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The Effect of Dead-Time and Damping Ratio on the Relative Performance of MPC and PID on Second Order Systems

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Abstract: Most industrial processes are regulated using PID control. However, many such processes often operate far from optimally because PID may not be the most suitable control method. Moreover, second-order models represent a large class of all controlled systems. This work studies the performance of some commonly used industrial PID controllers relative to MPC to understand when it is more suitable to use Model predictive control. MPC is used for this comparison because it has been the most successful industrial controller after PID. It can be concluded from the studies that improved performance can be achieved with MPC, even for modest dead time and when the damping ratio is relatively low. These improvements are prominent for dead-time dominant systems, whose dead-time to time-constant ratio is at least three.

Keywords: Model predictive control (MPC); PID; second order system; SOPDT; control performance

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1. Introduction

Proportional-Derivative-Integral (PID) control is the most popular and typically the default controller of choice by most practitioners for most industrial processes. This is especially true for Single-Input, Single-Output (SISO) processes. As such, advanced control schemes, such as MPC, are generally reserved for large Multi-Input Multi-Output (MIMO) systems. To justify the default adoption of PID control, there is often the argument that PID's popularity is due to its simplicity and ability to regulate most processes with a decent performance. However, some studies have shown that many industrial loops controlled using PID operate far from optimally [1,2]. This observation could be due to practitioners' default choice of PID, even on non-suitable processes. Therefore, we believe that advanced control could benefit low-performing loops if applied to suitable processes, even in the SISO case.

Model predictive control (MPC) is the most successful industrial advanced control scheme. The success of MPC has been attributed to factors, such as its inherent ability to deal with constraints, handle process interaction and accommodate dead time. Therefore, MPC is a good replacement when PID fails to give good performance. Much work has been done to propose MPC as a replacement for PID control [3–6]. Moreover, MPC has demonstrated improved performance for a certain class of SISO systems, such as dead-time dominant systems [7–9].

A significant of processes can be approximated using second-order models. Hence, the behavior of the Second Order Plus Deadtime (SOPDT) system models is crucial in studying control systems performance. In the control of SOPDT systems, two parameters affecting system response and performance are the process dead time and the damping ratio. Therefore, studying the effects of varying these two parameters with different control techniques could give insight into which control scheme is more suitable for specific parameter values.

Earlier works have focused on studying the relative performance of PID and MPC due to a single process parameter. For example, [7], PID and MPC are compared for different dead-time to time-constant ratio values. The study demonstrated that MPC outperformed PID in dead-time dominant systems, even in the SISO, non-constrained case. Similarly, it was demonstrated in [8] that, even for small dead-times when the process model was well defined, using a complex controller, such as MPC, was justifiable. However, it was argued that PID equipped with anti-windup for robust implementations provided comparable or superior performance. Salem [10] tuned PID controllers for five different systems (first, second, third, fourth and fifth order) using Bacterial Foraging Optimization (BFO) and compared them with results obtained using MPC. The work demonstrated improved performance with MPC in all the considered case studies. However, the results could not be generalized for any of the class of systems considered as the case studies used do not capture the entire spectrum of system model parameters. In [11], the relative performance of MPC and PID in Continuous Stirred Tank Reactor (CSTR) process control was studied. In this application, MPC improved in maintaining the reactor temperature at a predefined setpoint. The success of MPC was attributed to its anticipatory capability of MPC due to its inherent prediction capabilities.

Despite the recent adoption of MPC in diverse fields of application, it is still more prevalent in the process and industries. An area where MPC is finding more applications is the field of robotics and biomedical engineering. In [12], the performance of PID and MPC to control upper limb rehabilitation robots was studied. The results demonstrated comparable results for the two controllers. However, PID outperformed MPC with the introduction of disturbances. But the PID controller suffered some setbacks due to noise, which was effectively handled by the MPC using Kalman filter. In another work [13], PI, LQR and MPC controllers were applied to a laboratory-scale inverted pendulum. The results for the three controllers also demonstrated comparable performance. Because of the fast dynamics of the considered plant, the explicit form of MPC was used, and input constraints were accounted for in the formulation. The obtained results are expected since unconstrained MPC could be considered a form of LQR or state feedback control [14]. Similarly, the trio was compared in the control of a UAV in [15]. Considering system stability and robustness in this application, MPC outperformed LQR, which outperformed PID in simulation and real-time implementations on a Parrot Mambo Mini drone. Moreover, the tuning of LQR, in which stability is guaranteed by solving the algebraic Riccati equation, is less strenuous than PID. Additionally, LQR outperformed MPC in terms of setpoint tracking. Additionally, MPC tuning was also very strenuous for the UAV. Also, the computational requirements of MPC limited the choice of prediction horizon and sample interval. However, MPC proved very useful in accommodating control input and control rate constraints.

Most of the existing studies either compared the controllers of interest on a single process or a subset of processes that do not allow for a generalization of the obtained solutions. However, in [16], five different PID controllers were compared while varying the dead-time-to-time constant ratio and the damping ratio of second-order system models. This variation of parameters allowed for the comparison of controllers for a broad class that could generalize the obtained results. Three controllers were identified to perform better for different values of damping ratio and dead-time to-time-constant ratio. These are the internal model control (IMC) based PID proposed by Chien (IMC—Chien) [17], IMC with Maclaurin approximation of the dead-time (IMC—Mac) [18,19] and PID with specified closed-loop performance (CS—PID) [20]. Despite the widespread application of PID and thousands of publications on its tuning, there is still the argument that PID tuning is poorly understood [21]. This is evident by the continuous research in the field, the inaccessibility of tuning rules (often hidden in proprietary material), non-uniform notation and no uniformly agreed controller structure. However, the three controllers adopted in [17] are applied widely across the industry, making them suitable for this work. They have also been shown to give good performances.

