Supplementary material S1

The following non-dimensional parameters are used in this work.

$$X=\frac{x}{l},Y=\frac{y}{l},H=\frac{h}{l},P=\frac{P}{E'},\bar{\eta}=\frac{\eta}{\eta_0},\bar{\rho}=\frac{\rho}{\rho_0} \tag{1}$$

Where l is the length of the solution domain in x direction, $E^{'}$ is the equivalent Young's modulus. η_0 and ρ_0 are the ambient viscosity and density of the lubricant. The non-dimensional equations are as follows.

Non-dimensional Reynolds equation and the boundary condition at micro meshes:

$$\frac{\partial}{\partial X}\left(\varepsilon_x\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\varepsilon_y\frac{\partial P}{\partial Y}\right)=\frac{\partial\left(\bar{\rho}H\right)}{\partial X} \tag{2a}$$

where,

$$\varepsilon_x=\varepsilon_y=\frac{lE'}{12\eta_0 U}\frac{\bar{\rho}H^3}{\bar{\eta}} \tag{2b}$$

$$P\left(X_0,Y\right)=P\left(X_e,Y\right)=P\left(X,Y_0\right)=P\left(X,Y_e\right)=\frac{p_0}{E'} \tag{2c}$$

The non-dimensional film thickness equation is

$$\begin{aligned}H\left(X,Y\right)&=\frac{1}{l}\left[h_0+\delta_1\left(x,y\right)+\delta_2\left(x,y\right)+v\left(x,y\right)\right]\\&=H_0+\bar{\delta}_1\left(X,Y\right)+\bar{\delta}_2\left(X,Y\right)+V\left(X,Y\right)\end{aligned} \tag{3}$$

And the non-dimensional elastic deformation equation is

$$V=\frac{2}{\pi}\iint_{\Omega}\frac{P(\xi,\zeta)}{\sqrt{\left(X-\xi\right)^2+\left(Y-\zeta\right)^2}}d\xi d\zeta \tag{4}$$

The non-dimensional viscosity-pressure and density-pressure functions are

$$\bar{\eta}=\exp\left(\alpha E'P\right) \tag{5}$$

$$\bar{\rho}=1+\frac{0.6E'P}{1+1.7E'P} \tag{6}$$

Finally, the non-dimensional load balance equation is

$$W=\frac{w}{E'l^2}=\frac{1}{E'l^2}\iint_{\Omega}p\left(x,y\right)dxdy=\iint_{\Omega}P\left(X,Y\right)dXdY \tag{7}$$

The solution domain for micro meshes is $\left\{\left(X,Y\right)\middle|0\le X\le 1,0\le Y\le 1\right\}$. ΔX and ΔY are the grid space in X and Y directions, respectively, and ΔX=ΔY. i and j represent the nodes in X and Y, respectively. The density of grids is $M\times N$.

The discrete Eq(2a) is given as follows,

$$\begin{aligned}&\frac{1}{\Delta X^2}\left[\varepsilon^x_{i-1/2,j}P_{i-1,j}-\left(\varepsilon^x_{i-1/2,j}+\varepsilon^x_{i+1/2,j}\right)P_{i,j}+\varepsilon^x_{i+1/2,j}P_{i+1,j}\right]\\&\quad+\frac{1}{\Delta Y^2}\left[\varepsilon^y_{i,j-1/2}P_{i,j-1}-\left(\varepsilon^y_{i,j-1/2}+\varepsilon^y_{i,j+1/2}\right)P_{i,j}+\varepsilon^y_{i,j+1/2}P_{i,j+1}\right]\\&=\frac{\left(\bar{\rho}_{ij}-\bar{\rho}_{i-1,j}\right)H_{ij}}{\Delta X}+\frac{\bar{\rho}_{ij}\left(H_{ij}-H_{i-1,j}\right)}{\Delta X}\end{aligned}\tag{8}$$

The discrete film thickness equation is

$$H_{ij}=H_0+\bar{\delta}_{1i,j}+\bar{\delta}_{2i,j}+V_{i,j}\tag{9}$$

In order to discretize the non-dimensional elastic deformation equation Eq(4), Liu et al. [1] used parabolic-elliptic interpolation function to estimate the pressure distribution in one micro mesh element, and then utilized the interpolation function to discretize Eq(4) which turns into Eq(10).

$$V_{i,j}=\sum_{k=1}^{M-1}\sum_{l=1}^{N-1}D^{i,j}_{k,l}P_{k,l}\tag{10}$$

where D is the elastic deformation coefficient matrix, and it is only determined based on the specific micro meshes.

