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A New CRITIC-GRA Model for Stope Dimension Optimization Considering Open Stoping Stability, Mining Capacity and Costs

Chuanyu Li [1,2], Guangsheng Liu [1,2,*], Lijie Guo [1,2], Di Zheng [1,2] and Xuehao Yuan [1,2]

[1] Beijing General Research Institute of Ming and Metallurgy, Beijing 100160, China; lichuanyu2024@foxmail.com (C.L.); guolijie@bgrimm.com (L.G.); zhengdi@bgrimm.com (D.Z.); yuanxuehao@bgrimm.com (X.Y.)

[2] National Center for International Research on Green Metal Mining, Beijing 102628, China

* Correspondence: liuguangsheng@bgrimm.com

Abstract: In the long-hole stage open stoping with subsequent backfill mining of underground metal mines, the selection or optimization of stope dimension parameters is significant for safe and economic mining operations. To analyze the optimal stope sizes, the Mathews empirical graph method and FLAC3D numerical method can be used, but the analyzed safety results of the two methods are generally independent from each other. More importantly, economic indicators including production capacity and mining costs should be considered simultaneously to optimize the stope dimension which was mostly ignored in previous reports. In this paper, a new CRITIC-GRA model was proposed for the first time to build up a multi-factor quantitative optimization for stope dimension, which allows for a comprehensive analysis with preset influential safety and economic indicators. The indicators considered include the safety indicators such as stability probability for the side walls and roof of the open stope via the updated Mathews graph method, maximum displacement, plastic zones volume and maximum principal stress via FLAC3D simulations, as well as economic indicators such as mining costs and stope production capacity in mine operations. The model was then illustrated in an underground iron mine. With the given rockmass quality in the mine, the overall stability of the open stope can be improved instead of reduced to enlarge the single stage stope height (60 m) to a double stage height (120 m) by reducing the stope width from 20 m to 15 m, thereby significantly increasing the mineable ore amount and improving the stope safety. An integrated evaluation of open stope stability, mining capacity and costs objectively determined that scheme No. 10, with a slope length of 50 m, a width of 15 m and a height of 120 m, was the optimum out of the 20 preset schemes. The new CRITIC-GRA model offers a dependable reference tool for determining the optimal stope dimensions in similar underground mines.

Keywords: open stoping; stope dimension optimization; CRITIC; grey relational degree; FLAC3D; stability analysis; stope production capacity; mining cost

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1. Introduction

In order to meet the ever-increasing demand for mineral resources consumption and supply, increasing mining production capacity is an important consideration for designers and managers of nonferrous underground mines, however, adequate safety and reasonable mining operation costs must be guaranteed. Currently, long-hole stage open stoping with subsequent backfill is the most efficient mining method and represents the development direction of large-scale underground mining in nonferrous metal mines [1–3].

In this mining method, a traditional bottom-up mining sequence is usually adopted, in which the ore body is vertically divided into a number of mining stages, as illustrated in Figure 1a, where the stopes (1, 3, 5) in Stage-1 are excavated and then backfilled to provide a base platform for the mining of upper stopes (2, 4, 6) in Stage-2. Meanwhile, continuous stopes are divided along the horizontal direction of each stage, and the primary stopes (1, 2) are excavated preferentially and backfilled with cement to serve as artificial vertical pillars for the following excavations of secondary stopes (3, 4, 5, 6), which are mostly backfilled without adding cement to save costs. Undoubtedly, due to the higher efficiency of long-hole blasting, larger stope sizes can increase the stope production capacity and reduce the mining costs per excavated ore. But due to a larger exposed surface of the surrounding rockmass and backfill, it usually leads to a higher safety risk of stope failure. Therefore, the determination of reasonable dimension parameters (height, length and width) of the stopes is significant to the design of the mining process considering the stoping stability, capacity and costs.

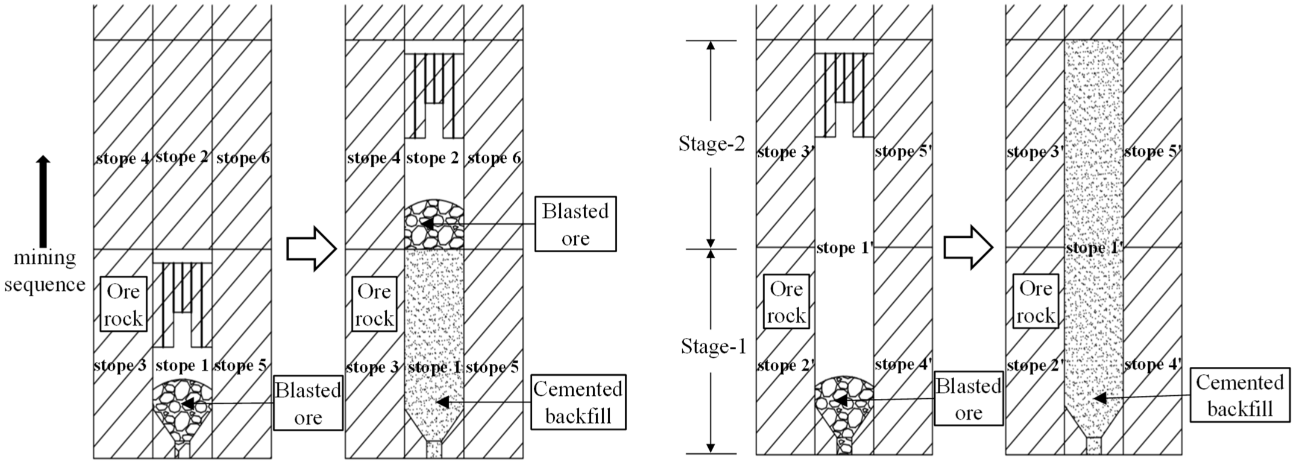

Figure 1. Typical excavating and filling sequence of (**a**) single stage and (**b**) double stage open stoping with subsequent backfill mining method.

To further increase the stope production capacity and reduce the cost of mined ore, the double stage open stoping for primary stopes (shown as Stope 1′ in Figure 1b) in some mines with high-dipping thick stable ore bodies can be employed to enlarge the advantages of mining efficiency and cost-saving [4,5]. The combination of vertically adjacent primary stopes (1, 2) saves construction costs to develop the entry drifts and ore drawing structures, and eliminates the waiting time of backfilling the lower stope void to build the platform for the upper stope. However, the combination of excavated primary stopes in the vertical direction would result in doubling the height of the primary stopes, thereby reducing the stability of side-exposed rock walls. This presents an even greater challenge in balancing stope stability and dimensions. Therefore, it can be seen that the benefits and safety management of stage open stoping mining is highly dependent on the optimization of stope dimensions and the corresponding indexes of mining capacity and costs, which is a hot topic in the field of underground mining with backfill.

Empirical charts, analytical models and numerical simulations are usually used to achieve the optimization and reasonable balance of stability and dimensions of open stopes, and the following fruitful research findings and typical applications are obtained.

The Mathews stability graph [6] is a world-wide method used to build up an empirical chart between the stability index and the shape factor with different dimension parameters of exposed side walls and roof around stope voids. Subsequently, Mawdesley [7] expanded the Mathews database and proposed an extended stability graph method based on logistic regression analysis which can calculate the failure probability of a stope void. Zhao et al. [8] further updated the extended Mathews stability graph by fitting a formula that can directly calculate the stope shape factors corresponding to a 95% stability probability for a specific underground mining project. Considering the effects of exposure time on the inclined stope stability in different sizes, Qiu et al. [9] proposed a modified stability synthetic graph for stope dimension selection.

Some analytical methods based on elasticity theoretical models can also be used for stability analysis of excavated stopes. For example, Gercek et al. [10] used the conformal mapping method to investigate the surface shear stress distributions as well as the maximum and minimum principal stress under different in situ rock stress conditions. The results can be used to assess the stability of surrounding rockmass around stope voids in different exposure sizes. Exadaktylos et al. [11] applied the complex variable function to calculate a closed-form elastic solution of the stresses and displacements of homogeneous isotropic surrounding rock, and then determined the stress distribution and concentration area of surrounding rock with different excavation sizes.

Numerical modeling is a widely used and cost-effective method for investigating the stability of excavated stopes affected by various influential parameters. Guo et al. [12] used ABAQUS to simulate the stope stability affected by initial and mining-induced stress conditions. Nordlund et al. [13] applied the variability and stochasticity of the rock parameters to a FLAC model and constructed a probabilistic model for predicting the stability of practical stopes under stochastic characteristics of the surrounding rock. Shahriyar et al. [14] built up a numerical simulation method with FLAC3D to assess the influences of stope dimension parameters generated using Monte-Carlo statistics on the failure probability of excavated stope voids.

