

Article

Research on Dynamic Characteristics of Flap Actuation System Considering Joint Clearance and Flexibility

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Abstract: The flight control effects and flight quality of the aircraft are influenced by the flap actuation system directly. The main motivation of this research is to develop a rigid-flexible model of the flap actuation system with clearance and to explore the coupling effects of clearance joints and flexible bodies on the system's dynamic characteristics. A modified contact force model is used to a joint with small clearance, heavy load, and large contact area. Then, the effectiveness of an embedded modeling method of this modified model is verified. Based on this method, a rigid-flexible coupling model of the flap actuation system with multi clearance joints is established. Moreover, the influence of system parameters, such as clearance size and clearance joint position, and coupling effects of the flexibility and joint clearance on dynamic responses, are analyzed. The results show that: (1) flexible bodies act as a suspension to reduce negative effects of joint clearance on dynamic responses of a clearance-contained system; (2) one flexible body can reduce the oscillation phenomenon of joint clearances, yet the suspension effect will be gradually weakened with the increase of the clearance joint number; (3) coupling effects of the elastic deformation owing to multi flexible bodies and the collision interaction between multi clearance joints make the system tend to chaos, and the system stability is reduced. This study can contribute to predicting the collision characteristics of the aircraft transmission system and improving the transmission accuracy, response speed, and stability of the aircraft.

Keywords: flap actuation system; joint clearance; flexibility; coupling effects



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1. Introduction

With the development of aerospace technology, the studies on the structural design, kinematic analysis, dynamic characteristics, and control performance of the flap actuation system of aircrafts have gradually attracted the attention of researchers. The flap actuation system is widely used in the control fields of launch vehicles, missiles, satellites, and various aircrafts, and its performance has an important influence on the attitude correction and flight path of the aircraft [1].

The linkage mechanism at the flap actuation system is an intermediate link that transmits the output force and torque of the actuator to the control surface to drive it to deflect. It usually consists of rods and lever arms connected with each other through joints. In the traditional modeling and analysis of mechanism dynamics, the influence of some factors at joints, such as tolerance [2], clearance [3], friction [4], local elastic deformation [5], lubrication [6], and wear [7], is ignored. However, clearances are inevitable in engineering, always resulting in passive impacts on the mechanical system. For example, due to the thermal deformation of the clearance joint of the solar panel, the active position of Hubble telescope deviates from the ideal design position [8]. In view of this, it is very important to investigate the influence of the joint clearance on system responses, which will contribute

to limiting the vibration, impact, friction, wear, and other phenomena caused by the clearance [9].

Many studies on the influence of flexibility on the system's performance with joint clearance were performed [10–12]. Taking into account the joint clearance and the flexibility of a connecting rod, Dubowsky et al. studied dynamic behaviors of a system and indicated that the joint clearance would amplify the force between the components, and the flexible connecting rod would significantly reduce impact forces [11]. Schwab et al. also investigated dynamic responses of a flexible mechanism with joint clearance and pointed out that the contact force curve became smooth owing to the flexible connecting rod [13]. Erkaya et al. performed experiments about a slider–crank mechanism and indicated that the system's output performance is affected by coupling effects of the clearance and flexibility. It was found that the flexibility of the small pivot could reduce the chaotic response of the mechanism caused by the joint clearance [14,15]. Li et al. built a rigid-flexible model of a solar array system. Coupling effects of panel flexibility and joint clearance on system dynamics are also studied [16]. Chen et al. proposed a modeling method to study the dynamics of a manipulator with clearances and a flexible beam [17]. Tan et al. established a rigid-flexible coupling model of a slider–crank mechanism with a clearance joint and indicated that the flexible components play an active role of intensifying the collision in clearance joint [18]. Dong et al. pointed out that the dynamic response of a mechanical system is influenced by the links' flexibility and demonstrated that the flexibility can cushion the impact between mixed clearances [19]. Li et al. analyzed the dynamics of the air rudder mechanism by a feasible simulation method, which considered the influences of the revolute joint clearance and the link flexibility. The pushing force and control surface angle with different loading moments and clearances are discussed [20]. Generally, the influence of component flexibility and joint clearance on the dynamic characteristics of a multi-body system is coupled and interacted. Therefore, it is necessary to consider the coupling effects of the joint clearance and flexibility on the dynamic performance of a transmission mechanism.

The structure of this paper is described. A mathematical model of the clearance joint is presented and a modified contact force model is compared in Section 2. Then, an embedded modeling method of this modified model is verified by the comparison results between simulations and experiments. In Section 3, a rigid model of the flap actuation system considering multiple clearance joints is developed, and the influence of clearance size and clearance joint position is studied. Then, a rigid-flexible system with multi clearance joints and multi flexible bodies is presented in Section 4, and the effects of the internal interaction between clearance and flexibility on the dynamic responses are investigated. Lastly, the main conclusions are given in Section 5.

2. Modeling of Clearance Joints

2.1. Mathematical Model

The clearance vector model is usually used to describe the relative motion and relative position changes of the journal and the bearing during the collision process.

As represented in Figure 1, a clearance joint model is given, where e is the clearance vector, which is represented by connecting the bearing center C_B and the journal center C_J .

$$e = r_J^O - r_B^O \quad (1)$$

where r_B^O and r_J^O are the position vectors of the bearing center C_B and the journal center C_J , respectively.

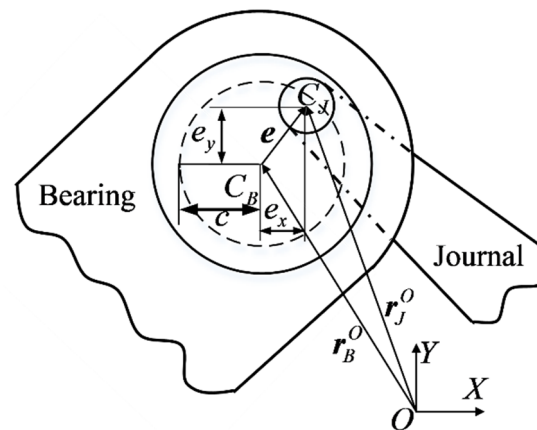


Figure 1. Clearance joint model.

The size e of the clearance vector e can also be called as the eccentricity and given as

$$e = \sqrt{e_x^2 + e_y^2} \quad (2)$$

As shown in Figure 2, when the clearance joint is in a collision state, the penetration depth vector δ at the collision position can be expressed as:

$$\delta = r_J^P - r_B^P \quad (3)$$

where r_B^P and r_J^P are the position vectors of the bearing contact point P_B and journal contact point P_J , respectively.

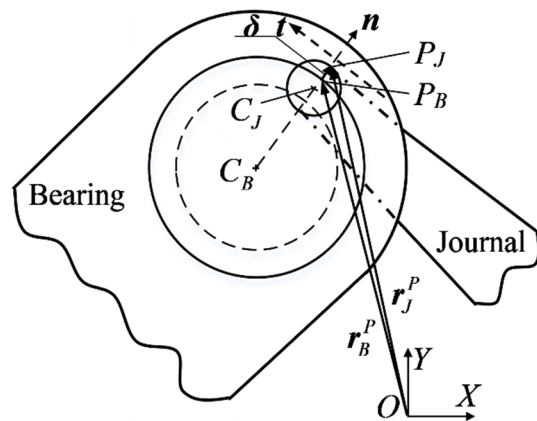


Figure 2. Collision model at the clearance joint.

The magnitude of the penetration depth vector can be obtained by the eccentricity e and clearance c :

$$c = r_B - r_J \quad (4)$$

$$\delta = e - c \quad (5)$$

where r_B and r_J are the radii of the bearing and journal.

2.2. Modified Normal Contact Force Model

According to the Gonthier model [21] and the Wang model [22], a modified normal contact force model [23] is established and briefly described as follows. In this model, the geometric shape of contact bodies, material properties, axial size of bearing, and energy

loss are considered. In addition, the model is applicable to different ranges of the restitution coefficient and is suitable for the collision process with a large contact area:

$$F_n = \frac{\pi}{2} L E^* \delta^n \left(\frac{1}{2(\Delta R + \delta)} \right)^{0.5} \left[1 + \frac{1 - c_r^2}{c_r} \frac{\dot{\delta}}{\dot{\delta}^{(-)}} \right] \quad (6)$$

where F_n is the normal contact force, L is the bearing's axial length, E^* is the composite modulus, δ is the penetration depth, ΔR is the radial clearance, c_r is the restitution coefficient, $\dot{\delta}^{(-)}$ is the journal's initial velocity, and n is equal to 1.5 for metallic contact [23]:

$$E^* = \left(\frac{1 - \nu_B^2}{E_B} + \frac{1 - \nu_J^2}{E_J} \right)^{-1} \quad (7)$$

where ν_B , ν_J , E_B , and E_J are Poisson's ratios and Young's moduli, and the symbols of B and J are for the bearing and the journal, respectively.

2.2.1. Comparison of Normal Contact Force Models

Several models including Hertz model [24] and other nonlinear spring–damping contact force models are compared and analyzed in [23]. The results show that the Hunt–Crossley model [25] and Lankarani–Nikravesh model [26] are applicable to the collision process with small clearance, high restitution coefficient, and low load. The Gonthier model [21] and Flores model [27] are not limited by clearance size and restitution coefficient. The curve of the Wang model [22] with the consideration of bearing's axial length is located between those two kinds of models. The Bai model [28] deviates slightly from other models because of its small nonlinear variable stiffness coefficient.

In view of these, on the basis of the Hertz model, the Lankarani–Nikravesh model, Gonthier model, Bai model, Wang model, and the modified normal contact force model, taking a collision between a journal and a bearing considering the clearance as an example, as illustrated in Figure 3, the comparison results are given. The parameters of colliding bodies are presented in Table 1, and the restitution coefficient is defined as 0.5 in this simulation.

