

Article Tilting-Pad Journal Bearings—Frequency-Dependent Dynamic Coefficients and Pivot Flexibility Effects

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Abstract: Tribologists have generally accepted that the dynamic modeling of tilting-pad journal bearings (TPJB) must consider the frequency dependency of the dynamic properties. Industrial compressors, turbines, and other rotating machines are subjected to instability drivers, such as blades, impellers, and seals, leading to dominant vibratory frequencies that are generally quite different from rotational frequency. Though the literature has provided related methods and numerical results, full understanding of the physics of TPJB frequency dependency is not generally available to the design community, and theorists and experimentalists are often not in agreement. This investigation hinges on a single-pad, two degree-of-freedom model that creates a basis for understanding the various geometries and operating conditions related to frequency dependency for a full bearing. The analytical results indicate that both stiffness and damping coefficients show frequency dependency, and that the dependency is primarily associated with the pad rotational damping and the flexibility of the pivot contact region that provides support for the pad. Understanding the role of pivot flexibility in combination with the fluid film provides a key to improving agreement between theory and experiment. This article is a revised and expanded version of the paper presented at the ASME 2019 Turbo Expo in Phoenix, Arizona from 17 to 21 June. The paper number was GT2019-90195 and it was titled "On the Frequency Dependency of Tilting-Pad Journal Bearings".

Keywords: bearings; tilting-pad; dynamic; coefficients; frequency; stiffness; damping; pivot; flexibility

1. Introduction

High-speed rotating machines with flexible rotors, such as compressors, turbines, and other machines supported on fluid-film bearings, are often subject to instability drivers, such as forces developed by annular-clearance seals, blade rows, or impellers. The accurate theoretical modeling of the bearing dynamic properties at the design stage is one step in the path toward the assurance of a stable machine in the field. The properties of the destabilizing elements usually cannot be determined with a confidence level on par with the properties of the bearings, so it is often desirable to include some stability margin in the chosen bearing design. Such a margin has traditionally been found with the tilting-pad journal bearing (TPJB).

Proper modeling, and even understanding, of tilting-pad journal bearing dynamic behavior has been a source of debate for several decades. A particularly vexing topic has been frequency dependency, an issue allied to rotordynamic stability. All theorists have concluded that frequency dependency is definite. Some experimentalists agree, but many say they are unable to find this dependency, especially the dependence of damping on frequency.

In the early days of computational rotordynamic stability analysis, when the dynamic effects of TPJBs were included, it was tacitly assumed that the bearing properties should be evaluated and measured at the frequency of rotation (i.e., at synchronous frequency) [1,2]. This focus is understandable, since most research at the time centered on unbalance excitation resulting in synchronous vibration. With continuing development of the analyses,



Citation: Wagner, L.F.; Allaire, P.E. Tilting-Pad Journal Bearings—Frequency-Dependent Dynamic Coefficients and Pivot Flexibility Effects. *Lubricants* **2022**, *10*, 20. https://doi.org/10.3390/ lubricants10020020

Received: 3 December 2021 Accepted: 29 January 2022 Published: 3 February 2022

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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). it became clear that TPJBs responded differently at different frequencies, leading to frequency dependency for TPJB dynamic properties (i.e., non-synchronous effects). Currently, non-synchronous analysis methods are in common industrial use, and are employed and recommended by the American Petroleum Industry Standard 617 [3]. The synchronous method can also be employed if involved parties agree, but it is noted in API 617 that the synchronous method predicts significantly higher stability margins than expected. In some cases, end users and/or OEMs are more familiar with, and are calibrated to, the synchronous method and prefer it.

In a 2011 paper, Dimond, Younan, and Allaire presented an excellent review of the tilting-pad journal bearing literature [4]. Theoretical and modeling issues were the primary concerns, with the note that some modeling options rely on the results of experimentation. A significant focus of this paper, either directly or indirectly, concerns the question of frequency dependency of tilting-pad bearing dynamic coefficients, concluding that the analysis must include the pad degrees of freedom (DOF) to show such dependency.

Warner and Soler published a paper in 1975 with one of the earliest mentions of TPJB frequency dependency [5]. Among their principal conclusions was: "Even when pad inertia is negligible, the spring and damping coefficients of the tilting pad bearing are frequency dependent."

In 1983, Parsell, Allaire, and Barrett [6] examined the effects of damped vibrational frequencies on the reduced stiffness and damping coefficients for tilting-pad bearings. Although frequency effects were evident, the use of synchronously reduced coefficients was claimed to be generally adequate for stability analysis with positively preloaded bearings. In the same year, Rouch [7] presented a pad-assembly model that would consider flexible pivots by the inclusion of a radial degree of freedom for each pad.

Within the next year, Springer published a book [8] containing a chapter on tilting-pad bearings providing the following statement: "... *the stiffness and damping coefficients of the oil film*... *are not influenced by the frequency of lateral shaft oscillations.*" However, at this point in time, 1984, there were few related experimental data available in the literature.

Noting that improving the understanding and computational accuracy of bearing performance leads directly to better rotordynamic modeling, particularly stability, a reference by Barrett et al. in 1988 [9] is of particular interest. This paper describes a pad-assembly and reduction effort for the determination of eight effective dynamic coefficients. The results show frequency dependency for both stiffness and damping. The effects of pivot flexibility were not considered.

In 1992, White and Chan [10] presented a theory with computed results for the dynamic properties of TPJBs. The results showed that for bearings with small preloads operating at high Sommerfeld numbers, the effective damping at subsynchronous frequencies is considerably lower than that predicted for synchronous vibration. Additionally, the stiffness was found to be affected by frequency. Increasing bearing preload or pad offset was found to attenuate frequency effects.

Allaire, Parsell, and Barrett [11] developed a new pad perturbation method for tiltingpad bearings in 1993. It described the evaluation of the dynamic coefficients with consideration of the frequency dependency of the dynamic coefficients. However, pad radial motion, for consideration of pivot flexibility, was not considered.

Ha and Yang [12] performed experiments to investigate the frequency effects of the excitation force on the linear stiffness and damping coefficients of a load on pad (LOP)-type five-pad TPJB. Their results showed that the variation in excitation frequency had a measurable effect on dynamic coefficients. The bearing stiffness coefficients decreased slightly as the excitation frequency increased, while the damping coefficients increased slightly, especially at a lower speed and higher load.

Dmochowski presented a theoretical and experimental examination [13] of the effects of frequency variation on the stiffness and damping characteristics of a TPJB. The study was principally aimed at the pivot flexibility, and the effects of frequency were also a general concern. The results showed that the frequency effects on the dynamic properties depend on the operating conditions and the geometric parameters of the bearing design. It was concluded that the pad inertia and pivot flexibility are behind the variations in the stiffness and damping properties with excitation frequency.