Considering that the stability of excavated stopes is affected by various uncertainties, including stope sizes, physical and mechanical properties of rockmass and backfill, stress conditions and excavating processes, etc., multi-factors evaluation methods have been increasingly employed as a new approach to analyze and optimize stope dimensions [15–17]. Li et al. [18] combined a set theory model in fuzzy mathematics with a hierarchical analysis method to select appropriate stope size parameters. Jin et al. [19] developed the CRITIC-TOPSIS (criteria importance through inter-criteria correlation and technique for order preference by similarity to ideal solution) evaluation model to optimize stope parameters and investigate the effect of mining depth.

In summary, empirical chart methods, analytical calculation methods and numerical simulation methods have been successively used for the stability analysis and dimension optimization of excavated stopes. However, the outcomes of safety indexes and stability conditions with the above three methods are usually independent of each other, which make it difficult for the designers or managers of a specific mine to make a final decision on the optimized stope sizes with uncoordinated results. More importantly, economic indicators including production capacity and mining costs of an excavated stope should be considered simultaneously to achieve an optimized stope size, since the expectation of profit is the only excuse to continue mining. Although the multi-factors evaluation methods have been applied on the stope dimension optimization, the influences of economic parameters are mostly neglected on the selection of stope size parameters in previous published papers. There are currently no reports for the stope dimension optimization using quantitative analysis methods that integrate the economic indicators, safety indexes and stability conditions.

To fill the gap, this paper proposes a new CRITIC-GRA (criteria importance through inter-criteria correlation and grey relation analysis) evaluation method for the optimization of stope dimension parameters considering the safety indexes and stability conditions of the long-hole stage open stoping mining, together with the economic indicators including stope production capacity and costs. The new method can be used for ordinary single stage open stoping with subsequent backfill mining, and is particularly suitable for extended double stage open stoping with larger stope size and higher mining capacity. The application is illustrated in detail based on a specific mining project in an underground iron mine, and may provide a novel reference for optimizing the stope dimension in other similar mining projects.

2. Newly Proposed Evaluation Method and Analysis Procedure

The Mathews stability graph method is supposed to be the earliest published empirical method for open stoping stability analysis with the preset schemes of different stope sizes, determined mainly by comparing the Stability Index M with the Shape Factor S. The Stability Index M represents the ability of surrounding rockmass of stope void to maintain unsupported self-stability under the classified rockmass quality grade and stress condition. The Shape Factor S is the hydraulic radius, calculated from the area and perimeter of the exposed walls or roof around the excavated stope void. When the Stability Index M is greater than the Shape Factor S, the Mathews stability graph method indicates the stable state of the side walls or roof around the stope void.

The calculation method of Stability Index M is shown in Equation (1).

$$M = 2.39e^{0.55 \ln N} \tag{1}$$

where, N is the rockmass quality index for the side walls or roof of the excavated stope void which can be calculated by Equation (3).

The calculation method of Shape Factor S is shown in Equation (2).

$$S = D/P \tag{2}$$

where, D is the area of the exposed planes and P is the corresponding perimeter. For the shape factor of the side walls around the excavated stope void, D and P are calculated with the height H and length L of the stope. And for the shape factor of the stope roof, D and P are calculated with the stope width W and length L.

$$N = Q' \times A \times B \times C \tag{3}$$

where, Q' is the modified Q value based on the rockmass quality rating system proposed by Barton [20]; A is the stress coefficient of the rockmass; B is the adjustment coefficient for the occurrence of structural planes within the rockmass; and C is the adjustment coefficient of gravity.

The tunneling quality index, also known as the Q system, is a tunneling data-based empirical classification system for surrounding rockmass that was presented by Barton. The system categorizes the rockmass into nine classes and the Q value can be calculated by Equation (4).

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF} \tag{4}$$

where, RQD is the rock quality designation, determined from the ratio of total length of the cored rock sample over 10 cm to the length of the whole core drilling, J_n is the number of joint sets, J_r is the roughness of the most unfavorable joint or discontinuity, J_a is the degree of alteration or filling along the weakest joint, J_w is the water inflow, and SRF is the stress condition given as the stress reduction factor, composed of loosening load in the case of shear zones and clay bearing rock, rock stress in competent rock, and squeezing and swelling loads in plastic incompetent rock. The value of Q' equals to the Q value when J_w/SRF = 1 in the Q classification system.

Mawdesley et al. further updated the Mathews stability graph by using a logit model to calculate the probability of stope stability instead of a fixed value, which is more reasonable for describing the safety condition of a stope void surrounded by practical rockmass with uncertainty and heterogeneity. The evaluated stable probability $f(z)$ of the open stoping is shown in Equation (5).

$$f(z) = 1/(1 + e^{-z}) \tag{5}$$

$$z = 2.9603 - 1.4427 \ln S + 0.7928 \ln N \tag{6}$$

The Mathews stability graph is widely used for stope stability analysis and dimension optimization [21–23]. In addition, to further investigate the stability conditions of the excavated stope voids, numerical simulations of the same stope are usually carried out to provide a supplementary validation against the quickly obtained results via the empirical stability graph. However, although the same open stoping conditions and dimension parameters are used, the results obtained from the Mathews graph and numerical methods usually show certain discrepancies [24,25], which creates confusion for mining designers or managers when making decisions on stope dimension optimization. Therefore, an integrated quantitative analysis combining the stability results from the Mathews empirical graph and the numerical simulations might be the way to go. Furthermore, economic indicators determined by open stoping capacity and mining costs should also be included in the stability analysis for optimal dimension parameters design. To fill the gap and achieve the above goals, a newly proposed evaluation method, the CRITIC-GRA model, is given below.

2.1. Typical Procedure of Newly Proposed CRITIC-GRA Evaluation Model

Figure 2 shows a typical procedure of the proposed CRITIC-GRA evaluation model for quantitatively selecting the optimal scheme with effects of multi-indicators. The procedure is divided into four parts: the identification of all possible influential indicators to optimize the desired scheme, the weight analysis of all selected indicators using the CRITIC method (criteria importance through inter-criteria correlation), the statistical analysis of the ranging conditions of each indicator using the GRA method (grey relation analysis) and the correlation analysis of all preset schemes with all the considered indicators and their corresponding value ranges to obtain the optimal one among the preset schemes.

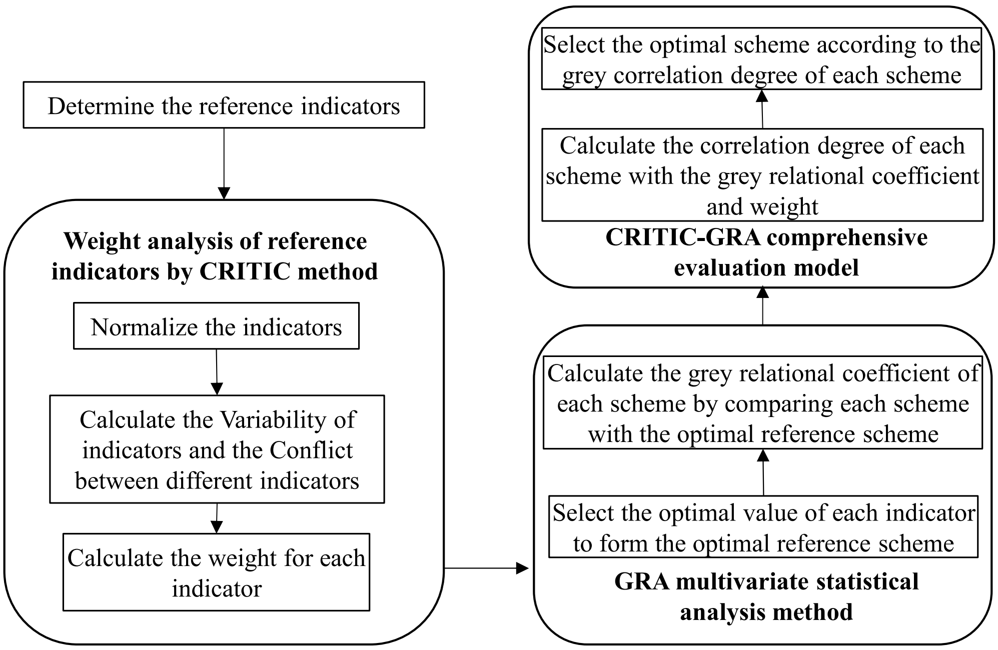

Figure 2. Typical procedure of CRITIC-GRA evaluation model to select the optimal scheme.

2.1.1. Weight Analysis of Safety and Economic Indicators via the CRITIC Method

The CRITIC method [26] allows one to objectively analyze the weight values of each considered safety and economic indicator, which represents different importance to the dimension optimization of open stopes, and to statistically calculate the Variability Index and Conflict Index for each indicator. This avoids the subjectivity of artificial selection of weights based on experience, thus obtaining a more reasonable and impersonal assessment within the assigned ranges of each considered indicator among the preset schemes.

The above-mentioned value of Variability Index [27] for each indicator is determined by calculating the standard deviation of their preset ranges. The value of the Variability Index is positively correlated with the weight value assigned to each indicator. In addition, the value of the Conflict Index, which is represented by the correlation among all the evaluated indicators, is negatively correlated with the weight value allocated to each indicator.

