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A New Evaluation Methodology for Quality Goals Extended by D Number Theory and FAHP

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Abstract: Evaluation of quality goals is an important issue in process management, which essentially is a multi-attribute decision-making (MADM) problem. The process of assessment inevitably involves uncertain information. The two crucial points in an MADM problem are to obtain weight of attributes and to handle uncertain information. D number theory is a new mathematical tool to deal with uncertain information, which is an extension of evidence theory. The fuzzy analytic hierarchy process (FAHP) provides a hierarchical way to model MADM problems, and the comparison analysis among attributes is applied to obtain the weight of attributes. FAHP uses a triangle fuzzy number rather than a crisp number to represent the evaluation information, which fully considers the hesitation to give a evaluation. Inspired by the features of D number theory and FAHP, a D-FAHP method is proposed to evaluate quality goals in this paper. Within the proposed method, FAHP is used to obtain the weight of each attribute, and the integration property of D number theory is carried out to fuse information. A numerical example is presented to demonstrate the effectiveness of the proposed method. Some necessary discussions are provided to illustrate the advantages of the proposed method.

Keywords: quality goals; belief function; D number theory; fuzzy analytic hierarchy process (FAHP); decision making

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1. Introduction

Quality goals play vital roles in the processes of quality management [1]. Quality goals can be defined as "something sought, aimed for, or related to quality", which are closely connected to management concepts [2,3]. The development of external environments, such as new information technology, new production and new requirements, will promote the improvement of management concepts [4]. There are five aspects related to quality goals in the ISO standard system. In other words, quality goals are embraced by the whole quality management system. It is necessary to improve the measure of quality management continuously [5]. One of the effective ways is to monitor and implement these appropriate management measures, which should have the biggest power to aim the planed quality goals. As there are many subjective and objective limitations, such as economy and energy, it is impossible to improve all the quality goals. How to choose the appropriate quality goals with the lowest effort to improve the management measures is an open issue. The selection of suitable quality goals can be viewed as the problem of rank of quality goals.

The experience of quality managers of companies is one of the commonly used methods for quality goals' ranking. However, it is full of subjective arbitrariness [6], and the uncertain information is unavoidable during the evaluation process of managers' experiences. How to measure quality goals in a quantitative way is still a challenge. It is clear that the quality is of subjectiveness, and it may contain multiplicity means that highly depend on human cognition [7]. That is, for the same quality, different people will generate different meanings based on their knowledge, experience and preference. Quality goals can be understood from different perspectives. In other words, quality goals can be depicted by several attributes or criteria. Furthermore, ranking quality goals is one of the typical multi-attribute decision-making (MADM) problems [8–13]. Recently, several methods have been proposed to solve MADM problems, such as the characteristic objects method (COMET) [14–16], the technique for order preference by similarity to ideal solution (TOPSIS) [17], Pythagorean fuzzy set thoery [18], inherent fuzzy entropy [19–21], and analytical network process (ANP) [22]. One of the key tasks in an MADM problem is to ascertain the relative importance of attributes. Among these methods, the analytic hierarchy process (AHP) is popular and widely used to solve the problem of weight due to its simplicity in concept and convenience in operation of hierarchy [23]. The AHP method provides a pairwise comparisons way to measure the degree of importance criteria in the same layer. The items used to represent the preference relationships are in the form of a positive integer. AHP is still unable to deal with these uncertain and imprecise information, and cannot indicate the hesitant information. Based on this, fuzzy AHP (FAHP) has emerged, which provides a way to deal with these fuzzy uncertain information with the aid of fuzzy set theory [24]. The framework of FAHP is similar to AHP, however the element of FAHP is in the form of triangular fuzzy numbers while positive integer in AHP, as detailed discussed later.

The evaluation process of quality goals is inevitably accompanied with uncertainty and impreciseness [25]. Several mathematical tools have been proposed to represent and deal with uncertain information, such as fuzzy set theory [26], intuitionistic fuzzy set [27–29], entropy [30–32], evidence theory [33,34], rough set [35], Z-numbers [36,37], R-numbers [38,39], probabilistic linguistic set [40,41], etc. [42–44]. Among them, Dempster–Shafer evidence theory, also named evidence theory, which can be regarded as the extension of the traditional Bayesian probability theory, has been widely used in many areas, such as data fusion [45,46], evaluation of nuclear safeguards [47], conflict management [48,49], uncertain measure [50], information quality evaluation [51], target recognition [52], fault diagnostics [53–57], reliability assessment [58–61], etc. [62–64]. However, there still exist some limitations while evidence theory is applied, such as mutual exclusion, exhaustive collectiveness, completeness constraint, highly computational complexity, "one-vote veto" mechanism, and independence of each other, as discussed later. Aiming to overcome the above mentioned deficiencies of evidence theory, D number theory has emerged in 2012, which provides a more flexible way to deal with uncertain information, and the above mentioned limitations are well addressed. Since the advantage of D number theory, it has been widely carried out to solve these problems, such as decision making [65–67], location selection [68], risk assessment [69], supply chain management [70], target recognition [71], environmental impact assessment [72,73], curtain grouting efficiency assessment [74] and so forth [75–77]. Besides, D number thoery can be together used with other measures, such as intuitionistic hesitant fuzzy set [78], decision-making trial and evaluation laboratory method (DEMATEL) [79], failure mode and effect analysis (FMEA) [80], to generate new measures for some real-life problems.