Dimond, Younan, and Allaire [14] noted that there was significant disagreement in the literature concerning the proper evaluation of the experimental identification and frequency response of TPJBs due to shaft excitation. Two models were developed to address the issue. The first was the KC model that explicitly considered all pads and pad degrees of freedom, with the possibility of reduction to the typical eight coefficients for a desired frequency. The second model, known as the KCM model, is based on curve fitting numerical or experimental force coefficient results, and is represented by a set of frequency-independent dynamic coefficients, including a "virtual mass". There are major differences between the results of the two approaches. The results indicate that the KCM method may not capture physically justifiable results. A similar theoretical study by Schmied, Fedorov, and Grigoriev [15] concurred with the KC model conclusions, and gave several examples.

Cloud, Maslen, and Barrett [16] compared rotor dynamic measurements for unbalance response and stability with modeling predictions. Predictions based on frequencydependent dynamic coefficients (i.e., the KC model), exhibited significantly better correlation with the stability measurements than those using synchronously reduced (thus frequency-independent) coefficients.

A series of experimental studies by Childs et al. and San Andres et al., some peripherally related, directed toward an understanding of TPJB modeling issues [17–24], examined different styles and configurations of bearings, and except for a few deviations, drew the conclusion that measured damping coefficients do not show frequency dependency, and that the frequency dependency of the impedance real part can be captured with a stiffness term and a term proportional to ω^2 (i.e., a virtual "mass").

Wilkes and Childs [21] presented a different theoretical and experimental study concerning frequency effects with TPJBs. The experimental measurements were principally associated with higher loads and lower speeds typical of many industrial applications. All results showed definite frequency dependency for the dynamic coefficients (both stiffness and damping), with additional significant effects of pivot and pad flexibility. Rotordynamic stability computations were made for models with synchronously reduced and subsynchronously reduced (including KC) coefficients. The KC and subsynchronously reduced coefficient models provided the best comparison to experiments.

An extensive study by Quintini et al. [23], although primarily concerned with the effects of preload and clearance variations, found stiffness and damping frequency dependence in computed results. Yang et al. [25] presented results for TPJBs with consideration of frequency effects that contrasted with many of the previous experimental efforts. The numerical results indicated that the direct stiffness coefficients decreased, while the direct damping coefficients increased with exciting frequency. Neither study reported mass coefficients.

Recently, Vinh Dang et al. [26] conducted theoretical and experimental studies on a five-pad TPJB that emphasized the effects of pivot flexibility on the effective dynamic coefficients. The experiments showed significant variations in both stiffness and damping as frequency changed. Pivot flexibility and, to a lesser extent, pad flexibility contributed greatly toward the property variability. It was further stated that a Hertzian contact model for the pivot over-emphasized the pivot contact stiffness.

It is clear that after many years of theoretical analysis, experimentation, and discussion, there is not complete closure on the topic of TPJB frequency dependency. Most researchers agree that such dependencies exist, but it is also clear that significant differences exist between the theoretically and experimentally derived dynamic properties required for accurate modeling of rotordynamic stability.

Current theoretical research has focused on numerical determination of the dynamic properties for a complete, multiple-pad TPJB. However, in practice, each pad operates independently, with little coupling to adjacent pads. Thus, the comprehensive study of a single tilting pad can provide a different perspective for understanding TPJB frequency effects.

The principal objective of this study is to determine the change in the TPJB stiffness and damping for arbitrary vibrational frequencies relative to these same properties evaluated at synchronous frequency. The analytical model used in this work, based on a single tilting pad, results in expressions for the dynamic coefficients that isolate the terms that affect TPJB frequency dependency. By working with such a model, the physical reasoning for this dependency will become more apparent.

2. Basic Geometry of a Tilting-Pad Bearing

Most readers are likely well familiar with the geometry and operation of tilting-pad journal bearings. These bearings are mechanically complex, having multiple pads with various loading arrangements, e.g., load-on-pad (LOP) or load-between-pads (LBP). They are available in many commercial variations, e.g., spherical pivots (point contact), rocker pivots (line contact), and flexure pivots.

The model used in this study provides each pad with a simple point or line contact, as shown in Figure 1a. The pads are positioned concentrically with the journal (this is a setup configuration, but is non-operational). The bearing is composed of multiple, identical rigid massless pads, each supported by a rigid point or line pivot (the rigidity will be relaxed later). The journal is of radius R and diameter D, and each pad surface is positioned further radially from R by a clearance C (machined clearance). Each pad is of active length L, and circumferential extent β , while the pivot is positioned at angle α relative to the pad leading edge. The leading edge is defined relative to journal rotational direction as shown in the figure.





Figure 1b shows a typical load configuration for a five-pad bearing operating with loadon-pad (LOP). The indication here is that the loaded pad develops the greatest pressures and thus encounters the largest percentage of the applied load W. This primary dominant pad load configuration will be considered throughout the analysis portion of this paper.

3. Preliminary Motivation

The first objective of any engineering study is to develop a broad understanding of the physics, and to this end, a long-standing axiom of practicing engineers has been to initially value simple, representative physical models over complex models. For example, while a complex finite element model may provide accurate localized stress information, a single equation for stress due to beam bending may provide an analyst with an understanding to support the direction of a project that is unavailable using the more complex method.

Consider a simple analog to the journal/pad model with an intermediate massless degree of freedom (DOF), vibrating harmonically at frequency ω , as shown in Figure 2. The mass is connected to a stiffness K and a damper B, and with a support stiffness K_S. The objective is to investigate the overall dynamic characteristics of the structure connecting the mass to ground. The key is to eliminate the massless DOF, and thus determine an effective, harmonically based connection impedance ($K_{eff} + i\omega B_{eff}$) between the mass and ground.



Figure 2. A two-DOF system with massless DOF.

Noting that this model is composed of two impedances in series, the effective dynamic coefficients can be easily determined, with the result,

$$K_{eff} = \frac{1}{\Delta} \left[KK_s(K + K_s) + \omega^2 K_s B^2 \right] \text{ and } B_{eff} = \frac{1}{\Delta} BK_s^2$$
(1)

with $\Delta = (K + K_s)^2 + \omega^2 B^2$. The supported mass is irrelevant to the result for the supporting structure impedance.

This procedure has effectively removed the internal DOF, and the result shows that the addition of damping causes the effective stiffness and damping terms to each become frequency-dependent. *Removing the damping removes the dependency*. While this result cannot be applied directly to a tilting-pad bearing model, it clearly implies that the addition of damping to such a coupled system must invariably lead to frequency dependency in the effective properties.

4. Single-Pad System

The single-pad system will be considered in two parts, with the first concerned primarily with the fluid film alone, and the second with the flexibility of the pivot that supports the pad. Individually, the film properties or the pivot stiffness both have the potential to dominate the make-up of the effective dynamic properties, particularly with regard to frequency dependency.

