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An Interval Pythagorean Fuzzy Multi-criteria Decision Making Method Based on Similarity Measures and Connection Numbers

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Abstract: Interval Pythagorean fuzzy set (IPFS), which can handle imprecise and ambiguous information, has attracted considerable attention in both theory and practice. However, one of the main difficulties under IPFSs is the comparison between interval numbers. To overcome this shortcoming, connection number theory is first introduced, and interval numbers are transformed into connection numbers in the operating process. Considering that similarity measures play an important role in assessing the degree between ideal and proposal alternatives in the decision making process, this paper aims to develop new similarity measures with IPFSs and apply them to multi-criteria decision making (MCDM) problems. The main contributions of this paper are as follows: (1) introduction of a comparison method through transforming interval numbers into connection numbers; (2) development of three new similarity measures with IPFSs based on the minimum and maximum operators, and investigation of their properties; (3) calculation of the similarity measures considering weights of membership and non-membership degrees; (4) establishment of an interval Pythagorean fuzzy decision making method applying the presented similarity measures. A case study on selecting a project delivery system is made to show the applicability of the proposed approach.

Keywords: interval Pythagorean fuzzy set; similarity measures; multi-criteria decision making; connection number

1. Introduction

The multi-criteria decision making (MCDM) method plays an important role in selecting the optimal criteria among a finite number of feasible alternatives evaluated according to multiple criteria in real-life decision making problems under uncertainty. To select a desirable alternative, the classical approaches attempt to obtain a degree of similarity between ideal and proposal alternatives.

The similarity measures are important and useful tools for determining the degree of similarity between two objects. Measures of similarity between fuzzy sets have gained attention from researchers for their wide applications in various fields, such as pattern recognition, machine learning, decision making, and image processing [1–4]. Fuzzy set theory, which was introduced by Zadeh [5], has been widely used to handle uncertainty in real-world applications. Atanassov [6,7] extended fuzzy sets to Atanassov's intuitionistic fuzzy sets (IFSs), whereas different similarity measures between IFSs have been investigated in the literature [8]. Meng and Chen [9] introduced a construction approach to obtain the similarity measure of Atanassov's intuitionistic fuzzy sets (IFSs), and then defined three Shapley-weighted similarity measures. Liu et al. [10] proposed the cosine similarity measures for intuitionistic fuzzy linguistic sets (IFLSs) and interval-valued intuitionistic fuzzy linguistic sets (IVIFLSs). Furthermore, the weighted cosine similarity measure and ordered weighted cosine similarity measure for IFLSs and IVIFLSs were introduced by considering the importance of each element. Based on the extension of the Hamming distance on fuzzy sets, Szmidt and Kacprzyk [11,12] developed a similarity measure between IFSs based on the Hamming distance. Hung and Yang [13] calculated the distance between IFSs based on the Hausdorff distance and generated similarity measures between IFSs. Chen and Chen [14] proposed a similarity measure based on the center of gravity of generalized fuzzy numbers; this similarity measure is complicated and inapplicable for general LR-type. Hung and Yang [15] proposed similarity measures between IFSs based on the Lp metric. Xu and Xia [16] defined the geometric distance and similarity measures of IFSs for group decision making problems. Ye [17] proposed the cosine similarity measure between IFSs. Hung [18] developed the likelihood-based measurement of IFSs for medical diagnosis and bacterial classification problems. Shi and Ye [19] further improved the cosine similarity measure of IFSs. Tian [20] proposed the cotangent similarity measure between IFSs for medical diagnosis. Rajarajeswari and Uma [21] further introduced the cotangent similarity measure, which considers the membership, non-membership, and hesitation degrees in IFSs. Furthermore, Szmidt [22] discussed the distances between IFSs and introduced a family of similarity measures, which consider the membership, non-membership, and hesitation degrees described in IFSs. Ye [23] proposed two new cosine similarity measures and weighted cosine similarity measures based on cosine function and the information carried by membership, non-membership, and hesitancy degrees in IFSs. Le and Phong [24] provided the intuitionistic vector similarity measures for medical diagnosis.

The Pythagorean fuzzy set (PFS) was originally developed by Atanassov [25–27] in 1999 using the name "intuitionistic fuzzy sets of second type", whereas further applications were made by Yager [28,29] in the decision making field since 2013. The PFS is also characterized by membership and non-membership degrees, whose sum of squares is less than or equal to 1. The PFS is equivalent to IFSs of second type and powerfully handles uncertain problems. Throughout the whole paper, we use the name "Pythagorean fuzzy sets" similar to various existing references [30,31].

Numerous researchers have conducted considerable work in the field of PFSs and interval Pythagorean fuzzy sets (IPFSs) [32–38]. Zhang and Xu [39] provided the detailed mathematical expression for PFSs and introduced the concept of the Pythagorean fuzzy number (PFN). Later, Peng and Yang [40] introduced the concept of IPFSs, which is a generalization of PFSs and interval-valued (IFSs). The fundamental characteristic of IVIFS is that the values of their membership and non-membership functions are intervals rather than exact numbers. Khan and Abdullah [41] introduced the concept of multiple-attribute group decision making (MAGDM) problems with interval-valued Pythagorean fuzzy and then presented the concept of the interval-valued Pythagorean fuzzy Choquet integral average (IVPFCIA) operator. Based on the IVPFCIA operator, Khan et al. [42] established an extension of the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method to deal with multi-attribute interval-valued Pythagorean fuzzy MAGDM problems. A multiple-attribute decision making method was proposed with incomplete weight information under PFSs and Pythagorean hesitant fuzzy sets in [43] and [44], respectively. Khan et al. [45] investigated a Pythagorean fuzzy-prioritized weighted average operator and Pythagorean fuzzy-prioritized weighted geometric operator and their properties. Then, a MAGDM approach based on the developed operators under a Pythagorean fuzzy environment was proposed.

The similarity measure is an important tool to assess the degree between ideal and proposal alternatives, with numerous studies focusing on the development of PFSs and IPFSs. Garg [46] proposed a novel correlation coefficient and weighted correlation coefficient formulation to measure the relationship between two PFSs. Biswas [47] presented the cosine similarity measure and weighted cosine similarity measure and then applied them to the multi-attribute decision making problem with a trapezoidal fuzzy neutrosophic environment. Xu et al. [48] proposed a variation coefficient similarity measure based on the extension of the Dice and a cosine similarity measure to show that the proposed method features better similarity identification and practicability; the similarity measures were applied to emergency group decision making problems. Zhang [49] first presented a novel distance measure for PFSs and discussed their desirable properties; then, a simple and effective Pythagorean fuzzy group decision making method was introduced and applied to a selection problem of photovoltaic cells. Wei and Wei [50] proposed 10 similarity measures between PFSs based on the cosine function by considering the membership and non-membership degrees in PFSs and applied these similarity measures and weighted similarity measures between PFSs for pattern recognition and medical diagnosis. Recently, Peng et al. [51] investigated the relationship between the distance measure, similarity measure, entropy, and inclusion measure for PFSs and suggested the systematic transformation of information measures (distance measure, similarity measure, entropy, and inclusion measure) and their new formulae.

The knowledge theory on similarity measures must be expanded. However, similarity measurement under IPFSs presents difficulty. The main obstacles and difficulties lie in the comparison between two interval numbers, although numerous comparison approaches are available. Additionally, the comparison method is the main operation for minimum and maximum operators. Therefore, there are two gaps that should be bridged: (1) the process of calculation in the existing similarity measures is too complex to apply to more practical fields; (2) a shortcoming for the development of similarity measures under IPFSs is to ignore the "true psychological" behavior and degree of confidence from decision experts. To overcome this shortcoming, the aim of this study is to build an interval Pythagorean fuzzy multi-criteria decision making method based on similarity measures and connection numbers. The main contributions of this paper are: (1) propose a comparison method between two interval numbers using the binary connection number in set pair analysis theory (SPAT) presented by Zhao [52], which measures the relationship of a set pair consisted of two interdependent sets from "identical", "discrepancy", and "contrary" features of the system, and transform the interval number to the connection number in the comparison process; (2) present similarity measures based on minimum and maximum operators which are simple and easy in the calculation process; (3) calculate the similarity measures considering weights of membership and non-membership degrees; (4) establish an interval Pythagorean fuzzy decision making method applying the presented similarity measures.

