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Simplified Vibration PSD Synthesis Method for MIL-STD-810

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Featured Application: In MIL-STD-810, a DP (damage potential) formula, a concise version of FDS (fatigue damage spectrum) formula, is used to synthesize vibration conditions. In this paper, a more simplified formula that yields the same synthesis results as the DP formula is proposed.

Abstract: In MIL-STD-810, the environmental engineering consideration and laboratory tests by US military, the DP (damage potential) formula is used to calculate the fatigue damage by vibration environments which is a simplified version of a FDS (fatigue damage spectrum) formula. DP, however, was originally made for comparison between different test standards and is not an optimized formula for the synthesis of vibration environments. This paper presents the GRS (Gaussian Random Synthesis) formula for only vibration test synthesis featuring simplified DP formula and produce the same result as DP. Although the GRS provides insight into the synthesis of vibration tests, it must be used with care because it inherits the constraints of the original FDS formula.

Keywords: fatigue damage spectrum (FDS); damage potential (DP); Gaussian random synthesis (GRS); power spectral density (PSD); MIL-STD-810

1. Introduction

Products must perform adequately under all environmental conditions associated with its service life. Laboratory environmental tests are used extensively instead of more time-consuming and less cost effective field exposure tests. The MIL-STD-810, Environmental Engineering Considerations and Laboratory Tests, establishes laboratory test methods that replicate the effects of environments on the equipment rather than imitating the environments themselves. Vibration test among dozens of environmental test methods are also performed to verify that materiel will function in and withstand the vibration exposures of a life cycle. The most dynamically accurate method to reproduce the full exposure would be to sequentially vibrate the product to all the individual, uncompressed events representing its full lifecycle. However, such an approach is generally not feasible from both schedule and economic perspectives, and some compromises must be made to realize the benefits of testing in the laboratory. Time compression techniques based on fatigue equivalency are typically employed such that vibration testing can be performed in a timely and economic manner. The objective of the laboratory vibration test is to generate the equivalent fatigue damage accumulated in the products by vibration throughout their lifetime [1].

This fatigue damage by vibration can be quantified by the fatigue damage spectrum (FDS) methods which is a spectral representation of fatigue damage as a function of the SDOF system's natural frequency [2]. The FDS can be computed in time domain [3] and in frequency domain [4–6]. In frequency domain, the formula calculating FDS can be obtained based on some assumptions. In case of a low damping SDOF system and Gaussian random input, in which the peak distribution of the response becomes a Rayleigh one, the FDS equation gets much simpler [7]. By simplifying this further, Henderson and Piersol proposed a descriptor for comparing random test conditions and named it as damage potential (DP) [7]. MIL-STD-810 uses this damage potential formula for synthesizing random vibration conditions. The purpose of DP is not to calculate the accurate fatigue damage, but to compare the relative fatigue damage between test specifications. However, as the synthesized vibration condition using DP is the same as using FDS, it is more efficient to use DP which is a much simpler formula. However, DP is not an optimized index for vibration condition synthesis. In this paper, we propose the slimmest formula in which unnecessary factors are removed from the DP to suit the purpose of vibration condition synthesis. This simplified equation, called GRS (Gaussian Random Synthesis), produce the same results as DP in vibration synthesis application only.

To introduce GRS, we first examine the concept of FDS thoroughly in time domain and how the FDS formula, which is the basis of DP, was derived in frequency domain, and then look at how to synthesize random vibration conditions under Miner's cumulative damage law in MIL-STD-810. Then the more simplified formula, GRS that is newly presented in this paper is derived. Actually, it reveals a concise expression in the process of composing the formula of itself. Finally, we present an example of synthesis using FDS, DP, and the new GRS equations and discuss the results.

GRS formula aims just to present a more convenient method for those who wish to create vibration test conditions by the method introduced in MIL-STD-810. Therefore, the constraints and problems that the method has inherited must be checked as well.

