc Bearing Type on the Cycloidal Speed Reducer Efficiency. *MATEC Web Conf.* **2021**, *343*, 01004. [\[CrossRef\]](#)
9. Botsiber, D.W.; Kingston, L. Design and Performance of the Cycloid Speed Reducer. *Mach. Des.* **1956**, *28*, 65–69.
10. Malhotra, S.K.; Parameswaran, M.A. Analysis of a Cycloid Speed Reducer. *Mech. Mach. Theory* **1983**, *18*, 491–499. [\[CrossRef\]](#)
11. Litvin, F.L.; Feng, P.H. Computerized Design and Generation of Cycloidal Gearings. *Mech. Mach. Theory* **1996**, *31*, 891–911. [\[CrossRef\]](#)
12. Blagojevic, M.; Marjanovic, N.; Djordjevic, Z. A New Design of a Two-Stage Cycloidal Speed Reducer. *ASME J. Mech. Des.* **2011**, *133*, 085001. [\[CrossRef\]](#)
13. Lin, W.S.; Shih, Y.-P.; Lee, J.-J. Design of a Two-stage Cycloidal Gear Reducer with Tooth Modifications. *Mech. Mach. Theory* **2014**, *79*, 184–197. [\[CrossRef\]](#)
14. Huang, C.H.; Hsieh, C.C.; Lee, J.-J.; Chiu, C.C. Two-Stage Differential Cycloidal Speed Reducer with a High Reduction Ratio. U.S. Patent No. 9,005,065 B2, 14 April 2015.
15. Pham, A.D.; Ahn, H.J. Rigid Precision Reducers for Machining Industrial Robots. *Int. J. Precis. Eng. Manuf.* **2021**, *22*, 1469–1486. [\[CrossRef\]](#)
16. Jang, D.J.; Kim, Y.C.; Hong, E.P.; Kim, G.S. Geometry design and Dynamic Analysis of a Modified of a Cycloid Reducer with Epitrochoid Profile. *Mech. Mach. Theory* **2021**, *164*, 104399. [\[CrossRef\]](#)
17. Li, X.; Tang, L.; He, H.; Sun, L. Design and Load Distribution Analysis of the Mismatched Cycloid-Pin Gear Pair in RV Speed Reducers. *Machines* **2022**, *10*, 672. [\[CrossRef\]](#)

18. Yang, Y.H.; Chen, C.; Wang, S.-Y. Response Sensitivity to Design Parameters of RV Reducer. *Chin. J. Mech. Eng.* **2018**, *31*, 49. [[CrossRef](#)]
19. Wang, H.; Shi, Z.; Yu, Y.B.; Xu, H. Transmission Performance Analysis of RV Reducers Influenced by Profile Modification and Load. *Appl. Sci.* **2019**, *9*, 4099. [[CrossRef](#)]
20. Tsay, D.M.; Chen, H.P.; Kou, T.C. Transmission Mechanism Having Eccentric Cam Assemblies. Taiwan Patent TW I431209 B, 21 March 2014.
21. Tsay, D.M.; Lin, B.J. Design and Machining of Globoidal Index Cams. *ASME J. Manuf. Sci. Eng.* **1997**, *119*, 21–29. [[CrossRef](#)]
22. Tsay, D.M.; Lin, B.J. Profile Determination of Planar and Spatial Cams with Cylindrical Roller-Followers. *J. Mech. Eng. Sci.* **1996**, *210*, 565–574. [[CrossRef](#)]
23. Spotts, M.F.; Shoup, T.E.; Hornberger, L.E. *Design of Machine Elements*, 8th ed.; Pearson Education Inc.: London, UK, 2004.
24. *Needle Roller Bearings* CAT No. E1419c; NSK Ltd.: Tokyo, Japan, 2007; p. B183.