The remainder of this paper is organized as follows. In the next section, we introduce the basic concepts related to interval numbers, PFSs, IPFSs, and connection numbers in SPAT and the relationship between interval and connection numbers. Section 3 proposes three new similarity measures (i.e., 1-type IPFS similarity measure, 2-type IPFS-weighted similarity measure, and 3-type IPFS-weighted similarity measure) between IPFSs based on minimum and maximum operators and investigates their properties. Section 4 proposes an interval Pythagorean fuzzy decision making method based on the 3-type IPFS-weighted similarity measure. Section 5 provides an example of selection of a project delivery system (PDS) to demonstrate the applications and effectiveness of the proposed decision making approach. Comparison analysis and discussion are given in Section 6. Section 7 provides conclusions and further research suggestions.

2. Preliminaries

This section introduces preliminaries on the concepts of PFS and IPFS and their operations. The concepts of PFSs originated from the work of Atanassov [25], with the name "intuitionistic fuzzy sets of second type" and with further development by Yager [28,29], similar to the latest research in decision making problems [30,31].

2.1. Concepts of PFS and IPFS

Definition 1 [29]. *Let X be a universe of discourse. A PFS P in X is given by the following:*

$$P = \{\langle x, u_P(x), v_P(x)\rangle | x \in X\} \tag{1}$$

where $u_P(x) : X \mapsto [0,1]$ *denotes the degree of membership, and* $v_P(x) : X \mapsto [0,1]$ *refers to the degree of non-membership of element* $x \in X$ *to P. Further,* $\pi_P = \sqrt{1 - u_P^2(x) - v_P^2(x)}$ *is called the Pythagorean fuzzy index of element* $x \in X$ *to P, representing the degree of indeterminacy of x to P. Furthermore,* $0 \le \pi_P \le 1$ *for every* $x \in X$.

Moreover, $u_P(x)$ and $v_P(x)$ satisfy the following condition:

$$0 \le u_P^2(x) + v_P^2(x) \le 1, \forall x \in X.$$

Definition 2 [29]. *A PFS* P_1 *is contained in another PFS* P_2*, i.e.,* $P_1 \subseteq P_2$*, if and only if*

$$u_{P_1}(x) \le u_{P_2}(x),\ v_{P_1}(x) \ge v_{P_2}(x)$$

for all $x \in X$.

Definition 3 [40]. *Let X be an ordinary finite nonempty set, and an* IPFS $\overline{P}$ *over X is an object with the following mathematic form:*

$$\overline{P} = \{\langle x, (\overline{u}_{\overline{P}}(x), \overline{v}_{\overline{P}}(x))\rangle | x \in X\},$$

where $\overline{u}_{\overline{P}}(x) = \left(\left[\overline{u}_{\overline{P}}^L(x), \overline{u}_{\overline{P}}^R(x)\right]\right) \subset [0,1]$ *and* $\overline{v}_{\overline{P}}(x) = \left(\left[\overline{v}_{\overline{P}}^L(x), \overline{v}_{\overline{P}}^R(x)\right]\right) \subset [0,1]$ *are interval values, and* $0 \le \left(\overline{u}_{\overline{P}}^R(x)\right)^2 + \left(\overline{v}_{\overline{P}}^R(x)\right)^2 \le 1$.

The interval indeterminacy degree is $\overline{\pi}_{\overline{P}}(x) = \left[\overline{\pi}_{\overline{P}}^L(x), \overline{\pi}_{\overline{P}}^R(x)\right]$ for all $x \in X$, where

$$\overline{\pi}_{\overline{P}}^L(x) = \sqrt{1 - \left(u_{\overline{P}}^R(x)\right)^2 - \left(v_{\overline{P}}^R(x)\right)^2}$$

$$\text{and } \overline{\pi}_{\overline{P}}^R(x) = \sqrt{1 - \left(u_{\overline{P}}^L(x)\right)^2 - \left(v_{\overline{P}}^L(x)\right)^2}.$$

Thus, if $\overline{u}_{\overline{P}}^L(x) = \overline{u}_{\overline{P}}^R(x)$ and $\overline{v}_{\overline{P}}^L(x) = \overline{v}_{\overline{P}}^R(x)$, then an IPFS reduces to a PFS; if $\overline{u}_{\overline{P}}^R(x) + \overline{v}_{\overline{P}}^R(x) \le 1$, then an IPFS reduces to Atanassov's IVIFS.

For simplicity, $\overline{P}(\overline{u}_{\overline{P}}(x), \overline{v}_{\overline{P}}(x))$ is called an IPFN denoted by $\overline{\beta} = \overline{P}\left(\overline{u}_{\overline{\beta}}, \overline{v}_{\overline{\beta}}\right)$, where $\overline{u}_{\overline{\beta}} = [\overline{u}_{\overline{\beta}}^L, \overline{u}_{\overline{\beta}}^R] \subset [0,1]$, $\overline{v}_{\overline{\beta}} = [\overline{v}_{\overline{\beta}}^L, \overline{v}_{\overline{\beta}}^R] \subset [0,1]$, and $0 \le \left(\overline{u}_{\overline{\beta}}^R\right)^2 + \left(\overline{v}_{\overline{\beta}}^R\right)^2 \le 1$. It is noted that the interval Pythagorean fuzzy number (IPFN) $\overline{\beta} = \overline{P}\left(\left[\overline{u}_{\overline{\beta}}^L, \overline{u}_{\overline{\beta}}^R\right], \left[\overline{v}_{\overline{\beta}}^L, \overline{v}_{\overline{\beta}}^R\right]\right)$ is called Atanassov's interval valued intuitionistic fuzzy number if $0 \le \overline{u}_{\overline{\beta}}^R + \overline{v}_{\overline{\beta}}^R \le 1$. Obviously, the space of the constraint condition of IPFN is usually greater than the space of the constraint condition of Atanassov's interval valued intuitionistic fuzzy number (IFN). An IFN must be an IPFN, but the converse is not true in general. For instance, $\beta = ([0.3, 0.5], [0.4, 0.6])$ is an IPFN but not an IFN, because $0.5 + 0.6 > 1$.

Definition 4 [40]. *For two IPFSs P_1 and P_2, i.e., $P_1 = \left(\left[u^L_{P_1}, u^R_{P_1}\right], \left[v^L_{P_1}, v^R_{P_1}\right]\right)$ and $P_2 = \left(\left[u^L_{P_2}, u^R_{P_2}\right], \left[v^L_{P_2}, v^R_{P_2}\right]\right)$, $P_1 \subseteq P_2$, if and only if*

$$u^L_{P_1}(x) \leq u^L_{P_2}(x),\ u^R_{P_1}(x) \leq u^R_{P_2}(x);$$

$$v^L_{P_1}(x) \geq v^L_{P_2}(x),\ v^R_{P_1}(x) \leq v^R_{P_2}(x),$$

for all $x \in X$, respectively.

Definition 5 [40]. *Let $\beta_1 = \left(\left[u^L_{\beta_1}, u^R_{\beta_1}\right], \left[v^L_{\beta_1}, v^R_{\beta_1}\right]\right)$ and $\beta_2 = \left(\left[u^L_{\beta_2}, u^R_{\beta_2}\right], \left[v^L_{\beta_2}, v^R_{\beta_2}\right]\right)$ be two IPFNs. Then,*

$$\mathrm{S}(\beta_1) = \frac{1}{2}\left[\left(u^L_{\beta_1}\right)^2 + \left(u^R_{\beta_1}\right)^2 - \left(v^L_{\beta_1}\right)^2 - \left(v^R_{\beta_1}\right)^2\right]$$

and

$$\mathrm{S}(\beta_2) = \frac{1}{2}\left[\left(u^L_{\beta_2}\right)^2 + \left(u^R_{\beta_2}\right)^2 - \left(v^L_{\beta_2}\right)^2 - \left(v^R_{\beta_2}\right)^2\right]$$

are the score functions of β_1 and β_2, respectively, and

$$\mathrm{W}(\beta_1) = \frac{1}{2}\left[\left(u^L_{\beta_1}\right)^2 + \left(u^R_{\beta_1}\right)^2 + \left(v^L_{\beta_1}\right)^2 + \left(v^R_{\beta_1}\right)^2\right]$$

and

$$\mathrm{W}(\beta_2) = \frac{1}{2}\left[\left(u^L_{\beta_2}\right)^2 + \left(u^R_{\beta_2}\right)^2 + \left(v^L_{\beta_2}\right)^2 + \left(v^R_{\beta_2}\right)^2\right]$$

are the accuracy degrees of β_1 and β_2, respectively.