2. Fatigue Damage Spectrum (FDS)

2.1. FDS in General

Fatigue Damage Spectrum represents the cumulative damage due to the random responses of a series of Single Degree of Freedom (SDOF) systems, with varying natural frequency, excited by random acceleration, both in the time and frequency domain [5]. So, the calculation of FDS is based on the response of SDOF system, which is the virtual specimen, to the input vibration. It has been shown that the velocity response is proportional to stress [7,8]. Once a stress time history is given, a stress histogram can be derived by the various counting methods and then damage index can be calculated for each stress level in case the slope of S-N (stress–number) curve in log–log plot is given and finally the total damage can be simply summed thanks to the cumulative damage law. The FDS of the frequency range of interest is derived by changing the resonant frequency of the virtual specimen and repeating the entire process as shown in Figure 1.

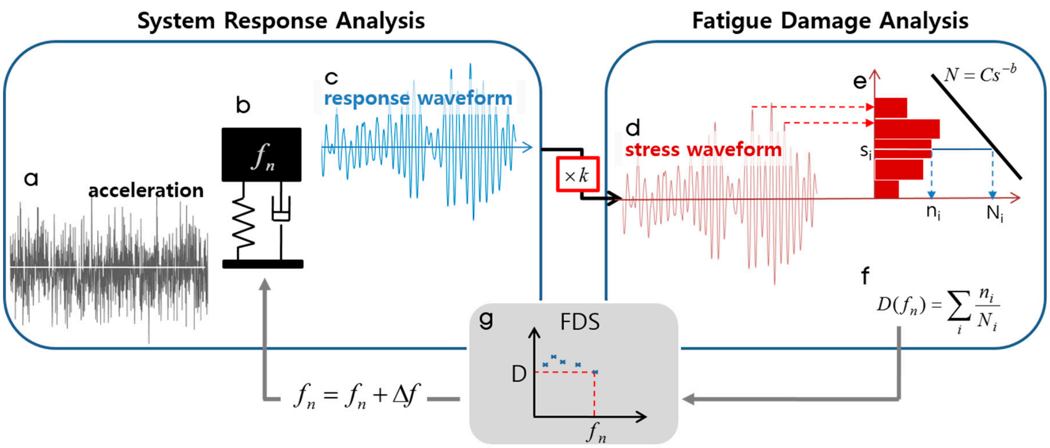

Figure 1. Flowchart of fatigue damage spectrum (FDS) calculations, (**a**) input acceleration, (**b**) Single Degree of Freedom (SDOF) system which is the virtual specimen, (**c**) velocity response, (**d**) stress waveform which is obtained by multiplying k to the velocity response, (**e**) stress counting and idealized stress–number (S-N) curve, (**f**) accumulation of damage index, (**g**) FDS.

The concept of FDS was explained in time domain above, but a lot of researchers calculate FDS in frequency domain. The paper cited in many FDS literature is that of Miles [9]. This is an article on the structural fatigue of aircraft, which describes the basic concept well, but the input is not vibrations but aerodynamic forces. The most extensive book on vibration fatigue is the Lalanne's five-volume books [5], which presents generalized formulas for accurate calculations and simplified equation based on some assumptions. McNeil described the application of FDS based on the pseudo-velocity response of the SDOF systems [3]. Halfpenny et al. discussed FDS to represent the cumulative fatigue damage caused by relative displacement response not velocity or pseudo velocity [4]. MIL-STD-810 describes the synthesis process of vibration conditions in detail from 'G w/Change1' version published in 2014 [1]. The DP (Damage Potential) formula used in the standard is from the paper by Henderson and Piersol [7]. They simplified the FDS formula and proposed the DP formula to compare the power spectral density and test duration of different excitation test specifications in terms of fatigue damage [7]. In case of the NATO environmental test specification, AECTP-200 (2006) [10] uses Lalanne's formula and AECTP-240 (2009) [2] uses Henderson and Piersol's Damage Potential.