It is assumed that all preset schemes and considered indicators form an $m \times n$ matrix as shown in Equation (7), where the number of all preset schemes to be evaluated is m and the number of all selected influential indicators is n.

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \tag{7}$$

where, x_{ij} represents the safety or economic value of the scheme i and the indicator j which affects the optimization of stope dimensions.

Due to the noncomparability of all the considered influential indicators which have different units, it is necessary to normalize the values of these indicators for integrated comparisons. Equation (8) gives a benefit-oriented normalization method for assessing parameters positively related to the open stoping stability or mining costs. Conversely, Equation (9) represents a cost-oriented normalization method applicable to the considered indicators that are inversely linked to the open stoping stability or mining costs.

$$X'_{ij} = \frac{X_j - X_{\min}}{X_{\max} - X_{\min}} \tag{8}$$

$$X''_{ij} = \frac{X_{\max} - X_j}{X_{\max} - X_{\min}} \tag{9}$$

The Variability Index S_j is represented by the standard deviation of Equation (10), which reflects the fluctuation of differences in the range of values taken by each indicator. The standard deviation is positively correlated with the weight value of the indicator.

$$S_j = \sqrt{\frac{\sum\limits_{i=1}^{m} \left(X_{ij} - \overline{X}_j\right)^2}{m-1}} \tag{10}$$

$$\overline{X}_j = \frac{1}{m}\sum_{i=1}^{m} X_{ij} \tag{11}$$

The Conflict Index R_j between indicators is measured by the correlation coefficient shown in Equation (12), which indicates the degree of association between indicators. The correlation coefficient exhibits a negative correlation with the weight value assigned to the indicators.

$$R_j = \sum_{i=1}^{n} (1 - r_{ij}) \tag{12}$$

where, r_{ij} represents the correlation coefficient between evaluated indicators i and j.

After that, by calculating the weight value w_j for each indicator using Equation (13), the weight values matrix W shown in Equation (14) can be obtained, which applies to all possible influential indicators considered in all the preset schemes to obtain an optimal scheme with suitable stope dimensions for the following GRA analysis.

$$w_j = \frac{S_j \times R_j}{\sum\limits_{j=1}^{n} (S_j \times R_j)} \tag{13}$$

$$W = (w_1 w_2 \cdots w_m) \quad (14)$$

2.1.2. GRA Multi-Variate Statistical Analysis Method

Grey relational analysis (GRA) [28,29] is a method for statistical analysis of multi-indicators. This method calculates the grey correlation degree of the comparative schemes by comparing the geometric relationship between the weight values of all the considered factors and the preset comparative schemes. The advantage of the GRA method lies in its ability to assess the quality of a plan based on the similarity of the geometry of the sequence curves, without being constrained by sample size or regularity.

The reference sequence is a hypothetical sequence containing the same number of indicators as the comparison sequence. The indicators in the reference sequence are derived from the optimal values of each indicator extracted from the scheme matrix.

Assuming that the reference sequence is represented by x_0 = {x_0 (j) | j = 1, 2, . . ., n} and the desired comparison sequence is shown by x_i = {x_i (j) | j = 1, 2, . . ., n. i = 1, 2, ..., m}, where m represents the number of preset schemes, and n represents the number of considered indicators, the same as m and n shown in Equation (7).

The grey correlation coefficient [30,31] between the comparative sequence and the reference sequence essentially reflects the disparity in their geometries, which can be quantified in terms of the magnitude of their differences. The grey correlation coefficient ζ (j) of sequence x_i with respect to the reference sequence x_0 on the indicator j is calculated by Equation (15).

$$\zeta_i(j) = \frac{\min_i \min_j |x_0(j) - x_i(j)| + \rho \max_i \max_j |x_0(j) - x_i(j)|}{|x_0(j) - x_i(j)| + \rho \max_i \max_j |x_0(j) - x_i(j)|} \quad (15)$$

where, ρ is the resolution coefficient which ranges from 0 to 1, normally takes 0.5 here.

2.1.3. CRITIC-GRA Multi-Factors Evaluation Model

The CRITIC method is used to determine the weight values matrix W shown in Equation (14) for all preset schemes and considered indicators, which is then applied to the GRA method to determinate the importance rank of the selected or calculated ranging values of safety and economic indicators. The GRA method allows for the quantitative analysis of various influential factors. Combining the outcomes with these two methods provides a unique evaluation criterion for the stope dimension optimization considering the open stoping stability, mining capacity and costs in the same model, and the criterion is the final output value of grey correlation degree r_i shown in Equation (16).

$$r_i = \sum_{j=1}^{n} w_i \zeta_i(j) \quad (16)$$

where, r_i is the grey correlation degree of scheme i, w_i is the weight value matrix of each factor shown in Equation (14), and ζ_i is the grey correlation coefficient of scheme i shown in Equation (15).

According to the magnitude of grey correlation degree r_i, the preset schemes can be prioritized, and a higher grey correlation degree indicates a better suggested stope dimension scheme.

The advantage of the proposed CRITIC-GRA method is that it completely eliminates the impact of subjective factors and finds the optimal scheme with the consideration of diverse factors. Moreover, the method imposes no constraints on the quantity or regularity of the considered indicators, which makes it possible to synthesize and analyze the open stoping stability index from the Mathews stability graph, the numerical simulated results and the economic indexes of mining capacity and costs.

2.2. Detailed Procedure of CRITIC-GRA Model to Optimize Stope Dimensions

A flow chart of the CRITIC-GRA model to quantitatively select the optimal dimension scheme with effects of multi-indicators is shown in Figure 2. In order to apply the newly proposed CRITIC-GRA model to optimize stope dimensions after considering multi-indicators such as open stope stability, production capacity and mining costs, the detailed procedure and all the influences considered are highlighted in Figure 3.

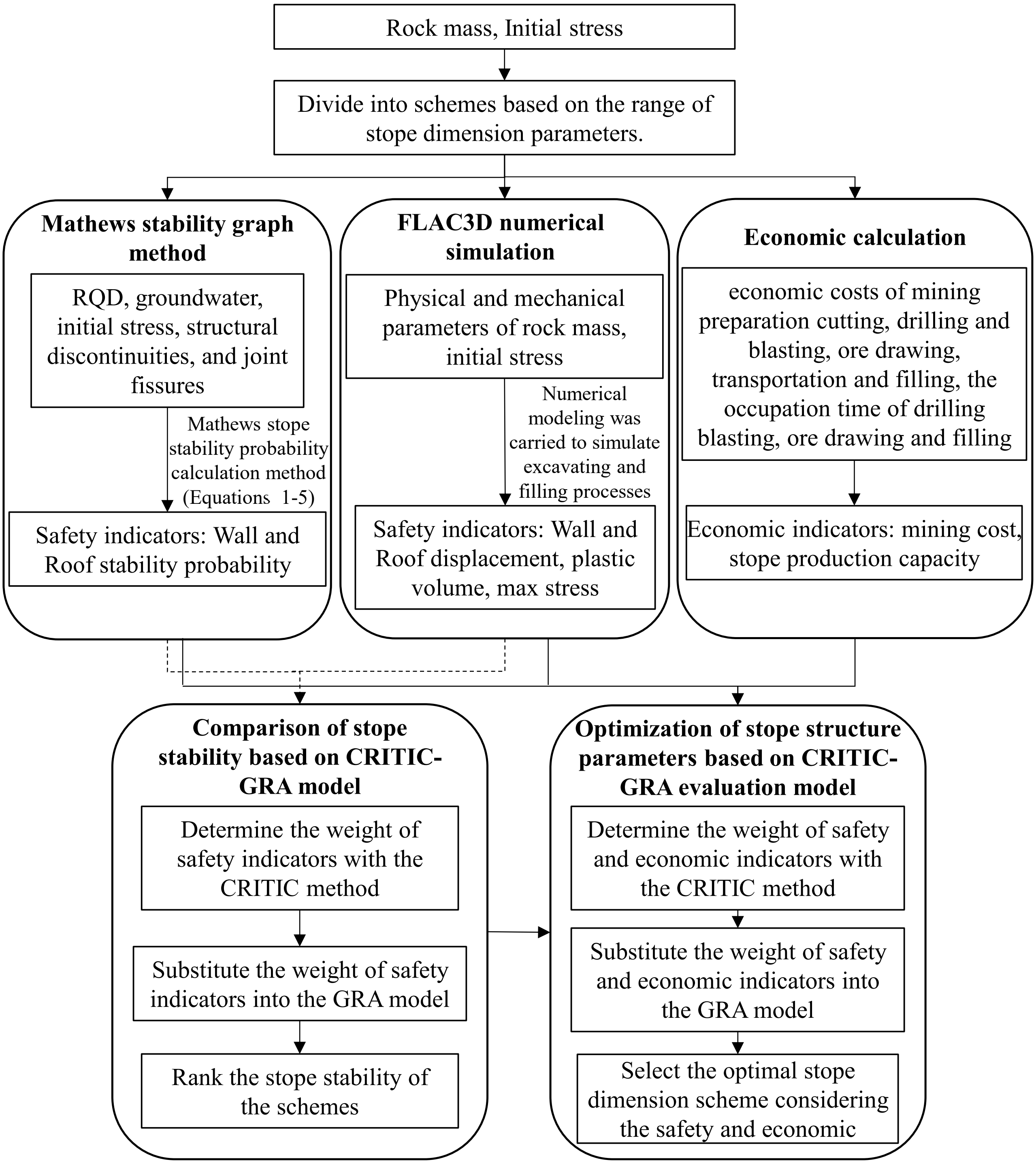

Figure 3. Detailed procedure of CRITIC-GRA model to optimize stope dimensions considering multi-indicators of open stope stability, production capacity and mining costs.