Recently, many methods have been put forward to solve the issue of evaluation of quality goals. For example, Li et al., proposed a method to evaluate in-flight service quality based on fuzzy AHP and 2-tuple fuzzy linguistic method [81]. Perçin proposed a combined fuzzy decision-making approach based on in DEMATEL, ANP and VIKOR to airline service quality evaluation [82]. Xu et al., proposed a method for evaluating service quality based on hesitant fuzzy linguistic information [83]. Cheng et al., proposed a method for evaluating the service quality of boutique tourist scenic spot based on TODIM [84]. In the previous study, Tadic et al. proposed a TOPSIS-FAHP method to evaluate quality goals based on TOPSIS and FAHP [85]. In their study, FAHP is used to obtain the weights of attributes with the aid of the distance between two triangular fuzzy numbers [86], and TOPSIS is used to rank quality goals [87]. In this paper, a new method of evaluation quality goals is proposed based on D number theory and FAHP, named D-FAHP method. A new measure of the probability degree of triangular fuzzy numbers is put forward to obtain the weights of attributes. D number theory is carried out to do the process of information fusion. Compared with the TOPSIS-FAHP method, the new proposed method is more intuitive and convenient, especially with the aid of the integration property of D number theory. The main contributions of this manuscript can be briefly summarized as follows.

(1) A new multi-attribute decision-making model to rank quality goals is built at the process level. (2) Triangular fuzzy numbers are adopted to represent and deal with uncertain information during the whole periods of evaluation. (3) D number theory and FAHP are synthesized for the decision making problem.

The rest of the manuscript is organized as follows. Section 2 gives some basic knowledge of fuzzy analytic hierarchy processes (FAHP), evidence theory and D number theory. Section 3 proposes the evaluation model of D-FAHP for quality goals. A numerical example is used to demonstrate detailed steps of the proposed D-FAHP method in Section 4. Some necessary discussions and contrastive analysis are provided in Section 5, which demonstrate the effectiveness and the advantages of the proposed method. A short conclusion is drawn in Section 6.

2. Preliminaries

In this section, some basic background knowledge of fuzzy analytic hierarchy processes, evidence theory and D number theory are recalled.

2.1. Fuzzy Analytic Hierarchy Process

The analytic hierarchy process (AHP) [23] was proposed by Saaty in 1986 as a typical method of multiple-criteria decision-making (MCDM), which has attracted much attention from decision makers for its ability to solve complex decision problems. The detailed steps of AHP can be abstracted in three phases as decomposition, pairwise comparison and synthesis of priorities. In spite of the popularity of AHP, the method is also often criticized for its incompetency to deal with these uncertain and imprecision information. Based on this, the fuzzy analytic hierarchy process (FAHP) method has emerged based on AHP [88]. FAHP is a synthetic extension of the classical AHP method while the fuzziness of the decision makers is considered. The previous study of FAHP was proposed by Laarhoven and Pedrycz, who extended the AHP method with triangular fuzzy numbers and used a logarithmic least squares method to derive fuzzy weights and fuzzy performance scores for ranking alternatives [89]. Apart from the triangular fuzzy number of FAHP, another FAHP method with a trapezoidal fuzzy number was also proposed [90]. Both the triangular fuzzy number and trapezoidal fuzzy number are the typical applications of fuzzy numbers. When the two points located in the topline of trapezium are the same, the trapezoidal fuzzy number will degenerate to a triangular fuzzy number. That is triangular fuzzy number is a special form of trapezoidal fuzzy number. If the three vertices of a triangle fuzzy number are identical, the triangular fuzzy number will reduce to a crisp number. In this paper, the FAHP proposed by Laarhoven and Pedrycz [89] is adopted, in which the elements are in the form of triangular fuzzy numbers. Some basic concepts of fuzzy set theory and FAHP are given as follows.

Definition 1. *A fuzzy set $\tilde{A}$ can be defined as follows [26].*

$$\tilde{A} = \{x, \mu_{\tilde{A}}(x) | x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\} \tag{1}$$

A fuzzy set $\tilde{A}$ is defined on a universe set $X \in R$ in general, and $\mu_{\tilde{A}}(x)$ is the membership function of fuzzy set $\tilde{A}$.

Definition 2. *A triangular fuzzy number (TFN) can be defined by a triplet number (l, m, u) as shown in Figure 1, where the membership can be obtained as follows:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, l \leqslant x \leqslant m \\ \frac{u-x}{u-m}, m \leqslant x \leqslant u \\ 0, \quad othersise \end{cases} \tag{2}$$

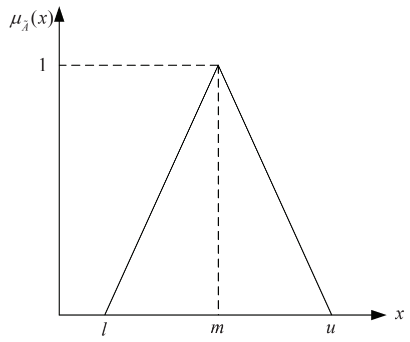

Figure 1. Membership function of a triangular fuzzy number $\tilde{A} = (l, m, u)$.

Here $l \leqslant m \leqslant u$, l and u indicate the lower and upper bounds of the TFN $\tilde{A}$, respectively. The gap between l and u reflects the ambiguity degree of $\tilde{A}$. A larger value of the difference between them indicates the vaguer the TFN $\tilde{A}$ is.

Definition 3. *Let $F_1 = (l_1, m_1, u_1)$ and $F_2 = (l_2, m_2, u_2)$ be two TFNs, then some operation laws of TFN are given as follows:*

$$\begin{aligned} &F_1 \oplus F_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \\ &\lambda \otimes F_1 = (\lambda l_1, \lambda m_1, \lambda u_1) \\ &F_1 \otimes F_2 = (l_1 l_2, m_1 m_2, u_1 u_2) \\ &F_1' = (\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1}) \end{aligned} \tag{3}$$

where $\oplus$, $\otimes$ and $'$ are the operations of plus, multiplication and reciprocal, respectively.

Similar to AHP method, some basic linguistic expressions should also be pre-provided in order to express opinions by experts during the evaluation process of FAHP method. In this paper, the fuzzy rating are described as five linguistic terms, which are modelled in the form of triangular fuzzy numbers, as shown in Table 1.