This work adopts the approach of [16]. It compares the performance of the best of the three PID controllers highlighted earlier (IMC—Chien, IMC—Mac and CS—PID) with systematically tuned MPC controllers. Therefore, the approach will give insights into the best choice of controller for regulating the process. Hence, this work aims to determine when using either a PID or MPC on a typical second-order system, represented by (4), based on dead-time and damping ratio is more appropriate. In view of the above, the contributions of the paper could be highlighted as follows:

1. The paper gives a systematic comparative study of industrially relevant PID controllers with an MPC formulated to simplify its tuning on SOPDT systems while considering the effects of dead time and damping. To the author's knowledge, such a study has not been undertaken before.
2. The development of guidance on the more appropriate controller to use (either PID or MPC) on second-order systems based on the combined effects of dead time and damping.
3. A departure from most works on such comparisons, this work focuses on using more realistic PID formulations routinely applied to real industrial processes. Moreover, the comparison is carried out with the best of the three PID controllers for any of the considered system models.

These results are important because second-order systems represent a good percentage of controlled systems. This is true because SOPDT transfer functions can adequately represent most systems. The paper outline is as follows. Section 2 presents the methodology employed in this research. The results are presented and discussed in Sections 3 and 4, respectively. Finally, the paper is concluded in Section 5.

2. Materials and Methods

2.1. PID Control

Over the years, several PID controller structures and tuning rules have been proposed. Three such structures that are commonly applied in the industry are given in Equations (1)–(3):

$$PID_1 = K_c\left(1 + \frac{1}{\tau_I s}\right)\left(\frac{\tau_D s + 1}{\alpha\tau_D s + 1}\right) \quad (1)$$

$$PID_2 = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s + 1}\right) \quad (2)$$

$$PID_3 = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s + 1}\right)\left(\frac{1}{\tau_f s + 1}\right) \quad (3)$$

where K_c represents the proportional gain; τ_I is the integral time; τ_D is the derivative time; and τ_f is the PID filter time constant. The variable alpha is a constant with a reasonable value of $\alpha = 0.1$. In this work, we adopt the process model defined by Equation (4):

$$G(s) = \frac{Ke^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1} \quad (4)$$

where K is the process gain; τ is the time constant; ζ is the damping ratio; and θ represents the dead time. For overdamped systems, let $-\tau_{P1}$ and $-\tau_{P2}$ represent the system poles such that $\tau_{P1} > \tau_{P2}$. Some examples of well-known second-order systems include armature-controlled DC motors, Servo Systems, Automotive Suspension Systems, pneumatic pressure Systems, Liquid level models, thermal wells and other heating systems, and hydraulic systems.

Details of the three PID controllers adopted in this study are given in [7]. However, we present a summary of the three PID controllers for completeness.

2.1.1. IMC—Chien

Chien [17] proposed a PID controller for SOPDT processes with the parameters presented in Equations (5) and (6) for underdamped and overdamped systems, respectively:

$$K_C = \frac{2\xi\tau}{K_p(\lambda+\theta)},\ \tau_I = 2\xi\tau,\ \tau_D = \frac{\tau}{2\xi} \quad (5)$$

$$K_C = \frac{\tau_{P1}+\tau_{P2}}{K_p(\lambda+\theta)},\ \tau_I = \tau_{P1}+\tau_{P2} \text{ and } \tau_D = \frac{\tau_{P1}\tau_{P2}}{\tau_{P1}+\tau_{P2}} \quad (6)$$

2.1.2. IMC—Mac

The PID with Maclaurin series approximation was proposed in [18,19]. IMC—Mac uses the parallel structure PID structure given in Equation (3), with the PID parameters defined as in Table 1:

Table 1. PID Parameters for IMC—Mac.

Process	K_C	τ_I	τ_D
Underdamped	$\frac{\tau_I}{K_p(2\lambda+\theta)}$	$2\xi\tau - \frac{2\lambda^2-\theta^2}{2(2\lambda+\theta)}$	$\tau_I - 2\xi\tau + \frac{\tau^2 - \frac{\theta^3}{6(2\lambda+\theta)}}{\tau_I}$
Critically damped	$\frac{\tau_I}{K_p(2\lambda+\theta)}$	$2\tau - \frac{2\lambda^2-\theta^2}{2(2\lambda+\theta)}$	$\tau_I - 2\tau + \frac{\tau^2 - \frac{\theta^3}{6(2\lambda+\theta)}}{\tau_I}$
Overdamped	$\frac{\tau_I}{K_p(2\lambda+\theta)}$	$(\tau_{P1}+\tau_{P2}) - \frac{2\lambda^2-\theta^2}{2(2\lambda+\theta)}$	$\tau_I - (\tau_{P1}+\tau_{P2}) + \frac{\tau_{P1}\tau_{P2} - \frac{\theta^3}{6(2\lambda+\theta)}}{\tau_I}$

2.1.3. Closed-Loop Specified PID

This PID tuning method was proposed in [20]. For the under-damped case, the PID has the structure of K_3 presented in Equation (3), with the following parameters:

$$K_C = \frac{\xi\tau}{K_p\theta},\ \tau_I = 2\tau\xi,\ \tau_D = \frac{\tau}{2\xi},\ \text{and } \tau_f = \alpha\tau_D \quad (7)$$