2.2. FDS in MIL-STD-810

There are various formulas calculating FDS, but here we thoroughly review the derivation process of the DP formula. As a special case, if the input is a random waveform featuring (1) Gaussian distribution and (2) constant PSD, and also (3) the damping coefficient of the SDOF system is low ($\zeta < 0.1$), then the response of the system is also normally distributed random waveform and the peak distribution shows a Rayleigh one [11] as depicted in Figure 2.

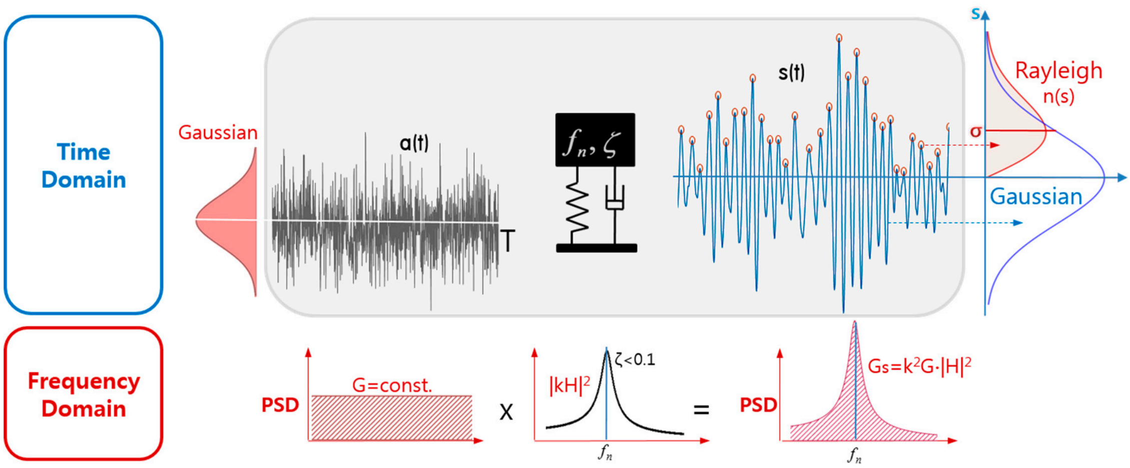

Figure 2. Velocity response of the SDOF system to Gaussian random acceleration input with constant power spectral density (PSD) shows Gaussian distribution. The peak distribution of the response becomes a Rayleigh function. The standard deviation of the response distribution matches the scale parameter of the Rayleigh function. a(t) is input acceleration and s(t) is stress waveform (above). Stress PSD is calculated by multiplying input PSD and squared stress amplitude of system's frequency response function (below).

This peak distribution corresponds to the histogram e in Figure 1, and is represented by the following equation.

$$n(s) = f_n T \frac{s}{\sigma^2} \exp\left(-\frac{s^2}{2\sigma^2}\right) = f_n T \cdot p(s) \tag{1}$$

$p(s)$ is the probability density function of the Rayleigh distribution, where σ is the scale parameter, which is equal to the standard deviation from the response waveform. Assuming an SDOF system with a low damping coefficient, the response waveform oscillates based on its natural frequency fn, so the total number of peaks during time T can be approximated by the product of fn and T. Multiplying the Rayleigh function by the total number of peaks yields the distribution function of peaks over stress, $n(s)$. The standard deviation, σ, of the normally distributed output random waveform is squared root of the area of the stress response PSD, $G_s(f)$, which can be obtained by multiplying the input PSD with the squared amplitude of the system's frequency response function (FRF), $H(f)$, in the frequency domain as shown in Equation (2). G is set to be a constant to enable the integration and k is conversion factor from velocity to stress. H is the FRF of pseudo velocity output to the acceleration input. These FRF relationships are well documented in ISO 18431-4 [12].