Further, the following conditions are true:

(1) If $\mathrm{S}(\beta_1) < \mathrm{S}(\beta_2)$, then $\beta_1 < \beta_2$.
(2) If $\mathrm{S}(\beta_1) = \mathrm{S}(\beta_2)$, then, the following are true.

 (a) If $\mathrm{W}(\beta_1) = \mathrm{W}(\beta_2)$, then $\beta_1 = \beta_2$;
 (b) If $\mathrm{W}(\beta_1) < \mathrm{W}(\beta_2)$, then $\beta_1 < \beta_2$;
 (c) If $\mathrm{W}(\beta_1) > \mathrm{W}(\beta_2)$, then $\beta_1 > \beta_2$.

2.2. Interval Number

First, the definition of the interval number $\overline{a}$ is given as $\overline{a} = \left[a^L, a^R\right] = \left\{a^L \leq \tilde{a} \leq a^R\right\}$, where superscripts a^L and a^R represent the lower and upper bounds of the interval number $\overline{a}$, respectively; the interval number also represents a closed bounded set of real numbers [53]. Especially, if $a^L = a^R$, then $\overline{a}$ is a real number.

For any two intervals $\left[a^L_1, a^R_1\right]$ and $\left[a^L_2, a^R_2\right]$, their arithmetic operations are as follows [53]:

(O1) $\left[a^L_1, a^R_1\right] + \left[a^L_2, a^R_2\right] = \left[a^L_1 + a^L_2, a^R_1 + a^R_2\right]$;
(O2) $\left[a^L_1, a^R_1\right] - \left[a^L_2, a^R_2\right] = \left[a^L_1 - a^R_2, a^R_1 - a^L_2\right]$;
(O3) $\left[a^L_1, a^R_1\right] \times \left[a^L_2, a^R_2\right] = [p, q]$, where $p = \min\left(a^L_1 a^L_2, a^L_1 a^R_2, a^R_1 a^L_2, a^R_1 a^R_2\right)$, and $q = \max\left(a^L_1 a^L_2, a^L_1 a^R_2, a^R_1 a^L_2, a^R_1 a^R_2\right)$;
(O4) $\left[a^L_1, a^R_1\right] \div \left[a^L_2, a^R_2\right] = \left[a^L_1, a^R_1\right] \times \left[\frac{1}{a^R_2}, \frac{1}{a^L_2}\right], 0 \notin \left[a^L_2, a^R_2\right]$.

2.3. Connection Number

For a given problem W, a set pair (A, B) consists of two interdependent sets A and B. The problem W analyzes the system on "identical", "discrepancy", and "contrary" features, and a connection number is set up. If the total number of features N, the identity and contrary features are denoted by S and P, respectively, such that $F = N - S - P$ is neither the identity nor contrary of sets A and B. The connection number is defined as below.

Definition 6 [52]. *Let A* and *B be two interdependent sets. The connection number $\mu(A, B)$ of sets A and B is represented as follows:*

$$\mu(A, B) = a + bi + cj$$

where $a = S/N$, $b = F/N$, and $c = P/N$ represents the "identity", "discrepancy", and "contrary" degrees, respectively. On the other hand, $0 < a, b, c \leq 1$, and $a + b + c = 1$; $i \in [-1, 1]$, and $j = -1$ are the coefficients of "discrepancy" and "contrary" degrees, respectively.

When $j = 0$, $\mu(A, B) = a + bi + cj$ is equal to $\mu(A, B) = a + bi$, where $i \in [-1, 1]$ is the coefficient of "discrepancy degree", $0 < a, b \leq 1$, and $a + b = 1$. The expression $\mu(A, B) = a + bi$ is called the binary connection number, which is a special form of $\mu(A, B) = a + bi + cj$. In the present study, we transform the interval number to a binary connection number and conduct a comparison between two interval numbers.

For any two connection numbers $\mu_1 = a_1 + b_1 i$ and $\mu_2 = a_2 + b_2 i$, the following are considered.

(I) If $a_1 = a_2$ and $b_1 = b_2$, then $\mu_1 = \mu_2$;
(II) If $a_1 > a_2$ and $a_1 - b_1 \geq a_2 + b_2$, then $\mu_1 \gg \mu_2$;
(III) If $a_1 > a_2$, then $\mu_1 > \mu_2$;
(V) If $a_1 = a_2$ and $b_1 > b_2$, then $\mu_1 \succ \mu_2$.

For an interval number $\left[a^L, a^R\right]$, its corresponding connection number is represented as follows:

$$\left[a^L, a^R\right] = a + bi = \left(a^L + a^R\right)/2 + \left(\left(a^R - a^L\right)/2\right)i. \tag{2}$$

Therefore, for two interval numbers $\bar{a}_1 = \left[a_1^L, a_1^R\right]$ and $\bar{a}_2 = \left[a_2^L, a_2^R\right]$,

(C1) if $\left(a_1^L + a_1^R\right)/2 > \left(a_2^L + a_2^R\right)/2$, then $\bar{a}_1 > \bar{a}_2$;
(C2) if $\left(a_1^L + a_1^R\right)/2 = \left(a_2^L + a_2^R\right)/2$, then,

if $\left(a_1^R - a_1^L\right)/2 = \left(a_2^R - a_2^L\right)/2$, then $\bar{a}_1 = \bar{a}_2$;
if $\left(a_1^R - a_1^L\right)/2 > \left(a_2^R - a_2^L\right)/2$, then $\bar{a}_1 \succ \bar{a}_2$.

3. Similarity Measures Between IPFSs

This section presents new similarity measures between IPFSs based on the minimum and maximum operators and investigates their properties.

Let X be a universe of discourse, a similarity measure between two sets P_1 and P_2 in X is a function defined as r: $X \otimes X \to [0, 1]$, where $\otimes$ is an operation corresponding to a specific question. In general, the similarity measure between two sets P_1 and P_2 satisfies the following properties:

(P1) $0 \leq \mathrm{r}(P_1, P_2) \leq 1$; (P2) $\mathrm{r}(P_1, P_2) = 1$ if $P_1 = P_2$; (P3) $\mathrm{r}(P_1, P_2) = \mathrm{r}(P_2, P_1)$;
(P4) $\mathrm{r}(P_1, P_3) \leq \mathrm{r}(P_1, P_2)$ and $\mathrm{r}(P_1, P_3) \leq \mathrm{r}(P_2, P_3)$ if $P_1 \subseteq P_2 \subseteq P_3$ for a set P_3.

Proposition 1. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, P_1 and P_2 be two IPFSs. Then, the 1-type IPFS similarity measure*

$$\mathrm{r}_1(P_1, P_2) = \frac{1}{2n} \sum_{i=1}^{n} \left(\frac{\min\left(u_{P_1}(x_i), u_{P_2}(x_i)\right)}{\max\left(u_{P_1}(x_i), u_{P_2}(x_i)\right)} + \frac{\min\left(v_{P_1}(x_i), v_{P_2}(x_i)\right)}{\max\left(v_{P_1}(x_i), v_{P_2}(x_i)\right)} \right) \tag{3}$$

should satisfy the following properties:

(P1) $0 \leq \mathrm{r}_1(P_1, P_2) \leq 1$; *(P2)* $\mathrm{r}_1(P_1, P_2) = 1$ *if* $P_1 = P_2$; *(P3)* $\mathrm{r}_1(P_1, P_2) = \mathrm{r}_1(P_2, P_1)$;
(P4) $\mathrm{r}_1(P_1, P_3) \leq \mathrm{r}_1(P_1, P_2)$ *and* $\mathrm{r}_1(P_1, P_3) \leq \mathrm{r}_1(P_2, P_3)$ *if* $P_1 \subseteq P_2 \subseteq P_3$ *for an IPFS* P_3.

Proof. By verification, $\mathrm{r}_1(P_1, P_2)$ satisfies properties (P1)–(P3) easily. Therefore, we only prove property (P4). Let $P_1 \subseteq P_2 \subseteq P_3$. Then, from Definition 4,

$$\begin{array}{c} u_{P_1}^L(x_i) \leq u_{P_2}^L(x_i) \leq u_{P_3}^L(x_i) \text{ and } u_{P_1}^R(x_i) \leq u_{P_2}^R(x_i) \leq u_{P_3}^R(x_i); \\ v_{P_1}^L(x_i) \geq v_{P_2}^L(x_i) \geq v_{P_3}^L(x_i) \text{ and } v_{P_1}^R(x_i) \geq v_{P_2}^R(x_i) \geq v_{P_3}^R(x_i) \end{array}$$

for every $x_i \in X$.