4.1. The Physical Model

The schematic picture of Figure 3 shows a journal rotating at frequency Ω , with one of the tilting pads that rotationally (θ) couples to the journal coordinates (X,Y) via the fluid film. The vibratory motion of these coordinate elements occurs at frequency ω , and there is no a priori requirement that $\omega = \Omega$. Without detracting from the arguments, the pad can be considered massless.



Figure 3. Single-pad model of a multiple-pad bearing (arbitrary pad orientation).

A more useful and still physically representative model can be developed by first aligning the pivot with the vertical *X* axis shown in Figure 3. In such a model, the *Y* coordinate can be ignored (tangential forces are small), and the problem is reduced to two degrees of freedom, translation *X*, and pad rotation θ . Once the equilibrium position

for this system has been established, the equation of motion about equilibrium can be written for the journal mass M, including consideration of the coupling between the shaft displacement X and the pad tilting rotation θ :

$$M\ddot{X} + B_{XX}\dot{X} + B_{X\theta}\dot{\theta} + K_{XX}X + K_{X\theta}\theta = f_X$$
(2)

where the *B* values indicate damping and the *K* values are stiffness. The *XX* subscripts represent values that are "direct", denoting, for example, the ratio of a change in *X* force to a change in *X* displacement or velocity. The $X\theta$ or "cross-coupled" subscripts describe the ratio of a change in *X* force to a change in θ rotational displacement. The force f_X is an externally applied force; most often, a force due to unbalance. A similar equation for the pad dynamics is found to be:

$$I_P \dot{\theta} + B_{\theta X} \dot{X} + B_{\theta \theta} \dot{\theta} + K_{\theta X} X + K_{\theta \theta} \theta = 0$$
(3)

The moment of inertia of the pad about the pivot is given by I_P , which is not significantly related to the following discussion, and will be assigned zero. The right-hand side of the equation is zero because external moments are not applied to the pad.

Once assumed that the forces and displacements are harmonic, with $f_X = F_X e^{i\omega t}$, $X = \overline{X}e^{i\omega t}$, and $\theta = \overline{\theta}e^{i\omega t}$, these equations of motion can be written in a more convenient form (see [11] for example):

$$\begin{bmatrix} Z_{XX} & Z_{X\theta} \\ Z_{\theta X} & Z_{\theta \theta} \end{bmatrix} \left\{ \begin{array}{c} \overline{X} \\ \overline{\theta} \end{array} \right\} = \left\{ \begin{array}{c} F_X + \omega^2 M \overline{X} \\ 0 \end{array} \right\}$$
(4)

The matrix on the left of this equation is the pad film coefficient matrix, composed entirely of the linearized complex impedances of the bearing fluid film. Each of these impedance terms is of the form $Z = K + i\omega B$, and all are associated with a particular operating condition of the bearing. In general, this matrix is positive definite and tends to be diagonally dominant, but is not symmetric.

Elimination of the rotational displacement $\overline{\theta}$ results in a single equation for the effective impedance relative to the journal DOF \overline{X} :

$$Z_{eff} = \frac{(Z_{XX}Z_{\theta\theta} - Z_{X\theta}Z_{\theta X})}{Z_{\theta\theta}} = K_{eff} + i\omega B_{eff}$$
(5)

The expanded forms for the effective stiffness and damping coefficients are:

$$K_{eff}(\omega) = \left(RK_{\theta\theta} + \omega^2 SB_{\theta\theta}\right)/\Delta \tag{6}$$

$$B_{eff}(\omega) = (SK_{\theta\theta} - RB_{\theta\theta})/\Delta$$
(7)

with,

$$R(\omega) = K_{XX}K_{\theta\theta} - K_{X\theta}K_{\theta X} - \omega^2(B_{XX}B_{\theta\theta} - B_{X\theta}B_{\theta X})$$
$$S = K_{XX}B_{\theta\theta} + B_{XX}K_{\theta\theta} - K_{X\theta}B_{\theta X} - B_{X\theta}K_{\theta X}$$

and $\Delta(\omega) = K_{\theta\theta}^2 + \omega^2 B_{\theta\theta}^2$.

These expanded forms show that frequency dependency arises in both the effective stiffness and damping terms, and is strongly associated with the damping influences. Removing the damping, particularly the rotational damping $B_{\theta\theta}$, eliminates the frequency dependency.

4.2. Determination of the Elements of the Pad Film Coefficient Matrix

The complex coefficients shown in the matrix on the left-hand side of Equation (4) must be computed via solution of the Reynolds' lubrication equation [27,28]. This 2D partial differential equation governs the distribution of pressure developed in the lubricating film

between the rotating journal and the non-rotating (but tilting) pad (i.e., the clearance space). Several significant assumptions concerning the lubricant flow in the radially thin clearance space have been applied for this study, the most important being laminar, incompressible flow, and constant lubricant viscosity. The film thickness is assumed constant over the axial length of each pad. Inclusion of turbulence would not be difficult, but this would require the introduction of a new parameter that would be unlikely to provide new insights. Temporal and convective inertia effects of the fluid film have been ignored, along with mechanical effects such as pad deformation and pivot flexibility, but pivot flexibility will be considered later. Although of consequence for accurate quantitative analysis, these effects cause complications that are unwarranted for the sole consideration of the fluid film mechanics.

Nondimensionalization of the selected form of Reynolds' equation and determination of the load capacity requirements exposes the only two significant dimensionless parameters required to completely define the bearing operational performance characteristics, namely, the aspect ratio L/D and the Sommerfeld number, given by,

$$S = \left(\frac{R}{C}\right)^2 \frac{\mu NLD}{W} \tag{8}$$

where μ is the lubricant absolute viscosity, and N is the rotational velocity in *rev/sec* (based on the traditional definition of S).

The Sommerfeld number is a generalized "inverse load" parameter, and in the sense of increasing force on a bearing, the Sommerfeld number must decrease. Once the Sommerfeld number and aspect ratio have been chosen, the solution of Reynolds' equation provides the results for the system equilibrium position: X_{eq} and θ_{eq} . The elements of the pad film coefficient matrix of Equation (4) are then determined relative to equilibrium by perturbing the journal and pad coordinates for both displacement and velocity. All of the resulting stiffness and damping coefficients are thus linear, and can be computed and displayed (in dimensionless form) solely as functions of L/D and the Sommerfeld number.

4.3. Analysis Using the Physical Model

As stated in the introduction, the principal objective of this study is to determine the change in the pad system stiffness and damping for arbitrary vibrational frequencies relative to these same properties evaluated at synchronous frequency ($\omega = \Omega$). The aim is thus not to show dimensional properties, or to determine more accurate properties, but to illustrate the property variations with changes in frequency.