For the over-damped case, the structure of K_1 in Equation (1) with the following parameters is used:

$$K_C = \frac{\tau_{P1}}{2K_p\theta},\ \tau_I = \tau_{P1},\ \tau_D = \tau_{P2} \quad (8)$$

With $\tau = \tau_{P1} = \tau_{P2}$ for critically damped systems. The parameter $\alpha = 0.1$ is used where applicable. The desired closed-loop time used for this PID controller depends on the dead time and time constant as defined in Equation (9).

$$\lambda = \max(0.25\,\theta,\ 0.2\tau) \quad (9)$$

2.2. Model Predictive Control

MPC uses predictions and estimates of the current state of the controlled plant to solve an optimal control cost function over a fixed horizon. The solution is an array of present and future control moves to push the plant towards optimal performance based on the adopted objective function. The first element of the array is used as the current control input, and the procedure is iteratively repeated. The most recent developments and findings in MPC are based on the state space formulation [22,23], but the implementation results are similar across various formulations. Furthermore, the state space formulation with a velocity setup offers several advantages, such as ensuring offset-free control [23].

Therefore, in this work, we adopt the discrete state space model with the augmented velocity model described in Equations (10) and (11), respectively.

$$(k+1) = A_m x_m(k) + B_m u(k) y(k) = C_m x_m(k) \quad (10)$$

$$y(k+1) = Ax(k) + B\Delta u(k) y(k) = Cx(k) \quad (11)$$

The state space matrices, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$, are defined according to Equation (12):

$$A = \begin{bmatrix} A_m & 0_p^T \\ C_m A_m & I_p \end{bmatrix}, \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}, C = \begin{bmatrix} 0_{n \times p} & I_p \end{bmatrix} \tag{12}$$

where the variables 0 and I represent matrices of zeros and ones or appropriate dimensions, respectively.

The optimization cost function adopted for the MPC allows for the penalization of the tracking error and change in control moves, as shown in Equation (13). However, we fix the penalty on the controlled variable so that only the manipulated variable is penalized, leading to a more precise tuning.

$$J = \sum_{i=1}^{P} \|r(k+i) - y(k+i)\|^2 + \sum_{i=1}^{M} \|\Delta u(k+i)\|_{r_w}^2 \tag{13}$$

The variable r_w is the input rate weighting factor; r is the setpoint; P is the prediction horizon; and M is the control horizon.

The prediction horizon, P, is set to the process settling time, including dead time, according to Equation (14) to capture process dynamics fully. The control horizon, M, is fixed to a value of $M = 3$ throughout this work. This choice is supported by the fact that $M = 3$ to 5 is sufficient for most systems [24], and setting the value higher than five offers no significant benefits. Pre-fixing the identified parameters reduces the tuning parameters from 4 to 1, making controller tuning transparent.

$$P = \frac{\theta + 5\tau}{T_s} \tag{14}$$

where the variable T_s is the sampling period. Therefore, the weighting r_w dictates the aggressiveness of control, hence its robustness. The weight r_w was initially varied over the range $0.001 \leq r_w \leq 100$. This value is adjusted for different system models, as highlighted in the simulation setup and relevant sections in the results section.

2.3. Simulation Setup

The three PID controllers discussed here were implemented on SOPDT process models with different gains and time constants. For the different process models simulated, the dead-time-to-time-constant ratio, $\frac{\theta}{\tau}$, was varied over the range $\begin{bmatrix} 0.01 & 0.1 & 1 & 3 & 5 & 7 & 10 \end{bmatrix}$ and the damping ratio, ξ, was varied over $\begin{bmatrix} 0.2 & 0.4 & 0.6 & 0.8 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}$. The process gain K_p and time constant were kept constant. The IAE for each of the tuned PIDs and MPC were obtained for both setpoint tracking and disturbance rejection. The IAE for the best performing PID controller amongst the three is then used to compare with MPC by computing the normalized IAE (IAE_N) according to Equation (15), and the results are tabulated. To further evaluate the performance of the controllers, the total variation (TV) was computed, and the normalized TV (TV_N), given in Equation (17), was also used to compare the controllers. Both setpoint response and disturbance rejection response were investigated using a unit step reference and unit step disturbance, respectively. The disturbance model is assumed to be the same as the process model. The process model is assumed to be perfect in all controller designs for both MPC and PID.

$$IAE_N = \frac{IAE}{IAE_{min}} \tag{15}$$

$$IAE = \int_{\theta}^{T_{ss}} |e(t)| dt = \sum_{i=\theta/T_s}^{T_{ss}/T_s} e(i) T_s \tag{16}$$

$$TV_N = \frac{TV}{TV_{min}} \tag{17}$$

$$TV = \sum_{i=1}^{T_{ss}} |u_i - u_{i-1}| = \sum_{i=1}^{T_{ss}} |\Delta u(i)| \quad (18)$$

where x_{min} is the value of x for the controller that has the minimum x; T_{ss} is the settling time; and θ is the dead time.

3. Results

The simulation setup discussed in Section 2.3 was applied to system models with different process gains and time constants, as indicated in subsequent sub-sections. The results were tabulated as follows.

3.1. Process 1: $k_p = 1,\ \tau = 1$

The simulation was initiated by setting both the gain and time constant of the SOPDT model to unity to obtain the following transfer function:

$$\frac{e^{-\theta s}}{s^2 + 2\xi s + 1} \quad (19)$$

Simulations were then carried out using the protocol defined in Section 2.3. The results obtained for setpoint response and disturbance rejection are tabulated in Tables 2–4. For disturbance rejection, MPC had a $TV_N = 1$ for all the tuned controllers. As such, the results are not presented in a table.