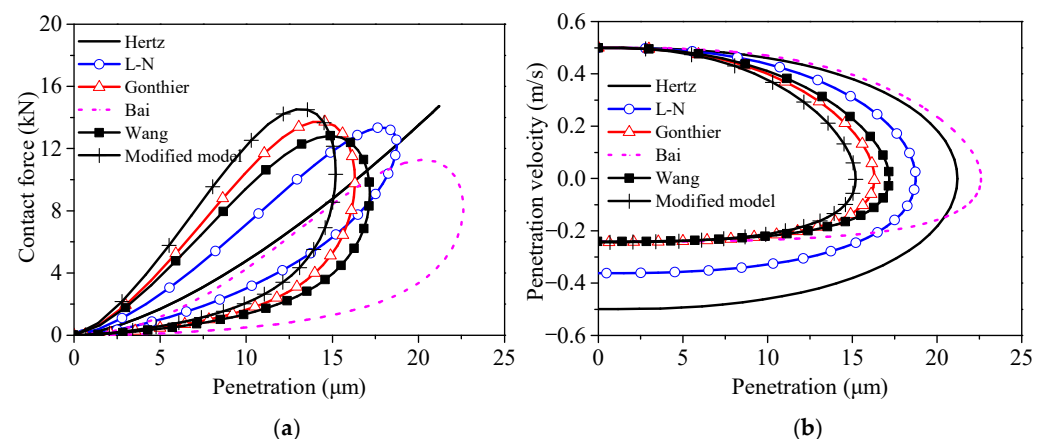


Figure 3. Dynamic responses based on the six models: (a) contact force; (b) penetration velocity.

Table 1. Parameters of the bearing and the journal.

Model	Radius (mm)	Young's Modulus (GPa)	Poisson's Ratio	Mass (kg)	Length (mm)	Initial Velocity (m/s)
Bearing	10	207	0.3	/	15	0
Journal	9.9	207	0.3	1	/	0.5

Based on the six models, the relative errors of the restitution coefficient are listed in Table 2. The actual restitution coefficient is defined as

$$c'_r = -\frac{\dot{\delta}^{(+)}}{\dot{\delta}^{(-)}} \quad 0 \leq c'_r \leq 1 \quad (8)$$

where c'_r is the actual restitution coefficient, and $\dot{\delta}^{(+)}$ is the journal's separation velocity after collision [23].

The relative error of the restitution coefficient can also be given

$$error = \left| \frac{c'_r - c_r}{c_r} \right| \times 100\% \quad (9)$$

where c_r is the restitution coefficient.

Table 2. Relative errors of the restitution coefficient.

	Hertz [24]	L-N [26]	Gonthier [21]	Bai [28]	Wang [22]	Modified Model [23]
Actual restitution coefficient	1	0.726	0.488	0.482	0.482	0.488
Relative errors	100%	45.2%	2.4%	3.6%	3.6%	2.4%

Compared with other models, the hysteresis loop area of the Lankarani–Nikravesh model is the smallest, and its restitution coefficient error is the largest because this model is applicable to the contact collision process with a high restitution coefficient, which can be observed in Figure 3 and Table 2. As shown in Figure 3, the dynamic response curves of the Gonthier model, Wang model, and the modified model are close. The restitution coefficient errors of the modified model and Gonthier model are equal to each other, both of which are 2.4%, which can be observed in Table 2. Moreover, compared with the Gonthier model, the modified model can consider the nonlinear contact stiffness coefficient related to the penetration depth and time. In conclusion, the modified model can be utilized to simulate the collision process of a clearance joint and can maintain a good response under the condition of a low restitution coefficient.

2.2.2. Embedded Modeling Method

The modified contact force is embedded into Adams environment by the following program codes:

```

if (gap < 0)
{
    E = 1/((1 - pow(v1,2))/E1 + (1 - pow(v2,2))/E2);
    K = pi*E*L*pow((pow((2*(R1 - R2 + gap)), -1), 0.5)/2;
    force[0] = max(0, K*pow(-gap, n)*(1 + (1 - pow(Cr, 2))^gapdot/(Cr*v)));
}
else force[0] = 0;

```

where *gap* and *gapdot* are system state variables provided by Adams for users to call, which can be used to obtain penetration depth and penetration velocity at contact, *K* is the contact stiffness coefficient, *R1* and *R2* are the radii of the bearing and journal, and the symbol definitions of *E*, *E1*, *E2*, *v1*, *v2*, *L*, *n*, and *Cr* in the codes are the same with the meanings of the previously mentioned symbols of *E**, *E_B*, *E_J*, *v_B*, *v_J*, *L*, *n*, and *c_r*, respectively.

The values of *L*, *R1*, *v*, and *n* are defined in the program, and the values of *E1*, *E2*, *v1*, *v2*, *R2*, and *Cr* can be transferred through the user-defined window under the MSC Adams environment, respectively.

2.2.3. Model Verification

The modified model is utilized to stimulate the collision process between the connecting rod and the slider at a slider–crank mechanism. The results comparison between simulations and experiments in [29] are discussed, which can verify the feasibility of the embedded modeling method. The mechanism parameters are presented in Table 3. The mechanism’s test rig is shown in Figure 4.

Table 3. Mechanism parameters [29].

	Length (m)	Mass (kg)	Moment of Inertial ($\text{kg}\cdot\text{m}^2$)
Crank	0.05	0.343	0.000216
Connecting rod	0.3	1.072	0.034
Slider	/	0.347	0.000115

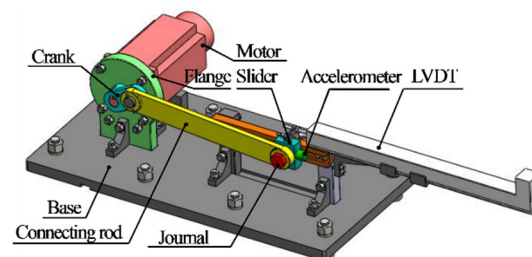
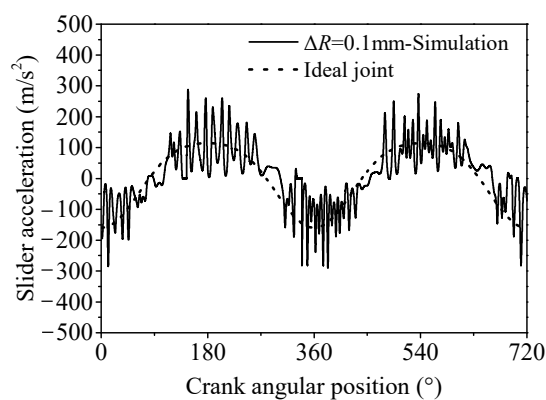
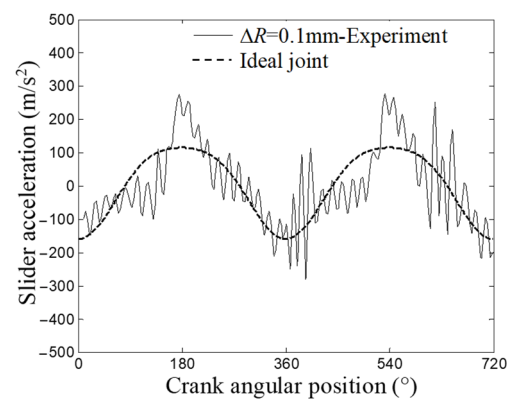


Figure 4. The test rig of the mechanism [29].

Defining the crank speed with 500 rpm, the clearance sizes are selected as 0.1, 0.3, and 0.5 mm, respectively. As represented in Figure 5, the comparison results between simulations and experiments indicate that, regardless of being simulation results or experimental results, the slider acceleration curves fluctuate around the ideal ones, and the oscillation degree is the largest near the pole position. The oscillation amplitude increases with the clearance size increasing. The relative errors of the oscillation peaks of the slider acceleration between simulation and experiment results are shown in Table 4. With the clearance size increasing, the relative errors between simulations and experiments exhibit an increasing trend, but they are all less than 10%. The similar fluctuation trend and magnitude of simulation and experiment results indicate that the embedded modeling method of the modified model is applicable to analyze the collision process of a clearance-contained system.



(a)



(b)

Figure 5. Cont.