Comparison of the coefficient results uses a coefficient ratio. The nonsynchronous results (K_{eff} and B_{eff}) are computed using Equations (6) and (7), then divided by the effective synchronous results (K_{Ω} , B_{Ω}), and plotted as stiffness and damping ratios (i.e., $\overline{K} = K_{eff}/K_{\Omega}$ and $\overline{B} = B_{eff}/B_{\Omega}$). Recall that these equations have been developed for a *single* tilting pad, evaluated at an equilibrium position defined by the Sommerfeld number S. Except as noted, the computations have used an arc angle of $\beta = 80^{\circ}$ (based on a four-pad TPJB), an aspect ratio (L/D) of unity, and a centrally located pivot ($\alpha/\beta = 0.5$). This will be referred to as the "datum" pad.

5. Single-Pad Results

The result plots shown in Figure 4 display the computed dynamic coefficient ratios as a function of the reduced frequency ratio $\Lambda = \omega/\Omega$ for four different operating conditions defined by the Sommerfeld number.



Figure 4. Single-pad effective coefficient ratio results: (a) stiffness; (b) damping.

The conclusions drawn from the plotted results indicate that reduced coefficients vary the least from synchronous coefficients in the lower Sommerfeld number range, but that significant differences are evident in the higher number range. If the "exciting" frequency is below synchronous, the effective stiffness is higher, but becomes lower for frequencies above synchronous. This trend is reversed for consideration of effective damping. Damping shows little variation with Sommerfeld number for the frequency ratios above synchronous. It should be clearly understood that these single-pad results apply to the film alone (rigid pivot). Pivot flexibility will be considered later.

In order to gain a different perspective, the same results plotted in Figure 4a,b have been replotted versus the Sommerfeld number in Figure 5 for two reduced frequency ratios generally significant to rotordynamicists: $\Lambda = 0.5$ (half-frequency whirl) and $\Lambda = 2.0$ (2X vibration). It is clear that as the "load" reduces, the stiffness values deviate further from synchronous stiffness, but the damping varies less in proportion to the synchronous damping.



Figure 5. Dynamic coefficient variations for two reduced frequencies.

5.1. Effect of Aspect Ratio (L/D)

In addition to the Sommerfeld number, the aspect ratio (L/D) is an independent dimensionless parameter that is often quite significant for the successful design of a bearing for dynamic operation. For a given journal loading, shortening a bearing tends to increase the levels of both stiffness and damping.

The computed results show that for a representative range of aspect ratios (L/D = 0.5 \rightarrow 1.5) and loading (S~0.2), the effective stiffness ratio varies less than about 5% relative to the trends shown for the datum pad. The effective damping ratios vary even less. Thus, changing the bearing aspect ratio does not markedly affect the sensitivity of a TPJB to frequency variation, and certainly does not affect the trending.

5.2. Effect of Pivot Position

It has been suggested by other researchers (e.g., see [16] or conclusions in [17]) that the circumferential position of the pivot could have a significant effect on frequency dependency. Early tilting-pad bearing designs typically operated with a central pivot. It was later found that moving the pivot somewhat further from the pad leading edge (e.g., $\alpha/\beta = 0.6$) resulted in improved performance, e.g., lower babbitt temperatures and losses, and greater minimum film thicknesses. It was also found to significantly affect the dynamic properties.

The results of offsetting the pivot using the single-pad dynamic coefficient model of Equations (6) and (7) are shown in Figure 6 for three offset values. It is clear that increasing the pivot offset tends to move the nonsynchronous results closer to the synchronous results for both the stiffness and damping coefficients, and for both of the reduced frequencies considered.



Figure 6. Effect of pivot offset (α/β) on effective (**a**) stiffness, (**b**) damping.

Figure 6 indicates that the operating condition for the bearing of interest completely loses frequency dependency at a Sommerfeld number S = 0.375 with α/β = 0.6. Using a 127 mm diameter bearing to provide some dimensional results, it is of interest to note that the dimensional value of $B_{\theta\theta}$ for the centered pivot is roughly of the same order as $B_{X\theta}$ and $B_{\theta X}$. However, for α/β = 0.6, $B_{\theta\theta}$ becomes an order of magnitude larger than $B_{X\theta}$ and $B_{\theta X}$, indicating that these coupling effects are declining. Note that removal of the cross-coupled terms in Equations (6) and (7) eliminates frequency dependency, and that K_{eff} and B_{eff} become equal, respectively, to K_{XX} and B_{XX} . It is clear that the pivot position, and thus the rotational film squeezing, has a significant effect on frequency dependency.

The results shown in Table 1 illustrate the computed dimensional effective coefficient results for varying frequency for two different pivot positions of the datum pad. Except for the pivot position, both results have used identical geometry and operating conditions (S = 0.375). Note that the results for the pivot offset of 0.6 show negligible variation with frequency.

It can be seen that a pivot offset is effectively involved in a dynamic pad balancing relative to rotational squeezing of the fluid film. Given a normally converging film wedge, as shown in Figure 7, the smaller film thicknesses and the generally higher pressures in the trailing edge region of the pad are relatively more effective for developing a moment about the pivot. Thus, moving the pivot closer to the trailing edge (starting from a central pivot position) tends toward a dynamic "balance" for the pad, and a rotationally stationary pad position for dynamics. The resulting coupling between journal translation and pad rotation will go to zero at a particular pivot position α , as would frequency dependency. This depends, of course, on the specific geometry and operating conditions. These statements do not imply that any particular pad position is superior for all applications. Such decisions should be made by the designer. Though the centrally pivoted pad results of Table 1 are frequency-dependent, the higher level of stiffness and/or damping may be desirable for a given application.

Excitation Eroquance	Pivot Offse	t, $\alpha/\beta = 0.5$	Pivot Offset, $\alpha/\beta = 0.6$				
(cpm)	K _{eff} (N/mm)	B _{eff} (N-s/mm)	K _{eff} (N/mm)	B _{eff} (N-s/mm)			
50	1.296×10^{6}	1985	$7.203 imes 10^5$	1810			
1250	$1.258 imes 10^6$	2033	$7.204 imes10^5$	1810			
2500	1.154×10^{6}	2163	$7.206 imes 10^5$	1810			
3750	1.010×10^{6}	2342	$7.211 imes 10^5$	1809			
5000 (synch)	8.546×10^5	2537	$7.216 imes 10^5$	1809			
6250	7.055×10^{5}	2723	$7.220 imes 10^5$	1808			
7500	5.729×10^{5}	2888	$7.225 imes 10^5$	1808			
8750	$4.597 imes 10^5$	3030	$7.228 imes 10^5$	1807			
10,000	$3.651 imes 10^5$	3148	$7.231 imes 10^5$	1807			

[ab]	le	1.	Com	parison	of	effective	TPJB	pro	perties	for two	o pivot	positions	(S =	= 0.375	5)
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Figure 7. Pivot offset and pad dynamic balance.

5.3. Effect of Pivot Flexibility

This study has thus far focused on the fluid film alone, with the journal and pad solely providing boundaries for the film. This is principally to highlight the physical mechanism that underlines pad rotational damping as governing frequency dependency. Additional significant dependencies on frequency are likely to arise due to the flexibility of the pad pivots, a consideration that must be recognized as a requirement for comparison to experimental data. Pad bending flexibility has an additional but smaller effect on the dynamic properties.