Based on the same interpolation function of pressure distribution, the non-dimensional load balance equation is given in Eq(11).

$$W=\sum_{i=1}^{m}\sum_{j=1}^{n}e_{i,j}P_{i,j}\tag{11}$$

where e is also the coefficient matrix. The calculation of D and e is given in work[1]. Eq(8) can be reformatted to Eq(12).

$$\alpha^{(s)}_{i,j}P^{(s+1)}_{i-1,j}+\beta^{(s)}_{i,j}P^{(s+1)}_{i,j}+\gamma^{(s)}_{i,j}P^{(s+1)}_{i+1,j}=b^{(s)}_{i,j}\tag{12}$$

where the superscript (s) means the (s)th iteration step, as the $\Delta X = \Delta Y$,

$$\alpha^{(s)}_{i,j}=\varepsilon^{x(s)}_{i-1/2,j}\tag{13a}$$

$$\beta^{(s)}_{i,j}=-\left(\varepsilon^{x(s)}_{i-1/2,j}+\varepsilon^{x(s)}_{i+1/2,j}+\varepsilon^{y(s)}_{i,j-1/2}+\varepsilon^{y(s)}_{i,j+1/2}\right)\tag{13b}$$

$$\gamma^{(s)}_{i,j}=\varepsilon^{x(s)}_{i+1/2,j}\tag{13c}$$

$$b^{(s)}_{i,j}=-\left(\varepsilon^{y(s)}_{i,j-1/2}P^{(s)}_{i,j-1}+\varepsilon^{y(s)}_{i,j+1/2}P^{(s)}_{i,j+1}\right)+\Delta X\left[\left(\bar{\rho}^{(s)}_{ij}-\bar{\rho}^{(s)}_{i-1,j}\right)H^{(s)}_{ij}+\bar{\rho}^{(s)}_{ij}\left(H^{(s)}_{ij}-H^{(s)}_{i-1,j}\right)\right]\tag{13d}$$

To a given j, the coefficient matrix of Eq(12) is tridiagonal. Thus chasing method can be used to solve the corresponding linear equation set.

However, when $h\rightarrow 0$,

$$\alpha^{(s)}_{i,j}=\varepsilon^{x(s)}_{i-1/2,j}\xrightarrow{h\rightarrow 0}0\tag{14a}$$

$$\beta_{i,j}^{(s)} = -\left(\varepsilon_{i-1/2,j}^{x(s)} + \varepsilon_{i+1/2,j}^{x(s)} + \varepsilon_{i,j-1/2}^{y(s)} + \varepsilon_{i,j+1/2}^{y(s)}\right) \xrightarrow{h \to 0} 0 \tag{14b}$$

$$\gamma_{i,j}^{(s)} = \varepsilon_{i+1/2,j}^{x(s)} \xrightarrow{h \to 0} 0 \tag{14c}$$

Therefore, the coefficient matrix of Eq(12) will lose its diagonal dominance. This is the reason why it is hard to get a converged solution under ultrathin lubricant film. Zhu and Hu [2, 3] utilized the elastic deformation equation Eq(10) to solve this problem. They transpose the RHS terms involving the unknown pressures in Eq(12), $P_{i-1,j}^{(s+1)}$, $P_{i,j}^{(s+1)}$, and $P_{i+1,j}^{(s+1)}$ to the left. In this manner, Eq(13) turns to follow

$$\alpha_{i,j}^{(s)} = \varepsilon_{i-1/2,j}^{x(s)} - \left(\bar{\rho}_{i,j}^{(s)} D_{i-1,j}^{i,j} - \bar{\rho}_{i-1,j}^{(s)} D_{i-1,j}^{i,j}\right)\Delta X - \left(\bar{\rho}_{i,j}^{(s)} D_{i-1,j}^{i,j} - \bar{\rho}_{i,j}^{(s)} D_{i-1,j}^{i-1,j}\right)\Delta X \tag{15a}$$

$$\beta_{i,j}^{(s)} = -\begin{pmatrix} \varepsilon_{i-1/2,j}^{x(s)} + \varepsilon_{i+1/2,j}^{x(s)} \\ +\varepsilon_{i,j-1/2}^{y(s)} + \varepsilon_{i,j+1/2}^{y(s)} \end{pmatrix} - \begin{pmatrix} \bar{\rho}_{i,j}^{(s)} D_{i,j}^{i,j} - \bar{\rho}_{i,j}^{(s)} D_{i,j}^{i-1,j} \\ +\bar{\rho}_{i,j}^{(s)} D_{i,j}^{i,j} - \bar{\rho}_{i-1,j}^{(s)} D_{i,j}^{i,j} \end{pmatrix}\Delta X \tag{15b}$$