By comparison rules (C1) and (C2) and given the arithmetic operations between two interval numbers, we can obtain the following:

$$\begin{array}{r} \min\left\{ \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right], \left[u_{P_2}^L(x_i), u_{P_2}^R(x_i)\right] \right\} = \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right]; \\ \max\left\{ \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right], \left[u_{P_2}^L(x_i), u_{P_2}^R(x_i)\right] \right\} = \left[u_{P_2}^L(x_i), u_{P_2}^R(x_i)\right]; \\ \min\left\{ \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \left[v_{P_2}^L(x_i), v_{P_2}^R(x_i)\right] \right\} = \left[v_{P_2}^L(x_i), v_{P_2}^R(x_i)\right]; \\ \max\left\{ \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \left[v_{P_2}^L(x_i), v_{P_2}^R(x_i)\right] \right\} = \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \end{array}$$

and

$$\begin{aligned} \mathrm{r}_1(P_1, P_2) = \; & \frac{1}{2n} \sum_{i=1}^{n} \left(\frac{\min\{u_{P_1}(x_i), u_{P_2}(x_i)\}}{\max\{u_{P_1}(x_i), u_{P_2}(x_i)\}} + \frac{\min\{v_{P_1}(x_i), v_{P_2}(x_i)\}}{\max\{v_{P_1}(x_i), v_{P_2}(x_i)\}} \right) \\ & = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \frac{\min\left\{ \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right], \left[u_{P_2}^L(x_i), u_{P_2}^R(x_i)\right] \right\}}{\max\left\{ \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right], \left[u_{P_2}^L(x_i), u_{P_2}^R(x_i)\right] \right\}} \right. \\ & \left. + \frac{\min\left\{ \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \left[v_{P_2}^L(x_i), v_{P_2}^R(x_i)\right] \right\}}{\max\left\{ \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \left[v_{P_2}^L(x_i), v_{P_2}^R(x_i)\right] \right\}} \right\} \\ & = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \frac{\left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right]}{\left[u_{P_2}^L(x_i), u_{P_2}^R(x_i)\right]} + \frac{\left[v_{P_2}^L(x_i), v_{P_2}^R(x_i)\right]}{\left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right]} \right\} \\ & = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \left[\frac{u_{P_1}^L(x_i)}{u_{P_2}^R(x_i)} + \frac{v_{P_2}^L(x_i)}{v_{P_1}^R(x_i)}, \frac{u_{P_1}^R(x_i)}{u_{P_2}^L(x_i)} + \frac{v_{P_2}^R(x_i)}{v_{P_1}^L(x_i)} \right] \right\}. \end{aligned} \tag{4}$$

Similarly, we obtain the following:

$$\begin{aligned} \mathrm{r}_1(P_1, P_3) & = \frac{1}{2n} \sum_{i=1}^{n} \left(\frac{\min\{u_{P_1}(x_i), u_{P_3}(x_i)\}}{\max\{u_{P_1}(x_i), u_{P_3}(x_i)\}} + \frac{\min\{v_{P_1}(x_i), v_{P_3}(x_i)\}}{\max\{v_{P_1}(x_i), v_{P_3}(x_i)\}} \right) \\ & = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \frac{\min\left\{ \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right], \left[u_{P_3}^L(x_i), u_{P_3}^R(x_i)\right] \right\}}{\max\left\{ \left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right], \left[u_{P_3}^L(x_i), u_{P_3}^R(x_i)\right] \right\}} \right. \\ & \left. + \frac{\min\left\{ \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \left[v_{P_3}^L(x_i), v_{P_3}^R(x_i)\right] \right\}}{\max\left\{ \left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right], \left[v_{P_3}^L(x_i), v_{P_3}^R(x_i)\right] \right\}} \right\} \\ & = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \frac{\left[u_{P_1}^L(x_i), u_{P_1}^R(x_i)\right]}{\left[u_{P_3}^L(x_i), u_{P_3}^R(x_i)\right]} + \frac{\left[v_{P_3}^L(x_i), v_{P_3}^R(x_i)\right]}{\left[v_{P_1}^L(x_i), v_{P_1}^R(x_i)\right]} \right\} \\ & = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \left[\frac{u_{P_1}^L(x_i)}{u_{P_3}^R(x_i)} + \frac{v_{P_3}^L(x_i)}{v_{P_1}^R(x_i)}, \frac{v_{P_3}^R(x_i)}{v_{P_1}^L(x_i)} + \frac{u_{P_1}^R(x_i)}{u_{P_3}^L(x_i)} \right] \right\} \end{aligned} \tag{5}$$

and

$$
\begin{aligned}
\mathrm{r}_1(P_2,P_3) &= \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\min\{u_{P_2}(x_i),u_{P_3}(x_i)\}}{\max\{u_{P_2}(x_i),u_{P_3}(x_i)\}}+\frac{\min\{v_{P_2}(x_i),v_{P_3}(x_i)\}}{\max\{v_{P_2}(x_i),v_{P_3}(x_i)\}}\right) \\
&= \frac{1}{2n}\sum_{i=1}^{n}\left\{\frac{\min\left\{\left[u^L_{P_2}(x_i),u^R_{P_2}(x_i)\right],\left[u^L_{P_3}(x_i),u^R_{P_3}(x_i)\right]\right\}}{\max\left\{\left[u^L_{P_2}(x_i),u^R_{P_2}(x_i)\right],\left[u^L_{P_3}(x_i),u^R_{P_3}(x_i)\right]\right\}}\right. \\
&\left.+\frac{\min\left\{\left[v^L_{P_2}(x_i),v^R_{P_2}(x_i)\right],\left[v^L_{P_3}(x_i),v^R_{P_3}(x_i)\right]\right\}}{\max\left\{\left[v^L_{P_2}(x_i),v^R_{P_2}(x_i)\right],\left[v^L_{P_3}(x_i),v^R_{P_3}(x_i)\right]\right\}}\right\} \\
&= \frac{1}{2n}\sum_{i=1}^{n}\left\{\frac{\left[u^L_{P_2}(x_i),u^R_{P_2}(x_i)\right]}{\left[u^L_{P_3}(x_i),u^R_{P_3}(x_i)\right]}+\frac{\left[v^L_{P_3}(x_i),v^R_{P_3}(x_i)\right]}{\left[v^L_{P_2}(x_i),v^R_{P_2}(x_i)\right]}\right\} \\
&= \frac{1}{2n}\sum_{i=1}^{n}\left\{\left[\frac{u^L_{P_2}(x_i)}{u^R_{P_3}(x_i)}+\frac{v^L_{P_3}(x_i)}{v^R_{P_2}(x_i)},\frac{v^R_{P_3}(x_i)}{v^L_{P_2}(x_i)}+\frac{u^R_{P_2}(x_i)}{u^L_{P_3}(x_i)}\right]\right\}.
\end{aligned}
\tag{6}
$$

For the proof of (P4), we only compare the right terms within the curly braces of Equations (4) and (5). From comparison rules (C1) and (C2), we can easily obtain the result by comparing the numerators or denominators in the corresponding terms. Therefore, $\mathrm{r}_1(P_1,P_3)\le \mathrm{r}_1(P_1,P_2)$.

Similarly, from Equations (5) and (6), we obtain $\mathrm{r}_1(P_1,P_3)\le \mathrm{r}_1(P_2,P_3)$.

Thus, $\mathrm{r}_1(P_1,P_2)$ satisfies property (P4). □

When we consider the importance of two terms (i.e., membership and non-membership degrees) in an IPFS, we should consider the weights of those terms in Equation (3). Therefore, we develop another similarity measure between IPFSs.