The most often cited reference presenting a reasonable physical model for pivot flexibility is that by Kirk and Reedy [29]. This model is based on Hertzian contact theory. Several studies have claimed that Hertzian theory grossly overestimates pivot stiffness [26,30]. This study will not use the Hertzian model, but will simply consider pivot stiffness as a multiple of the film stiffness at synchronous frequency.

Including the pivot flexibility effects into the single-pad model is simply a matter of assigning the effective coefficients (K_{eff} and B_{eff}) of Equations (6) and (7) as K and B in the model of Equation (1), with the pivot stiffness assigned the value K_s . Pivot damping is not considered.

The results of the pivot stiffness extension to the single-pad model are shown in Figure 8 for a given condition of operation of the datum bearing pad. Effective stiffness and damping are displayed over various frequencies for a rigid pivot and five different pivot stiffnesses as related to the effective film-alone stiffness for synchronous operation. The results are plotted dimensionally to better illustrate the variations in both magnitude and distribution of the coefficients over frequency. The fundamental character and shape of the curves has been found to hold well for other operating conditions representative of industry applications (e.g., $S = 0.05 \rightarrow 0.5$).



Figure 8. Effect of pivot on effective (a) stiffness, (b) damping.

The figures show that decreasing the pivot stiffness also monotonically decreases the effective stiffness for frequencies below synchronous, as do the effective damping coefficients over the entire frequency range considered. All curves also tend to "flatten", thus moderating the frequency effects. The stiffness curve variations for frequencies above synchronous become more complex.

6. Full Bearing

Prior to this point in the study, the analyses have been centered on the single-pad model. The following results will involve multiple-pad analyses that are more representative of the bearings used in practice. However, to more clearly display the effects related to the film alone, pivot flexibility effects are ignored in this section. The related effects on the full bearing follow the trends, but not the magnitudes, found for a single pad.

The single-pad model, as represented by Equation (4), was extended to include multiple pads and both X and Y journal DOF. The dynamic coefficient matrices and the reduced dynamic coefficients have been computed based on a pad-assembly method similar to that described in [1,7,11]. This model is of the KC type [7,14], with each pad dynamically independent.

In a complete TPJB, no single pad is likely to experience the full-bearing load, and this load must be distributed among the multiple pads. Since each pad may be loaded differently, frequency dependency of the dynamic coefficients will also involve a dependency on pad circumferential position within the housing, and thus on the direction of vibratory motion (X,Y, see Figure 3). In general, the most heavily loaded pads will dominate frequency dependency in the full-load direction (X), while pads more closely aligned with the orthogonal coordinate (Y) will additionally influence frequency dependency for Y-directed motion.

The full-bearing plots in this paper display results for X-directed vibratory motion. Since the Y-directed motion is related to the more lightly loaded pads, these pads will operate with higher pad Sommerfeld numbers, resulting in differences in the frequency dependency associated with the pad dynamic coefficients. All of the full-bearing result plots use Sommerfeld numbers based on the full-bearing loads. The complete bearing cross-coupled coefficients, generally being quite small, have been ignored.

6.1. Effect of Preload

Preload for a tilting-pad bearing is an important parameter that has the potential to markedly affect the bearing dynamic properties. The preload parameter "m" is defined as:

1

$$n = 1 - \frac{C'}{C} \tag{9}$$

where *C* is termed the "machined" radial clearance, and *C*′ is the "assembled" radial clearance. Preload applied to a TPJB generally results in a stiffer bearing having less damping. The preload is applied by initially aligning the pads concentrically with the journal so that there is a constant clearance *C* between the pads and the journal. The pads are then moved radially inward until the clearance at the pivot positions becomes *C*′. Thus, a bearing having a preload parameter *m* of zero (C' = C) is considered to have zero preload.

Under the condition of zero preload, some pads will be unloaded, thus applying no forces to the journal, and both static and dynamic behavior of the bearing will depend only on the loaded pads. As preload is increased, the unloaded pads will begin to attain load, until all pads are loaded.

The calculations in this study show that application of preload causes the effective stiffness coefficient ratios to diverge from the synchronous results for lower loading (higher S), and at higher loads, preload becomes less effective. The effective damping ratio is also affected by preload, particularly at frequencies above synchronous, but to a lesser degree than stiffness.

6.2. Effect of Number of Pads

The computed results indicate that the coefficients of a full bearing are less likely to be affected by frequency excitation variations, especially subsynchronous, as the number of pads increases. The four-pad bearing tends to be more likely to exhibit frequency dependency than five- or six-pad designs. A reduction in the stiffness ratio at half synchronous frequency of about 6% is associated with a change from four to five pads. This is not entirely unexpected, since circumferentially shorter pads should have less propensity to couple rotationally with the journal displacements. The trend is similar for both LOP and LBP configurations, but is not as evident for higher frequencies.

7. Comparison to Experiment

Most published measurements on TPJB dynamic coefficients prior to about 2010 involve conditions that are typical of lightly-loaded high-speed applications, while similar measurements involving higher-load bearings as used by larger steam and gas turbines have not been as prominent. However, an investigation by Kulhanek [30], with results also shown by San Andres [22], provided impedance results for higher unit loading of a five-pad, load-between-pads bearing. It is clear from these studies that the measured results must include pivot flexibility. The aim of this comparison is to show that the general trending of the frequency results agrees reasonably with the experiment.

The bearing under consideration had a diameter of 101.6 mm, an aspect ratio (L/D) of 0.6, central pivots, a machined (cold) pad clearance of 112 μ m, and a relatively small preload ratio of 0.27. The lubricant was ISO VG32, and the specific load (W/LD) was 1.723 MPa for the comparisons considered. Detailed pivot characteristics were not included in either of the references.

Comparisons were made between impedance results measured experimentally and computed results using the full-bearing KC isothermal model code previously mentioned, but permitting film turbulence. Two different speeds were used for the comparisons, 7000 and 16,000 rpm. The average oil temperatures, required for the isothermal analyses, were taken to be 80 °C and 100 °C for the two respective speeds, based on the published pad temperature data, resulting in respective assumed constant viscosities of 6.7 and 4.1 cp.

Pivot flexibility was included in the computational results; however, since these data were not known but estimated for the test bearing, the current study considers three pivot stiffness values applied to all pads. Two of the pivot stiffnesses were taken relative to the effective film stiffness at synchronous speed, namely, $2 \times$ film stiffness and $5 \times$ film stiffness, and the last used a rigid pivot. All pads and both speeds used the same pivot stiffness value for the computations. This was based on a film stiffness value for synchronous conditions at 7000 rpm (585 MN/m).

