

Article

# Model Predictive Control Method for Autonomous Vehicles in Roundabouts <sup>†</sup>

Zsófia Farkas <sup>1,\*</sup> , András Mihály <sup>2</sup>  and Péter Gáspár <sup>1,2</sup> 

<sup>1</sup> Department of Control for Transportation and Vehicle Systems, Budapest University of Technology and Economics, H-1111 Budapest, Hungary

<sup>2</sup> Systems and Control Laboratory, Institute for Computer Science and Control, Eötvös Loránd Research Network (ELKH), H-1111 Budapest, Hungary

\* Correspondence: farkas.zsofia@edu.bme.hu

<sup>†</sup> This paper is an extended version of our paper published in Farkas, Z.; Mihály, A.; Gáspár, P. MPC Control Strategy for Autonomous Vehicles Driving in Roundabouts. In Proceedings of the 2022 30th Mediterranean Conference on Control and Automation (MED), Vouliagmeni, Greece, 28 June–1 July 2022; pp. 939–944.

**Abstract:** This paper introduces a procedure for controlling autonomous vehicles entering roundabouts. The aim of the centralized controller is to define the velocity profile of each autonomous vehicle by which collisions can be avoided and traveling times can be minimized. To achieve these performances, a model predictive control is introduced based on the solution of an analytical calculation of traveling times spent in the roundabout and designing the autonomous vehicles' velocity profiles in order to avoid conflict situations while ensuring a time-optimal solution. By the application of the proposed procedure, safety of autonomous vehicles can be enhanced and the possibility of a forming congestion can be minimized. The operation of the proposed method is demonstrated by a few simulation examples in the CarSim simulation environment.

**Keywords:** roundabout control; autonomous vehicle control; connected autonomous vehicles



**Citation:** Farkas, Z.; Mihály, A.; Gáspár, P. Model Predictive Control Method for Autonomous Vehicles in Roundabouts. *Machines* **2023**, *11*, 75. <https://doi.org/10.3390/machines11010075>

Academic Editors: Kimon P. Valavanis, Maria Prandini, Andrea Monteriù and Alessandro Vittorio Papadopoulos

Received: 5 December 2022

Revised: 28 December 2022

Accepted: 4 January 2023

Published: 6 January 2023



**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

## 1. Introduction

### 1.1. Introduction

Autonomous vehicles (AVs) are becoming a new member of the road infrastructure. Both academia and private sectors are focusing on the development of AVs or their subsystems. With the increasing number of autonomous vehicles on the road, several problems occur, and they must be analyzed and considered. One of the key problems is the roundabout control of the AVs, and this study focuses on this problem. The six levels of automation are defined by the Society of Automobile Engineers (SAE) [1]. In Level 0, the vehicle does not have any automation features, and the driver has to perform the driving operation. For Level 1, driving assistance features assist the driving operation with lateral and longitudinal control, while the complete operation cannot be performed by the vehicle and the driver is necessary. Compared to Level 1, Level 2 has additional assistant features, and it is partially automated. The vehicle performs the combined automated functions, such as acceleration and steering, while the driver must be in the vehicle. Level 3 is conditional automation, and all driving features can be performed by the vehicle in certain conditions, while in a critical situation, the vehicle is informed and takes the driving operation from the vehicle. The driver is needed in Level 3, too. Level 4 and Level 5 do not require a driver. In Level 4, drivers can intervene if they want. The final level is Level 5, and it is full automation in all conditions.

The roundabout is a circular intersection or junction type in which traffic flows continuously in a direction around a central island. There are several types of roundabouts, depending on the country's standards. In the highways, circular intersections can have 2–6 lanes around the center, while they are usually small in urban environments [2]. Roundabouts are used to improve traffic safety on road networks in suburban and urban areas

due to their better performance proof regarding vehicle safety than the standard alt-grade intersections [3]. The reviews from 28 studies from different countries [4] depict that roundabouts reduce the number of injury accidents by 30% to 50% and fatal accidents are reduced by 50 to 70%. A similar study was carried out for European countries [5] and study has also been performed for different countries [6].

Roundabouts also eliminate the traffic signal control hardware and their electrical and maintenance costs; thus, they reduce operational costs. Therewith, they improve traffic operations compared to stopped-control or signalized intersections [7].

Note that the present paper addresses traffic scenarios in which fully automated vehicles are present with V2X communication ability [8], while conventional human-driven vehicles are neglected. Thus, mixed-traffic situations [9,10] are not considered, as an assumption is made for fully automated traffic. The present paper deals with a model predictive control (MPC) strategy based on an analytical calculation of traveling time and a velocity profile design that is proven to be efficient to accomplish safe driving of AVs [11]. Several constraints, such as speed regulations, acceleration limits, and maximal cornering velocities, are built in the control design to ensure safe operation. The main novelty of the proposed control strategy is that it guarantees safety of the AVs and an optimal travel time with a procedure which can be implemented in real-life applications due to its analytical calculation. Moreover, there are several advantages of the proposed method over other strategies found in the literature. Data-driven methods have to generate plenty of possible scenarios defined by the AVs position, speed, and acceleration. Hence, small variations in the AVs parameter space may lead to big changes in the accelerations of AVs, which can result in instability in the presence of sensor noises and uncertainties. Conventional MPC controllers are better in this aspect, but implementation can be problematic due to the high computational power required. The proposed strategy evaluates a simplified optimization method, resulting in sufficiently fast calculation for real-time applications. The contributions of the paper over [11] are a more extensive overview of already existing methods in the literature, a more detailed description of the MPC control process and trajectory tracking, while a new simulation is evaluated to demonstrate the effectiveness of the proposed method.

The paper is organized as follows: the motivation and problem description is defined in Section 2. Section 3 describes the proposed model predictive control method with the safety considerations. The operation of the proposed MPC strategy is presented through a CarSim simulation example in Section 4. The examination of the simulation results and the limitations of the proposed method along with the open research questions are detailed in Section 5. Finally, concluding remarks are given in Section 6.

## 1.2. Related Works

The public appearance of highly automated vehicles in traffic raises several safety challenges due to degree of social acceptance. The academic research of AVs in traffic scenarios, such as intersections, roundabouts, or on-ramps, poses the focus on the control design methods, motion control, and safety solutions for AVs.

There are several studies on the effect of AVs on traffic flow safety and stability. Cooperative adaptive cruise control systems are efficient in dampening oscillations and providing a stable flow [12]. The study [13] presents that the string stability of traffic flow is improved with AVs. The paper [14] shows that AVs improve traffic conditions, where the travel time and collision are reduced.

Centralized control methods are commonly used for the scenario of connected AVs on-ramp merging, for example, formulated as a biobjective optimization problem solved with Pontryagin's minimum principle [15]. A comprehensive review is given of the existing ramp merging strategies leveraging connected AVs, focusing on the latest developments in the field [16].

Plenty of control strategies have been designed for the collision-free driving of autonomous vehicles in mixed-traffic situations (with human participants and AVs), for

example, a trajectory tracking control algorithm based on the state estimation of vehicles in order to achieve the collision-free crossing of vehicles at roundabouts [10]. Several safety conditions are built in the designed methods for AVs to pass through the roundabout conflict areas, such as merging points. Control strategies have also been developed to guarantee safe navigation of AVs in one- and multilane roundabouts as well [17]. A vehicle-to-vehicle (V2V) communication and intersection control was proposed for AVs to avoid collisions in complex traffic scenarios, and the designed intersection protocols were also tested in roundabout situations. An optimization framework and an analytical solution that allows optimal coordination of vehicles at roundabouts was developed in a mixed-traffic environment, and the effect of penetration rates of connected and automated vehicles (CAVs) were analyzed [9].