Table 1. Fuzzy rating in the FAHP method.

Linguistic Term	Triangular Fuzzy Number (l, m, u)
equally important (EI)	(1, 1, 1)
moderately important (MI)	(1, 2, 3)
strongly important (SI)	(2, 3, 4)
very strongly important (VI)	(3, 4, 5)
most important (MOI)	(4, 5, 5)

The detailed steps of FAHP to obtain weights are given as follows.

Step 1. Modelling a hierarchical structure of the evaluation index system. There always contain at least three layers, named goal layer, attribute layer and candidate layer, as shown in Figure 2. The goal layer, which is the highest level, describes the purpose of the evaluation. The attribute layer is the decomposed criteria of a candidate, and evaluation by experts are mainly based on it. This layer can be decomposed to more sublayers, according to the actual situation. The candidate layer is the object of evaluation.

Step 2. Construct the judgment matrix by experts. Experts are asked to give their pairwise comparison evaluations to each attribute, using the pre-provided linguistic terms or triangular fuzzy numbers, as shown in Table 1. A simple example of a judgment matrix is shown in Table 2 based on Table 1. Since linguistic expression is more conformable to a human intuitive viewpoint, experts are recommended to adopt linguistic terms rather than triangular fuzzy numbers during the process of evaluation. As the weight obtained by FAHP is based on triangular fuzzy numbers, the linguistic terms should be converted to triangular fuzzy numbers before the process of computation weight. Then Table 2 can be transformed to Table 3, based on Table 1.

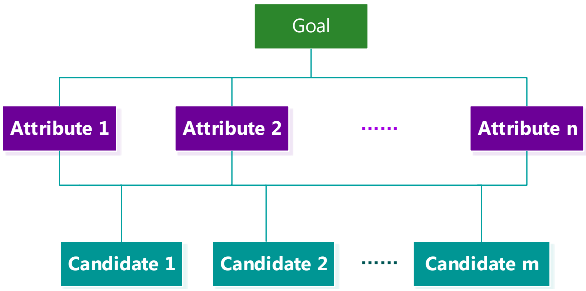

Figure 2. The hierarchical structure of a problem.

Table 2. A simple example of judgment matrix (linguistic terms).

	A1	**A2**	**A3**	**A4**
A1	EI	MI	SI	VI
A2	MI′	EI	MOI	MI
A3	SI′	MOI′	EI	VI
A4	VI′	MI′	VI′	EI

Table 3. A simple example of judgment matrix (triangular fuzzy numbers).

	A1	**A2**	**A3**	**A4**
A1	(1, 1, 1)	(1, 2, 3)	(2, 3, 4)	(3, 4, 5)
A2	(1/3, 1/2, 1)	(1, 1, 1)	(4, 5, 5)	(1, 2, 3)
A3	(1/4, 1/3, 1/2)	(1/5, 1/5, 1/4)	(1, 1, 1)	(3, 4, 5)
A4	(1/5, 1/4, 1/3)	(1/3, 1/2, 1)	(1/5, 1/4, 1/3)	(1, 1, 1)

Tables 2 and 3 indicate that the diagonal elements are the same as "EI" (linguistic terms form) or $(1,1,1)$ (TFN form). That is one compared with oneself is viewed as equally important. In addition, attribute i has one of the above items assigned to it when compared to attribute j, then j has the reciprocal value when compared with i. For example, $A1$ compared to $A3$ is $a_{13} = SI = (2,3,4)$, whereas $A3$ compared to $A1$ is $a_{31} = SI' = (1/4,1/3,1/2)$. If n experts participate in the evaluation, and there will obtain an n fuzzy judgment matrix. An operation of arithmetic mean should be adopted for this situation.

Step 3. Calculate the preliminary comprehensive fuzzy weight value.

$$D_i = \sum_{j=1}^{n} a_{ij} \div (\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}) \tag{4}$$

a_{ij} is the element in the judgment matrix, which is in the form of triangular fuzzy number. D_i represents the integrated fuzzy value of element i . It should be pointed out that D_i is still in the form of a triangular fuzzy number.

Step 4. Defuzzification. Assuming $F_1 = (l_1, m_1, u_1)$ and $F_2 = (l_2, m_2, u_2)$ are two TFNs, and the probability degree of $F_1 \geq F_2$ can be defined as

$$P(F_1 \geq F_2) = \begin{cases} 1 & m_1 \geq m_2 \\ \dfrac{l_2 - u_1}{(m_1 - u_1) - (m_2 - l_2)} & m_1 \leq m_2, u_1 \geq l_2 \\ 0 & otherwise \end{cases} \tag{5}$$

It should be pointed out that, considering there are often more than two elements (attributes) in the actual application, it is essential to compare multiple triangular fuzzy numbers. Thus, the probability degree of a TFN is greater than other TFNs and can be defined as follows.

$$d_i = P(F_i \geq F_1, \cdots, F_j, \cdots F_n) = min(P(F_i \geq F_n)) \tag{6}$$

where $i, j = 1, 2, \cdots n$, and $i \neq j$. d_i indicates the un-normalized weight of attribute i, and a normalized operation is provided to obtain the finally and unified weight of attribute i as follows.

$$w_i = d_i \div \sum_{i=1}^{n} d_i \tag{7}$$

where w_i represents the weight of the attribute i. It should be pointed out that if there exist more than one layer, then the above steps of FAHP should be carried out recursively in order to obtain the weight of attribute in the whole hierarchical structure.

2.2. Evidence Theory

Evidence theory was firstly proposed by Dempster in 1967 [33], and later extended by Shafer in 1976 [34]. Evidence theory is similar to Bayesian probability theory, since they both can transfer these discrete subjective information to a new comprehensive and brief information. Evidence theory is often regarded as an extension of Bayesian probability theory. There are two distinct different points among them. One is the prior information and is indispensable in Bayesian probability theory, which is not necessary in evidence theory. Another is that the information in Bayesian probability theory must be the form of a singleton, however it can be of the form of subset or singleton in evidence theory.