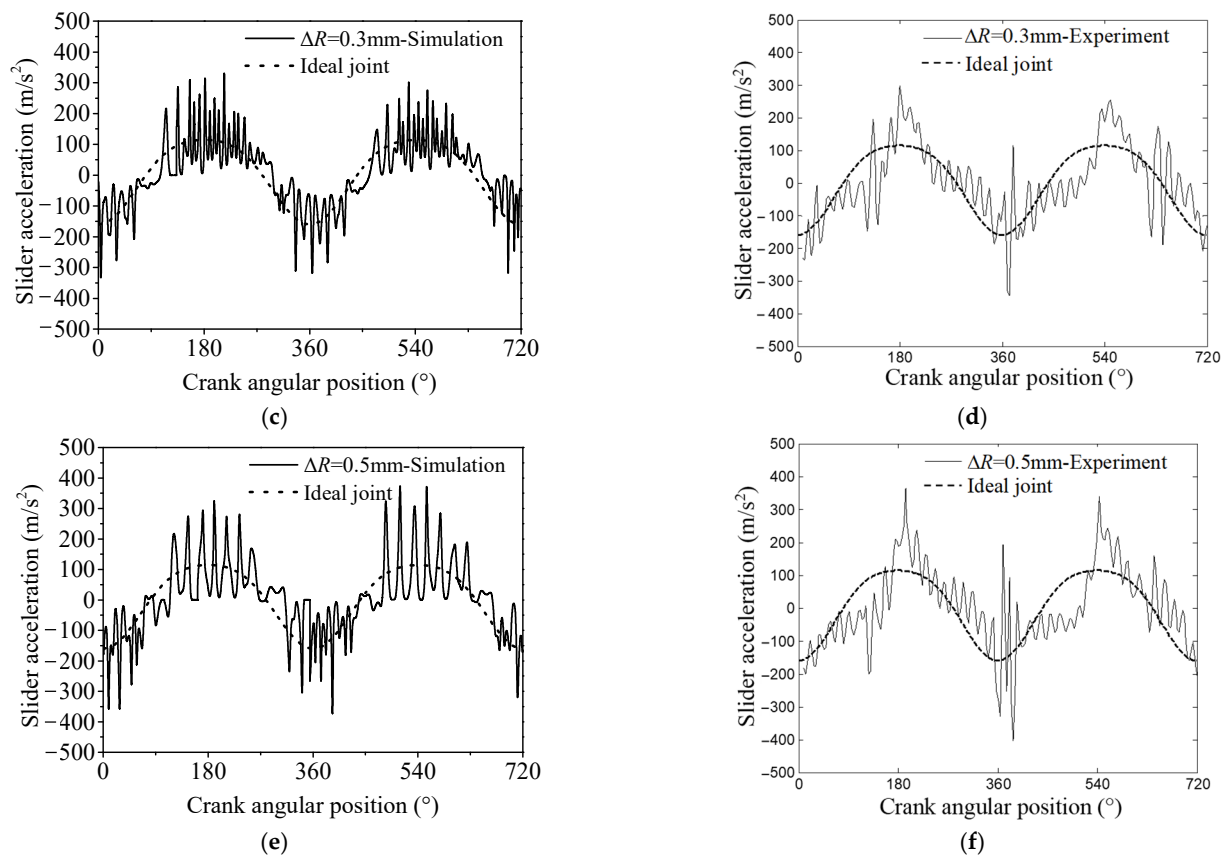


Figure 5. Comparison results of the slider acceleration with clearance sizes defining as 0.1, 0.3, 0.5 mm respectively: (a,c,e) simulations; (b,d,f) experiments [29].

Table 4. Relative errors of the oscillation peaks of the slider acceleration between simulation and experiment results.

Clearance Size (mm)	Oscillation Peak (m/s ²)			Relative Error
	Simulation	Experiment		
0.1	283.1	268.0		5.6%
0.3	318.2	297.7		6.9%
0.5	371.8	345.4		7.6%

2.3. Tangential Contact Force Model

Tangential contact force has an important influence on the analysis of dynamic characteristics of mechanisms with joint clearance. The Coulomb friction model is the simplest friction model describing dry contact surfaces. However, this friction model may lead to difficulties in numerical integration. In order to avoid this problem, some modified Coulomb friction models are proposed. In this paper, the Ambrósio friction model is utilized to simulate the friction behaviors of the multi-body system. The Ambrósio friction model can be expressed as [30]:

$$F_t = -c_f c_d F_n \text{sgn}(v_t) \quad (10)$$

where v_t is the relative tangential velocity, c_f is the friction coefficient, and c_d is the dynamic correction coefficient, which can be given as:

$$c_d = \begin{cases} 0 & v_t \leq v_0 \\ \frac{v_t - v_0}{v_1 - v_0} & v_0 \leq v_t \leq v_1 \\ 1 & v_t \geq v_1 \end{cases} \quad (11)$$

where v_0 and v_1 are given tolerances for the tangential velocity.

3. Modeling and Simulation of a Rigid Flap Actuation System with Clearance

As illustrated in Figure 6, the dynamic model of a rigid flap actuation system with four clearance joints is established. The linkage mechanism consists of an auxiliary lever arm, an auxiliary rod, and a lever arm. Joints connecting the auxiliary lever arm with the actuating rod, auxiliary rod, and bearing house 2 are regarded as joints 1, 2, and 3, respectively. The joint between the auxiliary rod and lever arm is defined as joint 4. The materials of the auxiliary lever arm, auxiliary rod, and lever arm are defined as the magnesium-aluminum alloy, which are defined in accordance with [23]. For details about the parameters of parts' material, please refer to [23]. The modified normal contact force model and Ambrósio friction model [30] are utilized in this simulation, and the simulation parameters are shown in Table 5.

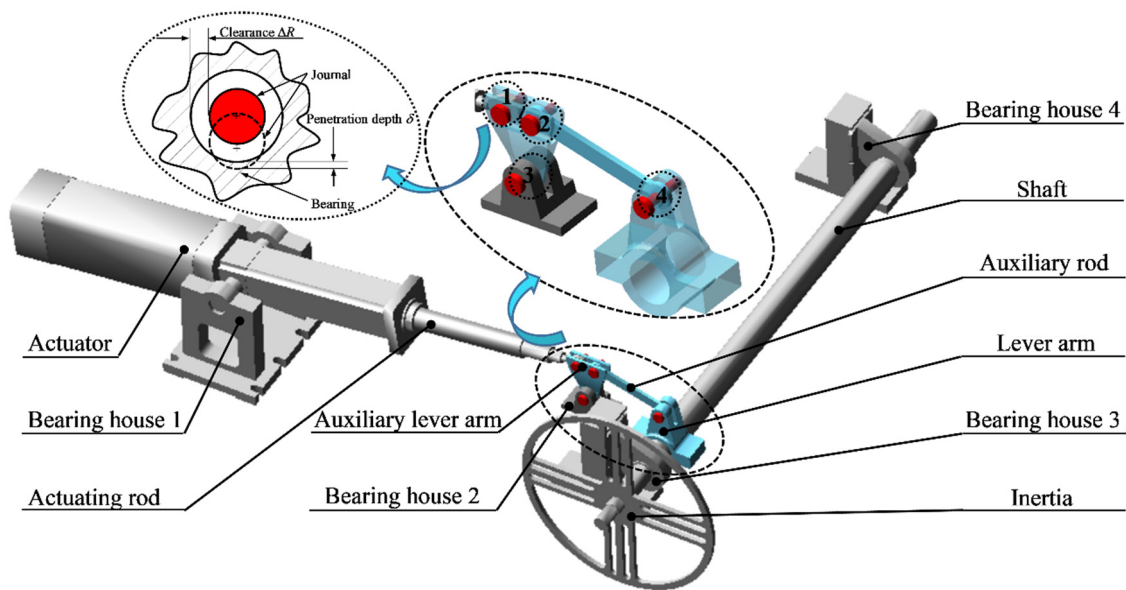


Figure 6. Dynamic model of a rigid flap actuation system with four clearance joints.

Table 5. Simulation parameters.

Parameter	Description	Value	Unit
x	Actuator's output displacement	10	mm
f	Actuator's output frequency	2	Hz
L	Bearing's length	15	mm
c_f	Friction coefficient	0.01	/
c_r	Restitution coefficient	0.46	/
v_0	Given tolerance for the tangential velocity	0.1	mm/s
v_1	Given tolerance for the tangential velocity	1	mm/s

3.1. Influence of Clearance Size

The clearance is exaggerated to observe the effects of joint clearance size on mechanism responses. Only the clearance at the joint 1 is considered, and other joints are regarded to be ideal. The clearance sizes at joint 1 are defined as 0.01, 0.05, 0.1, and 0.5 mm, respectively. The clearance effects on the system responses are shown in Figures 7–9.

As shown in Figure 7, the angle curves of the rigid flap actuation system with different clearance sizes almost coincide with the ideal curve, but Figure 7b shows that errors between them increase when the clearance size becomes large, which are 0.014%, 0.621%, 1.186%, and 1.931%, respectively.

Figure 8 indicates that a shaft's angular velocity curves oscillate because of joint clearance. When the size is defined as 0.01 mm, the angular velocity curve is almost consistent with the ideal one. However, as the size is chosen to 0.05 mm or 0.1 mm, the curve with joint clearance oscillates along the ideal curve, and the fluctuation amplitude becomes large with the clearance size increasing. Moreover, the fluctuation amplitude reaches the maximum value with the clearance size of 0.5 mm.

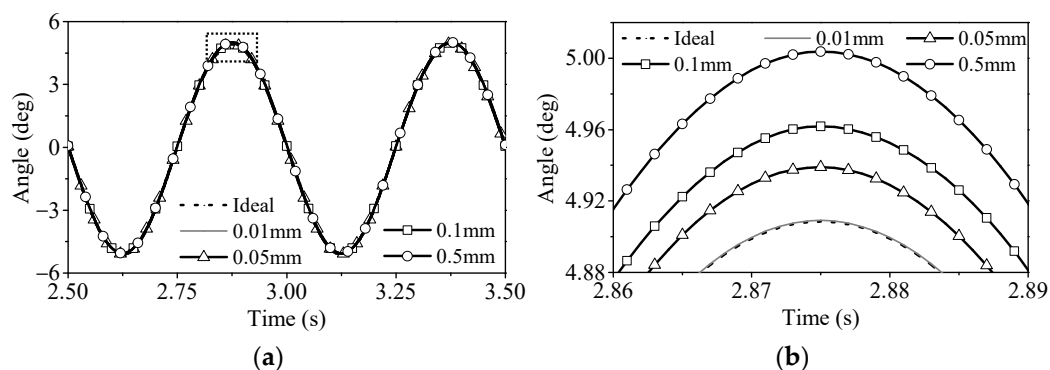


Figure 7. Angle of the rigid system with different clearance sizes: (a) angle; (b) partial enlarged drawing.