The main focus in the analysis of AVs in traffic systems is related to the behavior, intention, and motion of vehicles. The scenarios of roundabout crossing provide the opportunity to develop control methods for safe traffic. The behavior of human drivers can be analyzed to utilize the results in the design of coordination of AVs in roundabouts [18]. Based on driving data, a numerical optimization was performed for the minimization of travel time and comfort through motion planning and design of velocity profiles. The driving risks in roundabouts were also analyzed in order to apply the driving behavior for AVs, by which passenger comfort and traffic safety can be improved [19]. A machine learning model was also used and trained for the determination of the safe motion and possible exits of vehicles. An optimal control method was also developed with the aim to minimize travel time and increase energy efficiency, considering the constraints of collision avoidance in crossing roundabouts [20]. As a different approach, virtual platooning method for AVs handling complex traffic situations in roundabouts can be applied [21]. This approach combines the map-based concept with curvilinear coordinates framework to guarantee safe traffic between AVs and human-driven participants. As a contribution to the application of AVs in traffic systems, classification methods have also been designed. Based on dynamic Bayesian network, the classification identifies intentions of vehicles driving in a roundabout [22]. Decentralized coordination framework was also designed with virtual vehicles, in order to map states and interactions of AVs [23]. The control method contributes in creating balance between waiting times and velocities of the vehicles when passing through the roundabout. An adaptive tactical behavior planner was also developed, combining human behavior and tactical decision-making, in order to control AVs in roundabouts [24].

For the coordination of AVs in roundabouts, several researchers have designed control strategies including artificial intelligence (AI) approaches and models for the purpose of safe traffic. Support vector machine, linear regression, and deep learning algorithms have been compared in predicting vehicle speed and steering angle at different geometry roundabouts for drivers, and rules of action to be used have been generated for autonomous vehicles to perform roundabout maneuvers [25]. Learning methods have been applied for autonomous driving in different urban traffic scenarios as well [26,27]. Firstly, a control framework was designed combining state-of-the-art model-free reinforcement learning algorithms to replace complex manual designs for the crossing of roundabout by AVs. On the other hand, a learning framework was proposed, including safety control conditions to establish driving strategies for AVs crossing roundabouts. A roundabout scenario to develop an optimization embedded reinforcement learning in order to coordinate highly automated vehicles in these scenarios was also designed [28]. The control method analyzes the behaviors and decision-making of vehicles for the comparison of efficiency of the designed algorithm. Algorithms for motion prediction of vehicles are also used, combining dynamic Bayesian network and sequential neural network models in the framework [29]. Moreover, the adversarial multiagent reinforcement learning method is applied to coordinate the crossing of roundabouts by AVs by considering behaviors, e.g., human-driving baseline [30]. This method improves the performances of traveling time and average speed of the vehicles. A fuzzy-behavior-based algorithm for roundabout coordination was also designed to

calculate speed profiles for different vehicles, in order to achieve more comfortable driving profiles, as well to reduce congestion [31].

Several methods presented in the literature utilize game theory approaches to model the behavior and decision-making of autonomous vehicles at roundabouts, for example, an algorithm based on a game-theoretic model representing the interactions between the ego vehicle and an opponent vehicle [32], where online-estimated driver type of the opponent vehicle was also considered. The Prisoner's Dilemma game strategy [33] can also be chosen as a method for AVs decision-making, demonstrating that the roundabout entry problem can be handled optimally with reduced waiting times for AVs.

There are several papers that use the MPC method to control autonomous vehicles at roundabouts. The study [34] presented a controller for trajectory tracking control at the roundabout. The reference path is given and the decision layer, the MPC tracking controller, is used to test the effect of weight parameters and target speed on the performance of the tracking controller. The paper [35] proposed a method in order to solve the roundabout merging problem by considering a nominal trajectory generated through Bezier curves combined with the MPC method.

## 2. Problem Statement

Control methods connected to the application of AVs in traffic environments have become the focus of research interest. Traffic situations signify complex tasks in the design of coordination of AVs. Control solutions for the AVs to accomplish safe driving in intersections has been developed in recent years. The field of roundabouts is also experiencing increased relevance related to highly automated vehicles. Motion and trajectory planning, collision avoidance, energy efficiency, and passenger comfort are the most significant areas for analysis to adopt AVs in intelligent traffic systems. A simple roundabout scenario with four entrance/exit connections provides several conflict (collision) points between AVs, and a control algorithm needs to be designed to guarantee various performances considering the built-in control constraints. Nevertheless, it is necessary to develop control structures for the coordination of vehicles in complex (e.g., multilane) roundabouts to solve traffic situations by obvious, secure application of AVs.

The aim of the present paper is to introduce a novel model predictive control (MPC) method, by which the automated vehicles entering the intersection can be handled considering time-optimal and safety critical performances at the same time. The designed model predictive controller evaluates the calculation of AVs desired acceleration at each time step, assuming that velocity and position data are sent to the centralized controller. The architecture in the control strategy of the AVs requires enhanced infrastructure, e.g., V2V and V2I communications [36]. Autonomous vehicles are equipped with a several sensors which are reliable and accurate such as camera, GPS, light detection and ranging (LiDAR), and vehicle-to-everything (V2X) systems. These technologies allow AVs to gain necessary data for the exact velocity and position, communicate these signals to the centralized controller, and receive the necessary control signals. For AVs crossing the roundabout, the following performances are defined:

- Safety criteria: automated vehicles may be in an accident-prone situation when approaching the roundabout. Accordingly, the purpose of the roundabout controller is to ensure that incoming AVs enter the roundabout simultaneously, so that any collision can be eliminated.
- Traveling time and efficiency criteria: depending on the geometrical parameters of the roundabout and the road surface friction, the automated vehicles try to drive into the roundabout at the required maximum speed.

Operation of MPC Controller:

- Based on the initial vehicle entry data, the centralized model predictive controller defines the entering and exiting times of AVs entering the roundabout at each time step, assuming AVs are accelerating to the required maximum speed and decelerating with a constant value.

- Based on the comparison of the entry times obtained as a result of the above calculation, the latest entry vehicle is selected, the acceleration of which thus remains unchanged on the basis of the above calculation. Acceleration of additional vehicles will be reduced until their entry time is the same as the latest entry vehicle time.

### 3. Roundabout Control Method for Autonomous Vehicles

#### 3.1. Roundabout Scenario

In the introduced roundabout scenario, AVs can turn left by traveling three quarters in the roundabout, can turn right by traveling one quarter, head on straight by traveling two quarters, and turn back on the road by traveling four quarters in the roundabout. Hence, the possibility of collision may arise depending on the vehicle velocities and initial positions and their turning intentions. All vehicle data (position, velocity, driving intentions) are transmitted to the centralized controller, which calculates the optimal control input for all AVs. The purpose of the design is to ensure collision-free passage for the vehicles entering the roundabout with minimal traveling times, enhancing the safety and minimizing the risk of a forming congestion.

The introduced MPC method for roundabout control is founded on several preassumptions. A four-directional roundabout depicted in Figure 1 is considered, which is divided into dedicated sections.

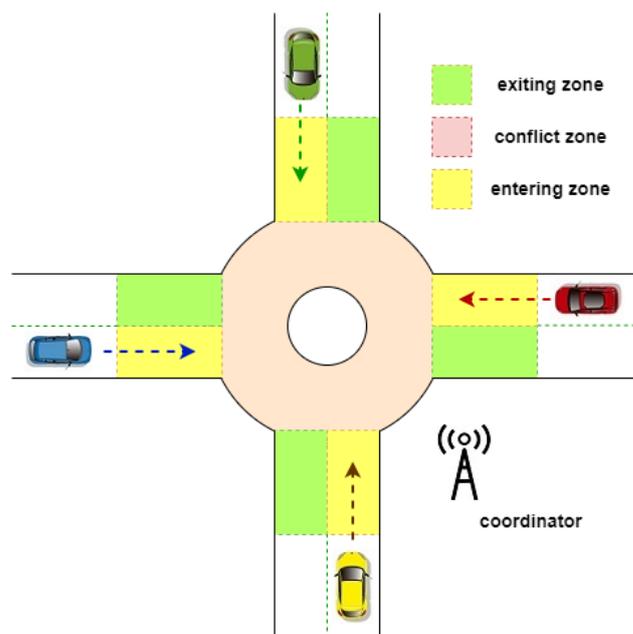


Figure 1. Roundabout scenario with the relevant zones for the control design.