2.2.1. Comparison of Stope Stability Indicators Based on CRITIC-GRA Model

All factors that may affect stope stability are typically categorized into rockmass quality factors and stope structural characteristics factors. Rockmass quality factors encompass the physical and mechanical properties of rockmass and geological conditions including the *RQD* value of rockmass, groundwater conditions, in situ stress, properties of geological joints and discontinuities. Stope structural characteristics factors include the length, width and height of the excavated stope void.

Although the application of the Mathews stability graph method and FLAC3D numerical simulations could separately accomplish the analysis of stope stability with different influential factors, the results of these two methods usually exhibit obvious differences. Therefore, the CRITIC-GRA model is recommended to integrate the results of the above two for stope dimension optimization.

2.2.2. Optimization of Stope Dimensions Based on the CRITIC-GRA Model

The dimension parameters of the excavated stopes have great influences on the key economic indicators of mining operations, including ore loss rate, ore dilution rate, mining efficiency, production capacity, mining costs, time cycle of stoping and backfilling, etc. The determination of stope dimension parameters is a crucial aspect in mining operations. Generally, larger stope sizes result in better economic indicators including annual production and mining costs, but with compromised stope stability. Therefore, it is of great significance to optimize the selection of stope dimension parameters in mining operations by comprehensively considering economic and safety factors.

The proposed CRITIC-GRA model is used to construct a step-by-step method for the optimization of stope dimension parameters. Firstly, the CRITIC method is used to calculate the weight values of the above selected safety and economic indicators, including the safety indicators such as the stability probability for side walls and roof via the Mathews graph, the safety indicators such as the maximum displacement, plastic zone volume and maximum principal stress calculated via FLAC3D simulations and the economic indicators such as the mining cost index and stope production capacity calculated with the mining design index and actually verified index in mining operations. Then, the weight matrix of each factor is used to calculate the ranking of each preset scheme with different combinations of length, width and height. After that, the grey correlation coefficient of each factor in each dimension scheme is assessed via the GRA method. Finally, the optimal selection on the stope dimension scheme is obtained based on the unique criterion of the grey correlation degree, i.e., the product of the weight values and the grey correlation coefficient for each factor in each dimension scheme.

3. Application and Illustration of the Proposed CRITIC-GRA Model

3.1. Project Overview

The proposed CRITIC-GRA model was illustrated in details based on its application to a newly built underground iron mine in Anhui Province, China. The 1# main orebody of the mine is a sedimentary-metamorphic iron deposit with main geological formations ranging from −200 m to −550 m, with an average dip of 85°, thickness of 35 to 70 m, and a strike length of more than 2 km. The ground surface of the mine is flat at an elevation of approximately +50 m. The long-hole stage open stoping with subsequent backfill mining method is the primary method to excavate the orebody, with the main levels designed to be mined at −500 m, −440 m, −380 m, −320 m, −260 m and −200 m in a bottom-up mining sequence. Currently, mining activities are concentrated on the primary stopes in the bottommost level −500 m~−440 m, while the other upper levels are under the engineering development period.

The intention of the double stage open stoping mining method for the primary stopes was originally proposed by the mine manager because the mining efficiency and stoping production capacity will be substantially improved if the stope height can be doubled from the current 60 m at the −500 m~−440 m level to 120 m at the −500 m~−380 m level. And the increased ore extraction volumes through the current mucking engineering structures at −500 m level will alleviate the financial pressure previously caused by the investment in mine construction. In addition, the following cemented backfill operations of the primary stope voids can build up a vertically intact man-made pillar instead of two separated parts at the middle −440 m level, which is beneficial to the adjacent secondary stopes mining.

Previous investigations into the rockmass quality of the mine have provided preliminarily evidence that the intact and stable rockmass (most regions with Grade I and partly Grad II classification) offers the possibility to enlarge the stope void height from 60 m in the design report to 120 m. In addition, a further expansion of the stope width from the previously designed 14 m could be considered. The expansions of the stope dimensions will affect the continuous stopes division layout along both the vertical and the strike directions of the orebody. Despite a predictable increase on stope production capacity and a reduction in tonnage costs under enlarged stope dimensions, a consequence of increased safety risk of the enlarged exposure on side walls and roof around the stope void could also occur. It is a key option for the mine to adjust the stope dimensions reasonably based on the practical rockmass properties to striking an optimal balance between enlarged stope stability, increased mining production capacity and costs.

Therefore, the typical stope dimension parameters including length (L = 40, 45, 50, 55, 60 m), width (W = 15, 20 m) and height (H = 60, 120 m) of the illustrated iron mine were combined to build up preset schemes to be input samples in the proposed CRITIC-GRA optimization process. As can been seen in Table 1, there are 20 preset schemes with different stope dimensions. The aim was to find out a relatively optimal scheme of them with an integrated stability probability via the updated Mathew graph and FLAC3D numerical simulations, and the estimated mining economic index for stoping production capacity and costs. The results can be an objective and reasonable reference for the decision-making of the mine designers or managers.

Table 1. Preset stope dimension schemes and the corresponding safety and economic indicators for CRITIC-GRA evaluation.

Preset Schemes	Stope Dimensions			Stability Probability		Simulated Safety Index with FLAC3D					Mining Economic Index	
	Length *L*	Width *W*	Height *H*	Wall Prob.	Roof Prob.	Wall Disp.	Roof Disp.	Wall Plastic Vol.	Roof Plastic Vol.	Max. Stress	Cost (CNY/t)	Capacity (t/Day)
1	40	15	60	0.91	0.90	0.70	0.80	38,357	10,846	37.65	54.03	521.21
2	40	15	120	0.88	0.90	1.00	0.80	84,220	16,573	38.78	48.89	795.48
3	40	20	60	0.91	0.87	0.70	0.90	34,943	13,851	38.05	52.41	552.20
4	40	20	120	0.88	0.87	1.00	0.90	69,045	21,630	39.81	47.50	813.20
5	45	15	60	0.90	0.89	0.80	0.85	46,401	11,004	37.63	53.37	534.49
6	45	15	120	0.87	0.89	1.10	0.85	96,484	17,937	37.62	48.24	803.29
7	45	20	60	0.90	0.86	0.80	0.90	39,114	16,190	38.04	51.90	563.29
8	45	20	120	0.87	0.86	1.20	0.90	84,839	24,768	38.62	47.00	819.29
9	50	15	60	0.89	0.89	0.85	0.85	58,041	12,875	37.47	52.84	545.60
10	50	15	120	0.85	0.89	1.20	0.85	110,462	18,526	36.66	47.71	809.64
11	50	20	60	0.89	0.86	0.90	1.10	45,333	17,388	37.93	51.49	572.47
12	50	20	120	0.85	0.86	1.25	1.10	105,128	26,903	37.63	46.60	824.21
13	55	15	60	0.89	0.89	0.95	0.95	40,591	13,548	37.21	52.40	555.03
14	55	15	120	0.84	0.89	1.40	0.95	109,289	19,661	36.63	47.28	814.90
15	55	20	60	0.89	0.85	0.95	1.20	64,314	19,724	37.64	51.15	580.19
16	55	20	120	0.84	0.85	1.40	1.20	136,156	29,812	36.71	46.27	828.28
17	60	15	60	0.88	0.88	0.95	0.95	38,727	14,071	37.03	52.04	563.13
18	60	15	120	0.83	0.88	1.40	1.00	121,240	20,988	36.68	46.93	819.33
19	60	20	60	0.88	0.85	1.00	1.20	53,346	19,720	37.49	50.87	586.79
20	60	20	120	0.83	0.85	1.50	1.20	126,167	28,619	36.86	46.00	831.70

3.2. Stability Index of Open Stoping

3.2.1. Stability Probability with the Updated Mathews Graph Method

An engineering geological investigation and evaluation of the geological structural planes of the rockmass around the field stopes in the iron mine was carried out. The diamond drilling and coring of the rockmass were performed to collect rock samples for lab testing of the rock's physical and mechanical properties. Based on the specific properties of the rockmass in the illustrated iron mine, the evaluated input data for the updated Mathew graph method are presented in Table 2.

Table 2. Rockmass properties of illustrated iron mine for Stability Index *M*.