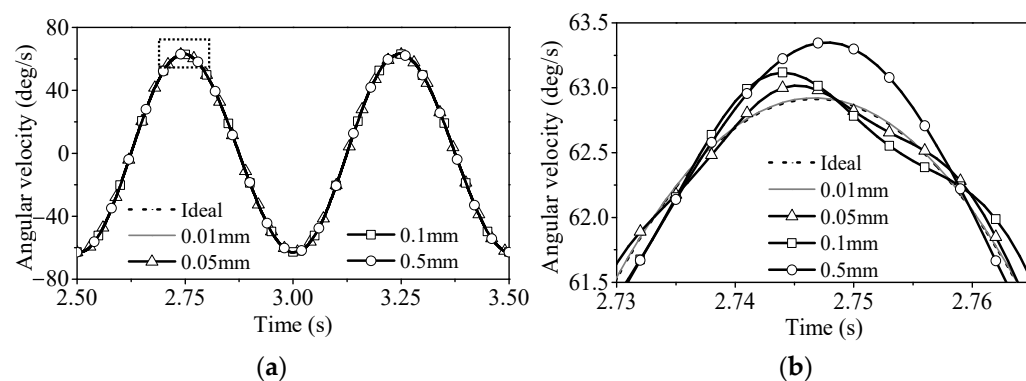


Figure 8. Angular velocity of the rigid system with different clearance sizes: (a) angular velocity; (b) partial enlarged drawing.

The system's angular acceleration curves with different clearance sizes oscillate strongly near the ideal curve, which are presented in Figure 9. The journal contacts with the bearing when they move in the same direction, and the collision frequency and peak value of the angular acceleration of the system are small. Conversely, when the system commutates, the journal and the bearing collide. The amplitude of angular acceleration reaches the maximum and then gradually decreases under the consumption of damping. With the clearance size changing from 0.01 to 0.5 mm, the oscillation amplitude and frequency of the system's angular acceleration increase and decrease, respectively.

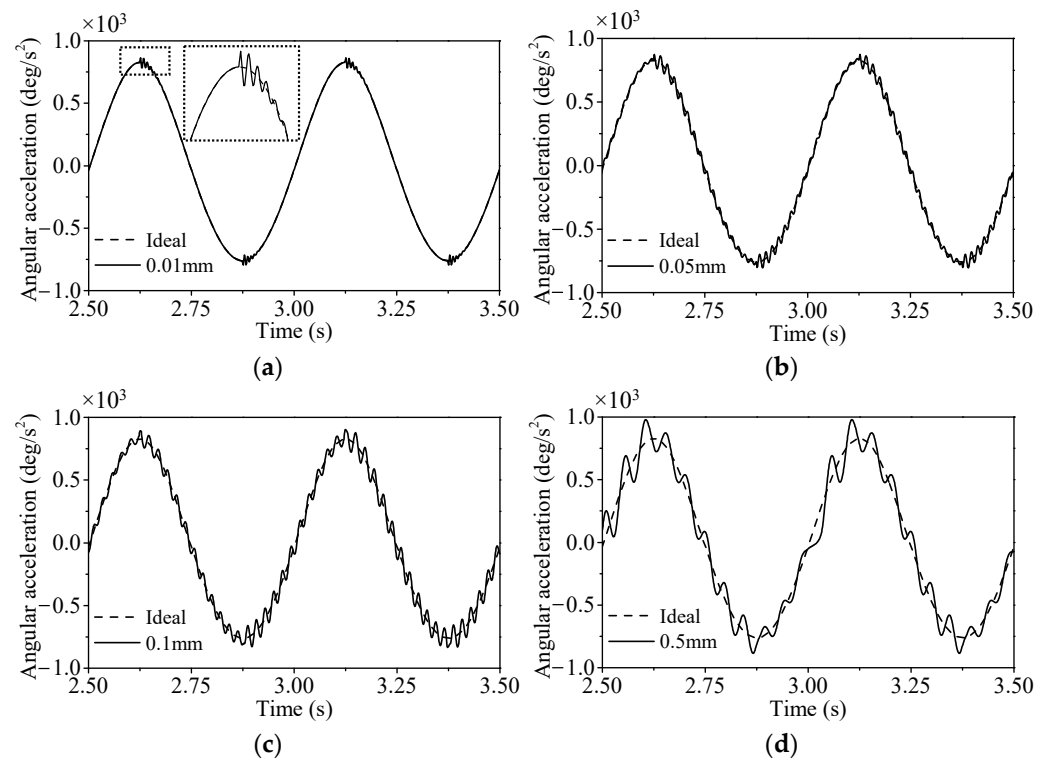


Figure 9. Angular acceleration of the rigid system with different clearance sizes: (a–d) clearance size = 0.01/0.05/0.10/0.50 mm.

3.2. Influence of Clearance Joint Position

As shown in Figures 10 and 11, the effects of clearance joint position on the responses of a rigid flap actuation system are investigated. The four revolute joints are regarded as clearance joints in different situations, respectively. The clearance size is defined as 0.1 mm in this simulation.

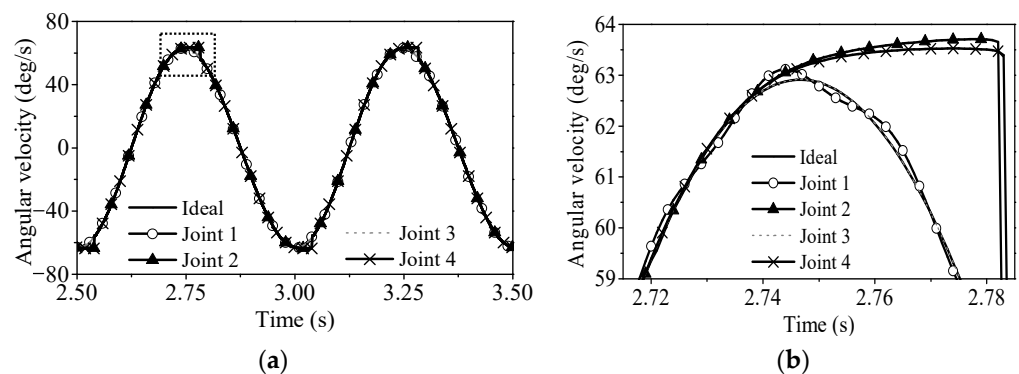


Figure 10. Angular velocity of the rigid system with different clearance joint position: (a) angular velocity; (b) partial enlarged drawing.

The dynamic response curves with clearance joint 1 fluctuate regularly up and down along the ideal curve. The simulation results of joint 2 and joint 4 are similar. They both maintain almost constant angular velocity and zero angular acceleration at the “zero position”, and then suddenly change in the form of pulse. The reason may be that the two clearance joints are located on both sides of the auxiliary rod. When the flexibility is not considered, the contact force at clearance joint 2 or clearance joint 4 directly interacts with the inertia force of the shaft and the driving force output by the actuator rod through the rigid auxiliary rod, resulting in a large deviation between the response curves with joint

clearance and ideal one. The system's angular velocity and angular acceleration curves with clearance joint 3 almost coincide with the ideal curve. The reason is that the bearing house 2 in joint 3 is fixed. However, Figure 11c shows that the angular acceleration only considering the clearance at joint 3 still exhibits weak oscillation during system commutation.

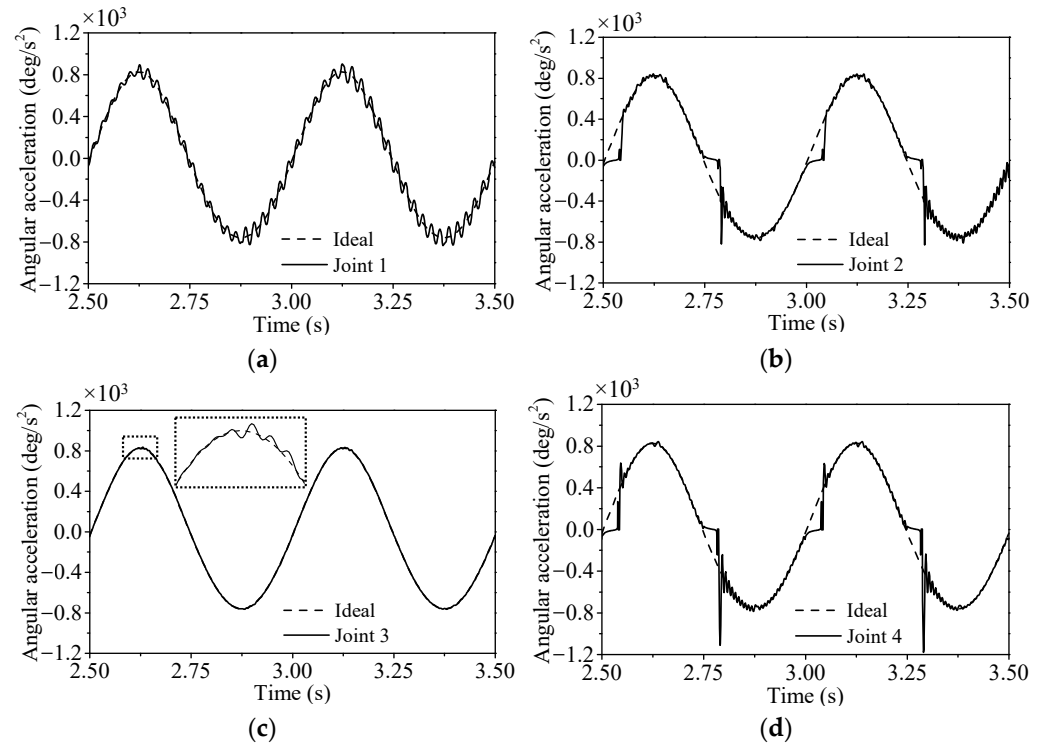


Figure 11. Angular acceleration of the rigid system with different clearance joint position: (a–d) joints 1,2,3,4.