Table 2. Normalized IAE for MPC (Processes 1, setpoint response).

		Damping Ratio (ξ)								
r_w	θ	0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	1.20	2.49	3.00	2.96	2.76	2.48	4.91	8.68	12.65
	0.1	1.00	1.85	2.26	2.47	2.55	2.97	3.57	4.18	4.84
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.12
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	2.20	3.47	3.82	3.74	3.76	4.59	10.79	21.52	33.49
	0.1	1.79	2.56	2.87	3.12	3.48	5.49	7.83	10.33	12.75
	1	1.00	1.00	1.00	1.00	1.00	1.28	1.82	2.40	2.94
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	2.23	4.21	5.78	6.43	6.78	9.55	21.72	38.73	54.00
	0.1	1.82	3.13	4.35	5.38	6.27	11.35	15.61	18.38	20.35
	1	1.00	1.00	1.01	1.08	1.33	2.62	3.59	4.22	4.64
	3	1.00	1.00	1.00	1.00	1.00	1.14	1.37	1.52	1.63
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	**0.01**	3.21	6.30	8.04	8.89	9.54	12.66	26.98	47.70	69.09
	0.1	2.65	4.68	6.05	7.43	8.80	15.00	19.42	22.84	26.44
	1	1.28	1.37	1.40	1.49	1.86	3.45	4.47	5.30	6.15
	3	1.00	1.00	1.00	1.05	1.14	1.50	1.71	1.91	2.16
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.14	1.29
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 3. Normalized TV for MPC (Process 1, setpoint response).

		Damping Ratio (ξ)								
r_w	θ	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	2.21	1.19	1.00	1.00	1.30	1.00	1.00	1.00	1.00
	3	4.26	2.79	2.23	1.95	3.42	1.28	1.00	1.00	1.00
	5	5.26	3.82	3.24	2.92	4.79	2.08	1.62	1.32	1.06
	7	5.91	4.56	4.00	3.69	5.79	2.80	2.24	1.83	1.48
	10	6.64	5.41	4.93	4.65	6.95	3.77	3.11	2.60	2.10
0.1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.89	1.42	1.19	1.05	1.78	1.00	1.00	1.00	1.00
	5	2.34	1.94	1.72	1.57	2.49	1.00	1.00	1.00	1.00
	7	2.63	2.32	2.13	1.99	3.01	1.18	1.00	1.00	1.00
	10	2.95	2.75	2.62	2.50	3.61	1.60	1.12	1.00	1.00
1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.38	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.56	1.27	1.01	1.00	1.19	1.00	1.00	1.00	1.00
10	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4. Normalized IAE for MPC (Process 1, Disturbance rejection).

		Damping Ratio (ξ)								
r_w	θ	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	**0.01**	4.95	4.10	3.69	5.40	3.23	26.04	28.35	30.72	34.06
	0.1	4.30	4.28	4.22	4.05	3.80	3.35	3.34	3.54	3.87
	1	1.00	1.00	1.07	1.12	1.20	1.26	1.26	1.29	1.34
	3	1.08	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	6.68	5.19	4.51	6.59	4.05	43.65	61.25	76.84	90.73
	0.1	5.55	5.26	5.05	4.84	4.66	5.33	6.70	8.17	9.52
	1	1.00	1.00	1.17	1.22	1.33	1.58	1.80	2.03	2.24
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.11	1.21	1.28
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.07
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	7.49	6.39	6.22	10.12	6.90	91.21	119.49	133.98	142.98
	0.1	6.22	6.43	6.83	7.21	7.62	10.57	12.52	13.77	14.61
	1	1.00	1.04	1.40	1.53	1.76	2.41	2.71	2.90	3.03
	3	1.04	1.00	1.00	1.00	1.00	1.11	1.41	1.50	1.54
	5	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.10	1.22
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4. *Cont.*

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
10	**0.01**	11.48	9.03	8.60	14.30	9.84	119.28	146.69	159.52	168.03
	0.1	9.40	8.91	9.25	9.99	10.66	13.65	15.26	16.33	17.12
	1	1.24	1.27	1.68	1.89	2.21	2.89	3.14	3.31	3.43
	3	1.24	1.02	1.03	1.06	1.12	1.25	1.55	1.64	1.68
	5	1.29	1.04	1.00	1.00	1.00	1.02	1.06	1.17	1.30
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

3.2. Process 2: $K_p = 5$ and $\tau = 1$

The simulation was carried out with the gain and time constant of the SOPDT model set to 5 and 1, respectively, to obtain the following transfer function:

$$\frac{5e^{-\theta s}}{s^2 + 2\xi s + 1} \tag{20}$$

Simulations were then carried out using the protocol defined in Section 2.3. The results obtained for setpoint response and disturbance rejection are tabulated in Tables 5–7. For disturbance rejection, MPC had a $TV_N = 1$ for all the tuned controllers. As such, the results are not presented in a table.

Table 5. Normalized IAE for MPC (Process 2, setpoint response).