The plots of Figures 9 and 10 show the real and imaginary parts of the bearing impedance as a function of the reduced frequency ratio. The real part of the impedance is the effective stiffness (K_{eff}) caused by joining the film and pivot. This is considered stiffness alone since a physical reason for including a mass effect does not exist (in general, the convective and temporal inertia of the film and pad radial and rotational inertias could be included, but these effects are typically not considered significant for the majority of bearings in industry). The imaginary part is the product of frequency and the effective damping coefficient (ωB_{eff}).



Figure 9. Comparison of impedance coefficient computation and test (7000 rpm) for (**a**) real part, (**b**) imaginary part.





The dashed lines indicate a rough fit to the measured data. The data extended only to synchronous frequency for the 16,000 rpm measurements. For these plots, the X coordinate is in the direction of the load, and is typically vertical, while the Y coordinate normally applies to horizontal motion.

The computational results for the real impedance coefficients ($\text{Re}(Z_x)$) at 7000 rpm, shown in Figure 9a, display the expected reduction in magnitude with increasing frequency when the pivots are rigid, when frequency dependency is entirely controlled by the film mechanics. As pivot flexibility increases, the frequency curves tend to flatten with the $2 \times$ film K curves, lining up well with experimental results. The related comparison for the magnitudes of the imaginary impedance coefficients of Figure 9b is also reasonably satisfactory.

At 16,000 rpm, the real impedance computed results for $2 \times$ film K of Figure 10a continue to agree well with the X coordinate measured results, but not as well with the Y results. Computation and measurement also do not agree well with the imaginary impedance results of Figure 10b. However, note that none of the imaginary impedance re-

sults, either measured or computed for either speed, show a linear variation with frequency. This implies that for these particular operating conditions for this bearing, the effective damping coefficients cannot be constant with frequency. Note also that, particularly for the 16,000 rpm imaginary impedance results, the behavior with frequency is different for the two journal coordinates. One must also consider that the computations do not account for all system properties, such as pad flexibility and damping due to material and pivot deformation.

The results for the 7000 rpm tests were found for a nominal Reynolds' number, based on clearance C, of 529, corresponding to laminar flow. The 16,000 rpm tests involved film turbulence at Re = 1914. It is possible, based on this higher Reynolds' number and the bearing clearance ratio, that film inertia effects could have significance. It is also possible that the differences can be attributed to thermal effects at the higher speed, giving rise to significant viscosity variations in the film and to thermal–mechanical deformations that affect clearances. Such effects cannot be captured using the present isothermal computational model. A final consideration may be that the below-synchronous frequencies at 16,000 rpm are generally higher than those at 7000 rpm, thus allowing pad inertial characteristics to be of greater significance over the broader speed range.

8. Discussion

The goal of this study has been to advance the general understanding of the role of tilting-pad bearings as elements in the operation and dynamic modeling of rotating machinery. The considerations discussed are most important for identification, in a computational vein, of the damped natural frequencies and modal damping parameters required for designing stable rotating machines, but this work will also guide experimentalists toward establishing trending behavior that may otherwise not be obvious.

The single-pad film model has been found convenient for study of the various design and operating parameters that affect the frequency dependency of a TPJB. The model shows that the film frequency dependency is strongly associated with the damping influences, and with rotational damping in particular. The equations derived for both K_{eff} and B_{eff} show functional dependence upon the square of the exciting frequency ω . This dependency is somewhat complicated, particularly for K_{eff} , and it does not appear possible to extract, in any exact manner, constant terms that could be perceived as an additional or virtual mass as used by some researchers.

While the mechanics of the fluid film alone tend to develop frequency dependency, the flexibility of the pivots tends to attenuate it. *This tendency to "flatten" the frequency effect curves due to pivot flexibility is potentially behind the inability of many experimentalists to uncover frequency effects.* Several experimentalists have claimed that TPJB dynamic properties, especially damping, are constant with frequency [17–19]. Such a conclusion cannot be drawn for general configurations based on experiments having little capacity to vary significant parameters (e.g., pivot flexibility). Additionally, increasing pivot flexibility will potentially result in greater prominence for the effects of pad inertia. This may be the case with the experimental results of Dang et al. [26] for a five-pad LOP configuration with spherical pivot that shows large variation in effective damping versus frequency.

One of the remaining related issues is the accuracy of the pivot stiffness computation. Current Hertzian contact theories tend to produce pivot stiffnesses greater than those measured [26]. A potential reason for this difference involves surface roughness for the pivot contact surfaces. Hertzian theory is based on linear elastic deformation for the ideally smooth surfaces of contacting bodies. Real surface contact involves smaller effective areas of contact relative to Hertzian results, because the deformations, both elastic and plastic, are primarily confined to the surface asperities. Along the same vein, real and Hertzian surfaces also differ in flexibility. Xi and Polycarpou [31] note that Hertzian-computed contact stiffness is roughly three times the actual stiffness at light loads, and the real stiffness is always less than the Hertzian result. This reference also shows that contact damping can be significant, but diminishes as load increases. Similar statements related to

rough surface contact deformation and stiffness are attributed to Jackson and Green [32] and Butt et al. [33].

It is worth mentioning that some past researchers have modeled the fluid film and pivot stiffness combination as springs in series. This is incorrect on two counts. Firstly, in this case, only *impedances* can be combined in such a manner, since the effective stiffness invariably includes damping contribution. Secondly, the springs in series assumption ignores the significant effect of the pad rotational dynamics.

The frequency dependencies of the effective coefficients for a TPJB are based primarily on the individual pad operating characteristics. The most heavily loaded pads are typically dominant. It is likely that a four- or five-pad bearing with LBP and light preload will have the coefficient frequency behavior controlled by the two loaded pads for both journal coordinates (X and Y). A five-pad bearing with LOP might be expected to exhibit different frequency behavior in the two journal coordinates.

Accurate modeling of a tilting-pad bearing system can be performed either by explicitly including the pad DOF in the model (i.e., the KC model), or by considering frequency-dependent stiffness and damping matrices. Approximate curve-fitting approaches (i.e., the KCM model) are also available for the description of experimental coefficient results. While useful in industry, these fitting methods are constrained to properties that are constant with frequency, including a "mass" term that attempts to account for the frequency variation in the measured impedance real part. The mathematical formulations of Equations (6) and (7) show this to be inaccurate, and defining such "mass" elements obscures the physics. Such curve-fitting schemes use the lower-degree frequency terms from a truncated Taylor series expansion. Experimentalists have shown no error estimation for such truncation.

The deviation of the nonsynchronous TPJB coefficients from the synchronous coefficients, particularly for $\Lambda = 0.5$, could be of concern for damped modal calculations and potentially for stability. Both the lowest damped natural frequency and the associated damping (log decrement) could be significantly inaccurate. This potential inaccuracy has been noted in several related studies (e.g., [16,21]). The accuracy issue may not be of concern for stability in some machines, because tilting-pad bearings do not inherently destabilize. However, if other destabilizing mechanisms happen to be present, the level of system damping may not be accurately assessed. Additionally, if accurate natural frequency determination is important, possibly for comparison to spectral data during system identification, errors in computed frequencies would make peak identity and separation difficult.