The phase diagram of the rigid system with different clearance joint position performs weak chaotic characteristics, as presented in Figure 12. The phase diagram only with the clearance at joint 3 is almost near the ideal state. When the clearance at joint 2 or joint 4 is considered, the system stability reaches the worst at the extreme speed and maintains good continuous motion stability at other times. The phase space trajectory with clearance joint 1 has the strongest chaos, that is, the clearance at joint 1 connected to the actuating rod has the greatest influence on the system stability, and this conclusion can also be acquired in [31]. Therefore, the dynamic characteristics of the revolute joint near the power source should be paid more attention to improve the system's stability and reliability.

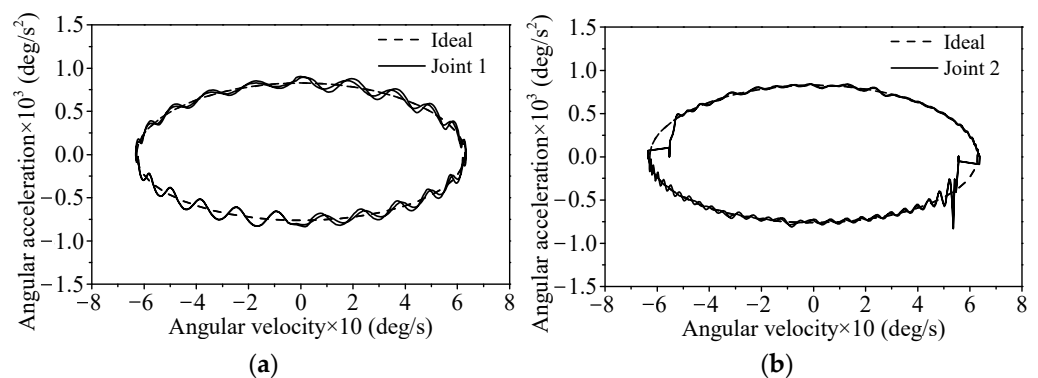


Figure 12. Cont.

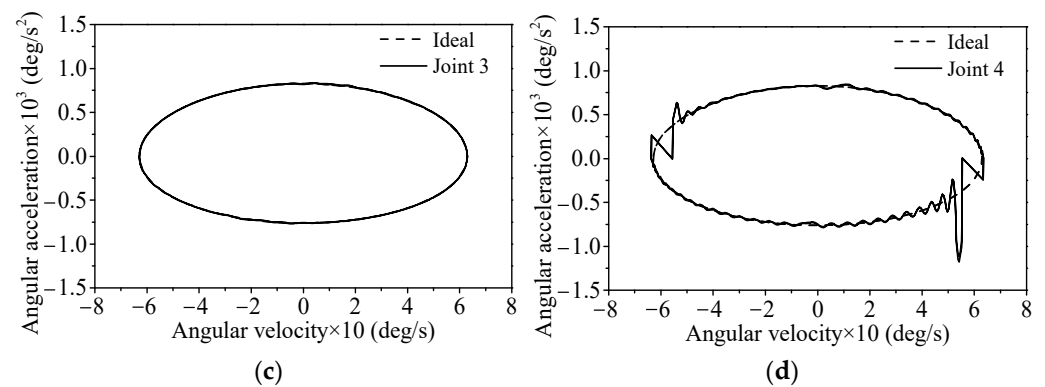


Figure 12. Angular acceleration-angular velocity phase diagram of the rigid system with different clearance joint position: (a–d) joints 1,2,3,4.

4. Modeling and Simulation of a Rigid-Flexible Flap Actuation System with Clearance

4.1. Modeling of Flexible Parts

The auxiliary lever arm and the lever arm bear tension, compression, bending, and torsion loads at the same time. In addition, the auxiliary rod is a slender connecting rod, which will be easy to produce large deformation. In view of these, the three parts should be considered to be flexible. The flexible models are established under the finite element software ANSYS, as shown in Figures 13–15. The first five typical natural frequencies of these three parts are shown in Table 6. These three flexible parts are generated into modal neutral files, which are successively imported into the rigid dynamic model in Section 3 according to different cases.

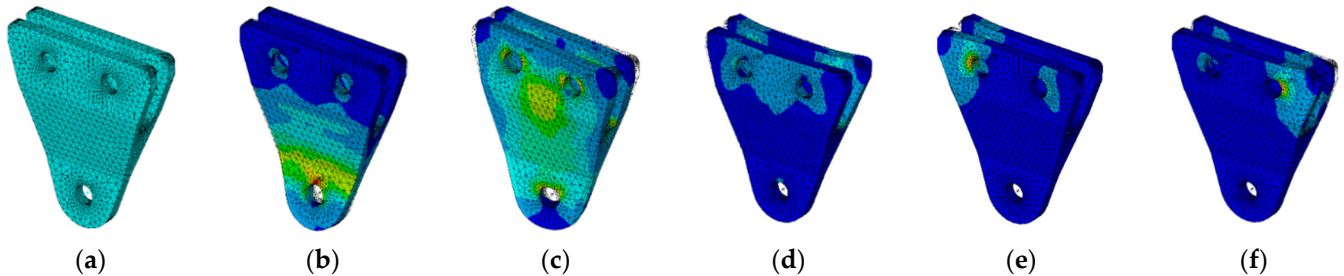


Figure 13. Finite element model of flexible auxiliary lever arm and modal diagrams of the first five modes: (a) finite element model; (b–f) first to fifth modes.

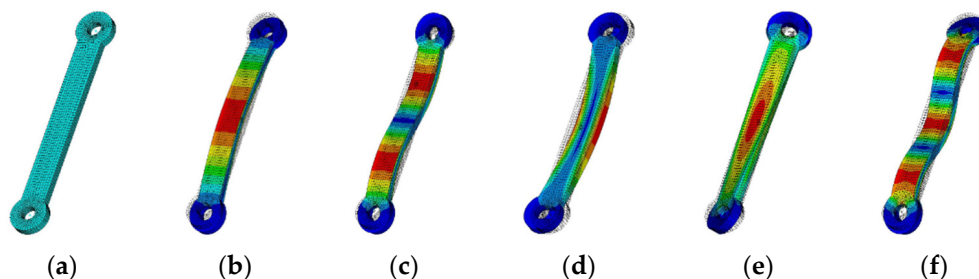


Figure 14. Finite element model of flexible auxiliary rod and modal diagrams of the first five modes: (a) finite element model; (b–f) first to fifth modes.

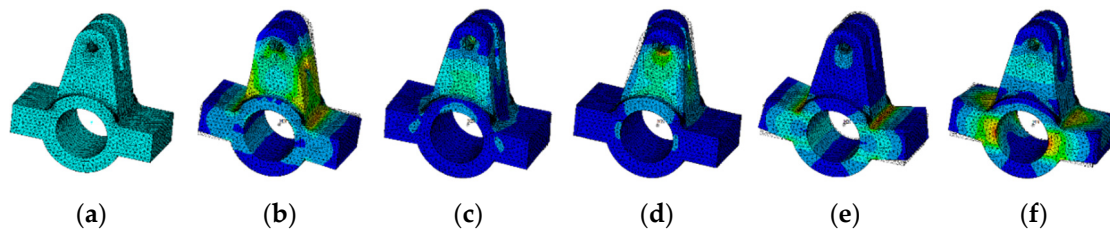


Figure 15. Finite element model of flexible lever arm and modal diagrams of the first five modes: (a) finite element model; (b–f) first to fifth modes.

Table 6. The eigenfrequencies of three flexible parts (Unit: Hz).

Parts	Eigenfrequencies				
	First	Second	Third	Fourth	Fifth
Auxiliary lever arm	3715.8	5437.1	7921.3	7946.1	7971.7
Auxiliary rod	690.40	1648.66	1869.88	2143.10	3588.54
Lever arm	5839.3	6201.3	7240.9	7976.3	9260.8

4.2. Influence of Flexible Body Number with Single Clearance Joint

Assuming that only joint 1 contains clearance, the clearance size is 0.1 mm. A different number of flexible bodies, including auxiliary rod, auxiliary lever arm, and lever arm, are considered in turn under different working conditions, as represented in Figures 16–18.

The angular velocity of the shaft with different flexible body number when considering a single clearance joint is represented in Figure 16. When only the clearance at joint 1 is considered, the system's angular velocity curves for the rigid model and the rigid-flexible coupling model are almost coincident, which indicates that the flexibility has little effect on the system's output characteristics with a single clearance joint. However, Figure 16b indicates that the fluctuation degree of the shaft's angular velocity curve gradually decreases and the curve tends to be smooth when the flexible body number increases. When the flexibility of those three parts is all considered, the collision effect caused by joint clearance almost disappears, and the shaft's angular velocity curve only deviates from the ideal curve to a certain extent with no obvious jitter.

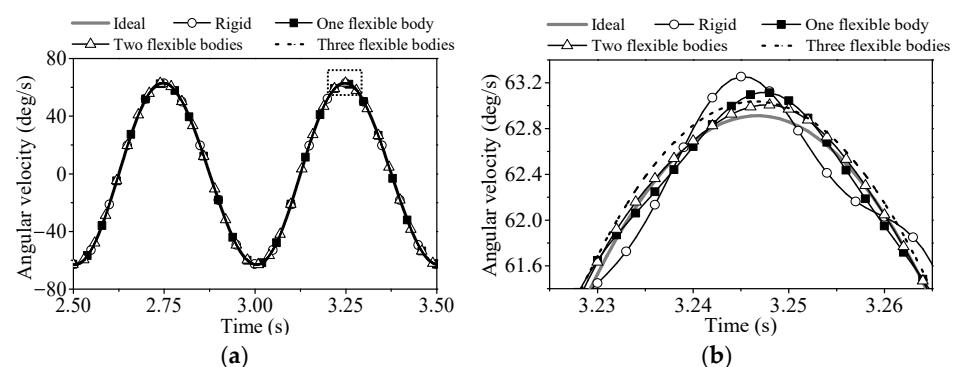


Figure 16. Angular velocity of the rigid-flexible system with different flexible body number when considering a single clearance joint: (a) angular velocity; (b) partial enlarged drawing.

