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Throughput Capacity of Selfish Wireless *Ad Hoc* Networks with General Node Density

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Abstract: In this paper, we study the throughput capacity of wireless networks considering the selfish feature of interaction between nodes. In our proposed network model, each node has a probability of cooperating to relay transmission. According to the extent of selfishness, we, by the application of percolation theory, construct a series of highways crossing the network. The transmission strategy is then divided into three consecutive phases. Comparing the rate in each phase, we find the bottleneck of rate is always in the highway phase. Finally, the result reveals that the node's selfishness degrades the throughput with a factor of square root of the cooperative probability, whereas the node density has trivial impact on the throughput.

Keywords: network capacity; selfish behavior; general node density; percolation theory

1. Introduction

Wireless *ad hoc* networks (WANETs) are an emerging networking technology, which are widely used in environmental monitoring, emergency communication, and military applications, *etc.* The unique feature of such networks is formed by the huge number of nodes. Each node communicates over a wireless channel without any centralized control. One of the problems in WANETs is the routing protocol. Much work has been done on this issue; for example, Zuhairi *et al.* [1] studied the routing protocol in Machine-to-Machine communication network, and the case of vehicular applications was investigated by Cho *et al.* [2]. The routing protocol in mobile WANETs was analyzed by Zaman *et al.* [3]. The other problem is the network capacity. Gupta and Kumar [4] gave an outstanding result on the scaling law of wireless network capacity. They considered n nodes randomly located in a unit network area, each node randomly selected a destination and derived a capacity upper bound of $O\left(\frac{1}{\sqrt{n}}\right)$ and a lower bound of $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$, respectively. (Given two functions $f(n)$ and $g(n)$: $f(n) = o(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = 0$; $f(n) = O(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = c < \infty$; if $g(n) = O(f(n))$, $f(n) = \Omega(g(n))$ *w.h.p.* ; if both $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$, $f(n) = \Theta(g(n))$; $f(n) = \widetilde{\Theta}(g(n))$ means $f(n) = \Theta(g(n))$ when logarithmic terms are ignored.) The results showed that the per-node rate decreases as the number of nodes increases. This pessimistic result is a milestone work of wireless network capacity proceeding. Motivated by [4], Franceschetti *et al.* [5] exploited percolation theory and variable transmission radius to construct a highway system in the network. Based on highways systems, a per-node rate of $\Omega\left(\frac{1}{\sqrt{n}}\right)$ was achieved. The result closed the capacity gap in Kumar's work [4]. Since mobility plays an important role in wireless networks, Grossglauser *et al.* [6] found that mobility can increase the throughput of networks. They demonstrated that a per-node rate of $\Theta(1)$ can be achieved while the transmission delay was up to $\Theta(n)$. Due to the results on network capacity, many researchers became devoted to solving the problem of capacity and delay; most of these works can be generalized into two categories: (1) Increase the number of simultaneous transmission; (2) Decrease the count of hops from source nodes to destination nodes. For the first case, some advanced technologies were employed, such as directional antennas (DA) [7,8], multi-packet reception (MPR) [9] and Multi-input Multi-output (MIMO) [10]. With regard to the second case, infrastructure or base station and nodes mobility were explored; for example, in [11,12], the authors added a base station into the networks. By constructing an optimized transmission scheme, the distance between source and destination can be decreased. Mobility can increase the throughput capacity to $\Theta(1)$, but at the cost of increasing the delay to $\Theta(n)$. Therefore, the work on the tradeoff between throughput and delay was elaborated in [13–16].

However, most of the previous works on network capacity of WANETs assumed that node was cooperative [17–19]. That is, whenever a node receives a request to relay traffic, it will forward the packets on no condition. This ignores the nodes' viewpoint. In addition, previous works either focused on dense or extended networks; that is, the node density is n or 1. They do not consider the impact of node density on the throughput. Therefore, the question remained: what is the throughput capacity of the network if the node exposed selfishness and the node density is general? In this work, we study the throughput of wireless *ad hoc* networks with selfish nodes under general node density. We denote these networks as SWANETs, which were widely researched in connectivity [20] and routing protocol [21]. Minho Jo [22] *et al.* proposed an easy and efficient cooperative of neighboring (COOPON) technique to detect the selfish cognitive radio attack. It is known that selfishness always comes with a price. The price may be tolerable in small-scale WANETs, but it may dominate the consumption of scarce network resources in large-scale WANETs. This situation makes the investigation of throughput with selfish characteristic in large scale WANETs an important open challenge.

In this paper, we consider n nodes randomly and independently distributed in an area with dimension of $\sqrt{A} \times \sqrt{A}$. The communication between source and destination nodes uses multi-hops transmission. we define $p(n)$ as the probability that a node will forward a packet. When $p(n) < 1$, in general, there is network performance degradation because we require all communications to operate on partial nodes, and some network resources (*i.e.*, the selfish nodes) cannot be utilized compared to WANETs. Therefore, it is natural to ask the following: What is the price of selfishness (performance degradation) we have to pay in SWANETs? We formally characterize the relation between the probability $p(n)$, node density ζ, and network performance. Then, we answer these questions with rigorous analysis based on reasonable assumptions on SWANETs.