Before entering the control zone, AVs are controlled individually by their own self-driving systems. When reaching the entering zone, the coordinator calculates an optimal velocity profile for the vehicle based on its distance from the roundabout center, its initial velocity, and planned trajectory in the roundabout, along with the same data acquired from the other AVs. Note that all communication between the vehicles entering the roundabout and the coordinator are established using V2I communication methods. The presented iterative calculation algorithm only considers four entering vehicles at the same time; in the case that a new vehicle enters the control zone, the calculation is repeated with new initial conditions when the preceding vehicle exits the roundabout. In this way, the introduced algorithm can be extended to deal with bigger traffic as well.

#### 3.2. Constraints for the Control Design

A safe cornering speed is specified for all AVs based on the geometry of the intersection and simplified vehicle dynamics for the cornering. Assuming that vehicle mass

$m$  is known for each AV and the side friction between the tire and the road surface  $\mu$  is estimated [37–39], we counterbalance the centrifugal force affecting the vehicle. Considering similar  $\mu$  friction at every wheel of the vehicle, the sum of the lateral forces is  $\sum F_y = mg\mu$ , where  $g = 9.81 \text{ m/s}^2$  is the gravitational constant. During the cornering in the roundabout, the dynamics of the vehicle is defined by the equilibrium of the two forces:

$$m \frac{v^2}{R} = mg\mu, \quad (1)$$

where  $R$  is the radius of the roundabout. Assuming that the geometry of the roundabout is known for the AVs by onboard devices such as GPS, a safe cornering velocity can be given by rearranging (1). Hence, the maximal safe velocity for the AVs in the given roundabout is calculated as follows:

$$v_{lim} = \sqrt{Rg\mu} \quad (2)$$

For a typical roundabout of 12.5 m radius with side friction of  $\mu = 0.8$ , the maximal safe cornering speed is  $v_{lim} \approx 35 \text{ km/h}$ . Moreover, minimal and maximal acceleration values are also defined to guarantee passenger comfort and to avoid wheel slip. In the paper, the thresholds  $a_{max} = 2.5 \text{ m/s}^2$  and  $a_{min} = -5 \text{ m/s}^2$  are selected [40].

### 3.3. Time-Optimal Roundabout Control Design

One of the performances of the proposed control design is to minimize the total traveling time of the vehicles  $T_{total}$  in order to avoid a congestion in the roundabout. For this reason, the coordinator prescribes acceleration values for the AVs with the intention to reach the highest possible speed. Thus, the control algorithm first calculates a constant acceleration  $a_{min} < a_i < a_{max} \ i \in [1 \dots n]$  for AVs reaching the roundabout entering zone, in order to achieve  $v_{lim}$  at the conflict zone:

$$a_i = \frac{v_{i,lim}^2 - v_{i,0}^2}{2s_{i,ent}} \quad (3)$$

where  $v_{i,0}$  is the beginning velocity, and  $s_{i,ent}$  is the primary distance of the AVs from the roundabout center.

In connection with the limits defined for acceleration,  $a_i = \{a_{max}; a_{min}\}$  replaces the acceleration values in (3) in case the limits are violated. Hence, the maximum velocity  $v_{lim}$  in the conflict zone is altered as given:

$$v_{i,max} = \sqrt{v_{i,0}^2 + 2a_i s_{i,ent}} \quad (4)$$

The coordination method is based on the comparison of the AVs traveling times. Hence, the goal is to prescribe an acceleration for each vehicle, by which the conflict zone can be reached at the same time in order to avoid a possible collision. For this purpose, after the calculation of maximal velocities and the corresponding accelerations defined by (3) and (4), the entering end exit times of each vehicle are calculated.

The entry time of each autonomous vehicle is defined by solving the next second-order equation:

$$\frac{1}{2}a_{i,0}t_{i,ent}^2 + v_{i,0}t_{i,ent} - s_{i,ent} = 0 \quad (5)$$

where  $t_{i,ent} \geq 0 \ i \in [1 \dots n]$  is the entry time.

In order to simplify the calculation of the entering times, it is assumed that vehicles select a constant acceleration when approaching the roundabout. Hence, using constant accelerations in the entering zone, (5) can be reduced as:

$$t_{i,ent} = \frac{s_{i,ent}}{(v_{i,max} + v_{i,0})/2} \quad (6)$$

The traveling time in the roundabout conflict zone is defined by assuming a fixed speed for the vehicles. This assumption is necessary, since accelerating in curves can induce unwanted dynamical issues which may affect the trajectory tracking capability of the vehicle. Selecting  $a_i = 0$  in the conflict zone, the traveling time is calculated as:

$$t_{i,con} = s_{i,con}/v_{i,max} \quad (7)$$

where  $s_{i,con}$  is the trajectory length in the roundabout, which depends on the turning intention of the vehicle assumed to be known by the centralized controller. Note that, based on the turning intention of each AV, the trajectory length  $s_{i,con}$  can be approximated as one, two, three, or four quadrant of the roundabout. Hence, by knowing the physical parameters of the roundabout,  $s_{i,con}$  can be directly derived from the turning intention given by the AV.

Finally, the travel time of each AV in the entering and control zone until exiting the roundabout is given as follows:

$$t_{i,fin} = t_{i,ent} + t_{i,con} \quad (8)$$

The computation is evaluated in an iterative manner as follows:

- First, the maximum vehicle speed  $v_{i,max}$   $i \in [1 \dots n]$  is defined for each vehicle, based on initial velocity and position, and the adhesion of the road and the roundabout geometry, along with the predefined acceleration limits. Corresponding acceleration values  $a_i$   $i \in [1 \dots n]$  are calculated for each vehicle.
- Next, entry time in the conflict zone  $t_{i,ent}$  and exit time  $t_{i,fin} \in [1 \dots n]$  are calculated for all vehicles using (6) and (8).
- The vehicle having the maximal entry time  $t_{max} = \max(t_{i,ent})$  is selected as benchmark, while the acceleration values of other AVs are decreased iteratively; their entry time given in (6) becomes equal to the maximal entry time, i.e.,  $t_{i,ent} = t_{max} \forall i \in [1 \dots n]$ .
- Lastly, in the case that additional vehicles approach the roundabout and the AVs inside the conflict zone exit, the procedure is repeated with new initial conditions for all vehicles. In the case that the conflict zone is still employed by AVs, the entry times of the new entering vehicles are set with the following constraint considered:  $t_{i,ent}^{new} \geq t_{i,fin}^{old} \forall i \in [1 \dots n]$ . Hence, the newly entered AVs might decrease their velocities in order not to conflict with the last AV exiting the roundabout.