Proposition 2. *Let $X=\{x_1,x_2,\ldots,x_n\}$ be a fixed set, P_1 and P_2 be two IPFSs. Then, the 2-type IPFS-weighted similarity measure*

$$
\mathrm{r}_2(P_1,P_2)=\frac{1}{n}\sum_{i=1}^{n}\left(\alpha\frac{\min(u_{P_1}(x_i),u_{P_2}(x_i))}{\max(u_{P_1}(x_i),u_{P_2}(x_i))}+\beta\frac{\min(v_{P_1}(x_i),v_{P_2}(x_i))}{\max(v_{P_1}(x_i),v_{P_2}(x_i))}\right) \tag{7}
$$

should satisfy the following properties:

(P1) $0\le \mathrm{r}_2(P_1,P_2)\le 1$; *(P2)* $\mathrm{r}_2(P_1,P_2)=1$ *if* $P_1=P_2$; *(P3)* $\mathrm{r}_2(P_1,P_2)=\mathrm{r}_2(P_2,P_1)$;
(P4) $\mathrm{r}_2(P_1,P_3)\le \mathrm{r}_2(P_1,P_2)$ *and* $\mathrm{r}_2(P_1,P_3)\le \mathrm{r}_2(P_2,P_3)$ *if* $P_1\subseteq P_2\subseteq P_3$ *for an* IPFS P_3*, where* α *and* β *are the weights of two elements (i.e., membership and non-membership) in an IPFS and* $\alpha+\beta=1$*. Especially, when* $\alpha=\beta=1/2$*, Equation (7) reduces to Equation (3).*

By means of the proof in Proposition 1, Proposition 2 can be proven.

Furthermore, if important differences are considered in the elements in a universe of discourse $X=\{x_1,x_2,\ldots,x_n\}$, the weight of each element x_i $(i=1,2,\cdots,n)$ needs to be considered. Let w_i be the weight for each element x_i $(i=1,2,\cdots,n)$ $(w_i\in[0,1],\sum_{i=1}^{n}w_i=1)$, and α,β $(\alpha,\beta\in[0,1],\alpha+\beta=1)$ be the weights of two terms (i.e., membership and non-membership). Then the 3-type IPFS-weighted similarity measure is defined as follows.

Proposition 3. *Let $X=\{x_1,x_2,\ldots,x_n\}$ be a fixed set, P_1 and P_2 be two IPFSs, the 3-type IPFS-weighted similarity measure*

$$
\mathrm{r}_3(P_1,P_2)=\sum_{i=1}^{n}w_i\left(\alpha\frac{\min(u_{P_1}(x_i),u_{P_2}(x_i))}{\max(u_{P_1}(x_i),u_{P_2}(x_i))}+\beta\frac{\min(v_{P_1}(x_i),v_{P_2}(x_i))}{\max(v_{P_1}(x_i),v_{P_2}(x_i))}\right) \tag{8}
$$

should satisfy the following properties:

(P1) $0\le \mathrm{r}_3(P_1,P_2)\le 1$; *(P2)* $\mathrm{r}_3(P_1,P_2)=1$ *if* $P_1=P_2$; *(P3)* $\mathrm{r}_3(P_1,P_2)=\mathrm{r}_3(P_2,P_1)$;

(P4) $r_3(P_1, P_3) \le r_3(P_1, P_2)$ *and* $r_3(P_1, P_3) \le r_3(P_2, P_3)$ *if* $P_1 \subseteq P_2 \subseteq P_3$ *for an* IPFS P_3, *where* α *and* β *are the weights of two elements (i.e., membership and non-membership, respectively) in an IPFS and* $\alpha + \beta = 1$. *Especially, when* $w_1 = w_2 = \cdots = w_n = 1/n$, *Equation (8) reduces to Equation (7).*

Proof of Proposition 3 can be obtained from the proof method of Proposition 1.

Example 1. *Assume that three IPFSs exist in a universe of discourse* $X = \{x_1, x_2, x_3\}$*:*

$$P_1 = \{\langle x_1, [0.2, 0.3], [0.6, 0.8]\rangle, \langle x_2, [0.3, 0.4], [0.7, 0.8]\rangle, \langle x_3, [0.4, 0.5], [0.6, 0.7]\rangle\}$$
$$P_2 = \{\langle x_1, [0.3, 0.5], [0.5, 0.6]\rangle, \langle x_2, [0.5, 0.6], [0.5, 0.7]\rangle, \langle x_3, [0.6, 0.7], [0.4, 0.5]\rangle\}$$
$$P_3 = \{\langle x_1, [0.7, 0.9], [0.1, 0.3]\rangle, \langle x_2, [0.7, 0.8], [0.3, 0.4]\rangle, \langle x_3, [0.8, 0.9], [0.2, 0.3]\rangle\}$$

and $P_1 \subseteq P_2 \subseteq P_3$. *By using Equation (2), the 1-type IPFS similarity measures are as follows:*

$$r_1(P_1, P_2) = \frac{1}{6}\left(\frac{[0.2,0.3]}{[0.3,0.5]} + \frac{[0.5,0.6]}{[0.6,0.8]} + \frac{[0.3,0.4]}{[0.5,0.6]} + \frac{[0.5,0.7]}{[0.7,0.8]} + \frac{[0.4,0.5]}{[0.6,0.7]} + \frac{[0.4,0.5]}{[0.6,0.7]}\right) = \left[\frac{147}{280}, \frac{83}{90}\right];$$
$$r_1(P_1, P_3) = \frac{1}{6}\left(\frac{[0.2,0.3]}{[0.7,0.9]} + \frac{[0.1,0.3]}{[0.6,0.8]} + \frac{[0.3,0.4]}{[0.7,0.8]} + \frac{[0.3,0.4]}{[0.7,0.8]} + \frac{[0.4,0.5]}{[0.8,0.9]} + \frac{[0.2,0.3]}{[0.6,0.7]}\right) = \left[\frac{307}{1008}, \frac{171}{336}\right];$$
$$r_1(P_2, P_3) = \frac{1}{6}\left(\frac{[0.3,0.5]}{[0.7,0.9]} + \frac{[0.1,0.3]}{[0.5,0.6]} + \frac{[0.5,0.6]}{[0.7,0.8]} + \frac{[0.3,0.4]}{[0.5,0.7]} + \frac{[0.6,0.7]}{[0.8,0.9]} + \frac{[0.2,0.3]}{[0.4,0.5]}\right) = \left[\frac{2201}{5040}, \frac{1367}{1680}\right].$$

By means of the comparison rules (C1) and (C2), we obtain the following

$$\frac{1}{2}\left(\frac{147}{280} + \frac{83}{90}\right) > \frac{1}{2}\left(\frac{307}{1008} + \frac{171}{336}\right) \text{ and } \frac{1}{2}\left(\frac{2201}{5040} + \frac{1367}{1680}\right) > \frac{1}{2}\left(\frac{307}{1008} + \frac{171}{336}\right).$$

Thus, $r_1(P_1, P_3) \le r_1(P_1, P_2)$ *and* $r_1(P_1, P_3) \le r_1(P_2, P_3)$ *are obtained.*

If the weight values of the two terms in an IPFS are $\alpha = 0.55$ *and* $\beta = 0.45$, *by applying Equation (7), the 2-type IPFS-weighted similarity measures are as follows:*

$$r_2(P_1, P_2) = \left[\frac{31927}{58800}, \frac{817}{900}\right];\ r_2(P_1, P_3) = \left[\frac{3113}{10080}, \frac{1793}{3360}\right];\ \text{and } r_2(P_2, P_3) = \left[\frac{7513}{16800}, \frac{2389}{3385}\right].$$

Then, $r_2(P_1, P_3) \le r_2(P_1, P_2)$ *and* $r_2(P_1, P_3) \le r_2(P_2, P_3)$.

Assume that the weight vector of the three criteria is $w = (0.4, 0.3, 0.3)$, *and the weight values of the two terms (i.e., membership and non-membership degrees) in an IPFS, are* $\alpha = 0.55$ *and* $\beta = 0.45$, *respectively. By applying Equation (8), the 3-type IPFS-weighted similarity measures are as follows:*

$$r_3(P_1, P_2) = \left[\frac{682773}{1960000}, \frac{39607}{56000}\right];\ r_3(P_1, P_3) = \left[\frac{2651}{14400}, \frac{8253}{15680}\right];\ r_3(P_2, P_3) = \left[\frac{71807}{168000}, \frac{46091}{56000}\right].$$

Therefore, $r_3(P_1, P_3) \le r_3(P_1, P_2)$ *and* $r_3(P_1, P_3) \le r_3(P_2, P_3)$.

4. Decision-making Method Using the Proposed Similarity Measures

In this section, we propose an MCDM method under IPFS by means of the proposed 3-type IPFS-weighted similarity measure.