Some basic concepts of evidence theory are introduced as follows.

A problem domain in evidence theory is denoted as a finite nonempty set H, also called the framework of discernment. Let $H = \{H_1, H_2, \cdots, H_n\}$ be a finite set of n mutually exclusive elements. Let 2^H denote the power set of H, and $2^H = \{\emptyset, H_1, \cdots, H_n, H_1 \cup H_2, H_1 \cup H_3 \cdots, H\}$.

Definition 4. *For a frame of discernment H, a basic probability assignment (BPA) is a mapping $m : 2^H \rightarrow [0, 1]$, which is also called the mass function, satisfying*

$$m(\emptyset) = 0, \sum_{A \in 2^H} m(A) = 1$$

where $\emptyset$ is an empty set and A is any element of 2^H. The mass function $m(A)$ reflects the degree of evidence supports to element A.

Support there are two BPAs m_1 and m_2, Dempster's combination rule [33] can be carried out to combine them and yield a new BPA.

Definition 5. *Dempster's rule of combination, also called orthogonal sum, denoted by $m = m_1 \oplus m_2$, is defined as follows:*

$$m(A) = \frac{1}{1-k} \sum_{B \cap C = A} m_1(B) m_2(C) \tag{8}$$

with

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{9}$$

where A, B and C are three elements of 2^H, and k is a normalization constant, called the conflict coefficient of two BPAs. The value of k reflects the consistency degree between two BPAs, the higher value of the k is, the more incompatible two BPAs are. $k = 0$ indicates that the two BPAs are totally consistent, which also means they are identical. A value of $k = 1$ indicates that the two BPAs are totally contradictory, and the combination rule *is invalid for this situation. It should be pointed out that the two BPAs m_1 and m_2 are assumed to be totally independent while Dempster's combination rule is implemented. The Dempster's rule of combination is the core of D-S theory, satisfying commutative and associative properties, i.e., (1) $m_1 \oplus m_2 = m_2 \oplus m_1$, and (2) $(m_1 \oplus m_2) \oplus m_3 = m_1 \oplus (m_2 \oplus m_3)$. Thus if there exist more than two BPAs, the combination operation among them can be carried out in a pairwise way with any order.*

2.3. D Number Theory

As mentioned in Section 2.2, evidence theory provides a flexible way to deal with uncertain information, which extends the application area of Bayesian probability theory. However, there still exist some limitations while evidence theory is applied. First, the element on the framework of discernment must be mutually exclusive and the set must be collective exhaustive. In many real-life situations, the hypothesis cannot always be satisfied. For example, when the hypothesis is presented in linguistic form, such as there may be some intersections among the linguistic terms "excellent", "good", "medium" and "poor". Second, the completeness constraint is that the sum of all mass functions in a BPA must be equal to 1. However, in real-life situation, due to the lack of adequate knowledge, it is feasible to obtain an incomplete BPA. For example, in an open world [91], the incompleteness of the framework of discernment will lead to incompleteness of representation and generate incomplete BPAs. Third, the combination rule of Dempster has a high computational complexity. When the number of element grows linearly, the time complexity of Dempster combination rule has exponential growth. Fourth, the "one-vote veto" mechanism of evidence theory, also called Zadeh paradox [92], that is once an element is vetoed, no matter how strongly other evidences support it, it will still keep the value of zero. Fifth, the BPAs must be independent of each other while Dempster combination rule is applied, which is hard to always meet in real-life situation. The above mentioned five limitations are inherent in evidence theory.

In order to overcome these shortcomings in evidence theory, a new mathematical tool to represent and handle uncertain information has emerged in 2012 [93], named D number theory. D number theory not only inherits the advantages of evidence theory but also overcomes the shortcomings of evidence theory. D number theory is often regarded as an extension of evidence theory, since it mirrors the framework of evidence theory, but weaker restrictions needed. Some basic concepts of D number are given as follows.

Definition 6. *Let U be a finite nonempty set, a D number is a mapping that D: U → [0,1], with*

$$\sum_{A \subseteq U} D(A) \leq 1, \quad D(\emptyset) = 0. \tag{10}$$

where ∅ is an empty set and A is any subset of U.

It is worth mentioning that different from evidence theory, the elements in set U do not require mutually exclusive and the completeness constraint is also not necessary in D number theory. If $\sum_{A \subseteq U} D(A) = 1$, the information is deemed as completeness. Otherwise, it is assumed to be incompleteness. It is very effective and convenient to use the form of D number to express uncertain information in the real world, especially for these incomplete information.

For a discrete set $U = \{b_1, b_2, \cdots, b_i, \cdots b_n\}$, where $b_i \in R$, a special form of D number can be expressed by

$$\begin{array}{l} D(b_1) = v_1 \\ D(b_2) = v_2 \\ \cdots \\ D(b_i) = v_i \\ \cdots \\ D(b_n) = v_n \end{array} \tag{11}$$

simply denoted as $D = \{(b_1, v_1), (b_2, v_2), \cdots, (b_i, v_i), \cdots (b_n, v_n)\}$, where $v_i > 0$ and $\sum_{i=1}^{n} v_i \leq 1$.

Similar to evidence theory, there are also some properties of D number, as follows.