The operation of the MPC roundabout control is depicted in Figure 2. The procedure is as follows: each autonomous vehicle reaching the roundabout transmits their turning intention  $d_i$ , along with their initial position and velocity  $s_{i,ent}(k)$ ,  $v_{i,ent}(k)$   $i \in [1 \dots n]$ , to the coordinator of the roundabout at a discrete time step  $k$  using a sampling time  $T_s$ . The time horizon of the optimization process is  $T = \max(t_{i,fin})$   $i \in [1 \dots n]$ . As the oncoming vehicles join the MPC optimization procedure only if all the former AVs leave the roundabout, a maximum of four AVs are coordinated simultaneously. The outcome of the control process are the input variables  $a_i(k+1)$   $i \in [1 \dots n]$  for the vehicles to follow before the consequent time step, when the above process is repeated with a forward-shifted horizon. Since prescribed accelerations are redefined at every time step for AVs inside the roundabout, a sampling time  $T_s = 0.1$  s is selected. It is also necessary to deal with unreliable communication links [41]. A robust management method can be used for this purpose based on position tracking, compensating the effects of model mismatch and disturbances [42]. Note that the introduced MPC method is inherently robust against

bounded initial errors in the measurement signals as data become more punctual when vehicles become closer to the origin of the roundabout.

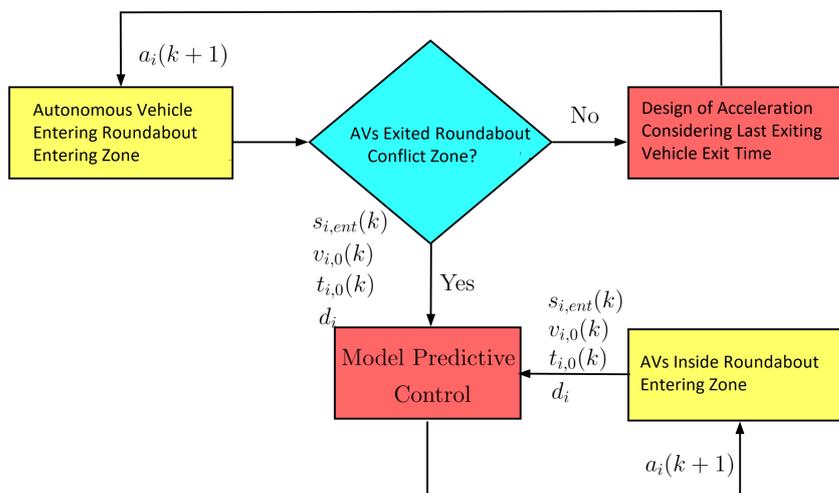


Figure 2. Roundabout control procedure.

### 3.4. Trajectory Tracking Control

The acceleration of the vehicles  $a_i \ i \in [1 \dots n]$  given with the proposed method is evaluated using a longitudinal drive force, utilizing the propulsion and brake system of the AVs. Hence, the following formula is applied to realize the necessary drive/brake forces for the vehicles:

$$F_{i,l} = m_i a_i + F_{i,d} \tag{9}$$

where  $m_i \ i \in [1 \dots n]$  stands for the vehicle mass,  $F_{i,l}$  denotes the control input, and  $F_{i,d}$  consists of the disturbances affecting the longitudinal dynamics such as aerodynamic drag, rolling resistance, and disturbance forces from the road slope [43].

The vehicle’s path-following control inside the roundabout is founded on the simplified bicycle model of the vehicle [43,44]. The differential equations in the planar plane are defined with the following equations:

$$\begin{aligned} \dot{X} &= v \cos(\psi) \\ \dot{Y} &= v \sin(\psi) \\ \delta &\cong \tan(\delta) = \frac{L}{R} \end{aligned} \tag{10}$$

where  $v$  is the speed of the vehicle,  $\psi$  denotes the yaw angle, and  $X$  and  $Y$  are coordinates of the AV in a global coordinate system. A simplified model is applied for the steering, where  $\delta$  stands for the steering angle and  $L$  is the wheelbase of the AV, while  $R$  is the curvature radius, as illustrated in Figure 3. Then, by applying the relationship  $\dot{\psi} = \frac{v}{R}$ , the motion equations of the AV are formulated as:

$$\begin{bmatrix} \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{L} \end{bmatrix} \tag{11}$$

Considering lateral velocity and alteration in the vehicle reference signals to be small ( $\dot{y}_{ref} \cong 0; \dot{\psi}_{ref} \cong 0$ ), Equation (11) is then rearranged as follows:

$$\dot{x} = Ax + Bu \tag{12}$$

where the state vector  $x = [e_\gamma \ e_\psi]^T$  contains the lateral position and yaw angle errors and  $u = \delta$  stands for the steer input. The aim of the design is to minimize the value of the signals given in  $x$  vector. Hence, an LQ controller is defined with the next cost function:

$$J = \int_0^\infty [x(t)^T Q(t)x(t) + u(t)^T r u(t)] dt \quad (13)$$

where  $Q$  and  $r$  are parameters of the control design, scaling the performances and the control input.

By solving the Ricatti equation, the feedback gain  $K$  is designed, which gives the steer input  $\delta$  for the AVs.

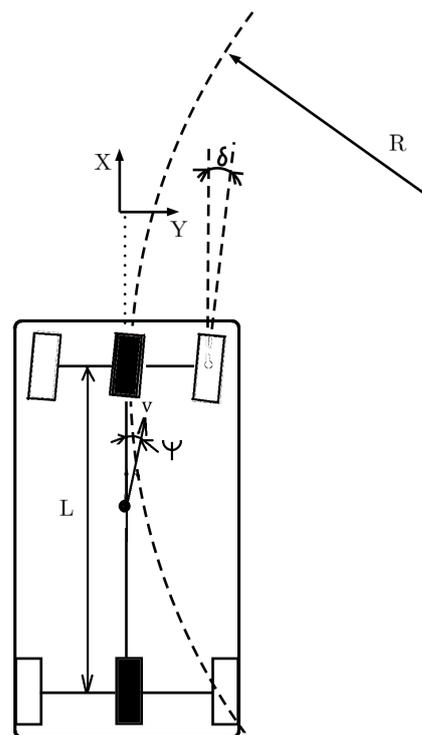


Figure 3. Two-wheeled bicycle model.

#### 4. Simulation Example

The operation of the proposed MPC control for AVs entering roundabouts is demonstrated with a simulation performed in CarSim environment using real geometry data of a typical roundabout. In the scenario, four autonomous vehicles arrive at the roundabout whose accelerations are determined by the centralized controller and the control force implemented by the vehicle model detailed in Section 3.4, as well as the trajectory tracking by the designed LQ steering control. The operation of the proposed MPC strategy in the CarSim simulation environment is illustrated in Figure 4.

The selected roundabout has a radius of 12.5 m and the coefficient of adhesion of  $\mu = 0.8$ , while the simulated vehicles have a total mass of 1600 kg, with a maximum acceleration of  $2.5 \text{ m/s}^2$  and a maximum deceleration of  $-5 \text{ m/s}^2$  (see [40]). During the simulation, AVs enter the roundabout from different distances and velocities: *Vehicle 1* (green) arrives from 66.9 m at 20 km/h and heads straight, i.e., it travels two quarters of the roundabout; *Vehicle 2* (red) arrives from 54.4 m at 30 km/h and turns left, i.e., it travels three quarters of the roundabout; *Vehicle 3* (yellow) arrives from 71 m at 40 km/h and turns right, traveling only one quarter of the roundabout; and *Vehicle 4* (blue) arrives from 59 m at 50 km/h and turns back on the road, i.e., it travels four quarters of the roundabout. The simulated vehicles intervene at all times based on the acceleration signal received from the MPC controller, while the lane tracking is implemented by a lateral steering controller.