Taking into consideration the effects of different flexible body number, the angular acceleration of the system with clearance at joint 1 is shown in Figure 17. The dynamic response represented by the red curve fluctuates up and down around the ideal curve violently. When the flexibility of one or two bodies is considered, the fluctuation degree of the shaft's angular acceleration weakens compared to that with rigid bodies, which can be observed in Figure 17b,c. In addition, when the flexibility of three components is considered, the output curve is almost close to the ideal one. Therefore, flexible bodies act

as a suspension to reduce adverse effects of joint clearance on shaft's responses, which can improve the motion stability.

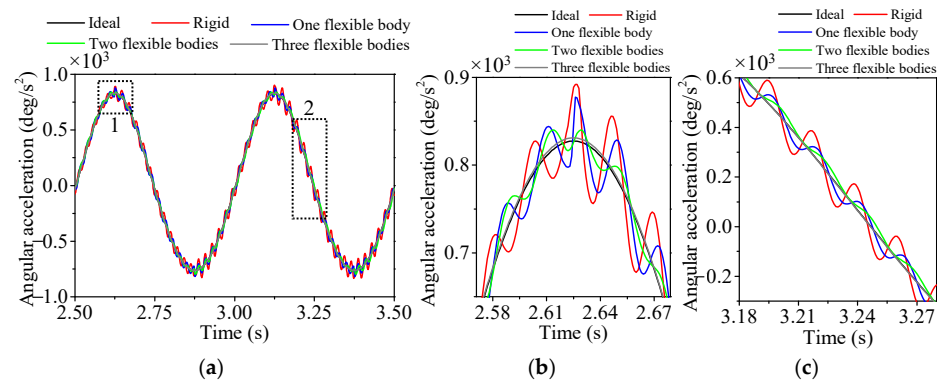


Figure 17. Angular acceleration of the rigid-flexible flap actuation system with different flexible body number when considering a single clearance joint: (a) angular acceleration; (b,c) partial enlarged drawings 1,2.

Figure 18 shows that the angular acceleration–angular velocity phase diagram of the rigid-body system has obvious chaotic characteristics compared with that of the rigid-flexible system. With the flexible body number increasing, the phase diagram gradually approaches the ideal state until it is almost consistent with the ideal one. In other words, flexible bodies can buffer and weaken the contact collision effects of the clearance joint in a multi-body system.

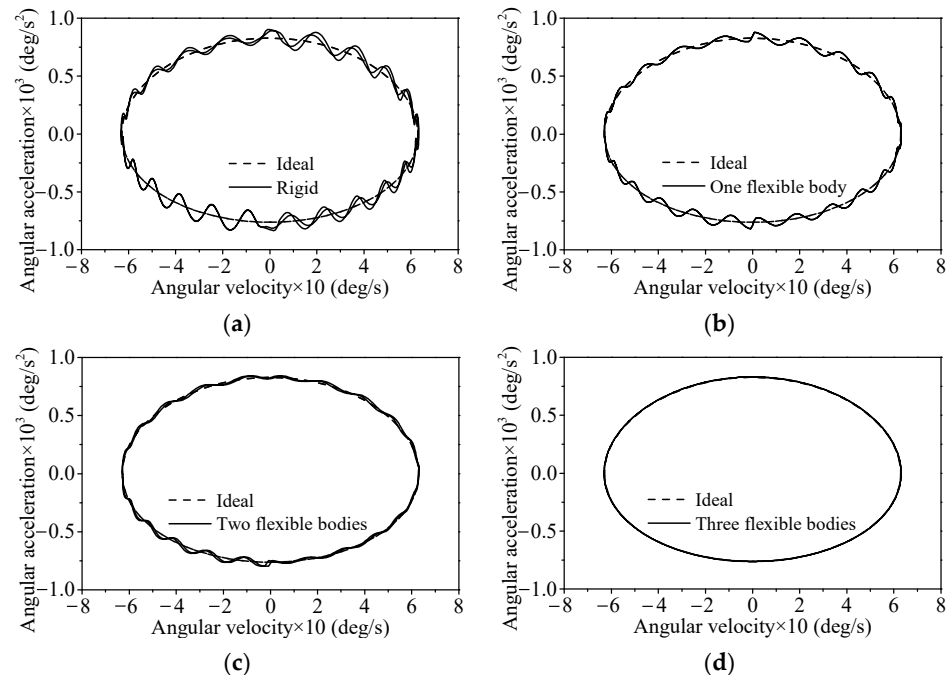


Figure 18. Angular acceleration-angular velocity phase diagram of the rigid-flexible flap actuation system with different flexible body number when considering the single clearance joint: (a) rigid; (b) one flexible body; (c) two flexible bodies; and (d) three flexible bodies.

4.3. Influence of Clearance Joint Number with Single Flexible Body

It is assumed that only the flexibility of the auxiliary rod is considered in this section. The clearance joint number is ordered as four cases, and the simulation results are shown in Figures 19–22.

As shown in Figures 19 and 20, whether or not the flexibility of the auxiliary rod is considered, the fluctuation peaks of shaft responses with two clearance joints are larger than those with only one clearance joint. In addition, whether the rigid system or the rigid-flexible coupling system, dynamic responses curves only with clearance at joint 1 are fluctuated along with ideal curves smoothly, while those with clearances at joint 1 and joint 2 oscillate violently with high pulse. This means that the system's dynamic responses will be strongly affected by the clearance joint number, whether or not the flexibility is considered in a multi-body system.

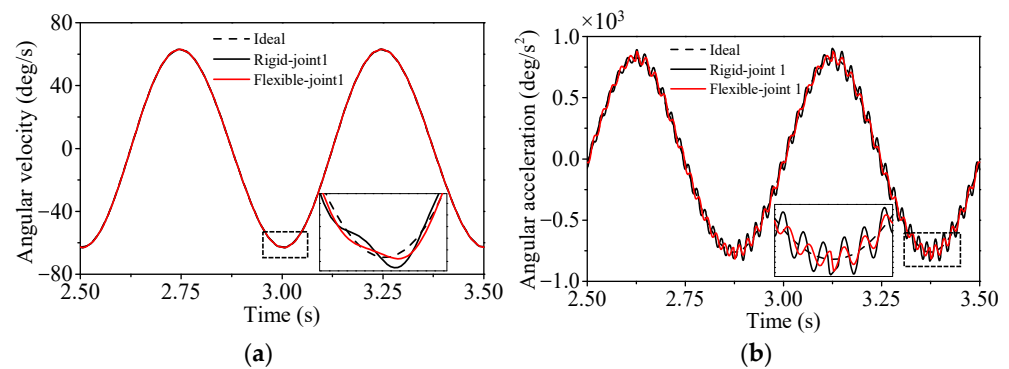


Figure 19. Effects of clearance at joint 1 when considering the flexibility of the auxiliary rod: (a) angular velocity; (b) angular acceleration.

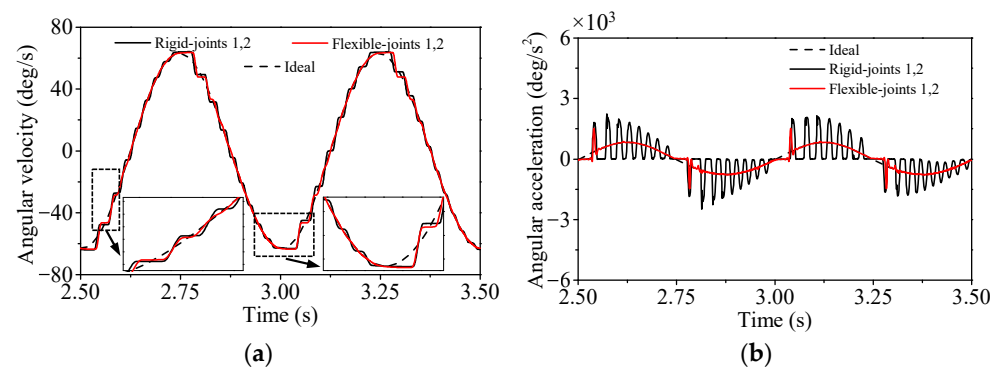


Figure 20. Effects of clearances at joints 1 and 2 when considering the flexibility of the auxiliary rod: (a) angular velocity; (b) angular acceleration.

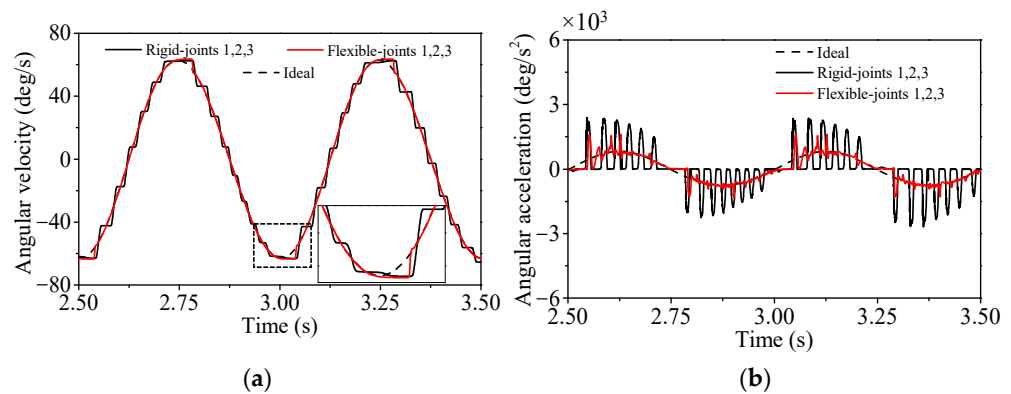


Figure 21. Effects of clearances at joints 1, 2, and 3 when considering the flexibility of the auxiliary rod: (a) angular velocity; (b) angular acceleration.