The charts showing the results of parameter variations are of value to bearing designers, not for direct use in design, but for understanding when frequency effects may be of importance. With this understanding, designers may be able to use the parameter charts to guide design processes that could potentially use frequency effects to enhance the dynamic stability of a machine.

Although most concern on the issue of frequency dependency in the literature has focused on half-frequency, frequencies above synchronous may often be of importance. Many phenomena in rotordynamics, such as misalignment, cracked rotors, or rotor–stator rubbing, may require consideration of higher frequencies, and TPJB modeling with synchronous coefficients in such cases would be technically unsupportable. Additionally, transient vibratory analysis involving TPJBs cannot generally be represented by synchronous coefficients since the response may contain arbitrary frequency content.

9. Conclusions

This study indicates that for physical understanding, for rotordynamic modeling purposes, and with concurrence of many researchers, tilting-pad bearings must be considered as inherently frequency-dependent components. Though some investigators claim constancy for damping, the dependency applies to both stiffness and damping, and both film mechanics and pivot flexibility share control. The support for these conclusions has grown-backed by experimental observation, but physical understanding has lagged. The principal physical reason for frequency dependency related to film mechanics is centered around the coupling between journal translation and pad rotation, and the associated damping, particularly pad rotational damping. Additionally, pivot flexibility generally provides significant reduction in both effective stiffness and damping, and also tends to provide a level of attenuation to the frequency variations, particularly for frequencies below synchronous.

Relative to the datum bearing configuration chosen for this study, the most significant results, particularly for the fluid film, found from the single-pad TPJB (rigid pivot) computational model are as follows:

- In general, the effective dynamic characteristics vary least from the synchronous coefficients at higher "loads" (lower Sommerfeld numbers), but differ significantly for lower levels of loading.
- Effective stiffness for below-synchronous frequencies is greater than synchronous stiffness, while effective damping is less. Results are reversed for above-synchronous frequencies.
- Circumferential position of the pivot provides the greatest variation in the effect of frequency over a wide range of Sommerfeld numbers. Mitigation or even elimination of frequency dependency at specific loads may be accomplished by judicial positioning of the pivot.
- Changes in bearing aspect ratio (L/D) do not strongly affect the nominal coefficient ratio results, and certainly not the trends.

Conclusions related to single-pad pivot flexibility are:

- Increasing pivot flexibility causes effective TPJB stiffness to reduce monotonically for below-synchronous frequencies, and over the entire frequency range considered for effective damping. Effective stiffness for above-synchronous frequencies is more complicated.
- Increasing pivot flexibility has the tendency to reduce variation or "flatten" all effective coefficient curves, particularly for synchronous frequencies and below.

The following conclusions apply to the multiple-pad, full-bearing results:

- Application of pad preload constrains the effective coefficients to remain closer to the synchronous values as the load decreases and the unloaded and lightly loaded pads begin to increase load proportion.
- Increasing the number of pads in the full bearing marginally reduces the sensitivity to frequency variation.
- Based on individual pad loading, frequency dependency will be different for each pad, resulting in frequency effects generally being different for the two journal degrees of freedom.
- To ensure accuracy for computed rotordynamics, particularly stability, it is suggested that analysts use TPJB dynamic property computations based on the pad-assembly technique (i.e., the KC method).

Author Contributions: L.F.W. performed the theoretical analysis and wrote the initial draft. P.E.A. checked the validity of the theory and revised the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not Applicable.

Informed Consent Statement: Not Applicable.

Data Availability Statement: Not Applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

- \overline{B} Effective equivalent damping ratio ($\overline{B} = B_{eff}/B_{\Omega}$)
- *B_{eff}* Effective damping coefficient
- B_{Ω} Effective damping coefficient at synchronous frequency
- C Set radial clearance or machined clearance
- C' Assembled radial clearance
- D Journal diameter
- i $\sqrt{-1}$
- \overline{K} Effective equivalent stiffness ratio ($\overline{K} = K_{eff}/K_{\Omega}$)
- K_{eff} Effective stiffness coefficient
- K_{Ω} Effective stiffness coefficient at synchronous frequency
- L Bearing axial length
- m Preload parameter (m = 1 C'/C)
- M Shaft mass
- N Rotational frequency (cyc/sec)
- R Journal radius
- Re Reynolds' number (Re = $\rho \Omega RC/\mu$)
- S Sommerfeld Number
- W Load on bearing
- X, Y Translational coordinates
- Z_{eff} Effective complex impedance ($Z_{eff} = K_{eff} + i\omega B_{eff}$)
- α Position of pivot relative to pad leading edge
- β Pad angular extent
- Λ Reduced frequency ratio ($\Lambda = ω/Ω$)
- μ Absolute viscosity
- θ Rotational coordinate
- ω Frequency of vibration (rad/sec)
- Ω Rotational frequency (rad/sec)
- DOF Degree of freedom
- KC TPJB model that explicitly includes individual pad DOF
- KCM TPJB model that uses constant values of stiffness, damping, and mass for a given operating condition
- LBP Load between pads
- LOP Load on pad
- OEM Original Equipment Manufacturer
- TPJB Tilting-pad journal bearing

References

- 1. Lund, J.W. Spring and Damping Coefficients for the Tilting-Pad Journal Bearing. ASLE Trans. 1964, 7, 342–352. [CrossRef]
- Hagg, A.C.; Sankey, G.O. Some Dynamic Properties of Oil-Film Journal Bearings with Reference to the Unbalance Vibration of Rotors. ASME J. Appl. Mech. 1958, 25, 141. [CrossRef]
- 3. American Petroleum Inst. *Axial and Radial Compressors and Expander-Compressors, Vibration Specifications,* 8th ed.; API STD 617; API Publishing Services: Washington, DC, USA, 2014.
- 4. Dimond, T.; Younan, A.; Allaire, P. A Review of Tilting Pad Bearing Theory. Int. J. Rotating Mach. 2011, 2011, 908469. [CrossRef]
- Warner, R.E.; Soler, A.I. Stability of Rotor-Bearing Systems with Generalized Support Flexibility and Damping and Aerodynamic Cross-Coupling. J. Lubr. Technol. 1975, 97, 461–469. [CrossRef]
- Parsell, J.K.; Allaire, P.E.; Barrett, L.E. Frequency Effects in Tilting-Pad Journal Bearing Dynamic Coefficients. A S L E Trans. 1983, 26, 222–227. [CrossRef]
- Rouch, K.E. Dynamics of Pivoted-Pad Journal Bearings, Including Pad Translation and Rotation Effects. A S L E Trans. 1983, 26, 102–109. [CrossRef]
- 8. Springer, H. Stiffness and Damping Characteristics of Tilting-Pad Journal Bearings. Dyn. Rotors 1984, 273, 45–64. [CrossRef]
- 9. Barrett, L.E.; Allaire, P.E.; Wilson, B.W. The Eigenvalue Dependence of Reduced Tilting Pad Bearing Stiffness and Damping Coefficients. *Tribol. Trans.* **1988**, *31*, 411–419. [CrossRef]
- 10. White, M.F.; Chan, S.H. The Subsynchronous Dynamic Behavior of Tilting-Pad Journal Bearings. J. Tribol. 1992, 114, 167–173. [CrossRef]
- 11. Allaire, P.; Parsell, J.; Barrett, L. A pad perturbation method for the dynamic coefficients of tilting-pad journal bearings. *Wear* **1981**, 72, 29–44. [CrossRef]