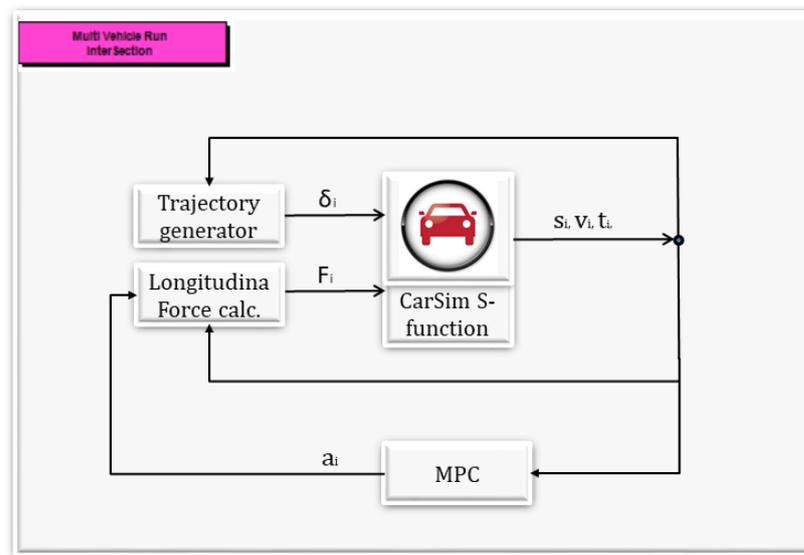


Figure 4. Real-time MPC control in CarSim environment.

The result of the simulation performed with the above parameters with the time-optimal intervention is shown in Figure 5a, where the collision avoidance criterion is not implemented. As can be seen, vehicles would have entered the roundabout at different times, which could have triggered a collision. Applying the condition of simultaneous entering depicted in Figure 5b, the entry and exit times for *Vehicles 2, 3, and 4* change in order to adapt to *Vehicle 1's* entry time. It is well demonstrated that the simultaneous entry of vehicles is taking place; thus, a possible accident has been eliminated.

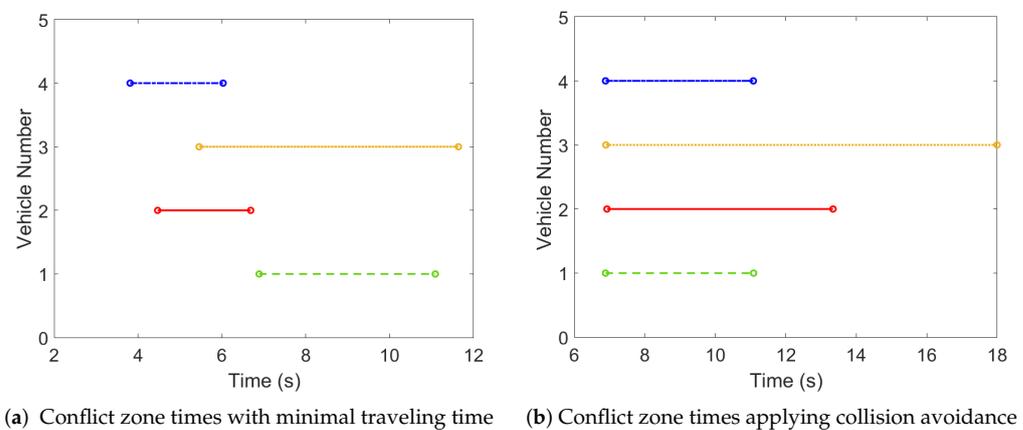
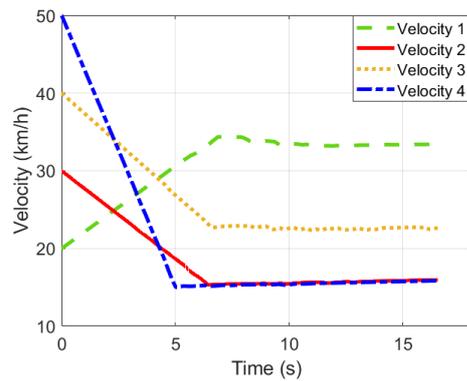


Figure 5. Traveling time of AVS inside the roundabout.

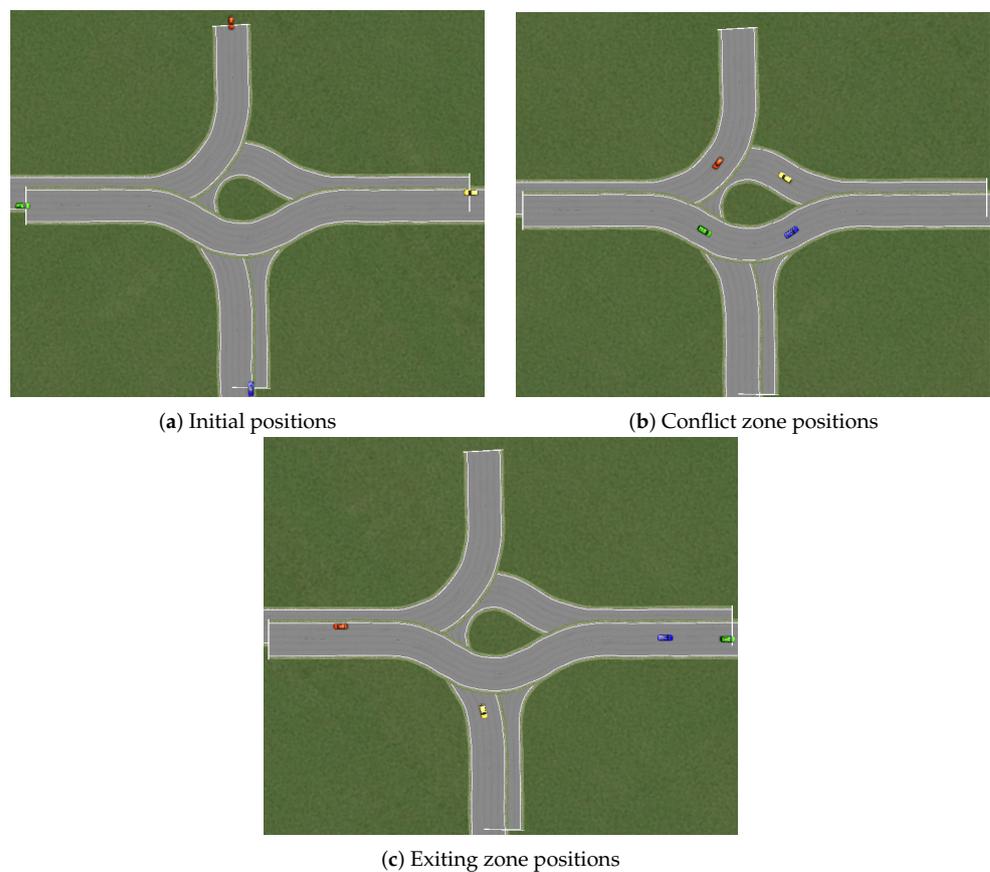
The velocity profiles of the AVs are shown in Figure 6. Note that in order to achieve the simultaneous entry in the conflict zone, *Vehicles 2, 3, and 4* have to decelerate, while *Vehicle 1* has to accelerate slightly.

The illustration of the simulation is given in Figure 7, showing the initial positions of the vehicle and the positions when traveling inside the roundabout.

Note that the proposed method can be implemented in real vehicles. These vehicles must be equipped with necessary autonomy functions, such as cruise control, steering control, and V2X communication systems. The proposed control method is feasible with the real-time application, while the controller can be run with hardware such as SpeedGoat or dSPACE. This implementation is depicted in Figure 8.



**Figure 6.** Velocities of AVs.



**Figure 7.** AVs motion in the roundabout using MPC control.

Next, in order to compare the results of the MPC roundabout controller, an offline simulation-based algorithm was designed in order to find acceleration values for AVs, by which the traveling time can be minimized. The constrained optimization designed in MATLAB and CarSim cosimulation is depicted in Figure 9. Note that in order to compare results with the proposed MPC method, the simulation was evaluated with similar initial conditions. The time-optimal optimization is set up as follows:

- Upper and lower bounds for the acceleration of AVs are given based on the predefined minimal and maximal acceleration values and the geometry of the roundabout. The latter defines the maximal velocities for the AVs, by which minimal and maximal accelerations are calculated, which guarantees that safe cornering velocities are not violated.
- The multivehicle simulation in CarSim was built with the same initial conditions described earlier. Note that the acceleration values  $a_i \in [1 \dots 4]$  are used as inputs for

the simulated vehicles. The iteratively running algorithm aims to find the acceleration values  $a_i$  for AVs, by which the traveling time, defined as the last vehicle exiting time from the roundabout, can be minimized.