$$\gamma_{i,j}^{(s)} = \varepsilon_{i+1/2,j}^{x(s)} - \left(\bar{\rho}_{ij}^{(s)} D_{i+1,j}^{i,j} - \bar{\rho}_{i-1j}^{(s)} D_{i+1,j}^{i,j}\right)\Delta X - \left(\bar{\rho}_{ij}^{(s)} D_{i+1,j}^{i,j} - \bar{\rho}_{ij}^{(s)} D_{i+1,j}^{i-1,j}\right)\Delta X \tag{15c}$$

$$b_{i,j}^{(s)} = -\left(\varepsilon_{i,j-1/2}^{y(s)} P_{i,j-1}^{(s)} + \varepsilon_{i,j+1/2}^{y(s)} P_{i,j+1}^{(s)}\right) + f_1 + f_2 \tag{15d}$$

where,

$$f_1 = \Delta X \begin{bmatrix} \bar{\rho}_{ij}^{(s)}\left(H_{ij}^{(s)} - H_{i-1,j}^{(s)}\right) - \left(\bar{\rho}_{i,j}^{(s)} D_{i,j}^{i,j} - \bar{\rho}_{i,j}^{(s)} D_{i,j}^{i-1,j}\right) P_{ij}^{(s)} \\ -\left(\bar{\rho}_{i,j}^{(s)} D_{i-1,j}^{i,j} - \bar{\rho}_{i-1,j}^{(s)} D_{i-1,j}^{i,j}\right) P_{i-1,j}^{(s)} - \left(\bar{\rho}_{i,j}^{(s)} D_{i-1,j}^{i,j} - \bar{\rho}_{i,j}^{(s)} D_{i-1,j}^{i-1,j}\right) P_{i-1,j}^{(s)} \end{bmatrix}$$

$$f_2 = \Delta X \begin{bmatrix} H_{ij}^{(s)}\left(\bar{\rho}_{ij}^{(s)} - \bar{\rho}_{i-1,j}^{(s)}\right) - \left(\bar{\rho}_{i,j}^{(s)} D_{i,j}^{i,j} - \bar{\rho}_{i-1,j}^{(s)} D_{i,j}^{i,j}\right) P_{ij}^{(s)} \\ -\left(\bar{\rho}_{ij}^{(s)} D_{i+1,j}^{i,j} - \bar{\rho}_{i-1j}^{(s)} D_{i+1,j}^{i,j}\right) P_{i+1,j}^{(s)} - \left(\bar{\rho}_{ij}^{(s)} D_{i+1,j}^{i,j} - \bar{\rho}_{ij}^{(s)} D_{i+1,j}^{i-1,j}\right) P_{i+1,j}^{(s)} \end{bmatrix}$$

Because element $D_{k,j}^{l,j}$ has the maximum value when $k = l$, following inequality always holds,

$$\left|\beta_{i,j}^{(s)}\right| > \left|\alpha_{i,j}^{(s)}\right| + \left|\gamma_{i,j}^{(s)}\right| \tag{16}$$

which means that the coefficient matrix of Eq(13) keeps diagonal dominance even under ultrathin lubricant film.

Eq(12) is solved iteratively by under relaxation method, the equation is

$$P_{new} = P_{old} + \omega\left(P - P_{old}\right) \tag{17}$$

where ω *is* the under-relaxation factor, which is smaller than one, and P_{new} is the pressure distribution values for the next iteration step.

The convergent criterion for pressure distribution

$$\frac{\sum\sum\left|P - P_{old}\right|}{\sum\sum\left|P_{old}\right|} \le \varepsilon_p \tag{18}$$

Where P is the newest pressure distribution, P_{old} is the pressure distribution obtained in the last iteration loop, and ε_p is a constant which is equal to 1×10^{-5} in this paper.

The convergent criterion for load capacity is

$$\frac{\sum\sum|W_{cal}-W|}{\sum\sum|W|}\leq\varepsilon_W \qquad (19)$$

where W_{cal} is the calculated load capacity, W is the applied load, and ε_W is a constant which equals to 1×10^{-5} in this paper.

Reference

1. Liu, S.B., J.J. Ma, and Y.X. Chen, *Elliptic-Paraboloid Method for Calculating Surface Elastic-Deformation in Ehl.* Tribology International, 1993. **26**(6): p. 443-448.
2. Hu, Y.Z. and D. Zhu, *A full numerical solution to the mixed lubrication in point contacts.* JOURNAL OF TRIBOLOGY-TRANSACTIONS OF THE ASME, 2000. **122**(1): p. 1-9.
3. Zhu, D. and Y.-Z. Hu, *The study of transition from elastohydrodynamic to mixed and boundary lubrication.* The advancing frontier of engineering tribology, Proceedings of the 1999 STLE/ASME HS Cheng Tribology Surveillance, 1999: p. 150-156.