$$\sigma^2 = \int_0^\infty G_s(f)df = \int_0^\infty k^2 G \cdot |H(f)|^2 df = k^2 G \int_0^\infty |H(f)|^2 df = \left(\frac{k}{2\pi f_n}\right)^2 \frac{G\pi f_n}{4\varsigma} \tag{2}$$

$$|H(f)|^2 = \left|\frac{pv}{a}\right|^2 = \frac{(1/2\pi f_n)^2}{\left(1-(f/f_n)^2\right)^2 + (2\zeta f/f_n)^2} \tag{3}$$

Since $n(s)$ has been completely defined, we calculate the damage index by stress and add them all up to get the cumulative damage index D with Equation (4). This corresponds to step f in Figure 1. Here, the Basquin's Equation (5) is used to represent $N(s)$ which is the number to failure.

$$D(f_n) = \int_0^\infty \frac{n(s)}{N(s)} ds = \int_0^\infty \frac{f_n T \cdot p(s)}{Cs^{-b}} ds = \frac{f_n T}{C} \int_0^\infty p(s) s^b ds \tag{4}$$

$$N(s) = Cs^{-b} \tag{5}$$

The last integral term in Equation (4) is the b-th moment of the Rayleigh function, $p(s)$, and can be expressed using the Gamma function [13] as below.

$$\int_0^\infty p(s) s^b ds = \sigma^2 2^{b/2} \Gamma[1+b/2] \tag{6}$$

Applying the results of Equations (2) and (6) to Equation (4) yields Equation (7).

$$D(f_n) = \frac{f_n T}{C} \left[\frac{G(f_n) k^2}{8\pi f_n \zeta}\right]^{b/2} \Gamma[1+b/2] \tag{7}$$

The equation above is a formula for calculating the fatigue damage index accumulated in a lightly damped SDOF system excited by Gaussian random vibration, and it can be called the FDS equation.

The constant terms are collected in Equation (7) as follows,

$$D(f_n) = \frac{k^b}{C(8\pi)^{b/2}} \Gamma[1+b/2] \cdot f_n T \left[\frac{G(f_n)}{f_n \zeta}\right]^{b/2} = const \cdot f_n T \left[\frac{G(f_n)}{f_n \zeta}\right]^{b/2} \tag{8}$$

DP is extracted from Equation (8) and defined as follows.

$$DP(f_n) = f_n T \left[\frac{G(f_n)}{f_n \zeta}\right]^{b/2} \tag{9}$$

Since the DP evolves from a proportional relationship with fatigue damage index, the acceleration PSD can be in the more common units of g^2/Hz. And, the value is not bounded by unity, as is the actual damage D estimated by Equation (7) [7].

3. Vibration PSD Synthesis

3.1. Vibration PSD Synthesis in MIL-STD-810

MIL-STD-810 states that since fatigue damage is based on a cumulative effect of various environments or conditions, a cumulative fatigue damage index can be calculated as the sum of the fatigue damage spectra for individual environments [1]. Thus

$$DP_{total}(f_n) = DP_1 + DP_2 + \ldots + DP_N = \sum_{i=1}^{N} DP_i(f_n) \tag{10}$$

The individual damage indexes, DP_i, obtained by Equation (9) are cumulated by Equation (10) to generate the total damage index DP_{total}.

Then the PSD is derived by Equation (11) which is the inverse form of Equation (9).

$$G_{test}(f_n) = f_n\zeta\left[\frac{DP_{total}(f_n)}{f_n T_{test}}\right]^{2/b} \tag{11}$$

Equation (11) reads that in order to apply damage, DPtotal, in Ttest seconds to the virtual specimen, the PSD must be G_{test}.