RQD (%)	J_n	J_r	J_a	J_w	*SRF*	A	Roof B	Wall B	Roof C	Wall C
90	4	3	1.5	1	1	0.45	0.8	0.5	1	8

According to Equation (1), the Stability Index *M* of the above illustrated rockmass properties was calculated to be a fixed value of 8.2 and 18.1 for the roof and wall of the considered open stopes, respectively. It should be mentioned that the rockmass was assumed to be homogeneous and isotropic here which led to a set of constant values of rockmass properties, and thus a constant Stability Index *M* in all the preset schemes. In addition, the Shape Factor *S* for the side walls and roof of the open stopes with different preset dimensions were directly calculated with Equation (2) considering different combinations for the roof with stope length and width and for the wall with stope length and height.

From Table 3, it can be seen that the roof is stable when the stope length *L* = 40~60 m and width *W* = 15~20 m. For the investigated rockmass quality and properties, the roof reached a critical stable state at a length of 91.3 m when the stope width was 20 m.

The side wall of the single stage stope (i.e., height *H* = 60 m) is stable when the stope length *L* = 40~60 m, which reaches the critical stable state at the length of 91.3 m.

The side wall of the double stage stope (i.e., height *H* = 120 m) reaches the critical stable state at the stope length of 51.8 m which implies that the stope wall fails when the stope length is greater than this critical length (Scheme No. 14, 16, 18, 20 of Table 3).

And then, the probability *f(z)* of the stability conditions for exposed side walls or roof (abbreviated as wall probability and roof probability) around the excavated stope void can be assessed with Equation (5) and shown in Table 3. These probability values can be used as basic samples for further synthesis with the stability results from FLAC3D simulations and mining economic indexes.

3.2.2. Stability Results with FLAC3D Simulations

According to the preset schemes in Table 1 of different stope dimensions and the mining sequence for typical primary and secondary stopes shown in Figure 1, the stability of the surrounding rockmass in side walls and roof of excavated stope voids was simulated using FLAC3D. An overview of the model and typical divided stopes can be seen in Figure 4, along with two typical cross sections along the stope width and stope length.

The Mohr–Coulomb criterion is widely recognized for its simplicity and is commonly employed. Consequently, the backfill and rock was modeled as an elasto-plastic material obeying the Mohr–Coulomb criterion.

For the boundary conditions of the numerical model shown in Figure 4c,d, the bottom boundary was fixed in all directions. The four lateral surfaces were applied with the stress boundaries in the normal directions based on the variation of the actual in situ stresses with depth *h* from the ground surface in the mine.

The investigation and field tests on the in situ stress were previously carried out in the mine from the levels of −160 m to −500 m. The obtained maximum horizontal principal stress $\sigma_{h\ max}$ (MPa), the minimum horizontal principal stress $\sigma_{h\ min}$ (MPa) and the vertical stress σ_v (MPa) of the mine given in Equations (17)–(19), respectively, increased linearly with the mining depth *h* from the ground surface.

$$\sigma_{h\ max} = 0.0569H - 2.1017 \tag{17}$$

$$\sigma_{h\ min} = 0.0363H - 1.8183 \tag{18}$$

$$\sigma_v = 0.03H - 1.1633 \tag{19}$$

Table 3. Stability analysis of open stopes with the updated Mathews graph method.

Preset Schemes	Stope Dimensions			Roof				Wall			
	Length *L*	Width *W*	Height *H*	Stability Index *M*	Shape Factor *S*	State	Stable Probability	Stability Index *M*	Shape Factor *S*	State	Stable Probability
1	40	15	60	8.2	5.5	stable	89.7%	18.1	12.0	stable	90.9%
2	40	15	120	8.2	5.5	stable	89.7%	18.1	15.0	stable	87.9%
3	40	20	60	8.2	6.7	stable	86.8%	18.1	12.0	stable	90.9%
4	40	20	120	8.2	6.7	stable	86.8%	18.1	15.0	stable	87.9%
5	45	15	60	8.2	5.6	stable	89.3%	18.1	12.9	stable	90.1%
6	45	15	120	8.2	5.6	stable	89.3%	18.1	16.4	stable	86.5%
7	45	20	60	8.2	6.9	stable	86.1%	18.1	12.9	stable	90.1%
8	45	20	120	8.2	6.9	stable	86.1%	18.1	16.4	stable	86.5%
9	50	15	60	8.2	5.8	stable	89.0%	18.1	13.6	stable	89.3%
10	50	15	120	8.2	5.8	stable	89.0%	18.1	17.6	stable	85.2%
11	50	20	60	8.2	7.1	stable	85.6%	18.1	13.6	stable	89.3%
12	50	20	120	8.2	7.1	stable	85.6%	18.1	17.6	stable	85.2%
13	55	15	60	8.2	5.9	stable	88.7%	18.1	14.3	stable	88.6%
14	55	15	120	8.2	5.9	stable	88.7%	18.1	18.9	fail	/
15	55	20	60	8.2	7.3	stable	85.1%	18.1	14.3	stable	88.6%
16	55	20	120	8.2	7.3	stable	85.1%	18.1	18.9	fail	/
17	60	15	60	8.2	6.0	stable	88.4%	18.1	15.0	stable	87.9%
18	60	15	120	8.2	6.0	stable	88.4%	18.1	20.0	fail	/
19	60	20	60	8.2	7.5	stable	84.7%	18.1	15.0	stable	87.9%
20	60	20	120	8.2	7.5	stable	84.7%	18.1	20.0	fail	/

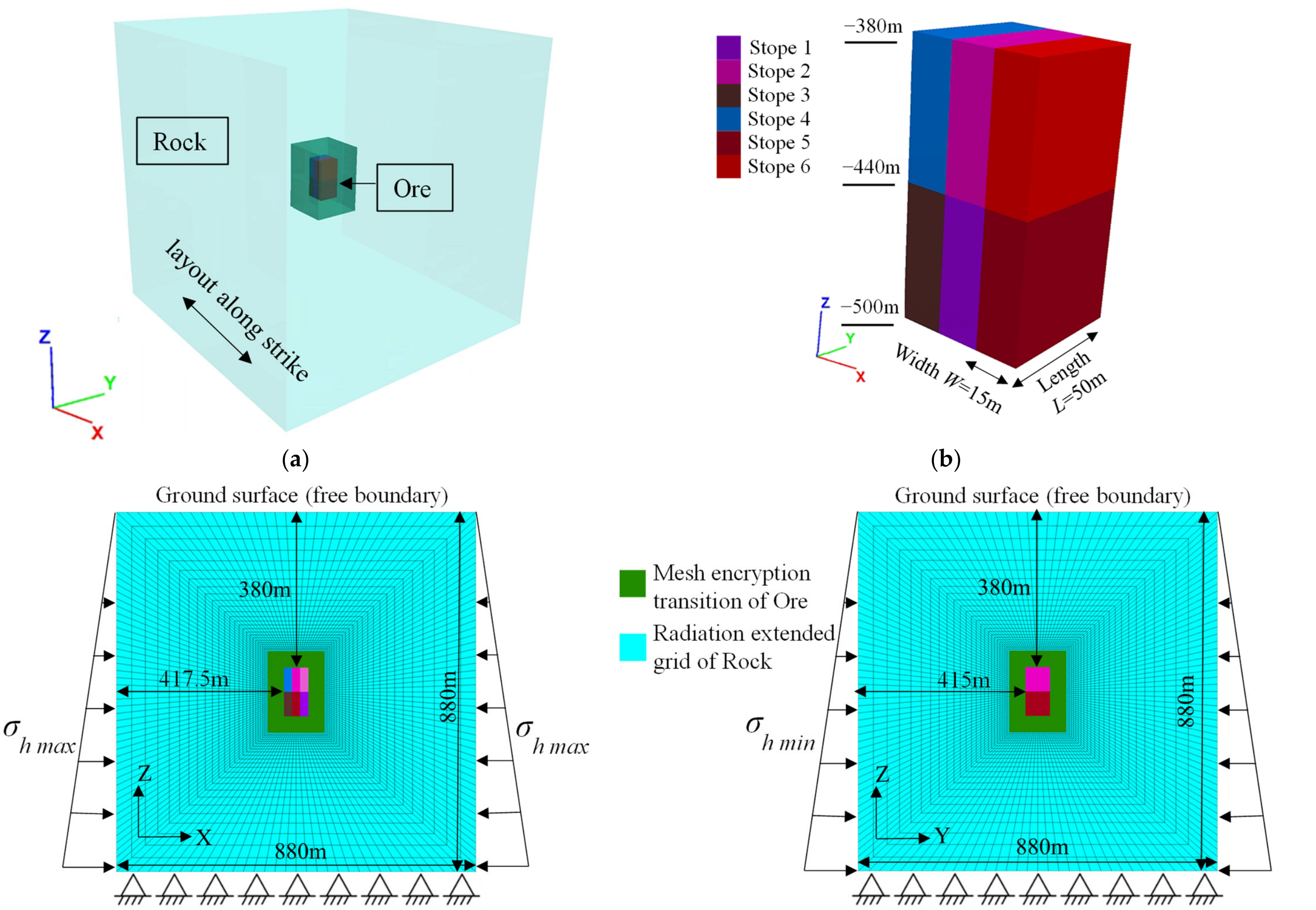

Figure 4. Numerical model constructed with FLAC3D for stability analysis of typical stopes in the illustrated iron mine: (**a**) overview; (**b**) mining sequence of primary and secondary stopes; (**c**) cross section along stope width; (**d**) cross section along stope length.