- In order to ensure collision avoidance, a 3 meter intervehicular distance among AVs is given as a constraint during the simulation. In practice, a large value is added to the measured simulation time in CarSim; hence, the optimization algorithm discards the result given by the actual input values.
- The constrained optimization is evaluated iteratively while it finds acceleration values for AVs, by which a local minimum for the traveling time is reached.

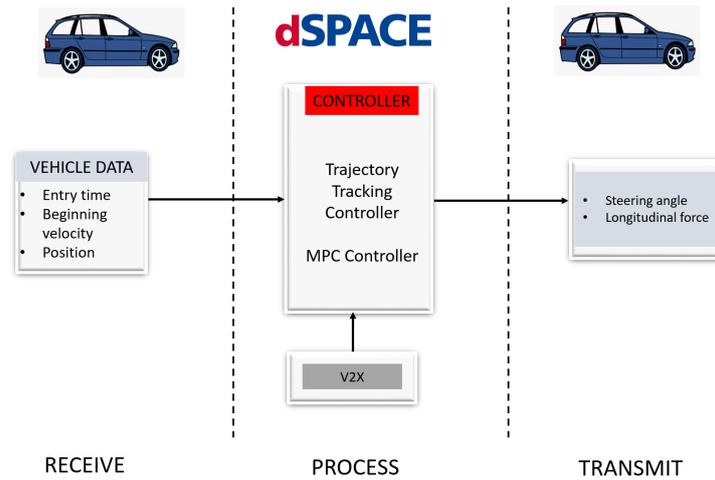


Figure 8. Real-time implementation.

The velocity profiles of the AVs given by the time-optimal optimization are depicted in Figure 10. It is well demonstrated that in this simulation case, each AV aims to reach the maximum possible velocity given by the geometry of the roundabout. Hence, *Vehicles 1 and 2* accelerate to reach this safe velocity in the roundabout, while *Vehicles 3 and 4* have to decelerate to achieve this velocity constraint. Note that total traveling time is only decreased slightly compared to the proposed MPC method.

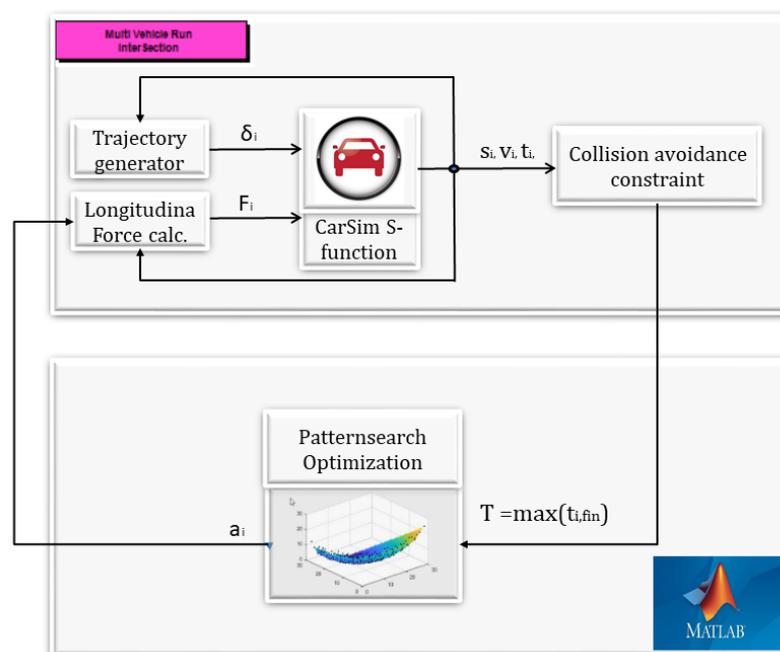


Figure 9. Offline time-optimal optimization in CarSim environment.

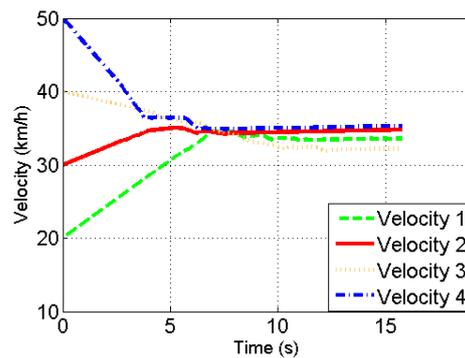


Figure 10. Velocities of AVs with time-optimal control.

## 5. Discussion

There are both open ethical and practical questions in the area of autonomous driving. The paper [45] defined the research questions in order to provide an overview of autonomous driving technology. The study answered the desirability of the AVs as the survey analyzed the acceptance of AVs and found that people are open to technology that outperforms human ability. Another important question regarding dealing with autonomous and nonautonomous vehicles and the unpredictability of human driving is answered by the enforcement of strict traffic laws. Moreover, the emergence of autonomous driving also opens questions which are related to several other fields in transportation and society as well. The appearance of AVs has a serious influence on several socially important areas, such as the improvement of air quality, the development of traffic safety, and corresponding health consequences. Developments of AVs have the greatest impact on the transformation of the transport infrastructure. The integration of automated vehicle technologies into the road infrastructure contributes significantly to the development of the intelligent city (see [46]). Travel needs will also be changed by the use of AVs becoming accessible to people who have been excluded from road transport, such as people with disabilities, the elderly, or young people, which may increase the traffic load [47]. This travel need, along with the need for parking infrastructure, may also change as carsharing services become more popular. The environmental load may increase despite energy efficiency, as the length and number of trips may increase due to convenience features. During the development of AVs, it is recommended to take into account the reduction of light pollution as well [48].

The present paper discussed the possibility of an effective roundabout controller for AVs, by which safety can be guaranteed. For the sake of simplification of the complex problem, it was assumed that every vehicle entering the roundabout is autonomous; thus, mixed-traffic situations were not considered. However, when assuming human-driven vehicles with appropriate sensors, the integration in the proposed method is possible. Several simulations were performed with different initial conditions for AVs, and a simulation was selected for demonstration, detailed in Section 4. The results showed the effectiveness of the proposed method, ensuring safety and resulting in a total traveling time close to the time optimal solution given by an offline optimization procedure performed with similar initial conditions. It is important to state that the major advantage of the proposed method over other solutions given in the literature review is that it does not require big computational power, which is relevant in real-time applications.

There are several open research questions, both practical and ethical, regarding autonomous driving. Several studies discussed these problems, especially in the view of roundabout control. The present research has the limitation of not considering mixed-traffic situations with human drivers involved. In addition, the effects of sensor faults and time-delays have to be studied in future work, given the crucial importance of punctual position and velocity data in the presented method. Note that the presented MPC method provides some kind of inherent stability over data loss and sensor faults, as the calculation algorithm

repeats the optimization at every time step; however, these questions and their potential answer should also be considered in future works.

## 6. Conclusions

The coordination of AVs at roundabouts is affected by multiple factors. In the presented MPC design, the main goals are the reduction of traveling times and guaranteeing safety of the AVs in the roundabout by avoiding collisions and skidding of the vehicles. This paper proposed a centralized MPC algorithm for a roundabout coordinator by which these performances can be fulfilled for the AVs. The operation of the presented method was validated by multiple simulations performed in the CarSim environment. It was shown that the designed centralized controller is able to handle AVs approaching the roundabout in a manner which ensures a safe passage for all vehicles. Future work should consider traffic scenarios with more lanes in the roundabout, mixed traffic with human-driven vehicles, or priority vehicles.