Property 1. *Permutation invariability: If there are two D numbers that* $D_1 = \{(b_1, v_1), \cdots, (b_i, v_i), \cdots (b_n, v_n)\}$ *and* $D_2 = \{(b_n, v_n), \cdots, (b_i, v_i), \cdots (b_1, v_1)\}$*, then* $D_1 \Leftrightarrow D_2$*, where "*$\Leftrightarrow$*" means "be equal to".*

Example 1. *If there are two D numbers:* $D_1 = \{(a, 0.1), (b, 0.4), (c, 0.5)\}, D_2 = \{(c, 0.5), (a, 0.1), (b, 0.4)\}$ *then* $D_1 \Leftrightarrow D_2$

Property 2. *Integration: For a D number,* $D = \{(b_1, v_1), \cdots, (b_i, v_i), \cdots (b_n, v_n)\}$*, the integration operation of D number theory is defined as*

$$I(D) = \sum_{i=1}^{n} b_i v_i \tag{12}$$

where $b_i \in R$*,* $v_i > 0$ *and* $\sum_{i=1}^{n} v_i \leq 1$*.*

Example 2. *Suppose there is a D number:* $D = \{(1, 0.1), (3, 0.1), (5, 0.1), (7, 0.1), (9, 0.6)\}$*, then* $I(D) = 1 \times 0.1 + 3 \times 0.1 + 5 \times 0.1 + 7 \times 0.1 + 9 \times 0.6 = 7$.

3. The Proposed Method

In this section, a new method for quality goals evaluation based on D number theory and FAHP, named D-FAHP method, is proposed. The flow of D-FAHP is shown in Figure 3. The processes of D-FAHP can be divided into five parts, as description (Part 1), weight (Part 2), evaluation (Part 3), information fusion (Part 4) and rank (Part 5).

- Description (Part 1). In this phase, the first thing is to introduce some background knowledge of the evaluation problem to experts. Then, the target of evaluation and correlative attributes should also be clearly put forward.
- Weight (Part 2). The task of this stage is to obtain the weight of attribute. Several measures can be applied to determine the weight. In D-FAHP method, FAHP is adopted. According to the flow of FAHP, triangular fuzzy numbers should be firstly provided to experts. In order to lessen the inconvenience of memorization the triangular fuzzy numbers, a set of corresponding linguistic terms is recommended. The detailed steps of FAHP to obtain weight are shown in Section 2.1.
- Evaluation (Part 3). Experts are asked to give their opinions to each candidate considering the criteria based on pre-defined information. In this phase, the qualitative or quantitative information is allowed.
- Information fusion (Part 4). The aim of this part is to handle the evaluation information provided by experts based on D number theory. If the information presented in Part 3 do not conform to

the requirements of D number theory, then the information should be transformed. After that, the integration property of D number theory will be carried out to fusion information.

- Rank (Part 5). According to the results of information of part 4, the rank of candidates will be presented to the organizer. Some necessary analyses and discussions will be provided, which may be included in a separated part.

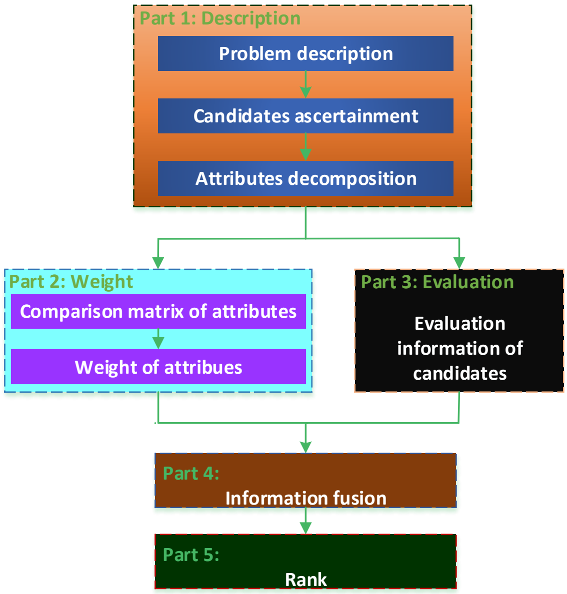

Figure 3. The model of D-FAHP.

4. Case Study

One industry association in Serbia want to investigate the situation of quality goals and to choose the finite quality goals to improve [85]. A total of 52 companies with similar business processes and with similar size are chosen as an example. A management team compound of association experts and quality managers of each company is set up to do the process of evaluation. As the similar companies are considered, it is realistic to assume that all the decisions made by management team are consensus and valid. The proposed D-FAHP method is carried out to solve the ranking problem of quality goal, as follows.

Step 1. Ascertain the candidate of quality goals. Based on the experience and some consensus of industry, the below 10 quality goals are proposed by the management team, as follows.

- Measure of process discrepancy (A1)
- Duration of production order realization (A2)
- Level of supplies in production (A3)
- Rate of complaints concerning production (A4)
- Level of capacity utilization (A5)
- Process capability (A6)
- Process effectiveness (A7)
- Effectiveness of corrective and preventive measures (A8)
- Level of application of methods and tools for process improvement (A9)
- Savings resulting from process improvement (A10)

Step 2. Ascertain the criteria of quality goals on the process level. The management team and some experts who have worked on projects concerning quality system implementation in more than 150 organizations are together invited to analyse the process level of quality goals, and the following criteria are chosen as follows [94].

- *Conformity with overall quality goal (C1)* is one of the most important criteria for quality goals' evaluation on the process level, which is of benefit to enable the elimination of possible conflicts between quality goals and other business goals.
- *Reflection of the state of a process (C2)* is an important criterion especially in those occasions when an urgent decision is necessary.
- *Measurability (C3)* means the demand of process measure which is compulsory. Quality goals' measurement is of diversity even during the same process. It provides the possibility to monitor and measure some quality goals automatically
- *Reflection of the outcomes of a process (C4)* is based on the requirements of a quality management system measuring the outcomes of a process. If the outcomes of a process is contained in some process goals, then the outcomes are highly supported in the process goal.
- *Relation to hierarchical process structure (C5)* indicates the level of goal importance and its correlation among other things, which emphasizes the complexity and structure of the process goal.
- *Reasonable for employees (C6)* reflects the realization for a process, where exists a demand in theory and practice to direct processes towards goals that should be recognizable and generally accepted. The goal is reasonable, which is one of the preconditions.
- *Controllability (C7)* indicates the possibility of process change in relation to new demands, which provides the power of dynamic adjustment to management towards goals.
- *Effort for implementation (C8)* is a considerable criterion, which means the subjective possibility of quality implementation.