In addition to Equation (9), the damage index can be calculated by various methods as discussed in Section 2. The important thing is that the formula calculating FDS and the inverse formula calculating PSD must be consistent. If the Equation (7) is used instead to calculate FDS, the PSD for testing shall be obtained using the following formula. The 'const' is the same as used in Equation (8).

$$G_{test}(f_n) = \frac{8\pi f_n\zeta}{k^2}\left[\frac{D_{total}(f_n)\cdot C}{f_n T_{test}\cdot\Gamma[1+b/2]}\right]^{b/2} = f_n\zeta\left[\frac{D(f_n)}{const\cdot f_n T}\right]^{2/b} \tag{12}$$

In order to create a test specification, in summary, by synthesizing vibration environment conditions according to the MIL-STD-810, calculate each damage index with the Equation (9) and add them to calculate the cumulative damage index, and then substitute it into the Equation (11) along with the test time T. T and G will be adjusted to prevent the G from becoming too large or the test time from being unrealistically extended.

3.2. Vibration PSD Synthesis Made Simpler

Originally, the concept of damage potential (DP) was proposed to describe the potential damage delivered to the virtual test items, which are the SDOF systems with different natural frequencies, by random vibration test [7]. So, DP itself is meaningful for comparison purpose. In the combination process, however, DP is an intermediate process, not a final output. Therefore, the Equation (9) that produce the individual DPs and the Equation (10) that combines them is substituted into the Equation (11) that calculates the test PSD, which can be summarized as follows.

$$G_{test}(f_n) = f_n\zeta\left[\frac{DP_{total}(f_n)}{f_n T_{test}}\right]^{\frac{2}{b}} = f_n\zeta\left[\sum_{i=1}^{N}\frac{T_i}{T_{test}}\left(\frac{G_i(f_n)}{f_n\zeta}\right)^{b/2}\right]^{2/b} = \left(\sum_{i=1}^{N}\frac{T_i(G_i(f_n))^{b/2}}{T_{test}}\right)^{2/b} \tag{13}$$

This formula can be rewritten as follows to look simple,

$$\text{for each } f_n, G_{test} = \left(\sum_{i=1}^{N}\frac{T_i G_i^{b/2}}{T_{test}}\right)^{2/b} = \left(\frac{T_1 G_1^{b/2} + T_2 G_2^{b/2} + \ldots + T_N G_N^{b/2}}{T_{test}}\right)^{2/b} \tag{14}$$

This can be rearranged as follows in a symmetric form which shows the ultimately simple relationship between field conditions and test condition in fatigue damage perspective when input PSDs are Gaussian waveform.

$$T_{test}G_{test}^{b/2} = T_1G_1^{b/2} + T_2G_2^{b/2} + \ldots + T_NG_N^{b/2} \tag{15}$$

Equations (15) shows that we do not need natural frequency and damping ratio in combining the PSDs. Now we can remove the unnecessary parameters and define a new index for only synthesizing process as follows and named it as Gaussian random synthesis (GRS) to emphasize its inherent constraints and usage at the same time,

$$GRS(f_n) = T{\cdot}G(f_n)^{b/2} \tag{16}$$

The inverse form is

$$G(f_n) = \left(\frac{GRS(f_n)}{T}\right)^{2/b} \tag{17}$$

The same results as Equations (14) and (15) are obtained from other FDS equation. The nCode, fatigue and durability analysis software, uses the FDS formula under the assumption that the relative displacement response instead of the velocity response of the virtual specimen is proportional to stress [4]. The equation below shows the process that the constant terms are collected in the FDS as in Equation (8) when DP is obtained.

$$D(f_n) = f_nT \cdot \frac{k^b}{C}\left[\frac{Q{\cdot}G(f_n)}{2(2\pi f_n)^3}\right]^{b/2}\Gamma\left[1+\frac{b}{2}\right] = const{\cdot}f_nT\left[\frac{Q{\cdot}G(f_n)}{{f_n}^3}\right]^{b/2} \tag{18}$$

Here, $Q = 1/2\zeta$, The inverse of Equation (18) calculating the test PSD is

$$G_{test}(f_n) = \frac{{f_n}^3}{Q}\left[\frac{DP_{total}(f_n)}{const{\cdot}f_nT_{test}}\right]^{\frac{2}{b}} = \frac{{f_n}^3}{Q}\left[\sum_{i=1}^{N}\frac{T_i}{T_{test}}\left(\frac{Q{\cdot}G_i(f_n)}{{f_n}^3}\right)^{\frac{2}{b}}\right]^{\frac{2}{b}} = \left(\sum_{i=1}^{N}\frac{T_i(G_i(f_n))^{\frac{2}{b}}}{T_{test}}\right)^{\frac{2}{b}} \tag{19}$$

The last expression is same as Equation (14) again which means we would get the same synthesized PSD using both FDS formula based on different FRF.