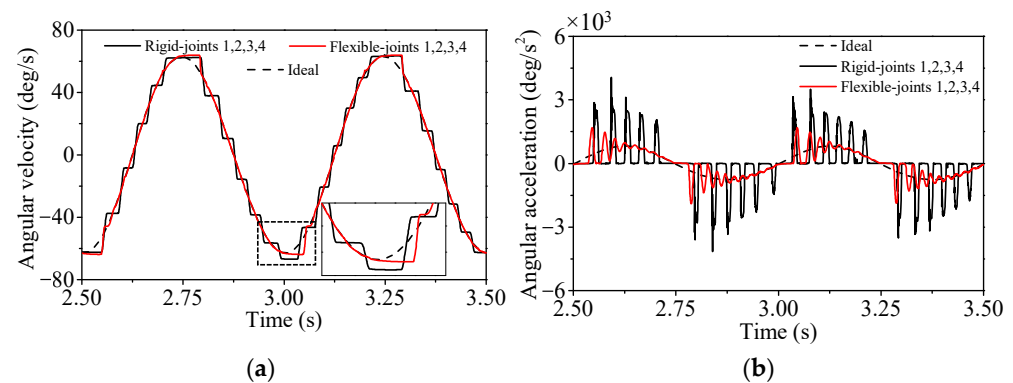


Figure 22. Effects of clearances at joints 1, 2, 3, and 4 when considering the flexibility of the auxiliary rod: (a) angular velocity; (b) angular acceleration.

The flexible auxiliary rod has a certain weakening effect on the fluctuation degree of shaft's curves including the angular velocity and angular acceleration, which can be observed from Figures 19–22. The fluctuation trend of angular acceleration curves of a rigid-flexible coupling system with different clearance joint number is similar to those of a rigid body system, but the fluctuation peak of the former is lower than that of the latter. The shaft's angular acceleration reaches the maximum during commutation and gradually tends to be stable under the action of flexible components in different cases. To sum up, the flexible body will weaken the contact collision phenomenon due to the joint clearance and reduce the fluctuation degree of output responses. However, with the increase of the clearance joint number, the suspension effect will be gradually decreased.

4.4. Coupling Effects of Four Clearance Joints and Three Flexible Bodies

It is assumed that the flexibility of the auxiliary lever arm, auxiliary rod, and lever arm is considered in this section. At the same time, the joints 1, 2, 3, and 4 are regarded as imperfect joints with clearances.

When the three parts are all considered as flexible bodies, the shaft's angular velocity curve with one clearance joint is shown in Figure 16 and that with four clearance joints is represented in Figure 23a. The angular velocity curve in Figure 23a fluctuates along the ideal one with a few pulses, and the curve in Figure 16 is not much different from the ideal curve. In addition, comparing the shaft's angular acceleration with four clearance joints shown in Figure 23b to that with one clearance joint shown in Figure 17, the pulse fluctuation of the former decreases gradually under the action of flexible damping, and the latter fluctuates up and down slightly along the ideal curve.

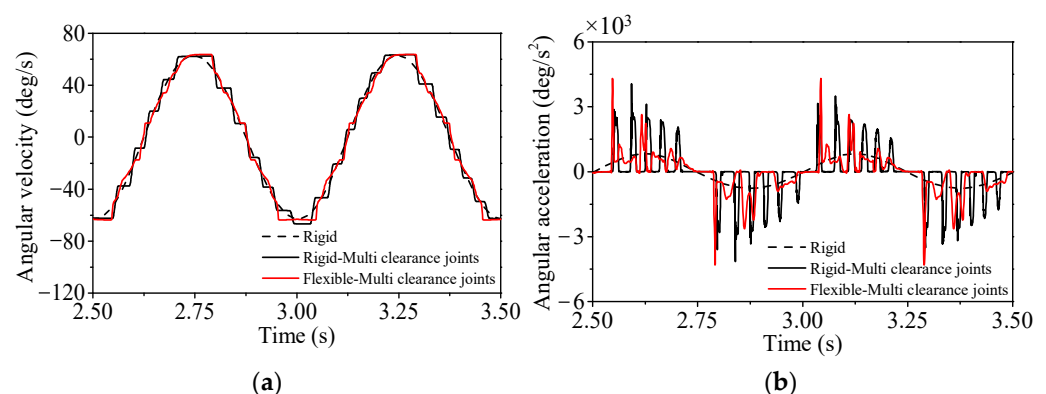


Figure 23. Coupling effects of four clearance joints and three flexible bodies: (a) angular velocity; (b) angular acceleration.

When the clearances existed in the four joints are all considered, the output curves of the shaft with three flexible bodies shown in Figure 23 to that with one flexible body shown in Figure 22 can be compared. The comparison results indicate that the angular acceleration of the former one oscillates more violently with higher amplitude. The reason for that is the coupling effects between the elastic deformation caused by the flexible bodies and the contact collision due to the joint clearances will reduce the suspension effect of the flexible bodies on the collision at the clearance joint.

As shown in Figure 24, the main frequencies of the shaft's angular acceleration with different cases are basically maintained at 2 Hz, and the amplitudes are relatively close, which are 805.15 deg/s^2 , 792.37 deg/s^2 , 798.90 deg/s^2 , and 821.83 deg/s^2 , respectively. The amplitude is the smallest for the shaft's angular acceleration with one clearance joint and three flexible bodies, while when four clearance joints and three flexible bodies are considered at the same time, the amplitude is the largest. The coupling effects between the interaction of multiple clearance joints and elastic deformation caused by multiple flexible bodies make the system tend to chaos, while the collision phenomenon of one clearance joint is gradually consumed under the suspension effect owing to flexible bodies, which makes the system tend to be stable.

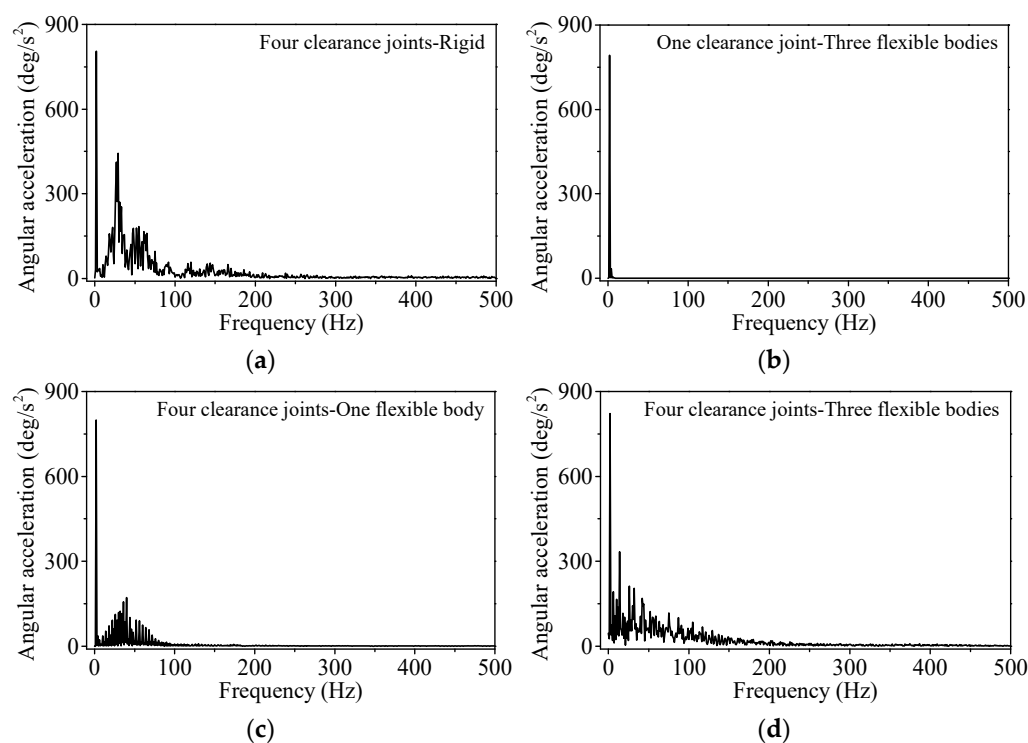


Figure 24. FFT analysis of the angular acceleration of shaft with different cases: (a) four clearance joints-rigid; (b) one clearance joint-three flexible bodies; (c) four clearance joints-one flexible body; (d) four clearance joints-three flexible bodies.

Due to the interacted collision–separation–free motion modes of multiple clearance joints, the system presents a high oscillation peak with low frequency, which can be seen from Figure 24a,c,d. In other frequency bands of the angular acceleration spectrum of the shaft, the peak value only considering four clearance joints is higher than that considering four clearance joints and one flexible body at the same time because flexible bodies can reduce the passive effects of joint clearances on the shaft's response. However, under the action of the multiple clearance joints and multiple flexible bodies together, its contribution of other frequency bands is higher than that with only one flexible body, as shown in Figure 24c,d. Comparing Figure 24b,d, when the three parts are all considered to be flexible,

the greater the clearance joint number, the greater the main frequency amplitude and the contribution of different frequency bands.

5. Conclusions

Coupling effects of multi clearance joints and multi flexible bodies on the dynamic behaviors of the flap actuation system are studied, which is helpful for the development of the system with ultra-precision, great efficiency, and good reliability.