Let $O = \{o_1, o_2, \ldots, o_n\}$ be a set of alternatives and $C = \{c_1, c_2, \ldots, c_m\}$ be a set of criteria. Assume that the weight of criterion c_j $(j = 1, 2, \ldots, m)$ is w_j, $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, the weights of the two terms (i.e., membership and non-membership degrees), are α and β in an IPFS, respectively, which are given by the decision maker. In this case, the characteristic of alternative o_i $(i = 1, 2, \ldots, n)$ is represented as: $o_i = \{c_j, u_{o_i}(c_j), v_{o_i}(c_j) | c_j \in C\}$, where $u_{o_i}(c_j) = \left[u_{o_i}^L(c_j), u_{o_i}^R(c_j)\right] \subset [0, 1]$ and

$v_{o_i}(c_j) = \left[v_{o_i}^L(c_j), v_{o_i}^R(c_j)\right] \subset [0,1]$ are intervals, and $\left(u_{o_i}^R(c_j)\right)^2 + \left(v_{o_i}^R(c_j)\right)^2 < 1$ for $c_j \in C$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

For convenience, the terms $u_{o_i}(c_j)$ and $v_{o_i}(c_j)$ in the IPFS are denoted by $a_{ij} = \{u_{ij}, v_{ij}\} = \left\{\left[u_{ij}^L, u_{ij}^R\right], \left[v_{ij}^L, v_{ij}^R\right]\right\}$ $(i = 1,2,\ldots,n; j = 1,2,\ldots,m)$, which is usually derived from the evaluation of an alternative o_i with respect to a criteria c_j by the expert or decision maker. Hence, we can establish an interval Pythagorean fuzzy decision matrix

$$O = \begin{pmatrix} o_{11} & o_{12} & \cdots & o_{1m} \\ o_{21} & o_{22} & \cdots & o_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ o_{n1} & o_{n2} & \cdots & o_{nm} \end{pmatrix}.$$

In an MCDM setting, the concept of an interval Pythagorean fuzzy ideal solution has been used to identify the best alternative in decision making problems, although no interval Pythagorean fuzzy ideal solution usually exists in the actual decision making process. In other words, the interval Pythagorean fuzzy ideal solution vector O^* is usually a not-feasible alternative, that is, $O^* \notin O$. Otherwise, the Pythagorean fuzzy ideal solution vector O^* is the optimal alternative vector of the decision making problem.

For the evaluation values, we only assume two kinds of criteria: benefit and cost types. An ideal solution can be identified using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives.

For brevity, the ideal alternative is rewritten as $o_j^* = \left\{c_j, \left(u_j^*, v_j^*\right) | j = 1,2,\ldots,m\right\}$.

From the above analysis, in the following sections, we propose a practical algorithm based on the 3-type IPFS-weighted similarity measure between the ideal alternative and each alternative. The algorithm can be described by the following steps, and the process is shown in Figure 1.

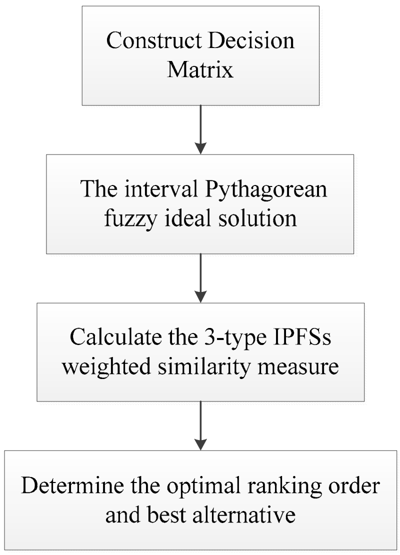

Figure 1. Process of a project delivery system (PDS) selection model based on the 3-type interval Pythagorean fuzzy set (IPFS)-weighted similarity measure.

Step 1: For an MCDM problem with IPFNs, we construct the decision matrix $O_{n\times m} = \left(o_{ij}\right)_{n\times m}$, where $o_{ij} = \mathrm{p}\left(\left[u_{ij}^L, u_{ij}^R\right], \left[v_{ij}^L, v_{ij}^R\right]\right)$ denotes the evaluation value of the alternative $o_i \in O$ with respect to criterion $c_j \in c$.

Step 2: We employ the following equations to identify the interval Pythagorean fuzzy ideal solution O^*.

For benefit criteria

$$O^* = \{o_1^*, o_2^*, \ldots, o_m^*\},\ o_j^* = \left\{c_j, \max_i\{o_{ij}\} | j = 1, 2, \ldots, m\right\}. \tag{9}$$

For cost criteria

$$O^* = \{o_1^*, o_2^*, \ldots, o_m^*\},\ o_j^* = \left\{c_j, \min_i\{o_{ij}\} | j = 1, 2, \ldots, m\right\}. \tag{10}$$

Step 3: By applying Equation (8), the 3-type IPFS-weighted similarity measure between an alternative o_i and ideal alternative O^* are written as follows:

$$\mathrm{r}_4(o_i, O^*) = \sum_{j=1}^{m} w_j \left(\alpha \frac{\min\left(u_{ij}, u_j^*\right)}{\max\left(u_{ij}, u_j^*\right)} + \beta \frac{\min\left(v_{ij}, v_j^*\right)}{\max\left(v_{ij}, v_j^*\right)} \right) \tag{11}$$

which provides global evaluation for each alternative regarding to all criteria.

Step 4: According to the results obtained from Step 3, the bigger the measure value $\mathrm{r}_4(o_i, O^*)$ $(i = 1, 2, \ldots, n)$, the better the alternative o_i is. Therefore, the ranking order of all alternatives can be determined, and the best one can be easily selected.

5. Practical Example

In this section, we apply the proposed model to a real-world infrastructure project. Nanjing Metro Co. Ltd. wants to alternate and innovate its PDS. The owner intends to select the most applicable from four delivery systems including design-build (DB), engineering-procurement-construction (EPC), construction management at risk method (CM at-Risk), and design-bid-build (DBB) delivery systems. The decision making criteria are Cost (C), Schedule (S), Quality (Q), Complexity (Com), Scope Change (SC), Experience (E), Financial Guarantee (FG), Risk Management (RM), Uniqueness (U), and Project Size (Size), as shown in Figure 2.

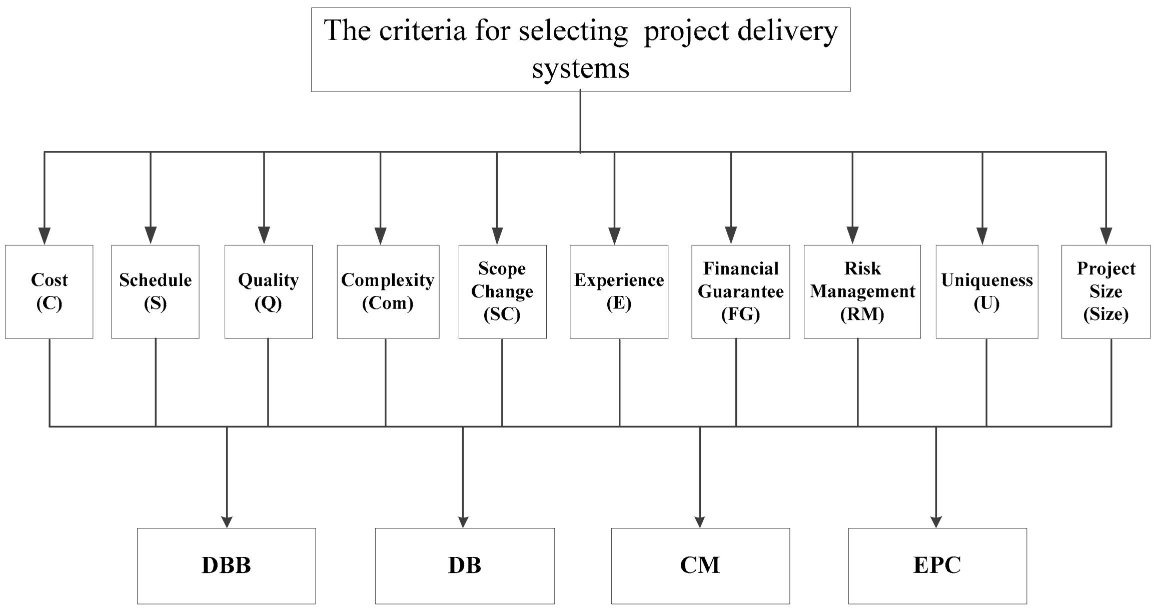

Figure 2. Decision making framework for selection of project delivery systems. DBB: design-bid-build; DB: design-build; CM: construction management; EPC: engineering-procurement-construction.