Step 3. Construct the judgment matrices of criteria. After selection of the above mentioned criteria, it is necessary to evaluate the relative importance degree of each criterion. Uncertainties are inevitable during the process of evaluation, because of various background of knowledge, experience and preferences. In this paper, the triangular fuzzy numbers are adopted to model uncertainties described in the form of linguistic expressions. The importance of the criteria of evaluation quality goals are not always be regarded as the same, however they can be regarded as unchangeable during a period of time. The subjective judgments and the individual preferences of quality managers are highly involved during the processes of evaluation. In order to decrease the complexity of the evaluation and more habitually to human expression, the fuzzy rating is described by linguistic expressions which can be represented as the triangular fuzzy numbers, as shown in Table 1. After that, the management team is asked to do the evaluation of importance of criteria of quality goals, using the linguistic terms as shown in Table 1. The evaluation results for the comparison among criteria are listed in Table 4. As the elements in FAHP must be in the form of triangular fuzzy numbers, the elements of judgment matrices provided by management team can be transferred to the form of triangular fuzzy numbers, as shown in Table 5.

Table 4. Relative importance of each pair of the considered criteria (linguistic terms).

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
C1	EI	SI′	SI	MI	MI	SI	SI	MOI
C2	SI	EI	MI	EI	MI	MI	MI	VI
C3	SI′	MI′	EI	MI	MI	MI	EI	MI
C4	MI′	EI	MI′	EI	MI	MI	MI′	MI
C5	MI′	MI′	MI′	MI′	EI	MI′	SI′	MI′
C6	SI′	MI′	MI′	MI′	MI	EI	EI	MI′
C7	SI′	MI′	EI	MI	SI	EI	EI	MI
C8	MOI′	VI′	MI′	MI′	MI	MI	MI′	EI

Table 5. Relative importance of each pair of the considered criteria (TFN).

	C1	C2	C3	C4	C5	C6	C7	C8
C1	(1, 1, 1)	(1/4, 1/3, 1/2)	(2, 3, 4)	(1, 2, 3)	(1, 2, 3)	(2, 3, 4)	(2, 3, 4)	(4, 5, 5)
C2	(2, 3, 4)	(1, 1, 1)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 4, 5)
C3	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)
C4	(1/3, 1/2, 1)	(1, 1, 1)	(1/3, 1/2, 1)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(1/3, 1/2, 1)	(1, 2, 3)
C5	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1, 1, 1)	(1/3, 1/2, 1)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)
C6	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1, 2, 3)	(1, 1, 1)	(1, 1, 1)	(1/3, 1/2, 1)
C7	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(1, 1, 1)	(1, 2, 3)	(2, 3, 4)	(1, 1, 1)	(1, 1, 1)	(1, 2, 3)
C8	(1/5, 1/5, 1/4)	(1/5, 1/4, 1/3)	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1, 2, 3)	(1, 2, 3)	(1/3, 1/2, 1)	(1, 1, 1)

All the elements of diagonal in Table 5 are the same as $(1, 1, 1)$, which means that one compared with oneself is regarded as the *equally important (EI)*. Elements on the both sides of the diagonal are with reciprocal relations, which provide simplified operations to the evaluation processes.

Step 4. Calculate the weights of each criterion of quality goals. After the judgment matrices of criteria of quality goals are obtained, next is to calculate the weights of criteria based on FAHP method, as shown in Section 2.1. Based on Equations (4)–(7), the weights of the eight criteria of quality goals are obtained as shown in Table 6.

Table 6. Weight of each criterion.

Criterion	C1	C2	C3	C4	C5	C6	C7	C8
Weight	0.2418	0.2213	0.1497	0.1307	0.0259	0.0652	0.0827	0.0827

Step 5. Evaluate each quality goal by experts. The below seven linguistic terms as shown in Table 7 are utilized to evaluate the eight criteria on the ten quality goals, and the initial results are shown in Table 8.

Table 7. Fuzzy rating in the evaluation process.

Linguistic Term	Triangular Fuzzy Number (l, m, u)	Scale
low value (L)	(1, 1, 2)	1
rather low value (RL)	(1.5, 2, 2.5)	2
fairly moderate value (FM)	(2, 3, 4)	3
moderate value (M)	(3.5, 5, 6.5)	5
highly moderate value (HM)	(6, 7, 8)	7
high value (H)	(7.5, 8, 8.5)	8
very high value (VH)	(8, 9, 9)	9

Table 8. The initial evaluation results consider the criteria for each quality goal.

	C1	C2	C3	C4	C5	C6	C7	C8
A1	HM	HM	VH	VH	HM	H	HM	H
A2	M	H	H	HM	M	HM	M	VH
A3	L	M	HM	FM	FM	M	H	M
A4	RL	FM	M	RL	RL	FM	FM	FM
A5	FM	M	FM	M	M	RL	FM	M
A6	L	FM	HM	FM	FM	RL	FM	FM
A7	FM	HM	M	M	H	M	FM	HM
A8	RL	M	RL	H	RL	RL	FM	M
A9	L	FM	FM	FM	FM	RL	M	RL
A10	L	M	HM	M	VH	FM	RL	H

Step 6. Information fusion. The integration property of D number theory as shown in Equation (12) will be carried out to fuse the information provided by management team. However, the integration property of the D number can only function under special conditions. Table 8 can be converted based on the third column of Table 7. The degree of certainty is not provided, in other words, which can be

regarded as the same value of 0.1250, since each quality goal is evaluated by eight criteria. Then Table 9 can be obtained based on Tables 7 and 8 as follows.