The main contributions of this paper can be summarized as follows:

(1) Comparing to previous research, we firstly consider the model of selfish wireless *ad hoc* network, which is more realistic.
(2) We derive the asymptotic throughput capacity of the network combining selfish feature and general node density, which is different from the previous works of dense or extended networks.
(3) We observe that the selfishness degrades the achievable throughput with a factor $\sqrt{p(n)}$, where $p(n)$ is the probability of forwarding transmission. In addition, the node density impacts the throughput trivially.

The roadmap of the paper is as follows. In Section 2 we introduce the network model in detail. The achievable rate is derived in Section 3. In Section 4, we discuss the results and conclude the paper in Section 5.

2. System Assumption

In this paper, we construct a random SWANET with the general node density $\zeta \in [1, n]$. The general node density includes the case of random dense networks where $\zeta = n$ and random extended networks where $\zeta = 1$. The other features of the system model we considered are as follows:

(1) We assume n static nodes uniformly and independently placed over an area $\mathbb{A} = [0, \sqrt{A}] \times [0, \sqrt{A}]$, where $A = n/\zeta$.
(2) The node is classified by whether it will forward other transmission. As shown in Figure 1, if a node will forward other transmission, we define the node as an altruistic node (AN, empty points in Figure 1). Otherwise the node is a Selfish Node (SN, solid points in Figure 1). In this work, we assume each node may be selfish, and define the probability that a node i will forward other transmission as $p_i(n)$. For simplicity, we assume the probability that each node forwards other transmission is $p(n)$.

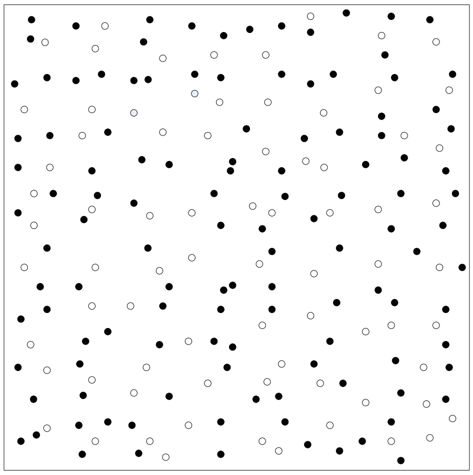


Figure 1. Network model of selfish wireless *ad hoc* networks (SWANETs). Solid points are selfish nodes, and empty points denote altruistic nodes.

(3) Each node randomly chooses a destination node and each node is the destination of exactly one node.
(4) The transmission model we adopted is General Physical Model [4], where the channel gain ignores shadowing and fading, and only depends on the distance between the transmitter and receiver. Let S denote the subset of nodes transmitting simultaneously. Based on the point-to-point coding and decoding [4], the transmission rate $R_{i,j}$ between node i to node j is:

$$R_{i,j} = \log\left(1 + \frac{P_i \cdot d_{ij}^{-\alpha}}{N_0 + \sum_{k \in S \setminus \{i\}} P_k \cdot d_{kj}^{-\alpha}}\right) \text{bit/s} \tag{1}$$

where P_i is the transmission power of node i. We assume each employs identical power P to transmit. d_{ij} denotes the distance between an arbitrary pair of nodes i and j. N_0 is the ambient noise power at the receiver. α is the path loss exponent, and $\alpha > 2$. The notations of this paper are summarized in Table 1.
(5) We say that the throughput capacity of a network [4] is of the order $O(f(n))$ bits per second if there is a deterministic constant $c_1 < +\infty$ such that

$$\liminf_{n \to +\infty} \mathrm{Pro}(T(n) = c_1 f(n) \text{ is feasible}) < 1$$

and is of order $\Theta(f(n))$ bits per second if there are deterministic constants $0 < c_2 < c_3 < +\infty$ such that

$$\liminf_{n \to +\infty} \mathrm{Pro}(T(n) = c_2 f(n) \text{ is feasible}) = 1,$$

$$\liminf_{n \to +\infty} \mathrm{Pro}(T(n) = c_3 f(n) \text{ is feasible}) < 1$$

Table 1. Notations.

Notation	Definition
n	Total number of nodes in the network.
ζ	Node density.
A	Network area; $A = n/\zeta$.
$p(n)$	Probability that a node will forward other transmission.
S	Set of simultaneous nodes.
$R_{i,j}$	Point to point rate.
P	Power of transmission.
d_{ij}	The distance between node i and j.
N_0	Power of noise.
α	Path loss exponent.
$T(n)$	The achievable throughput.

3. Achievable Rate

In order to derive the per-node rate of the networks, we leverage the routing strategy illustrated in [5] and show that there also exists a routing scheme in SWNETs.

Without loss of generality, the strategy of obtaining per-node achievable rate operates as follows: Firstly, using percolation theory, we construct a backbone network which is composed of many horizontal and vertical highways. Then, according to the density of AN, we partition the network area dynamically to ensure each square contains an AN *w.h.p.*, such that the transmission would not be terminated by the selfish nodes. Since we begin with percolation theory in the SWNETs model, we will take a brief look at it here.