**Author Contributions:** Conceptualization, methodology, validation: Z.F.; conceptualization, methodology, software: A.M.; supervision: P.G. All authors have read and agreed to the published version of the manuscript.

**Funding:** The research was supported by the Ministry of Innovation and Technology NRD Office within the framework of the Autonomous Systems National Laboratory Program.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Glossary

$\mu$	Road surface
$g$	Gravity constant
$R$	Radius of roundabout
$m$	Mass
$v$	Velocity of the vehicle
$v_{lim}$	Maximal safe velocity
$v_{i,0}$	Beginning velocity
$a$	Acceleration
$s$	Distance
$s_{i,ent}$	Primary distance of AVs from the roundabout center
$t_{i,ent}$	Entry time of each AV
$t_{i,con}$	Traveling time
$t_{i,fin}$	Travel time of each AV in the entering and control zone
$i$	ID of the vehicle
$t_{max}$	Maximum entry time
$T$	Time horizon
$T_s$	Sampling time
$\psi$	Yaw angle
$X$ and $Y$	Coordinates of AV
$L$	Wheelbase of AV
$F_{i,l}$	Control input
$F_{i,d}$	Disturbances affecting the longitudinal dynamics
$e$	Error
$J$	Const function
$Q$ and $r$	Parameters of the control design
$\delta$	Steer input
$K$	Feedback gain

## References

1. SAE International. *Taxonomy and Definitions for Terms Related to Driving Automation Systems for On-Road Motor Vehicles*; SAE International: Warrendale, PA, USA, 2018. Available online: [https://www.sae.org/standards/content/j3016\\_202104/](https://www.sae.org/standards/content/j3016_202104/) (accessed on 3 January 2023).
2. Rastelli, J.P.; Peñas, M.S. Fuzzy logic steering control of autonomous vehicles inside roundabouts. *Appl. Soft Comput.* **2015**, *35*, 662–669. [\[CrossRef\]](#)
3. Deluka Tibljaš, A.; Giuffrè, T.; Surdonja, S.; Trubia, S. Introduction of Autonomous Vehicles: Roundabouts design and safety performance evaluation. *Sustainability* **2018**, *10*, 1060. [\[CrossRef\]](#)
4. Elvik, R. Effects on road safety of converting intersections to roundabouts: review of evidence from non-US studies. *Transp. Res. Rec.* **2003**, *1847*, 1–10. [\[CrossRef\]](#)
5. Ambros, J.; Novák, J.; Borsos, A.; Hóz, E.; Kieć, M.; Machcíník, Š.; Ondrejka, R. Central European comparative study of traffic safety on roundabouts. *Transp. Res. Procedia* **2016**, *14*, 4200–4208. [\[CrossRef\]](#)
6. Rodegerdts, L.; Blogg, M.; Wemple, E.; Myers, E.; Kyte, M.; Dixon, M.P.; List, G.; Flannery, A.; Troutbeck, R.; Brilon, W.; et al. *Appendixes to NCHRP Report 572: Roundabouts in the United States*; Technical Report; Transportation Research Board: Washington, DC, USA, 2007.
7. Mohebifard, R.; Hajbabaie, A. Connected automated vehicle control in single lane roundabouts. *Transp. Res. Part Emerg. Technol.* **2021**, *131*, 103308. [\[CrossRef\]](#)
8. Shi, Y.; Pan, Y.; Zhang, Z.; Li, Y.; Xiao, Y. A 5G-V2X Based Collaborative Motion Planning for Autonomous Industrial Vehicles at Road Intersections. In Proceedings of the 2018 IEEE International Conference on Systems, Man, and Cybernetics (SMC), Miyazaki, Japan, 7–10 October 2018; pp. 3744–3748. [\[CrossRef\]](#)
9. Zhao, L.; Malikopoulos, A.; Rios-Torres, J. Optimal Control of Connected and Automated Vehicles at Roundabouts: An Investigation in a Mixed-Traffic Environment. *IFAC-PapersOnLine* **2017**, *51*, 73–78. [\[CrossRef\]](#)
10. Wang, L.; Huang, W.; Liu, X.; Tian, Y. Vehicle collision avoidance algorithm based on state estimation in the roundabout. In Proceedings of the 2012 Third International Conference on Intelligent Control and Information Processing, Dalian, China, 15–17 July 2012; pp. 407–412. [\[CrossRef\]](#)
11. Farkas, Z.; Mihály, A.; Gáspár, P. MPC Control Strategy for Autonomous Vehicles Driving in Roundabouts. In Proceedings of the 2022 30th Mediterranean Conference on Control and Automation (MED), Vouliagmeni, Greece, 28 June–1 July 2022; pp. 939–944. [\[CrossRef\]](#)
12. Ma, F.; Yang, Y.; Wang, J.; Li, X.; Wu, G.; Zhao, Y.; Wu, L.; Aksun-Guvenc, B.; Guvenc, L. Eco-driving-based cooperative adaptive cruise control of connected vehicles platoon at signalized intersections. *Transp. Res. Part Transp. Environ.* **2021**, *92*, 102746. [\[CrossRef\]](#)
13. Talebpour, A.; Mahmassani, H.S. Influence of connected and autonomous vehicles on traffic flow stability and throughput. *Transp. Res. Part Emerg. Technol.* **2016**, *71*, 143–163. [\[CrossRef\]](#)
14. Li, D.; Wagner, P. Impacts of gradual automated vehicle penetration on motorway operation: A comprehensive evaluation. *Eur. Transp. Res. Rev.* **2019**, *11*, 1–10. [\[CrossRef\]](#)
15. Min, H.; Fang, Y.; Wu, X.; Wu, G.; Zhao, X. On-ramp merging strategy for connected and automated vehicles based on complete information static game. *J. Traffic Transp. Eng. (Engl. Ed.)* **2021**, *8*, 582–595. [\[CrossRef\]](#)
16. Zhu, J.; Easa, S.; Gao, K. Merging control strategies of connected and autonomous vehicles at freeway on-ramps: a comprehensive review. *J. Intell. Connect. Veh.* **2022**, *5*, 99–111. [\[CrossRef\]](#)
17. Masi, S.; Xu, P.; Bonnifait, P. A Curvilinear Decision Method for Two-lane Roundabout Crossing and its Validation under Realistic Traffic Flow. In Proceedings of the 2020 IEEE Intelligent Vehicles Symposium (IV), Las Vegas, NV, USA, 19 October–13 November 2020; pp. 1290–1296. [\[CrossRef\]](#)
18. Sackmann, M.; Leemann, T.; Bey, H.; Hofmann, U.; Thielecke, J. Multi-Step Training for Predicting Roundabout Traffic Situations. In Proceedings of the 2021 IEEE International Intelligent Transportation Systems Conference (ITSC), Indianapolis, IN, USA, 19–22 September 2021; pp. 1982–1989.
19. Deveaux, D.; Higuchi, T.; Uçar, S.; Wang, C.H.; Härrí, J.; Altintas, O. Extraction of Risk Knowledge from Time To Collision Variation in Roundabouts. In Proceedings of the 2021 IEEE International Intelligent Transportation Systems Conference (ITSC), Indianapolis, IN, USA, 19–22 September 2021; pp. 3665–3672.
20. Xu, K.; Cassandras, C.G.; Xiao, W. Decentralized Time and Energy-Optimal Control of Connected and Automated Vehicles in a Roundabout. In Proceedings of the 2021 IEEE International Intelligent Transportation Systems Conference (ITSC), Indianapolis, IN, USA, 19–22 September 2021; pp. 681–686.
21. Masi, S.; Xu, P.; Bonnifait, P. Adapting the Virtual Platooning Concept to Roundabout Crossing. In Proceedings of the 2018 IEEE Intelligent Vehicles Symposium (IV), Changshu, China, 26–30 June 2018; pp. 1366–1372. [\[CrossRef\]](#)
22. Trentin, V.; Artuñedo, A.; Godoy, J.; Villagra, J. Interaction-Aware Intention Estimation at Roundabouts. *IEEE Access* **2021**, *9*, 123088–123102. [\[CrossRef\]](#)
23. Debada, E.; Makarem, L.; Gillet, D. A virtual vehicle based coordination framework for Autonomous Vehicles in heterogeneous scenarios. In Proceedings of the 2017 IEEE International Conference on Vehicular Electronics and Safety (ICVES), Vienna, Austria, 27–28 June 2017; pp. 51–56. [\[CrossRef\]](#)