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	1.00	1.00	1.00	1.00	1.00	1.02	1.95	3.14	4.12
	0.1	1.00	1.00	1.00	1.00	1.00	1.24	1.43	1.53	1.60
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	1.00	1.81	2.40	2.51	2.43	2.18	4.05	6.70	9.29
	0.1	1.00	1.36	1.81	2.10	2.24	2.60	2.92	3.23	3.58
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	1.93	3.22	3.55	3.39	3.17	3.45	7.64	14.44	22.17
	0.1	1.58	2.39	2.67	2.83	2.92	4.12	5.52	6.96	8.54
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.28	1.62	1.98
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	**0.01**	2.34	3.71	4.62	5.16	5.43	7.41	18.07	33.61	48.10
	0.1	1.89	2.75	3.48	4.32	5.02	8.83	12.96	16.03	18.30
	1	1.00	1.00	1.00	1.00	1.07	2.05	2.99	3.69	4.19
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.14	1.33	1.47
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 6. Normalized TV for MPC (Process 2, setpoint response).

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	4.60	2.46	1.66	1.27	1.01	1.00	1.00	1.00	1.00
	0.1	5.29	2.82	1.93	1.46	1.16	1.00	1.00	1.00	1.00
	1	25.81	13.35	9.00	6.71	11.16	2.64	1.54	1.17	1.00
	3	49.55	31.55	23.23	18.36	29.43	7.96	4.67	3.59	2.88
	5	61.52	43.36	33.66	27.36	41.17	12.87	7.73	6.06	4.82
	7	69.15	51.81	41.58	34.72	49.66	17.32	10.65	8.42	6.72
	10	77.69	61.53	51.09	43.72	59.61	23.36	14.81	11.77	9.60
0.1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	3.99	1.85	1.22	1.00	1.64	1.00	1.00	1.00	1.00
	3	7.67	4.36	3.14	2.55	4.33	1.66	1.26	1.01	1.00
	5	9.52	5.99	4.55	3.80	6.06	2.68	2.08	1.71	1.50
	7	10.70	7.16	5.62	4.82	7.31	3.60	2.86	2.37	2.09
	10	12.02	8.50	6.90	6.07	8.78	4.86	3.98	3.31	2.99
1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	2.40	1.84	1.53	1.33	2.30	1.00	1.00	1.00	1.00
	5	2.98	2.53	2.22	1.99	3.22	1.36	1.00	1.00	1.00
	7	3.35	3.02	2.74	2.52	3.88	1.83	1.26	1.00	1.00
	10	3.76	3.59	3.37	3.17	4.66	2.46	1.76	1.35	1.08
10	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.68	1.31	1.03	1.00	1.27	1.00	1.00	1.00	1.00
	7	1.89	1.57	1.28	1.08	1.53	1.00	1.00	1.00	1.00
	10	2.12	1.86	1.57	1.36	1.84	1.00	1.00	1.00	1.00

Table 7. Normalized IAE for MPC (Process 2, Disturbance Rejection).

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	2.05	1.76	1.71	2.75	1.77	17.69	19.51	19.69	19.66
	0.1	2.06	2.11	2.21	2.27	2.29	2.43	2.44	2.43	2.42
	1	1.00	1.00	1.00	1.00	1.00	1.11	1.11	1.11	1.11
	3	1.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	3.67	3.25	3.09	4.76	2.90	23.91	24.83	25.23	26.23
	0.1	3.33	3.50	3.62	3.59	3.46	3.11	2.98	2.99	3.08
	1	1.00	1.00	1.01	1.06	1.15	1.22	1.20	1.20	1.21
	3	1.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	6.30	4.91	4.24	6.13	3.63	32.99	42.13	51.64	61.82
	0.1	5.28	5.01	4.77	4.49	4.21	4.13	4.75	5.66	6.66
	1	1.00	1.00	1.14	1.17	1.26	1.38	1.48	1.63	1.79
	3	1.01	1.00	1.00	1.00	1.00	1.00	1.01	1.08	1.13
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 7. *Cont.*

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
10	**0.01**	7.05	5.68	5.25	8.31	5.46	71.79	99.71	117.07	128.92
	0.1	5.84	5.75	5.82	5.94	6.12	8.45	10.56	12.13	13.25
	1	1.00	1.00	1.27	1.37	1.55	2.07	2.40	2.65	2.82
	3	1.00	1.00	1.00	1.00	1.00	1.02	1.31	1.42	1.47
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.05	1.18
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

3.3. Process 3: $K_p = 1$ and $\tau = 3$

The simulation was carried out with the gain and time constant of the SOPDT model set to 1 and 3, respectively, to obtain the following transfer function:

$$\frac{e^{-\theta s}}{9s^2 + 6\xi\tau s + 1} \tag{21}$$

Simulations were then carried out using the protocol defined in Section 2.3. The results obtained for setpoint response and disturbance rejection are tabulated in Tables 8–11.

Table 8. Normalized IAE for MPC (Process 3, setpoint response).

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	1.01	1.81	1.77	1.77	1.00	1.00	1.00	1.10	1.33
	0.1	1.22	1.82	1.82	2.02	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	1.14	1.91	1.92	2.04	1.00	1.00	2.44	3.70	4.71
	0.1	1.37	1.93	1.98	2.32	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	1.00	2.07	2.74	3.46	1.00	2.04	5.28	6.92	7.79
	0.1	1.19	2.10	2.82	3.95	1.00	1.00	1.00	1.07	1.19
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	**0.01**	1.40	3.19	3.85	4.74	1.00	2.59	6.19	7.74	8.48
	0.1	1.70	3.23	3.96	5.40	1.00	1.00	1.02	1.20	1.29
	1	1.21	1.23	1.21	1.26	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 9. Normalized TV for MPC (Process 3, setpoint response).