Table 9. The converted evaluation information consider the criteria for each quality goal.

	C1	C2	C3	C4	C5	C6	C7	C8
A1	(7, 0.1250)	(7, 0.1250)	(9, 0.1250)	(9, 0.1250)	(7, 0.1250)	(8, 0.1250)	(7, 0.1250)	(8, 0.1250)
A2	(5, 0.1250)	(8, 0.1250)	(8, 0.1250)	(7, 0.1250)	(5, 0.1250)	(7, 0.1250)	(5, 0.1250)	(9, 0.1250)
A3	(1, 0.1250)	(5, 0.1250)	(7, 0.1250)	(3, 0.1250)	(3, 0.1250)	(5, 0.1250)	(8, 0.1250)	(5, 0.1250)
A4	(2, 0.1250)	(3, 0.1250)	(5, 0.1250)	(2, 0.1250)	(2, 0.1250)	(3, 0.1250)	(3, 0.1250)	(3, 0.1250)
A5	(3, 0.1250)	(5, 0.1250)	(3, 0.1250)	(5, 0.1250)	(5, 0.1250)	(2, 0.1250)	(3, 0.1250)	(5, 0.1250)
A6	(1, 0.1250)	(3, 0.1250)	(7, 0.1250)	(3, 0.1250)	(3, 0.1250)	(2, 0.1250)	(3, 0.1250)	(3, 0.1250)
A7	(3, 0.1250)	(7, 0.1250)	(5, 0.1250)	(5, 0.1250)	(8, 0.1250)	(5, 0.1250)	(3, 0.1250)	(7, 0.1250)
A8	(2, 0.1250)	(5, 0.1250)	(2, 0.1250)	(8, 0.1250)	(2, 0.1250)	(2, 0.1250)	(3, 0.1250)	(5, 0.1250)
A9	(1, 0.1250)	(3, 0.1250)	(3, 0.1250)	(3, 0.1250)	(3, 0.1250)	(2, 0.1250)	(5, 0.1250)	(2, 0.1250)
A10	(1, 0.1250)	(5, 0.1250)	(7, 0.1250)	(5, 0.1250)	(9, 0.1250)	(3, 0.1250)	(2, 0.1250)	(8, 0.1250)

Since the weight information is involved in this study, the integration property of D number theory as shown in Equation (12) should be adjusted appropriately, as follows.

$$I(D) = \sum_{i=1}^{n} w_i b_i v_i \tag{13}$$

where $b_i \in R^+$, $v_i > 0$, $w_i > 0$, $\sum_{i=1}^{n} v_i \leq 1$, $\sum_{i=1}^{n} w_i \leq 1$ and w_i is the weight factor.

Then the integration property of D number as shown in Equation (13) will be carried out to integrate the information. Taking $A1$ for example, based on Tables 6 and 9, the result of information integration can be obtained as follows.

$$\begin{aligned} I(A1) = \sum_{i=1}^{8} w_i b_i v_i &= 0.2418 \times 7 \times 0.1250 + 0.2213 \times 7 \times 0.1250 \\ &+0.1497 \times 9 \times 0.1250 + 0.1307 \times 9 \times 0.1250 + 0.0259 \times 7 \times 0.1250 \\ &+0.0652 \times 8 \times 0.1250 + 0.0827 \times 7 \times 0.1250 + 0.0827 \times 8 \times 0.1250 \\ &= 0.9636 \end{aligned} \tag{14}$$

Analogously, the results of information integration of remainder quality goals can be obtained, as shown in the first three columns of Table 10.

Table 10. The final results of evaluation quality goals by D-FAHP method and TOPSIS-FAHP method.

Qualtity Goals	D-FAHP Method		TOPSIS-FAHP Method [85]	
	Results	Rank	Results	Rank
A1	0.9636	1	0.8849	1
A2	0.8545	2	0.7124	2
A3	0.5334	5	0.4017	4
A4	0.3626	9	0.1390	10
A5	0.4820	6	0.3752	5
A6	0.3813	8	0.2040	8
A7	0.6296	3	0.4331	3
A8	0.4724	7	0.2669	7
A9	0.3169	10	0.1974	9
A10	0.5382	4	0.2894	6

Step 7. Rank. The first three columns in Table 10 indicate that the rank of quality goals is presented as $A1 \succ A2 \succ A7 \succ A10 \succ A3 \succ A5 \succ A8 \succ A6 \succ A4 \succ A9$, where "$\succ$" means "be prior to".

5. Discussion

The rank by D-FAHP method, as shown in Table 10, indicates which quality goals have the most influence on process realization in companies. The quality managers should pay more effort to these quality goals of the top order of list, aiming to improve the competitiveness of companies. That is, the measure of process discrepancy (A1), duration of production order realization (A2) and process effectiveness (A7) are the top three most important quality goals, which should be prioritized and recommended to quality managers. These quality goals such as savings resulting from process improvement (A10), level of supplies in production (A3), level of capacity utilization (A5), and effectiveness of corrective and preventive measure (A8) are mostly related to production process management. Since the effects of them cannot be directly observed in the results of business, they have a lower influence. The remaining quality goals of process capability (A6), level of application of methods and tools for process improvement (A9) and rate of complaints concerning production (A4) are given the lowest priority.

In order to verify the effectiveness of proposed D-FAHP method, the TOPSIS-FAHP method is adopted [85], and the results are shown in the last two columns of Table 10. The evaluation results by D-FAHP and TOPSIS-FAHP method are figured in Figure 4. Table 10 indicates that the rank lists by D-FAHP and TOPSIS-FAHP are almost the same, which demonstrates that the new proposed D-FAHP method is effective. The minor difference mainly locates in the difference of weights, which are not the selfsame in different methods because of different mechanisms. However, the rank of weights of criteria are also almost the same. TOPSIS-FAHP method computes the weights by the distance of two triangular fuzzy numbers, and the weights of criteria are obtained as $(0.224, 0.210, 0.136, 0.1250, 0.025, 0.067, 0.132, 0.081)$. The measure of probability degree is adopted by D-FAHP method to calculate the weights, as shown in Table 6.