For the simulations of horizontal initial stresses in the numerical models, the field-tested maximum principal stress $\sigma_{h\,max}$ and minimum principal stress $\sigma_{h\,min}$ were applied to the external lateral boundaries, where the direction of the maximum principal stress $\sigma_{h\,max}$ was perpendicular to the strike of the ore body and the direction of the minimum principal stress $\sigma_{h\,min}$ was orthogonal to $\sigma_{h\,max}$. In the vertical direction, the whole model extended to the ground surface and there was no stress boundary at the top surface boundary which led to an initial vertical stress based on the gravity simulations of the whole model.

In addition, as shown in Figure 4c,d, the external expansions from side walls of the stopes to the outer side boundaries were set to be more than 400 m to eliminate the domain effect in numerical simulations. After a series of sensitivity analysis, the mesh for the central stopes was set as a uniform cube grid of 1 m on each side, and the outward mesh was radial to reduce the total number of grids.

Based on the lab test results of rock samples which were further calibrated with the geological structures survey and quality evaluation of rockmass, the physical and mechanical parameters of the rockmass used in the numerical models are listed in Table 4, which were consistent with the input parameters for the Mathews graph method. The properties of the cemented backfill were also tested with the cured samples by tailings and cement based on the designed backfill recipe in the illustrated iron mine.

Table 4. Physical and mechanical parameters of the rockmass and backfill used in models.

Item	Density (kg/m^3)	Young's Modulus (GPa)	Poisson Ratio	Cohesion (MPa)	Friction Angle (°)	Tensile Strength (MPa)
Rockmass	2847	10.50	0.28	1.60	41.6	0.62
Backfill	1950	0.80	0.30	0.70	30.0	0.40

As shown in Figure 4b, in the sequence of primary and secondary stopes in the single stage open stoping mining method illustrated in Figure 1a, the primary stope-1 between level −500 m and level −440 m was first excavated, and the stope void was then filled with the cemented backfill to provide the platform for the upper mining of the primary stope-2 between level −440 m and level −380 m. All stability indexes required for CRITIC-GRA model were collected around the excavated stope void which was 60 m in height. Moreover, for the simulations of mining procedure with double stage open stoping shown in Figure 1b, the primary stope-1′ between level −500 m and level −380 m was excavated at one time, and the corresponding stability indexes of side walls and roof for different dimension schemes were then collected around the stope void which was 120 m in height. The practical cemented filling began after the excavation was fully completed for the two stages.

Numerical simulations were performed in FLAC3D for the 20 preset schemes with different stope dimensions given in Table 1. The maximum displacement (m) of the roof and side walls (abbreviated as Roof Disp. and Wall Disp.), the volume of the plastic zones in the roof and side walls (abbreviated as Roof Plastic Vol. and Wall Plastic Vol.) and the maximum principal stress (abbreviated as Max. stress, MPa) around the excavated stope voids are summarized in Table 1.

Typical stability results for the No. 1 scheme in Table 1 are shown in Figure 5 with the double stage open stopes with a length of L = 40 m, a width of W = 15 m and a height of H = 60 m. Because of the limited space of the manuscript, the simulated results for the 20 schemes could not be fully presented here.

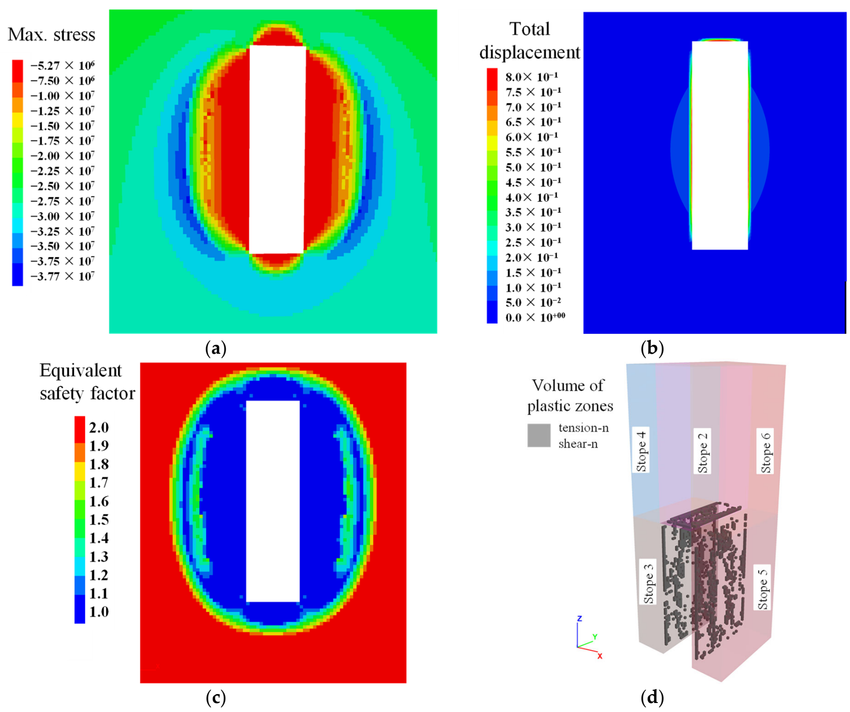

Figure 5. Numerical simulated results of typical stope dimension with width 15 m, length 40 m, height 60 m: (**a**) Maximum stress, Pa; (**b**) total displacement, m; (**c**) equivalent factor of safety; and (**d**) volume of plastic zones.

Figure 5a shows the stress release from the initial stress state and redistribution around the slope void after the excavation of primary Stope-1. A significant concentration of the maximum principal stress was observed around the excavated void with a maximum value of 37 MPa at a distance about 15 m from the stope boundaries.

From Figure 5b, the maximum total displacement of the rockmass occurred in the shallow parts of the stope roof at about 0.8 m. And the total displacement in the sidewalls was located in the part of middle height reaching about 0.7 m.

Figure 5c illustrates the equivalent factor of safety which was higher than 1.0 in the roof and side walls around the excavated stope void. This indicated that after the excavation of Stope-1, the surrounding rockmass could still maintain a relatively stable state.

It can be observed from Figure 5d that the plastic zones were developed in some parts of the surface areas in the roof and side walls around the stope void which meant the overall stable state of the excavation in the given dimensions.

3.2.3. Stope Stability Analysis with CRITIC-GRA Evaluation Model

To investigate the stope stability of single stage and double stage open stoping, the stability probability of the roof and side walls was evaluated via the updated Mathew graph method. The simulated maximum displacement and the volume of plastic zones with FLAC3D were seamlessly brought into the CRITIC-GRA model together to carry out an integrated analysis on the stope stability under different dimensions which were initially performed parallel in the two methods with no interactions.

Firstly, the prepared initial data shown in Table 1 were substituted into Equations (7)~(14) to calculate the weight values of all the considered influential factors on the stope stability, as shown in Table 5.

Table 5. Weight values for influential factors on stope stability with the CRITIC model.

Factors	Wall Prob.	Roof Prob.	Wall Disp.	Roof Disp.	Wall Plastic Vol.	Roof Plastic Vol.
Weight values	0.14	0.22	0.15	0.21	0.17	0.11

Then, the weight values were substituted into Equations (15) and (16) to calculate the values of the grey correlation degree of each preset scheme with different stope dimensions, as shown in the column 'Only Safety Index' in Table 6, where the economic indexes for open stope mining have not been considered yet.

In these three columns, the stability conditions only for the side wall, only for the roof, and for the overall stope void were considered to reveal their specific relationships with stope dimensions (length L, width W, height H), which were innovatively accomplished by the stability results combining the empirical graph method with the numerical simulation method. In addition, the corresponding grey correlation degree for the side walls, for the roof and for the overall stope void which varied with stope dimensions were further graphed in Figure 6 to illustrate their integrated stability conditions.

Figure 6a shows the calculated values of grey correlation degree with the CRITIC-GRA model to illustrate the stability conditions of the side walls as a function of the stope length, width and height. A greater value of grey correlation degree means a better stability condition. It can be seen that the stability of the side walls in the single stage open stope was obviously greater than the double stage open stope. For the single stage open stope, when the length was less than 50 m, the stability of side wall with a width of 15 m and 20 m was almost the same. However, when the length was more than 50 m, the stability of the side wall with a width of 20 m showed a continuous decreasing trend that was much lower than that of the single stage open stope with a width of 15 m.

Table 6. Grey correlation degree for preset schemes with different stope dimensions via CRITIC.GRA.