		Damping Ratio (ξ)								
r_w	**θ**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	2.67	1.78	1.40	1.14	3.94	1.29	1.14	1.00	1.00
	3	5.15	4.30	3.67	3.15	10.12	2.99	2.36	2.49	2.45
	5	6.48	6.04	5.49	4.87	13.59	4.78	2.44	3.04	3.20
	7	7.28	7.26	6.88	6.21	15.77	6.43	3.25	2.73	3.52
	10	8.04	8.64	8.52	8.01	17.84	8.66	4.54	2.71	2.58
0.1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.72	1.08	1.00	1.00	1.69	1.00	1.00	1.00	1.00
	3	3.32	2.60	1.93	1.49	4.32	1.00	1.00	1.00	1.00
	5	4.17	3.65	2.89	2.30	5.80	1.32	1.00	1.00	1.00
	7	4.69	4.38	3.62	2.93	6.74	1.77	1.00	1.00	1.00
	10	5.18	5.22	4.48	3.78	7.62	2.39	1.18	1.00	1.00
1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.47	1.00	1.00	1.00	1.09	1.00	1.00	1.00	1.00
	5	1.85	1.18	1.00	1.00	1.46	1.00	1.00	1.00	1.00
	7	2.08	1.42	1.01	1.00	1.70	1.00	1.00	1.00	1.00
	10	2.29	1.69	1.25	1.00	1.92	1.00	1.00	1.00	1.00
10	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.04	1.00	1.00	1.00	1.00

Table 10. Normalized IAE for MPC (Process 3, Disturbance rejection).

		Damping Ratio (ξ)								
r_w	**θ**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	**0.01**	3.47	3.94	3.49	3.19	1.00	18.60	51.93	65.96	77.78
	0.1	3.78	4.01	3.80	3.55	1.00	1.00	2.32	3.62	4.83
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	3.47	3.94	3.49	3.19	1.00	18.60	51.93	65.96	77.78
	0.1	3.78	4.01	3.80	3.55	1.00	1.00	2.32	3.62	4.83
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	3.69	4.68	4.80	5.09	1.11	43.74	110.36	121.58	126.88
	0.1	4.04	4.76	5.19	5.58	1.00	2.28	4.83	6.59	7.80
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 10. *Cont.*

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
10	**0.01**	5.87	6.77	6.63	7.07	1.56	55.42	128.81	135.54	137.80
	0.1	6.40	6.83	7.11	7.70	1.10	2.87	5.62	7.33	8.45
	1	1.00	1.00	1.14	1.25	1.00	1.00	1.00	1.00	1.00
	3	1.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11. Normalized TV for MPC (Process 3, Disturbance rejection).

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	3.28	3.72	3.22	2.79	1.00	9.41	18.75	20.33	23.01
	0.1	3.59	3.80	3.51	3.13	1.00	1.00	1.00	1.17	1.49
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.40	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.43	1.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00

3.4. Process 4 $K_p = 7$ and $\tau = 5$

The simulation was carried out with the gain and time constant of the SOPDT model set to 7 and 5, respectively, to obtain the following transfer function:

$$\frac{7e^{-\theta s}}{25s^2 + 10\xi s + 1} \tag{22}$$

Simulations were then carried out using the protocol defined in Section 2.3. The results obtained for setpoint response and disturbance rejection are tabulated in Tables 12–14. For disturbance rejection, MPC had a $TV_N = 1$ for all the tuned controllers. As such, the results are not presented in a table.

Table 12. Normalized IAE for MPC (Process 4, Setpoint response).

		Damping Ratio (ξ)								
r_w	**θ**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	**0.01**	1.00	1.00	1.40	1.52	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.30	1.54	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	1.14	1.88	2.08	1.99	1.00	1.00	1.00	1.00	1.00
	0.1	1.26	1.84	1.93	2.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	1.42	2.07	2.22	2.11	1.00	1.00	1.00	1.00	1.00
	0.1	1.57	2.02	2.05	2.12	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	**0.01**	1.40	2.11	2.45	2.63	1.00	1.00	1.00	1.58	2.33
	0.1	1.54	2.05	2.27	2.65	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 13. Normalized TV for MPC (Process 4, Setpoint response).

		Damping Ratio (ξ)								
r_w	**θ**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	**0.01**	2.33	1.32	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	3.12	1.54	1.05	1.00	1.00	1.00	1.00	1.00	1.00
	1	15.67	7.12	4.68	3.63	12.32	5.22	6.13	7.23	7.84
	3	30.28	16.91	12.25	10.01	33.47	14.78	9.78	7.46	12.63
	5	37.78	23.61	18.06	15.30	43.86	23.17	15.90	11.54	8.68
	7	42.24	28.12	22.47	19.44	49.91	30.56	21.68	15.99	12.09
	10	46.59	33.22	27.59	24.79	55.28	40.07	29.84	22.48	17.14
0.1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	4.96	3.45	2.84	2.47	8.82	3.56	3.91	4.14	3.93
	3	9.58	8.18	7.44	6.81	23.97	10.07	6.25	4.28	6.34
	5	11.95	11.43	10.97	10.40	31.41	15.79	10.16	6.61	4.35
	7	13.36	13.61	13.65	13.22	35.75	20.82	13.86	9.16	6.06
	10	14.74	16.08	16.76	16.86	39.59	27.30	19.07	12.88	8.60
1	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	3.36	2.31	1.88	1.55	5.13	1.48	1.19	1.08	1.00
	3	6.50	5.49	4.91	4.27	13.95	4.19	1.91	1.11	1.56
	5	8.10	7.67	7.25	6.52	18.28	6.57	3.10	1.72	1.07
	7	9.06	9.14	9.01	8.29	20.80	8.66	4.22	2.39	1.49
	10	9.99	10.79	11.07	10.58	23.04	11.35	5.81	3.35	2.11