The new proposed method has less computational effort, since the advantage of representation to represent uncertain information by D number theory. The application of the integration property of D number theory will further simplify the process of information fusion. Besides, D-FAHP does not involve a cumbersome mathematical operation.

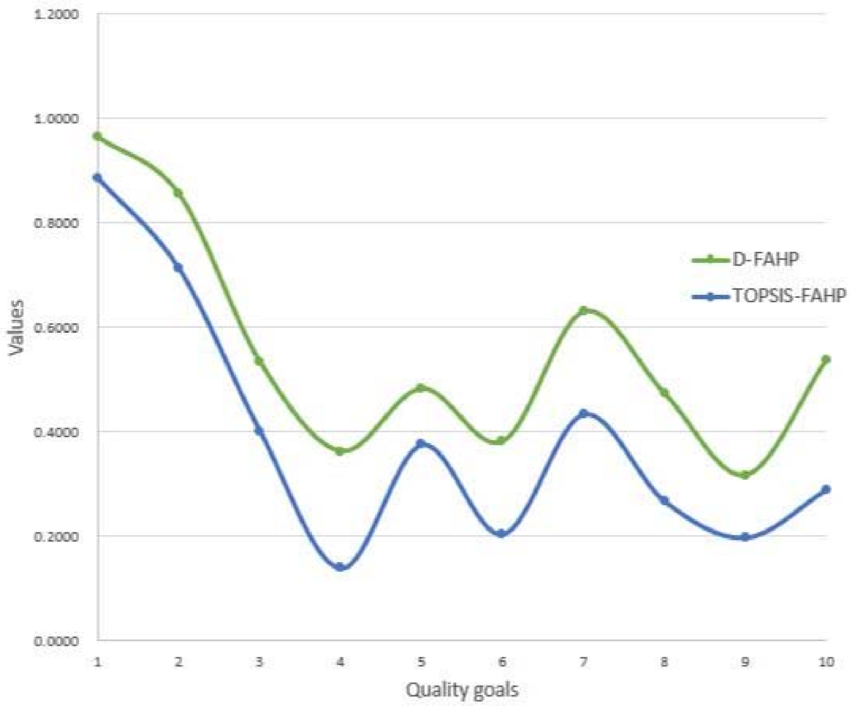

Figure 4. The compared analysis between D-FAHP and TOPSIS-FAHP methods.

It should be pointed out that there exists a transformation processes between linguistic information to crisp value during the part of candidate evaluation. It is clear that the final result is strongly related to the crisp value of linguistic information. That is, when the corresponding crisp value of linguistic information is changed, the final result and the rank order will also be modified.

6. Conclusions

With the development of technology and business environment, the organization and management of the companies are also changed, and process management approaches are updated accordingly. The process management approach, as one of the fundamental characteristics of ISO standard, has been widely adopted in recent years. Aiming to improve a process within a short time and at least cost, the quality manager should identify and pay more attention to these which have more influence on process efficiency among quality goals. The problem of evaluation of quality goals are inevitable with uncertain and imprecise information, and numerical values are not suitable for these situations directly. Linguistic terms are appropriate to describe the uncertainties, since they are accommodated to human habits. The fuzzy set theory is the foundation of linguistic terms. In this paper, the initial information of evaluation to criteria and candidates are all modelled in the form of triangular fuzzy numbers.

FAHP and D number theory are integrated in this paper, and a new method named D-FAHP is proposed for the evaluation of quality goals. The relative importance of criteria are obtained by pairwise comparison by linguistic terms which are in the form of triangular fuzzy numbers. FAHP method is carried out to solve the weight problem, involving the steps of comparison matrix, defuzzification, etc. The crisp values of weight are presented. In order to better represent information, linguistic terms are also adopted in evaluation of candidates of quality goals, which provide flexible and natural ways for information expression. The integration property of D number theory is carried out to fuse the information. A numerical example is used to demonstrate the detailed steps of the proposed method. The rank by D-FAHP method is consistent with other methods, such as TOPSIS-FAHP method, which demonstrates the effectiveness of the new proposed method.

As with all studies, the present research is not without limitations. First, the evaluation criteria of quality goals were selected from experience and some consensus of industry, the selected criteria system is not inclusive of all influences to evaluate quality goals and it only assumes the criteria is static excluding these dynamic and updated factors. Therefore, different methodologies should be considered to identify other factors influencing the quality goals and not only these static criteria but also dynamic criteria should also been involved to enrich the research content. Second, determining the most appropriate sample size of case study is a never-ending quandary for researchers, which is also the same with this research. The results are obtained from relatively small samples, which may have resulted in sample selection bias. A larger and more complicated sample that brings more explanatory power would have allowed more sophisticated evaluation analysis. In the case study of this manuscript, it only considers one layer of criteria to illustrate the application of the proposed method. In the future study, expanding the number of layers according to the real-life situation is necessary. Third, an arithmetic mean is involved during the process of FAHP, which means the important degree of viewpoints provided by experts with different background and experiences are viewed as the same. Further, the weight information of experts in the process of candidates evaluation is also omitted. In the follow-up studies, not only the weight of attribute should be considered, but also the weights of experts during the process of FAHP and candidates evaluation should also be involved. Finally, this study employs FAHP and D number theory to develop an evaluation model that is useful to help managers understand the critical factors in promoting quality goals. Other multi-criteria decision making methods to estimate the relative weights of the influences on quality goals also should be attempted in the future studies. It should be pointed out that the proposed D-FAHP method provides a generalized framework, which can be easily extended to other decision making problem areas, including but not limited to industrial engineering research areas.

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