24. Rodrigues, M.; McGordon, A.; Gest, G.; Marco, J. Autonomous Navigation in Interaction-Based Environments - A Case of Non-Signalized Roundabouts. *IEEE Trans. Intell. Veh.* **2018**, *3*, 425–438. [[CrossRef](#)]
25. Garcia Cuenca, L.; Sanchez-Soriano, J.; Sanz, E.; Andrés, J.; Aliane, N. Machine Learning Techniques for Undertaking Roundabouts in Autonomous Driving. *Sensors* **2019**, *19*, 2386. [[CrossRef](#)]
26. Chen, J.; Yuan, B.; Tomizuka, M. Model-free Deep Reinforcement Learning for Urban Autonomous Driving. In Proceedings of the 2019 IEEE Intelligent Transportation Systems Conference (ITSC), Auckland, New Zealand, 27–30 October 2019; pp. 2765–2771. [[CrossRef](#)]
27. Chen, J.; Yuan, B.; Tomizuka, M. Deep Imitation Learning for Autonomous Driving in Generic Urban Scenarios with Enhanced Safety. In Proceedings of the 2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Macau, China, 3–8 November 2019; pp. 2884–2890. [[CrossRef](#)]
28. Zhang, Y.; Gao, B.; Guo, L.; Guo, H.; Chen, H. Adaptive Decision-Making for Automated Vehicles Under Roundabout Scenarios Using Optimization Embedded Reinforcement Learning. *IEEE Trans. Neural Netw. Learn. Syst.* **2021**, *32*, 5526–5538. [[CrossRef](#)]
29. Mehran, Z.A.; Nasser, L.A. On-line Situational Awareness for Autonomous Driving at Roundabouts using Artificial Intelligence. *J. Mach. Intell. Data Sci.* **2021**, *2*, 17–24. [[CrossRef](#)]
30. Chalaki, B.; Beaver, L.E.; Remer, B.; Jang, K.; Vinitzky, E.; Bayen, A.M.; Malikopoulos, A.A. Zero-Shot Autonomous Vehicle Policy Transfer: From Simulation to Real-World via Adversarial Learning. In Proceedings of the 2020 IEEE 16th International Conference on Control Automation (ICCA), Singapore, 9–11 October 2020; pp. 35–40. [[CrossRef](#)]
31. Bosankic, I.; Banjanovic-Mehmedovic, L. Cooperative intelligence in roundabout intersections using hierarchical fuzzy behavior calculation of vehicle speed profile. *MATEC Web Conf.* **2016**, *81*, 01008. [[CrossRef](#)]
32. Tian, R.; Li, S.; Li, N.; Kolmanovsky, I.; Girard, A.; Yildiz, Y. Adaptive Game-Theoretic Decision Making for Autonomous Vehicle Control at Roundabouts. In Proceedings of the 2018 IEEE Conference on Decision and Control (CDC), Miami, FL, USA, 17–19 December 2018; pp. 321–326. [[CrossRef](#)]
33. Banjanovic-Mehmedovic, L.; Halilovic, E.; Bosankić, I.; Kantardzic, M.; Kasapovic, S. Autonomous Vehicle-to-Vehicle (V2V) Decision Making in Roundabout using Game Theory. *Int. J. Adv. Comput. Sci. Appl.* **2016**, *7*, 292–298. [[CrossRef](#)]
34. Cao, H.; Zoldy, M. MPC Tracking Controller Parameters Impacts in Roundabouts. *Mathematics* **2021**, *9*, 1394. [[CrossRef](#)]
35. Hidalgo, C.; Lattarulo, R.; Pérez, J.; Asua, E. Hybrid trajectory planning approach for roundabout merging scenarios. In Proceedings of the 2019 IEEE International Conference on Connected Vehicles and Expo (ICCVE), Graz, Austria, 4–8 November 2019; pp. 1–6.
36. Wuthishuwong, C.; Traechtler, A. Vehicle to infrastructure based safe trajectory planning for Autonomous Intersection Management. In Proceedings of the 2013 13th International Conference on ITS Telecommunications (ITST), Tampere, Finland, 5–7 November 2013; pp. 175–180.
37. Gustafsson, F. Slip-based tire-road friction estimation. *Automatica* **1997**, *33*, 1087–1099. [[CrossRef](#)]
38. Li, K.; Misener, J.A.; Hedrick, K. On-board road condition monitoring system using slip-based tyre-road friction estimation and wheel speed signal analysis. *Automatica* **2007**, *221*, 129–146. [[CrossRef](#)]
39. Alvarez, L.; Yi, J.; Horowitz, R.; Olmos, L. Dynamic Friction Model-Based Tire-Road Friction Estimation and Emergency Braking Control. *J. Dyn. Syst. Meas. Control* **2005**, *127*, 22–32. [[CrossRef](#)]
40. Bichiou, Y.; Rakha, H.A. Real-time optimal intersection control system for automated/cooperative vehicles. *Int. J. Transp. Sci. Technol.* **2019**, *8*, 1–12. [[CrossRef](#)]
41. Chohan, N. Robust Trajectory Planning of Autonomous Vehicles at Intersections with Communication Impairments. Master's Thesis, Aalto University, Espoo, Finland, 2019.
42. Khayatian, M.; Mehrabian, M.; Shrivastava, A. RIM: Robust Intersection Management for Connected Autonomous Vehicles. In Proceedings of the 2018 IEEE Real-Time Systems Symposium (RTSS), Nashville, TN, USA, 11–14 December 2018. [[CrossRef](#)]
43. Rajamani, R. *Vehicle Dynamics and Control*; Springer: Berlin/Heidelberg, Germany, 2005.
44. Kiencke, U.; Majjad, R.; Kramer, S. Modeling and performance analysis of a hybrid driver model. *Control. Eng. Pract.* **1999**, *7*, 985–991. [[CrossRef](#)]
45. Parekh, D.; Poddar, N.; Rajpurkar, A.; Chahal, M.; Kumar, N.; Joshi, G.P.; Cho, W. A Review on Autonomous Vehicles: Progress, Methods and Challenges. *Electronics* **2022**, *11*, 2162. [[CrossRef](#)]
46. Lim, H.S.M.; Taeihagh, A. Algorithmic Decision-Making in AVs: Understanding Ethical and Technical Concerns for Smart Cities. *Sustainability* **2019**, *11*, 5791. [[CrossRef](#)]
47. Ryan, M. The Future of Transportation: Ethical, Legal, Social and Economic Impacts of Self-driving Vehicles in the Year 2025. *Sci. Eng. Ethics* **2020**, *26*, 1185–1208. [[CrossRef](#)] [[PubMed](#)]
48. Stone, T.; Santoni de Sio, F.; Vermaas, P. Driving in the Dark: Designing Autonomous Vehicles for Reducing Light Pollution. *Sci. Eng. Ethics* **2020**, *26*, 1–17. [[CrossRef](#)]

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.