In this selection process, the four PDSs (i.e., DBB, DB, CM, and EPC) form the set of delivery options, which is written as $O = \{o_1, o_2, o_3, o_4\}$. Similarly, the 10 criteria (i.e., C, S, Q, Com, SC, E, FG, RM, U, and Size) make up the set of criteria $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}\}$, where the weights are represented as $w_1 = w_2 = \cdots = w_{10} = 0.1$, as obtained by averaging weight for convenience. The weights of the membership and non-membership degrees are assumed as $\alpha = 0.55$ and $\beta = 0.45$. We assume that (u_{ij}, v_{ij}) $(i = 1, \ldots, n,\ j = 1, 2, \ldots, m)$ is the evaluation value under c_j with respect to delivery option o_i, as shown in Table 1.

Table 1. Interval Pythagorean fuzzy evaluation values.

	C	S	Q	Com
DBB	p([0.6,0.7],[0.4,0.5])	p([0.4,0.6],[0.5,0.7])	p([0.5,0.7],[0.3,0.5])	p([0.5,0.7],[0.6,0.7])
DB	p([0.7,0.8],[0.3,0.4])	p([0.5,0.7],[0.4,0.5])	p([0.5,0.7],[0.3,0.5])	p([0.5,0.7],[0.5,0.6])
CM	p([0.3,0.5],[0.6,0.8])	p([0.5,0.7],[0.3,0.4])	p([0.2,0.3],[0.3,0.6])	p([0.2,0.4],[0.6,0.8])
EPC	p([0.8,0.9],[0.2,0.3])	p([0.5,0.7],[0.1,0.2])	p([0.6,0.7],[0.2,0.4])	p([0.1,0.2],[0.6,0.8])
	SC	**E**	**FG**	**RM**
DBB	p([0.6,0.7],[0.4,0.6])	p([0.5,0.7],[0.6,0.7])	p([0.6,0.7],[0.3,0.5])	p([0.5,0.7],[0.6,0.8])
DB	p([0.5,0.7],[0.3,0.5])	p([0.5,0.7],[0.4,0.6])	p([0.5,0.7],[0.4,0.6])	p([0.6,0.7],[0.5,0.6])
CM	p([0.6,0.7],[0.4,0.5])	p([0.2,0.3],[0.6,0.8])	p([0.4,0.6],[0.6,0.8])	p([0.4,0.5],[0.6,0.7])
EPC	p([0.2,0.4],[0.3,0.4])	p([0.8,0.9],[0.1,0.2])	p([0.7,0.8],[0.3,0.4])	p([0.6,0.8],[0.3,0.4])
	U	**Size**		
DBB	p([0.6,0.7],[0.5,0.6])	p([0.4,0.6],[0.5,0.6])		
DB	p([0.5,0.7],[0.4,0.5])	p([0.5,0.7],[0.4,0.5])		
CM	p([0.6,0.7],[0.2,0.3])	p([0.2,0.3],[0.6,0.8])		
EPC	p([0.2,0.3],[0.6,0.8])	p([0.6,0.8],[0.2,0.3])		

Step 1: From the evaluation values in Table 1, we construct the evaluation matrix:

$$U_{4\times 10} = \left(\begin{array}{ccc} ([0.6,0.7],[0.4,0.5]) & ([0.4,0.6],[0.5,0.7]) & ([0.5,0.7],[0.3,0.5]) \\ ([0.7,0.8],[0.3,0.4]) & ([0.5,0.7],[0.4,0.5]) & ([0.5,0.7],[0.3,0.5]) \\ ([0.3,0.5],[0.6,0.8]) & ([0.5,0.7],[0.3,0.4]) & ([0.2,0.3],[0.3,0.6]) \\ ([0.8,0.9],[0.2,0.3]) & ([0.5,0.7],[0.1,0.2]) & ([0.6,0.7],[0.2,0.4]) \end{array} \right.$$

$$\begin{array}{ccc} ([0.5,0.7],[0.6,0.7]) & ([0.6,0.7],[0.4,0.6]) & ([0.5,0.7],[0.6,0.7]) \\ ([0.5,0.7],[0.5,0.6]) & ([0.5,0.7],[0.3,0.5]) & ([0.5,0.7],[0.4,0.6]) \\ ([0.2,0.4],[0.6,0.8]) & ([0.6,0.7],[0.4,0.5]) & ([0.2,0.3],[0.6,0.8]) \\ ([0.1,0.2],[0.6,0.8]) & ([0.2,0.4],[0.3,0.4]) & ([0.8,0.9],[0.1,0.2]) \\ ([0.6,0.7],[0.3,0.5]) & ([0.5,0.7],[0.6,0.8]) & ([0.6,0.7],[0.5,0.6]) \\ ([0.5,0.7],[0.4,0.6]) & ([0.6,0.7],[0.5,0.6]) & ([0.5,0.7],[0.4,0.5]) \\ ([0.4,0.6],[0.6,0.8]) & ([0.4,0.5],[0.6,0.7]) & ([0.6,0.7],[0.2,0.3]) \\ ([0.7,0.8],[0.3,0.4]) & ([0.6,0.8],[0.3,0.4]) & ([0.2,0.3],[0.6,0.8]) \end{array}$$

$$\left. \begin{array}{c} ([0.4,0.6],[0.5,0.6]) \\ ([0.5,0.7],[0.4,0.5]) \\ ([0.2,0.3],[0.6,0.8]) \\ ([0.6,0.8],[0.2,0.3]) \end{array} \right).$$

Step 2: By means of Equations (9) and (10), we determine the ideal option as follows:

$$\begin{aligned} O^* = \quad & \{([0.3,0.5],[0.6,0.8]),([0.5,0.7],[0.1,0.2]),([0.6,0.7],[0.2,0.4]), \\ & ([0.1,0.2],[0.6,0.8])([0.2,0.4],[0.3,0.4])([0.8,0.9],[0.1,0.2]) \\ & ([0.7,0.8],[0.3,0.4])([0.6,0.8],[0.3,0.4])([0.2,0.3],[0.6,0.8]) \\ & ([0.6,0.8],[0.2,0.3])\}. \end{aligned}$$

Step 3: Calculating the weighted similarity measure between each alternative and ideal option using Equation (11) with $w_1 = w_2 = \cdots = w_{10} = 0.1$ and $\alpha = 0.55$ and $\beta = 0.45$, we can obtain the following:

$$\begin{aligned} \mathrm{r}_5(o_1, O^*) = \quad & 0.1 \times \Big(0.55 \cdot \tfrac{[0.3,0.5]}{[0.6,0.7]} + 0.45 \cdot \tfrac{[0.4,0.5]}{[0.6,0.8]} + 0.55 \cdot \tfrac{[0.4,0.6]}{[0.5,0.7]} + 0.45 \cdot \tfrac{[0.1,0.2]}{[0.5,0.7]} \\ & + 0.55 \cdot \tfrac{[0.5,0.7]}{[0.6,0.7]} + 0.45 \cdot \tfrac{[0.2,0.4]}{[0.3,0.5]} + 0.55 \cdot \tfrac{[0.1,0.2]}{[0.5,0.7]} + 0.45 \cdot \tfrac{[0.6,0.7]}{[0.6,0.8]} \\ & + 0.55 \cdot \tfrac{[0.2,0.4]}{[0.6,0.7]} + 0.45 \cdot \tfrac{[0.3,0.4]}{[0.4,0.6]} + 0.55 \cdot \tfrac{[0.5,0.7]}{[0.8,0.9]} + 0.45 \cdot \tfrac{[0.1,0.2]}{[0.6,0.7]} \\ & + 0.55 \cdot \tfrac{[0.6,0.7]}{[0.7,0.8]} + 0.45 \cdot \tfrac{[0.3,0.4]}{[0.3,0.5]} + 0.55 \cdot \tfrac{[0.5,0.7]}{[0.6,0.8]} + 0.45 \cdot \tfrac{[0.3,0.4]}{[0.6,0.8]} \\ & + 0.55 \cdot \tfrac{[0.2,0.3]}{[0.6,0.7]} + 0.45 \cdot \tfrac{[0.5,0.6]}{[0.6,0.8]} + 0.55 \cdot \tfrac{[0.4,0.6]}{[0.6,0.8]} + 0.45 \cdot \tfrac{[0.2,0.3]}{[0.5,0.6]} \Big) \\ & \approx [0.4639, 0.8745]. \end{aligned}$$

Similarly, the weighted similarity measures between the other three options and ideal option are as follows:

$$\mathrm{r}_5(o_2, O^*) \approx [0.4710, 0.9032]; \ \mathrm{r}_5(o_3, O^*) \approx [0.3854, 0.8314];$$

$$\mathrm{r}_5(o_4, O^*) \approx [0.6534, 1.4094].$$

Thus, the ranking order of the four options is $o_4 \succ o_2 \succ o_1 \succ o_3$, that is, EPC $\succ$ DB $\succ$ DBB $\succ$ CM. Therefore, EPC is the best choice among the four options. From the results, we can determine that the ranking order is acceptable for practical applications.