Table 13. *Cont.*

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
10	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.84	1.05	1.00	1.00	1.36	1.00	1.00	1.00	1.00
	3	3.56	2.50	1.76	1.28	3.69	1.00	1.00	1.00	1.00
	5	4.45	3.49	2.59	1.95	4.84	1.47	1.00	1.00	1.00
	7	4.97	4.16	3.23	2.48	5.51	1.94	1.03	1.00	1.00
	10	5.48	4.91	3.96	3.16	6.10	2.55	1.42	1.00	1.00

Table 14. Normalized IAE for MPC (Process 4, Disturbance Rejection).

r_w	θ	Damping Ratio (ξ) 0.2	0.4	0.6	0.8	1	2	3	4	5
0.01	**0.01**	1.00	1.21	1.29	1.37	1.00	1.00	2.33	4.83	5.95
	0.1	1.01	1.26	1.44	1.54	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	**0.01**	2.73	3.04	2.83	2.61	1.00	1.00	2.76	5.80	7.48
	0.1	2.68	3.03	3.03	2.86	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	**0.01**	4.24	4.16	3.54	3.10	1.00	1.00	4.37	11.45	17.79
	0.1	4.09	4.10	3.76	3.38	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	**0.01**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

3.5. Summary of Results

This section summarizes the results obtained for the four different process models studied.

3.5.1. Process 1

Generally, MPC gives better performance for underdamped systems and systems with larger time delays. On the other hand, the PID controllers give better performance when the time delays are smaller and the process is overdamped, when both TV and IAE are considered. For disturbance rejection, MPC moved relatively less across all values of r_w. Hence, both aggressive and conservative MPC controllers moved less than the corresponding PID controllers, which is suitable for actuators and controller robustness. The IAE values for disturbance rejection show that the MPC had better performance when $\frac{\theta}{\tau}$ is at least 3. However, there is a noticeable drop in performance from left to right of the table for $\frac{\theta}{\tau} < 3$, i.e., with decreasing damping ration ζ. For the setpoint response, the TV values when $0.01 \leq r_w \leq 0.1$ indicate that the MPC controller moved more when $\frac{\theta}{\tau} > 3$ for all values of ζ or when $\frac{\theta}{\tau} > 1$ and $\zeta < 1$. The corresponding IAE values show that the MPC controller had lower IAE values for $\frac{\theta}{\tau} \geq 1$. For $\frac{\theta}{\tau} \leq 1$, there is a noticeable increase in IAE values from left to right of the table, i.e., with increasing damping. The boundary of the performance improvement in terms of IAE is seen to gradually move from $\frac{\theta}{\tau} \geq 1$ for $r_w = 0.01$ to $\frac{\theta}{\tau} \geq 5$ for $r_w = 10$.

Hence, MPC tends to give better performance than PID when $\frac{\theta}{\tau} > 1$, irrespective of the damping ratio. But when $\frac{\theta}{\tau} \leq 1$, the PID controller tends to give a better performance. It is also observed that the relative performance of MPC increases as damping is decreased. Hence, even for lower values of $\frac{\theta}{\tau}$, MPC is likely to perform better if the system is underdamped.

3.5.2. Process 2

The results obtained here are somewhat similar to those obtained with process 1. Again, MPC gives better performance for underdamped systems and systems with larger time delays. On the other hand, the PID controllers give better performance when the time delays are smaller and the process is overdamped, when both TV and IAE are considered. For disturbance rejection, MPC moved relatively less across all values of r_w. Hence, both aggressive and conservative MPC controllers moved less than the corresponding PID controllers, which is good for actuators and controller robustness. The IAE values for disturbance rejection show that the MPC had better performance when $\frac{\theta}{\tau}$ is at least 3. However, there is a noticeable drop in performance from left to right of the table for $\frac{\theta}{\tau} < 3$, i.e., with decreasing damping ratio ζ. For the setpoint response, the TV values when $0.01 \leq r_w \leq 0.1$ indicate that the MPC controller moved more when $\frac{\theta}{\tau} > 1$ for all values of ζ or when $\frac{\theta}{\tau} > 1$ and $\zeta < 1$. However, as the controller became more cautious at $r_w = 10$, the controller performance boundary moved to around $\frac{\theta}{\tau} \geq 5$ and $\zeta \leq 0.8$ making the MPC controller move more towards the left bottom corner of the table. The corresponding IAE values show that the MPC controller had lower IAE values for $\frac{\theta}{\tau} \geq 1$ for all values of ζ. For $\frac{\theta}{\tau} \leq 1$, there is a noticeable increase in IAE values from left to right of the table, i.e., with increasing damping. Looking specifically at the values for $1 \leq r_w \leq 10$, there is a noticeable decrease in MPC performance for low values of $\frac{\theta}{\tau}$ when ζ exceeds 2 to 3. Here, the boundary of performance improvement in terms of IAE is seen to remain around $\frac{\theta}{\tau} \geq 1$ over the range of r_w.