Table 1 is a simple example synthesizing two PSDs by using FDS based on pseudo velocity, DP, and GRS. The frequency is fixed at 300 Hz, and b = 4, ζ = 0.05, C = 1, and k = 1 is given then the intermediate calculation results are different but the final test PSDs to generate the equivalent fatigue damage index for 10 min test time are exactly the same, 30 (m/s^2)^2/Hz.

Table 1. PSD synthesis with FDS, damage potential (DP), and GRS [1].

Events				Calculations		
Event	**Frequency**	**Time**	**PSD**	**FDS Equation (7)**	**DP Equation (9)**	**GRS Equation (16)**
1	300 Hz	2 h	5 (m/s^2) ^{2}/Hz	7.6×10^2	2.4×10^5	1.8×10^5
2	300 Hz	1 h	10 (m/s^2) ^{2}/Hz	1.5×10^3	9.6×10^5	3.6×10^5
Synthesis		SUM (Event 1 + 2)		2.3×10^3	12.0×10^5	5.4×10^5
		test PSD for test T = 10 min		Equation (12) 30 (m/s^2) ^{2}/Hz	Equation (11) 30 (m/s^2) ^{2}/Hz	Equation (17) 30 (m/s^2) ^{2}/Hz

[1] b = 4, ζ = 0.05.

It should be noted that the intermediate calculation values have no physical meanings and that the unit of the test PSD follows the unit of the input PSDs. Equation (15) is the best representation showing these characteristics in synthesis.

4. Conclusions

In this paper, the DP formula used to synthesize vibration test conditions in MIL-STD-810 was further simplified and GRS formula which is the most concise form for vibration synthesis is proposed. GRS is used under the assumption that vibration conditions are Gaussian random, constant PSD, and the response system has low damping ratio (less than 0.1).

DP is presented as a descriptor for comparing random test conditions in their fatigue damage potential perspective, which MIL-STD-810 uses to synthesize a Gaussian random vibration environment. The results of combining PSDs with DP are no different from the results by DP's original FDS formula. But in the same context, the same synthesis results can be obtained with the simplified formula, GRS. The numerical verification was done through simple example.

However, both DP and GRS methods are subject to the same constraints as of the original FDS as mentioned. The assumption that the damping coefficient is less than 0.1 when synthesizing vibration using DP or GRS in MIL-STD-810 is valid if the material of the component considering fatigue damage is metal. However, the effectiveness of this method should be reconsidered for synthetic resins with high damping coefficients for example. In addition, it is not valid when the input vibration includes a lot of shocks and is not Gaussian distribution. Otherwise, if the DP can be used, it is efficient to use the simplified GRS formula instead for synthesis of random vibration condition. Lastly, it is emphasized that it would be helpful to have an intuitive understanding of the relationship between PSDs through Equation (15).

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Nomenclature

$n(s)$	peak distribution	a	acceleration
f_n	natural frequency	$N(s)$	Number of stress cycles to failure
T	duration	b	fatigue parameter
s	stress	C	constant of proportionality
σ	standard deviation	Γ	gamma function
$p(s)$	Rayleigh distribution function	D	cumulative damage index
G_s	PSD of stress	DP	damage potential
G	PSD of input acceleration	T_i, G_i	exposure time and PSD of event i
k	proportional constant	$Ttest$	exposure time for test
H	frequency response function	$Gtest$	PSD for test
ζ	damping ratio	Q	amplification factor
pv	pseudo-velocity	GRS	Gaussian random synthesis

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