6. Comparison Analysis and Discussion

This section states the advantage of the proposed model through comparison with the existing methods.

The comparative methods we chose were the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [54] and Multi-Objective Optimization by Ratio Analysis (MULTIMOORA) [55], which are multi-criteria decision making methods. The main principle of the TOPSIS method is that the optimal alternative should have the shortest distance measure from the positive ideal solution and the farthest distance measure from the negative one. Using the line of the classical TOPSIS method, we calculated the case study in Section 5. In order to compare convenience, we assumed that the weights of ten criteria were $w_1 = w_2 = \cdots w_{10} = 0.1$. For applying the steps of the TOPSIS method, the ranking result of the four PDSs was EPC $\succ$ CM $\succ$ DB $\succ$ DBB. Similarly, the ranking result was EPC $\succ$ DB $\succ$ DBB $\succ$ CM based on the steps of MULTIMOORA. Additionally, the order of the four PDSs using the proposed method was: EPC $\succ$ DB $\succ$ DBB $\succ$ CM. It was shown that EPC is the best option for this project using the three methods.

Ranking results for the TOPSIS method, MULTIMOORA method, and the proposed method are visually shown in Figure 3. From the ranking order in Figure 3, both the MULTIMOORA method and the proposed method are totally the same, and the result of the TOPSIS method was different to a small extent. Although the ordering results appear to have a slight difference among the three methods in ranking orders, the best alternative PDS is completely consistent with them, that is, the EPC PDS is always at the first rank. Besides, the worst alternative PDS provided by the proposed method is CM, and by the TOPSIS method is DBB. The results of the proposed method and the TOPSIS method are also consistent with the practical situation. It illustrates the reliability and feasibility of the proposed method. The differences of rank are delivered based on different basic theories to some extent. However, apart from the discrepancy of basic theories, some superiorities of the proposed approach are as follows:

(1) Calculating the interval number is one of the difficulties when using the interval Pythagorean fuzzy set, although there are many methods to deal with it. This research gives a valuable

and easy solution to calculate the interval number by transforming the interval number to the connection number, at the same time reducing the loss of information.

(2) The similarity measure is an important tool to judge the degree between the ideal alternative and the proposal alternative. However, the existing similarity measures under interval Pythagorean fuzzy settings are generally complex due to the tedious operation of the Pythagorean fuzzy setting, which restricts the practical application of IPFS. The proposed similarity measures based on minimum and maximum operators are simple and easy in the calculation process.

(3) The major difference between the proposed method and the existing decision making method is that the proposed decision making method considers not only the weights of criteria, but also the weights of membership and non-membership degrees. This method accurately describes the true psychological behavior of experts when judging the decision making problem, that is, the expert is determinant or indeterminant about their judging. It makes the decision making result more reasonable and reliable.

(4) Interval Pythagorean fuzzy set, as an extension of intuitionistic fuzzy set, is more flexible and suitable in dealing with uncertainty and complex decision making information in practical situations. The proposed decision making method developed under IPFSs has very extensive application fields with decision making under uncertainty.

(5) To make sure that the method is better or at least it is not worse than the other existing methods, it is appropriate to apply several related approaches to compare their ranking results for the same problem. Accordingly, an illustrative example has been presented to fulfill the task. It is encouraging that the results have shown great similarity to other methods. This fact can be considered to be one of the advantages of the novel approach.

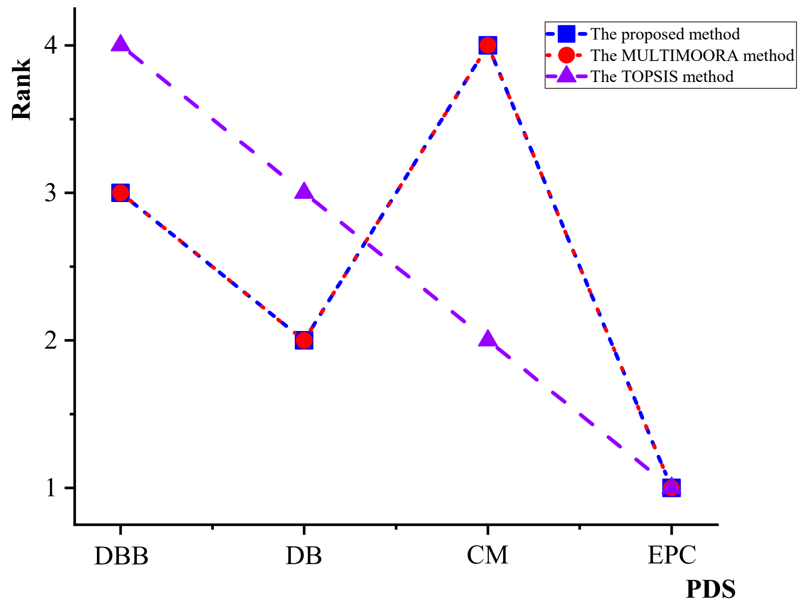

Figure 3. The results of rank for the three methods. MULTIMOORA: Multi-Objective Optimization by Ratio Analysis; TOPSIS: Technique for Order of Preference by Similarity to Ideal Solution.

As previously discussed, using the proposed similarity measures makes the process of decision making simple, intuitive, and an easy operation. Therefore, it gives better enrichment and expansion for the knowledge theory for the decision making method efficiently. In practice, the decision problem was implemented under a high level of complexity and uncertainty. Therefore, the requirement of highly efficient and easy operating methods is on the increase, and the development and application of the proposed method is stated to enrich the theory knowledge and practice from a reference view.

7. Conclusions

The complexity of the objective world and ambiguity of human thinking are widespread in real-life decision making problems. IPFSs show power in dealing with uncertainties, in which one of the primary challenges is the comparison of two interval numbers. To overcome these shortcomings, this paper transformed interval numbers into connection numbers in SPAT in the operating process and proposed three new similarity measures. Then, an interval Pythagorean fuzzy MCDM method based on the proposed similarity measures was built. An example of the selection of a PDS was given to demonstrate the applications and effectiveness of the proposed decision making method. Finally, comparison analysis of results between the proposed and existing methods was given to show the superiority of the former. The proposed method would consider the degree of confidence from the evaluators rather than deal with decision making problems similar to that in existing methods.

The main contributions of this paper are as follows: (1) This study introduced a comparison method by bringing in the binary connection number in SPAT, which transforms interval numbers into connection numbers in the process of comparison. It is a good way to complete the comparative operation and reduce the loss of information. (2) It developed three new similarity measures with IPFSs based on the minimum and maximum operators, and investigation of their properties. (3) Through considering the membership and non-membership degrees simultaneously, three new similarity measures (i.e., 1-type IPFSs similarity measure, 2-type IPFS-weighted similarity measure, 3-type IPFS-weighted similarity measure) based on the minimum and maximum operators were constructed, which have the characteristics of simple thinking and easy operation. (4) This study established an interval Pythagorean fuzzy decision making method by applying the presented similarity measures that consider the degrees of membership and non-membership from the decision experts. In practical decision making problems, the determinacy degrees from experts are very important for the result of decision making. Therefore, it is necessary to consider completely the "true psychological" behavior and degree of confidence of decision experts. It makes the decision making result more reasonable and reliable under uncertainty.

Through the whole process of research and practice, we realized that the development of interval number theory was important in obtaining precise results. Thus, in the future, on the one hand, other areas of applications of similarity measures, such as pattern recognition, clustering analysis, and image processing between IPFSs must be investigated. On the other hand, we should conduct further studies on the comparison of intervals.

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