Therefore, as with process 1, MPC tends to give better performance than PID when $\frac{\theta}{\tau} > 1$, irrespective of the damping ratio. But when $\frac{\theta}{\tau} \leq 1$, the PID controller tends to give a better performance. It is also observed that the relative performance of MPC increases as damping is decreased, i.e., along the values $1 \leq \frac{\theta}{\tau} \leq 1$ as $\zeta \geq 3$. Therefore, even for lower values of $\frac{\theta}{\tau}$, MPC is likely to perform better if the system is underdamped as with the first process.

3.5.3. Process 3

The results for this system are similar to those obtained for the first two systems. The performance boundary is even more apparent here. For disturbance rejection, in terms of TV, MPC gave a value of 1 for all values of r_w, $\frac{\theta}{\tau}$ and ζ. The same result was obtained in setpoint response for $1 \leq r_w \leq 10$. For $0.01 \leq r_w \leq 0.1$, MPC had a TV of 1 when $\frac{\theta}{\tau} < 1$ for all values of ζ. In terms of IAE, MPC had the best performance in setpoint response when $\frac{\theta}{\tau} \geq 1$ for $0.01 \leq r_w \leq 0.1$ and $\frac{\theta}{\tau} \geq 3$ for $1 \leq r_w \leq 10$. For disturbance rejection, MPC had the best IAE when $\frac{\theta}{\tau} \geq 1$ for all tunings. Hence, MPC could be said to perform better in most cases when $\frac{\theta}{\tau} \geq 3$ overall values of values. The trend for lower values $\frac{\theta}{\tau}$ is similar to the first two processes, with MPC performing relatively better as damping decreases. This boundary is $\frac{\theta}{\tau} \geq 1$ for aggressively tuned MPC, i.e., $0.01 \leq r_w \leq 0.1$.

3.5.4. Process 4

From the results obtained, when $\frac{\theta}{\tau} \geq 1$, MPC generally performs better than PID both in terms of TV and IAE, irrespective of the damping. However, when $\frac{\theta}{\tau} < 1$, a general conclusion cannot be reached as to when MPC outperforms PID and vice versa, as the simulations indicate some conflicting results.

4. Discussion

All simulation results for the four case studies demonstrate that MPC outperforms the best of the three PID controllers when the dead-time-to-time-constant ratio exceeds three. For some parameters of the studied systems or tuning, this value changes to around $\frac{\theta}{\tau}$. This is especially true for the aggressively tuned systems or controllers with $0.1 \leq r_w \leq 0.01$ although there is no strict boundary defining the parameter values that define the performance regions. The results can guide practitioners on when it is likely to benefit from replacing PID controllers with MPC. The results obtained are as expected because MPC is seen to avoid excessive movements in dead-time dominant systems and the underdamped system. This could be attributed to MPC's ability to anticipate such systems' behavior because of its predictive capabilities. Hence, the results depend heavily on the availability of fairly accurate models that can provide good predictions. The speed of response is also expected to affect results because MPC implementation may be computationally expensive for very fast dynamics, depending on the computational capabilities of the control algorithm implementation hardware. However, MPC implementations such as explicit MPC could be explored when process speed is a concern. Another factor that may affect the results is how noisy the process is. This is because the derivative action of the PID controllers is likely to amplify any noise. Moreover, using observers or Kalman filters, integral to MPC, can naturally solve the problem of process noise.

5. Conclusions

This paper presented studies on the effect of the process time delay and damping ratio on the performance of MPC and PID controllers. The study has shown that, for the four second-order systems investigated in this paper, the performance of MPC was better than PID in terms of both normalized TV and IAE for all damping ratios when the time delay is approximately three times the time constant. Furthermore, for systems with a low dead-time-to-time-constant ratio, MPC also showed improved performance for underdamped systems with a damping ratio of less than 1. The MPC controller was compared with the best of three commonly used industrial PID control algorithms, and steps were taken to make the MPC controller tuning transparent. However, the method adopted for comparison does not quantify the improvements provided by MPC. As such the results may need to be more definitive, especially since MPC comes with additional costs related to plant testing and software. However, since the low TV demonstrated by some of the systems directly translates to actuator wear, this could further support the conclusions of this work. Moreover, the MPC formulation adopted for this work uses few tuning parameters, and information embedded in routine plant data could easily be harnessed for controller design. More importantly, the work guides when to consider deploying MPC, which presents a good starting point for further analysis and studies. Practitioners can leverage these findings to anticipate plants that may or may not benefit from MPC implementation. The results obtained can thus be summarized in Table 15. Note that the boundary presented in Table 15 is not crisp, and zones may infringe into an adjoining zone. Therefore, our results suggest that, if steps are taken to adopt more advanced controllers on second-order processes that may not be well-suited for PID, then performance improvements are possible. However, this work did not quantify the amount of control improvements. Also, nonstandard PID schemes may offer improved performance over the standard PID techniques considered in this study. Therefore, quantifying the improvements and considering nonstandard PID schemes are possible directions for future studies.

Table 15. Preferred controller for second order system.

θ	Damping Ratio (ξ)								
	0.2	**0.4**	**0.6**	**0.8**	**1**	**2**	**3**	**4**	**5**
0.01	–	–	–	–	PID	PID	PID	PID	PID
0.1	–	–	–	–	PID	PID	PID	PID	PID
1	–	–	–	–	PID	PID	PID	PID	PID
3	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC
5	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC
7	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC
10	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC	MPC

Another dimension which is critical to MPC is that of constraint handling and multivariable systems. These are outside the scope of this work as it is well-known that MPC is more suitable for large, multivariable constrained systems. Hence this work focused on SISO unconstrained processes.

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