- Ha, H.C.; Yang, S.H. Excitation Frequency Effects on the Stiffness and Damping Coefficients of a Five-Pad Tilting Pad Journal Bearing. J. Tribol. 1999, 121, 517–522. [CrossRef]
- 13. Dmochowski, W. Dynamic Properties of Tilting-Pad Journal Bearings: Experimental and Theoretical Investigation of Frequency Effects due to Pivot Flexibility. J. Eng. Gas Turbines Power 2006, 129, 865–869. [CrossRef]
- Dimond, T.W.; Younan, A.A.; Allaire, P. Comparison of Tilting-Pad Journal Bearing Dynamic Full Coefficient and Reduced Order Models Using Modal Analysis (GT2009-60269). J. Vib. Acoust. 2010, 132, 051009. [CrossRef]
- 15. Schmied, J.; Fedorov, A.; Grigoriev, B.S. Non-Synchronous Tilting Pad Bearing Characteristics. In Proceedings of the 8th IFToMM International Conference on Rotor Dynamics, Seoul, Korea, 12–15 September 2010; pp. 143–148.
- Cloud, C.H.; Maslen, E.H.; Barrett, L.E. Influence of Tilting Pad Journal Bearing Models on Rotor Stability Estimation. In Proceedings of the 8th IFToMM International Conference on Rotordynamics, Seoul, Korea, 12–15 September 2010; pp. 94–102.
- Childs, D.W.; Carter, C.R. Rotordynamic Characteristics of a Five Pad, Rocker-Pivot, Tilting Pad Bearing in a Load-on-Pad Configuration; Comparisons to Predictions and Load-Between-Pad Results. *J. Eng. Gas Turbines Power* 2011, 133, 082503. [CrossRef]
- Childs, D.W.; Delgado, A.; Vannini, G. Tilting-Pad Bearings: Measured Frequency Characteristics of Their Rotordynamic Coefficients. In Proceedings of the 40th Turbomachinery Symposium, Houston, TX, USA, 12–15 September 2011.
- Delgado, A.; Vannini, G.; Ertas, B.; Drexel, M.; Naldi, L. Identification and Prediction of Force Coefficients in a Five-Pad and Four-Pad Tilting Pad Bearing for Load-on-Pad and Load-Between-Pad Configurations. *J. Eng. Gas Turbines Power* 2011, 133, 092503. [CrossRef]
- Delgado, A.; Librashi, M.; Vannini, G. Dynamic Characterization of Tilting Pad Journal Bearings from Component and System Level Testing. In Proceedings of the ASME Turbo Expo 2012: Turbine Technical Conference and Exposition, Copenhagen, Denmark, 11–15 June 2012; pp. 1007–1016. [CrossRef]
- 21. Wilkes, J.C.; Childs, D.W. Tilting Pad Journal Bearings—A Discussion on Stability Calculation, Frequency Dependence, and Pad and Pivot. J. Eng. Gas Turbines Power 2012, 134, 122508. [CrossRef]
- 22. San Andres, L.; Tao, Y.; Li, Y. Tilting Pad Journal Bearings: On Bridging the Hot Gap Between Experimental Results and Model Predictions. *ASME J. Eng. Gas Turbines Power* **2014**, 137, 2.
- Quintini, J.C.R.; Pineda, S.; Matute, J.A.; Medina, L.U.; Gómez, J.L.; Diaz, S.E. Determining the Effect of Bearing Clearance and Preload Uncertainties on Tilting Pad Bearings Rotordynamic Coefficients. In Proceedings of the ASME Turbo Expo 2014: Turbine Technical Conference and Exposition, Dusseldorf, Germany, 16–20 June 2014. [CrossRef]
- 24. Tschoepe, D.P.; Childs, D.W. Measurements Versus Predictions for the Static and Dynamic Characteristics of a Four-Pad, Rocker-Pivot, Tilting-Pad Journal Bearing. *J. Eng. Gas Turbines Power* **2014**, *136*, 052501. [CrossRef]
- Yang, Q.; Li, W.; Luo, J.; Zhao, W.; Zheng, X.; Wang, K.; Wang, J. Dynamics modeling and analysis for tilting-pad fluid film bearing considering perturbation frequency. In Proceedings of the 2015 International Conference on Fluid Power and Mechatronics (FPM), Harbin, China, 5–7 August 2015; pp. 1257–1262. [CrossRef]
- 26. Dang, P.V.; Chatterton, S.; Pennacchi, P. The Effect of the Pivot Stiffness on the Performances of Five-Pad Tilting Pad Bearings. *Lubricants* **2019**, *7*, 61. [CrossRef]
- 27. Pinkus, O.; Sternlicht, B.; Saibel, E. Theory of Hydrodynamic Lubrication. J. Appl. Mech. 1962, 29, 221–222. [CrossRef]
- Adams, M.L., Jr. Bearing and Seal Rotor Dynamics, Rotating Machinery Vibration, From Analysis to Troubleshooting, 2nd ed.; CRC Press, Inc.: Boca Raton, FL, USA, 2010.
- Kirk, R.G.; Reedy, S.W. Evaluation of Pivot Stiffness for Typical Tilting-Pad Journal Bearing Designs. J. Vib. Acoust. 1988, 110, 165–171. [CrossRef]
- Kulhanek, C.D.; Childs, D.W. Measured Static and Rotordynamic Coefficient Results for a Rocker-Pivot, Tilting-Pad Bearing With 50 and 60% Offsets. J. Eng. Gas Turbines Power 2012, 134, 052505. [CrossRef]
- Shi, X.; Polycarpou, A.A. Measurement and Modeling of Normal Contact Stiffness and Contact Damping at the Meso Scale. J. Vib. Acoust. 2005, 127, 52–60. [CrossRef]
- 32. Jackson, R.L.; Green, I. A statistical model of elasto-plastic asperity contact between rough surfaces. *Tribol. Int.* **2006**, *39*, 906–914. [CrossRef]
- Butt, S.U.; Antoine, J.-F.; Martin, P. Simplified stiffness model for spherical rough contacts. *Tribol. Mater. Surf. Interfaces* 2015, 9, 63–70. [CrossRef]