Firstly, a modified contact force model is compared with several typical models to indicate that it can be applied to a clearance-contained actuation system. In addition, based on a slider–crank mechanism, the effectiveness of an embedded modeling method about this modified model is verified by comparing the simulation with experiments.

Then, the modified model is compiled and embedded to a rigid flap actuation system with four clearance joints. The factors including clearance size and clearance joint position are discussed. (1) The angular acceleration of the shaft is the most sensitive to the joint clearance. The larger the clearance size, the greater the fluctuation amplitude, and the lower the frequency of the angular acceleration curve around the ideal one. (2) Compared with other clearance joint positions, the clearance at the joint connected with the actuating rod has the greatest impact on the system stability and the strongest influence on the chaos of the phase space trajectory of the system.

Finally, a rigid-flexible flap actuation system with four clearance joints and three flexible bodies is presented. Coupling effects and the internal relationship between clearance and flexibility are analyzed. (1) The flexible body can reduce negative effects owing to joint clearance and improve the motion stability. When the flexible body number is increasing, the fluctuation degree of the output curve gradually decreases without obvious jitter; (2) The flexible auxiliary rod acts as a suspension to weaken the collision phenomenon caused by joint clearance, but this weakening effect will gradually decrease with the clearance joint number increasing; (3) coupling effects between the elastic deformation generated by multi flexible bodies and the impact caused by multi clearance joints will reduce the suspension effect of the flexible body on the collision at the clearance joint.

The main contribution of this research is to establish a rigid-flexible model of the flap actuation system with multi clearance joints and multi flexible bodies and to analyze coupling effects of the clearance and flexibility. This study of coupling effects on the dynamic characteristics of the flap actuation system will help with the design, optimization, and analysis of the transmission system.

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References

1. Arriola, D.; Thielecke, F. Model-based design and experimental verification of a monitoring concept for an active-active electromechanical aileron actuation system. *Mech. Syst. Signal Proc.* **2017**, *94*, 322–345. [\[CrossRef\]](#)
2. Geng, X.Y.; Wang, X.J.; Wang, L.; Wang, R.X. Non-probabilistic time-dependent kinematic reliability assessment for function generation mechanisms with joint clearances. *Mech. Mach. Theory* **2016**, *104*, 202–221. [\[CrossRef\]](#)
3. Li, Y.Y.; Chen, G.P.; Sun, D.Y.; Gao, Y.; Wang, K. Dynamic analysis and optimization design of a planar slider-crank mechanism with flexible components and two clearance joints. *Mech. Mach. Theory* **2016**, *99*, 37–57. [\[CrossRef\]](#)
4. Marques, F.; Flores, P.; Claro, J.C.P.; Lankarani, H.M. A survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems. *Nonlinear Dyn.* **2016**, *86*, 1407–1443. [\[CrossRef\]](#)

5. Chen, Y.; Sun, Y.; Chen, C. Dynamic analysis of a planar slider-crank mechanism with clearance for a high speed and heavy load press system. *Mech. Mach. Theory* **2016**, *98*, 81–100. [[CrossRef](#)]
6. Daniel, G.B.; Machado, T.H.; Cavalc, K.L. Investigation on the influence of the cavitation boundaries on the dynamic behavior of planar mechanical systems with hydrodynamic bearings. *Mech. Mach. Theory* **2016**, *99*, 19–36. [[CrossRef](#)]
7. Li, P.; Chen, W.; Li, D.S.; Yu, R.F.; Zhang, W.J. Wear analysis of two revolute joints with clearance in multibody systems. *J. Comput. Nonlinear Dyn.* **2016**, *11*, 011009. [[CrossRef](#)]
8. Cheng, C.; Huang, Q.G. An accurate determination of the Hubble constant from baryon acoustic oscillation datasets. *Sci. China-Phys. Mech. Astron.* **2015**, *58*, 599801. [[CrossRef](#)]
9. Flores, P.; Ambrósio, J.; Claro, J.C.P.; Lankarani, H.M.; Koshy, C.S. A study on dynamics of mechanical systems including joints with clearance and lubrication. *Mech. Mach. Theory* **2006**, *41*, 247–261. [[CrossRef](#)]
10. Zheng, E.L.; Zhu, R.; Zhu, S.H.; Lu, X.J. A study on dynamics of flexible multi-link mechanism including joints with clearance and lubrication for ultra-precision presses. *Nonlinear Dyn.* **2016**, *83*, 137–159. [[CrossRef](#)]
11. Dubowsky, S.; Moening, M.F. An experimental and analytical study of impact forces in elastic mechanical systems with clearances. *Mech. Mach. Theory* **1978**, *13*, 451–465. [[CrossRef](#)]
12. Erkaya, S.; Doğan, S.; Ulus, S. Effects of joint clearance on the dynamics of a partly compliant mechanism: Numerical and experimental studies. *Mech. Mach. Theory* **2015**, *88*, 125–140. [[CrossRef](#)]
13. Schwab, A.L.; Meijaard, J.P.; Meijers, P. A comparison of revolute joint clearance model in the dynamic analysis of rigid and elastic mechanical systems. *Mech. Mach. Theory* **2002**, *37*, 895–913. [[CrossRef](#)]
14. Erkaya, S. Experimental investigation of flexible connection and clearance joint effects on the vibration responses of mechanisms. *Mech. Mach. Theory* **2018**, *121*, 515–529. [[CrossRef](#)]
15. Erkaya, S.; Doğan, S.; Şefkathioğlu, E. Analysis of the joint clearance effects on a compliant spatial mechanism. *Mech. Mach. Theory* **2016**, *104*, 255–273. [[CrossRef](#)]
16. Li, Y.Y.; Wang, C.; Huang, W.H. Dynamics analysis of planar rigid-flexible coupling deployable solar array system with multiple revolute clearance joints. *Mech. Syst. Signal Proc.* **2019**, *117*, 188–209. [[CrossRef](#)]
17. Chen, Z.Q.; Qian, L.F.; Chen, G.S.; Nie, S.C.; Yin, Q.; Yue, C.C. Dynamics of luffing motion of a hydraulically driven shell manipulator with revolute clearance joints. *Def. Technol.* **2022**, *18*, 689–708.
18. Tan, X.; Chen, G.P.; Shao, H.B. Modeling and analysis of spatial flexible mechanical systems with a spherical clearance joint based on the LuGre friction model. *J. Comput. Nonlinear Dyn.* **2020**, *15*, 011005. [[CrossRef](#)]
19. Dong, X.Y.; Sun, Y.; Wu, X.Z.; Wang, R.D. Dynamic modeling and performance analysis of toggle-linkage presses considering mixed clearances and flexibility. *Int. J. Non-Linear Mech.* **2022**, *147*, 104243. [[CrossRef](#)]
20. Li, Y.T.; Quan, Q.Q.; Li, H.; Tang, D.W.; Li, Z.H.; Fan, W.Y.; Deng, Z.Q. Air rudder mechanism dynamics considering two elements: Joint clearance and link flexibility. *J. Mech. Sci. Technol.* **2017**, *31*, 3189–3197. [[CrossRef](#)]
21. Gonthier, Y.; Mcphee, J.; Lange, C.; Piedboeuf, J.C. A regularized contact model with asymmetric damping and dwell-time dependent friction. *Multibody Syst. Dyn.* **2004**, *11*, 209–233. [[CrossRef](#)]
22. Wang, X.P.; Liu, G.; Ma, S.J. Dynamic analysis of planar mechanical systems with clearance joints using a new nonlinear contact force model. *J. Mech. Sci. Technol.* **2016**, *30*, 1537–1545. [[CrossRef](#)]
23. Wan, Q.; Liu, G.; Song, C.Y.; Zhou, Y.; Ma, S.J.; Tong, R.T. Study on the dynamic interaction of multiple clearance joints for flap actuation system with a modified contact force model. *J. Mech. Sci. Technol.* **2020**, *34*, 2701–2713. [[CrossRef](#)]
24. Hertz, H. Über die berührung fester elastischer körper. *J. Reine Angew. Math.* **1881**, *92*, 156–171.
25. Hunt, K.H.; Crossley, F.R.E. Coefficient of restitution interpreted as damping in vibroimpact. *J. Appl. Mech.-Trans. ASME* **1975**, *42*, 440–445. [[CrossRef](#)]
26. Lankarani, H.M.; Nikravesh, P.E. A contact force model with hysteresis damping for impact analysis of multibody systems. *J. Mech. Des.* **1990**, *112*, 369–376. [[CrossRef](#)]
27. Flores, P.; Machado, M.; Silva, M.T.; Martins, J.M. On the continuous contact force models for soft materials in multibody dynamics. *Multibody Syst. Dyn.* **2011**, *25*, 357–375. [[CrossRef](#)]
28. Bai, Z.F.; Zhao, Y. A hybrid contact force model of revolute joint with clearance for planar mechanical systems. *Int. J. Non-Linear Mech.* **2013**, *48*, 15–36. [[CrossRef](#)]
29. Wang, X.P.; Liu, G.; Ma, S.J.; Tong, R.T. Study on dynamic responses of planar multibody systems with dry revolute clearance joint: Numerical and experimental approaches. *J. Sound Vibr.* **2019**, *438*, 116–138. [[CrossRef](#)]
30. Ambrósio, J. Impact of rigid and flexible multibody systems: Deformation description and contact models. *Virtual Nonlinear Multibody Syst.* **2002**, *103*, 57–81.
31. Ma, J.; Qian, L.F. Modeling and simulation of planar multibody systems considering multiple revolute clearance joints. *Nonlinear Dyn.* **2017**, *90*, 1907–1940. [[CrossRef](#)]