Preset Scheme	Length *L* (m)	Width *W* (m)	Height *H* (m)	Grey Correlation Degree					
				Only Safety Index			Integrated Safety and Economic Index		
				Only Wall Stability	Only Roof Stability	Overall Stability	Overall Stability + Cost	Overall Stability + Capacity	Overall Stability + Cost + Capacity
1	40	15	60	0.97	1.00	0.99	0.77	0.75	0.70
2	40	15	120	0.53	0.83	0.75	0.65	0.71	0.69
3	40	20	60	1.00	0.69	0.82	0.67	0.65	0.63
4	40	20	120	0.58	0.53	0.58	0.57	0.63	0.63
5	45	15	60	0.84	0.89	0.85	0.69	0.67	0.65
6	45	15	120	0.48	0.74	0.65	0.64	0.69	0.67
7	45	20	60	0.89	0.57	0.71	0.62	0.60	0.59
8	45	20	120	0.48	0.49	0.52	0.57	0.61	0.62
9	50	15	60	0.72	0.80	0.77	0.65	0.63	0.61
10	50	15	120	0.43	0.70	0.61	0.68	0.73	0.71
11	50	20	60	0.77	0.46	0.59	0.55	0.53	0.53
12	50	20	120	0.42	0.38	0.42	0.55	0.59	0.61
13	55	15	60	0.76	0.69	0.71	0.63	0.61	0.59
14	55	15	120	0.38	0.59	0.53	0.63	0.68	0.67
15	55	20	60	0.64	0.41	0.51	0.51	0.49	0.49
16	55	20	120	0.35	0.34	0.37	0.58	0.61	0.63
17	60	15	60	0.76	0.66	0.69	0.63	0.61	0.59
18	60	15	120	0.37	0.55	0.49	0.62	0.66	0.66
19	60	20	60	0.61	0.39	0.48	0.51	0.49	0.49
20	60	20	120	0.34	0.34	0.36	0.58	0.60	0.63

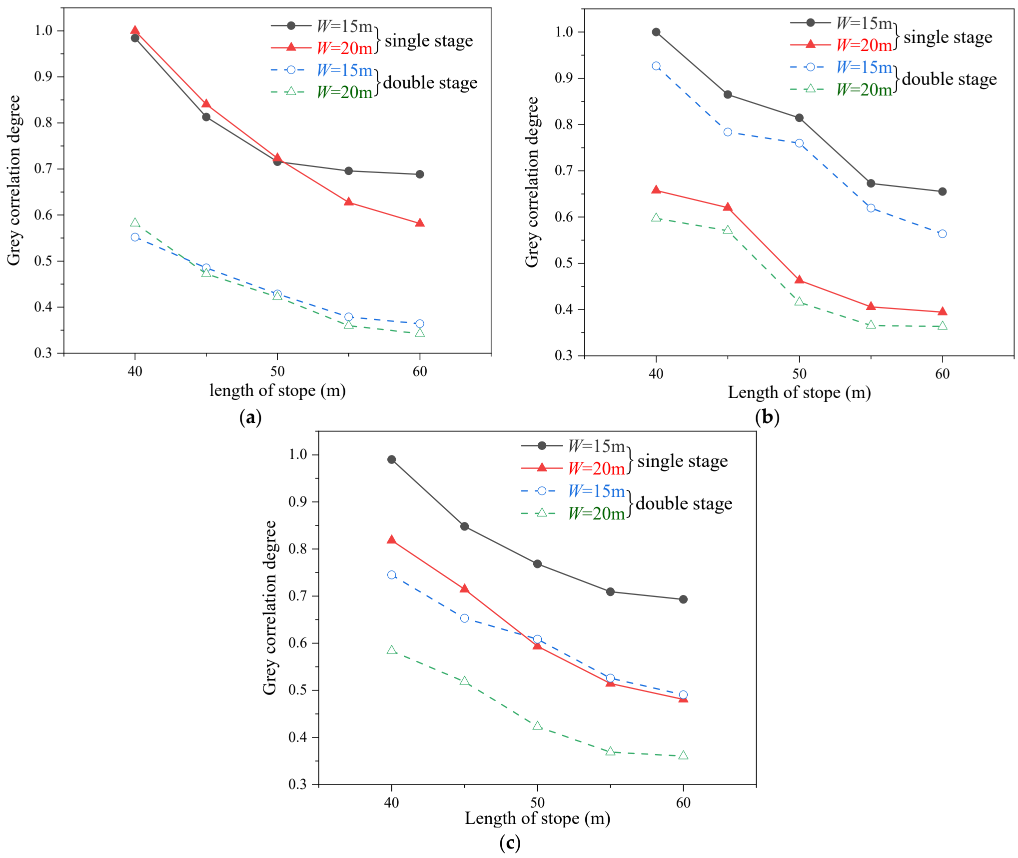

Figure 6. Grey correlation degree about stability conditions for preset schemes of different stope dimensions: (**a**) only for the side walls; (**b**) only for the roof; and (**c**) for the overall stope void predicted by the CRITIC-GRA model without considering economic indicators.

In addition, from Figure 6a, the stability of the side walls with a width of 15 m and 20 m for the double stage open stope reduced with the increase in stope length, and the stability of the side wall of the 15 m width stope was slightly higher than that of the 20 m width. Therefore, when the stope length was larger than 50 m, it was wise to select a stope with a width of 15 m to ensure the integrity of the side walls. The value of stope height has the greatest effect on the wall stability, followed by stope length and width.

It can be seen from Figure 6b that the values of grey correlation degree representing the stability conditions of the stope roof decreased with the increase in stope length. For both single stage and double stage open stopes, the roof stability with a stope width of 15 m was much greater than that of the 20 m width. Specifically, for the stopes with a width of 15 m or 20 m, the roof stability of the single stage stope was a little higher than the double stage stope. Stope width has the greatest influence on the roof stability, followed by the stope length and height. Therefore, it is the key consideration to optimize the stope width to achieve satisfactory roof stability.

Figure 6c gives the overall stability around the stope void quantitatively represented by the grey correlation degree, which meant all the safety indexes for stope roof and side walls computed by the Mathews empirical graph method and the numerical simulation method were wholly input into the CRITIC-GRA model. It is interesting to note that when the stope length varies from 40 m to 48 m, the overall stability of the single stage stope with a 20 m width is superior to that of the double stage stope with a 15 m width. However, when the stope length falls in the range of 48 m to 60 m, the double stage stope with the 15 m width has the whip hand in stability. Therefore, when the stope length exceeds 48 m with the given rockmass quality, the double stage open stope with the 15 m width prevails over the single stage open stope with the 20 m width.

It can be further obtained from Figure 6 that the overall stability of the open stope can be improved instead of reduced to enlarge the single stage stope height 60 m to a double stage height of 120 m by slightly reducing the stope width from 20 m to 15 m. This will dramatically increase the blasted ore volume with long-hole mass mining and obtain a desirable overall stability in a stope with a width of 15 m and a height of 120 m compared to a stope with a width of 20 m and a height of 60 m.

3.3. Estimation of Economic Indicators of the Open Stoping

Discussions on stope dimension optimization would lose much of their value if the economic indicators and the stability conditions were not considered together. In this paper, there were a total of 20 preset schemes for the illustrated iron mine. The economic indicators including the unit costs for typical mining process and the production capacity of a typical open stope were collected and are shown in Tables 7 and 8, respectively.

Table 7. The unit costs for typical mining process in open stoping of the illustrated iron mine.

Item	Preparation Cutting (CNY/m^3)	Ore Drilling (CNY/m^3)	Ore Blasting (CNY/t)	Ore Mucking (CNY/t)	Ore Transportation (CNY/t)	Stope Backfilling (CNY/m^3)	Stope Ventilation and Drainage (CNY/t)
unit costs	400	80	8	7	8	20	7

Table 8. The production capacity of a typical open stope in the illustrated iron mine.

Item	Rate of Ore Recovery (%)	Rate of Ore Dilution (%)	Ore Drilling Ability (m/Day)	Ore Mucking Ability (t/Day)	Stope Backfilling (m^3/Day)
Values	90	11	120	900	720

In Table 7, the unit costs for a typical mining process in open stoping of the illustrated iron mine cover the open stope preparation cutting, ore drilling, ore blasting, ore mucking, stope backfilling, stope ventilation and drainage. In Table 8, the values of the production capacity of a typical open stope in the illustrated iron mine were collected and calculated by evaluating the occupation time of drilling, blasting, mucking and filling, and these factors further determine the rates of ore recovery, ore dilution, ore drilling and mucking ability and the stope void backfilling capacity. These unit data can be used to calculate the economic indicators for open stopes with the different stope dimensions listed in Table 1.

3.4. Optimization of Stope Dimensions with CRITIC-GRA Model Considering Stability and Economic Indexes

To pick out the optimal scheme among the preset 20 schemes with different stope dimensions (length, width and height), the stope stability and corresponding economic indexes were jointly brought into the CRITIC-GRA model for integrative analysis. The stope stability indexes consist of the stability probability using the updated Mathews graph method, and the typical safety indexes simulated with FLAC3D for each scheme of stope sizes. Furthermore, the economic indexes include the mining cost and production capacity for the typical open stope in different schemes.

Firstly, all the prepared initial data shown in Table 1 were substituted into Equations (7)–(14) to calculate the weight values of all the considered influential factors on stope stability and economic indexes, as displayed in Table 9.

Table 9. Weight values for influential factors on stope stability, mining capacity and costs.

Factors	Updated Mathews Graph		Simulated Safety Indexes with FLAC3D				Economic Indexes	
	Wall Prob.	**Roof Prob.**	**Wall Disp.**	**Roof Disp.**	**Stope Overall Plastic Vol.**	**Max. Stress (MPa)**	**Capacity (t/d)**	**Cost (CNY/t)**
Weight values	0.111	0.111	0.108	0.106	0.110	0.110	0.181	0.163

According to data in Table 9, the weight values of the economic indexes were greater than those of safety indexes, which meant that the economic indexes of open stope in the preset schemes weighed more on the integrated analysis and optimal scheme selection. The calculated weight values in the CRITIC model gave an objective and quantitative evaluation of the importance of each considered index on the optimization of stope dimension.

Then, the calculated weight values in Table 9 were substituted into Equations (15) and (16) to calculate the values of grey correlation degree of each preset scheme, which are listed in the columns of 'Integrated Safety and Economic Index' in Table 6 after considering the economic indexes for open stope mining. In addition, the integrated analysis about the 'overall stability + cost', 'overall stability + capacity' and the 'overall stability + cost + capacity' were separately given in detail for different considerations such as mining cost, mining capacity and their combination with the overall stability for the sake of stope dimension optimization.

To deepen the exploration, Figure 7 plots the grey correlation degree of the integrated model that combines stability and economic indexes across all the preset schemes to illustrate the stope dimension optimization from multiple perspectives.

Combining the information in Table 6 and Figure 7, the preset scheme No. 10 with the length of L = 50 m, width of W = 15 m and height of H = 120 m was the objectively selected optimal stope dimension using the proposed CRITIC-GRA model after comprehensively considering the open stoping stability, mining capacity and costs. With the optimized stopes dimension, the iron mine adjusted the stope design and carried out the practical mining operations which proved that the double stage open stoping for primary stopes with the above dimensions could obviously increase the mining production efficiency with a safe open stoping stability. The illustrated CRITIC-GRA model is a reliable reference tool for selecting the optimal stope dimension based on the designed schemes, rockmass quality, and the technical and economic indexes of mining process in different mines.

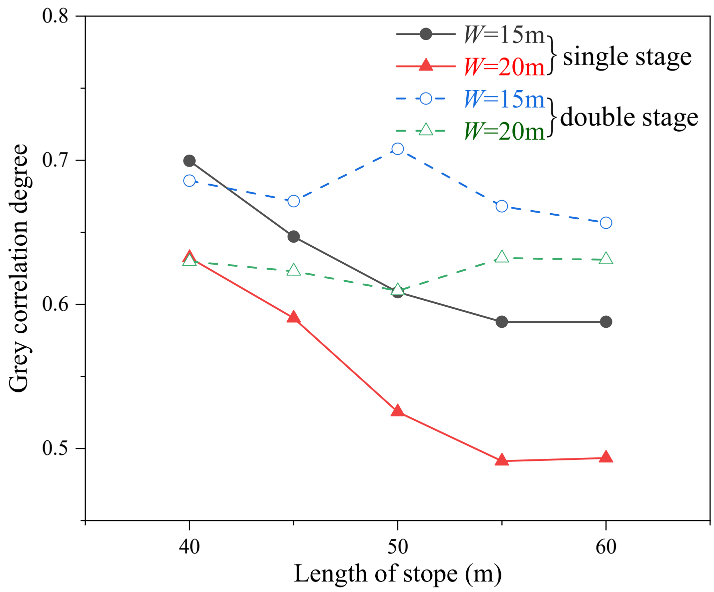

Figure 7. Grey correlation degree of open stope with different dimensions considering the overall stability and economic indexes.

For double stage open stopes with widths of 15 m and 20 m, the grey correlation degree of comprehensive evaluation shows little variation with increasing stope length. However, at the length of 50 m, the double stage open stope with a width of 15 m demonstrates better production capacity and safety, thus indicating a relative increase in the grey correlation degree for this scheme.

In addition, Figure 7 shows that with the increase in stope length, the grey correlation degrees for single stage open stope schemes decreased greatly while that of the double stage open stope schemes had no obvious reduction, which meant the effect of length variation was more sensitive to the single stage open stopes. For the double stage open stopes, the stope width has a significant influence on the stope dimension optimization, but the linear trend of the stope width has not been witnessed, after considering the stability and economic indexes at the same time.

4. Limitations and Discussion

A new CRITIC-GRA model has been proposed for stope dimension optimization considering open stoping stability, mining capacity and costs, but there are still some limitations of the method.

The properties of the rockmass used for the Mathews stability graph and FLAC3D simulations in the CRITIC-GRA model were assumed to be homogeneous and isotropic. The nonhomogeneous properties of the rockmass can lead to statistical variations for the calculated safety indexes of open stopes which will further affect the dimension optimization. In the next step, the research will focus on the variations and distributions of rockmass properties considering the variations of rockmass quality in different regions of the mined ore body.

Many critical factors had been taken into account in the CRITIC-GRA model including rockmass quality, stope dimensions, excavation and backfill process etc., but there are still some practical factors that have not been considered, including the safety indicators of stope blasting and groundwater and the economic indicators of mining equipment efficiency and ventilation.

In the proposed CRITIC-GRA method, the weight values of different influential factors were calculated based on the preset schemes of different stope dimensions. In practical mines, the weight value of a critical value might be more important in different conditions.

The reasonable evaluation method of these weight values requires more considerations under different practical circumstances.

5. Conclusions

In order to achieve optimal selection of stope dimension parameters in long-hole stage open stoping with the subsequent backfill mining method, a new CRITIC-GRA model was developed and illustrated for the first time in this paper in an iron mine, which innovatively integrated the stope stability, mining capacity and costs in the analysis. The key conclusions regarding the model and its application are listed as follows.

The CRITIC-GRA model proposed for selecting the optimal scheme considering the effects of multi-indicators consists of four key modules: the determination of all the considered possible influential indicators; analysis on the weight values of all the considered indicators using the CRITIC method; statistical analysis on the ranging conditions of each indicator using the GRA method to obtain the grey correlation coefficient in each scheme; and calculations on the grey correlation coefficient of each preset scheme with the weight values of all the considered indicators to obtain the grey correlation degree for each scheme. The obtained grey correlation degree serves as the only quantitative criterion to evaluate the optimum out of all the preset schemes.

In the process of applying the CRITIC-GRA model to optimize stope dimensions, the key influential indicators on open stoping stability, mining capacity and costs consist of the safety indicators including stability probability for the side walls and the roof via the updated Mathews graph method, the maximum displacement, plastic zones volume and maximum principal stress via FLAC3D simulations, and the economic indicators including the mining costs and stope production capacity adopted in the mining design and practically verified in the mine operations.

In the model illustration of an iron mine, the proposed CRITIC-GRA was firstly used to analyze the stability conditions of the open stope. The primary influence on stope wall stability is attributed to stope height, followed by stope length and width. The roof stability is primarily influenced by stope width, with stope length and height playing secondary roles. With the given rockmass quality in the illustrated iron mine, when the stope length exceeds 48 m, the double stage open stope with a 15 m width shows a better overall stability than the single stage open stope with a 20 m width. The overall stability of the open stope can be improved instead of reduced to enlarge the single stage stope height of 60 m to a double stage height of 120 m by reducing the stope width from 20 m to 15 m, which can significantly increase the mineable ore amount and improve the stope safety.

The proposed CRITIC-GRA was employed to optimize the stope dimensions considering the stope safety and economic indicators synchronously in the demonstration iron mine. The objectively calculated weight values assigned to the economic indexes surpassed those of safety indexes, indicating that the economic factors of open stope within the preset schemes exerted a more pronounced influence on the integrated analysis and optimal selection of varying stope dimensions, which were mostly ignored in the previous analysis on stope dimension optimization. Discussions on the stope dimension optimization lose much of their value if the economic indicators are not considered along with the stability conditions.

After applying the CRITIC-GRA model to the iron mine for an integrated evaluation of open stope stability, mining capacity and costs, scheme No. 10 with the stope length of L = 50 m, width of W = 15 m and height of H = 120 m was objectively identified as the optimal stope dimension among all the preset 20 schemes.

Based on the illustration and verification of the proposed model in the mine, the proposed CRITIC-GRA model offers a dependable reference tool for determining the optimal stope dimensions in similar underground mines, considering multiple factors of designed dimension schemes, rockmass quality and technical and economic indicators of stope mining.

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