Percolation theory [23,24] is a field of mathematics and statistical physics that provides models of phase transition phenomena. For example, assume that water is poured on top of a porous stone—will the water be able to make its way from hole to hole and reach the bottom? By modeling the stone as a square grid, each edge can be open and traversed by water with probability p, or closed with probability $1-p$, and they are assumed to be independent. For a given p, what is the probability that an open path exists from the top to the bottom? That is, is there a path of connected points of infinite length through the network? In fact, there exists a critical p_c below which the probability is always 0 and above which the probability is always 1. In some cases p_c may be calculated explicitly; for example, in a two-dimensional square lattice $\mathbf{Z}^2$, when $p > 1/2$, water percolates through the stone with a probability of one. One can then ask at what rate the water percolates and how it depends on p. In other words, how rich in disjoint paths are the connected component of open edges? To maximize the information flow, we want to operate the network at $p > 1/2$, above the percolation threshold, so that we can guarantee the existence of many disjoint paths that traverse the network.

In this paper, we merely consider the case of $\frac{1}{n} \leq p(n) \leq 1$. When $p(n) < \frac{1}{n}$, following the Chernoff Bound, we know that there are few ANs in the network, and each node transmits the packets directly to the destination or via constant hops by increasing the power. In this case, the asymptotic throughput is similar to the broadcast capacity [25], which is $\Theta\left(\frac{1}{n}\right)$.

3.1. Construction of the Backbone Network

According to percolation theory, a square is said to be open if it contains at least one AN, and closed otherwise. To construct the backbone of the network, we divide the network area into squares of dimensions $l(n) \times l(n)$, where $l(n) = c_0\sqrt{\frac{1}{\zeta p(n)}}$. By appropriately choosing the constant c_0, we can adjust the probability that a square contains at least one AN:

$$P(\text{a square contains at least one AN}) = 1 - e^{-c_0^2} \equiv p_o \tag{2}$$

Note that squares are open (closed) with probability p_o, related to $p(n)$ and ζ and independently of each other. As Figure 2 shows, in each square we draw a horizontal edge (thick lines) across it, and in the vertical direction as well. An edge is said to be open if there exists at least one AN in the square and closed otherwise. On the basis of construction, we establish a bond percolation model [23]. Consequently, the probability that an edge is open is p_o. Since the number of nodes located in a given square follows Poisson random process with parameter $\frac{1}{\zeta p(n)}$, we can get Lemma 1:

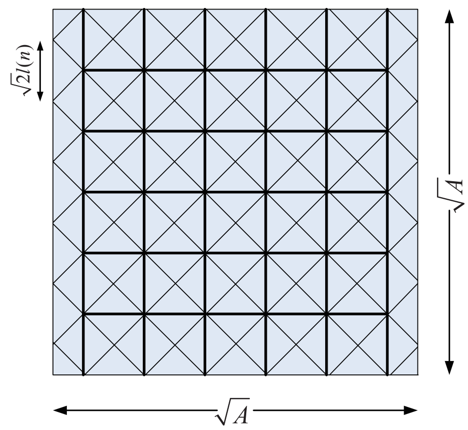


Figure 2. A new square system. We partition the network with side length $l(n)$, and the thick line represents that there is at least one altruistic node in the square.

Lemma 1. *Let N_i denote the number of node contained in a given square s_i. Let E_i be the event $\frac{1}{8p(n)} \le N_i \le \frac{2}{p(n)}, \forall i$. Then,*

$$p_a(n) \equiv P(E_i) > 1 - 16\,(2/e)^{1/4p(n)} \tag{3}$$

Proof. The lemma can be proven simply by applying the Chernoff bound and the Union bound. □

According to the backbone construction, we use $m(n)$ to denote the number of horizontal (vertical) edges which compose the side length of the area $\mathbb{A}$. Then,

$$m(n) = \frac{\sqrt{A}}{\sqrt{2}l(n)} = \sqrt{c_1 n p(n)} \tag{4}$$

Notice that $m(n) \to \infty$ as $n \to \infty$, and c_1 is a constant.

Next we divide the area $\mathbb{A}$ into horizontal rectangular slabs with dimensions $\sqrt{A} \times \sqrt{2}l(n)\,(\kappa \log m(n) + \epsilon_n)$, where $\kappa > 0$, as shown in Figure 3. Let R_n^i denote the i-th slab, where $i \le \frac{m(n)}{\kappa \log m(n) + \epsilon_n}$. The parameter ϵ_n is the smallest nonnegative number, as the number of rectangular slabs $\frac{m(n)}{\kappa \log m(n) + \epsilon_n}$ is an integer.

As proven in Theorem 1 of [5], many open paths from left to right exist inside each slab. Let C_N^i be the maximal number of disjoint left-to-right crossing paths in rectangle R_n^i and $N_n = \min_i C_n^i$. The following lemma shows that there exists a large number of crossing paths in each horizontal rectangular slab of $\mathbb{A}$.

Lemma 2. *For all $\kappa > 0$, there exists a δ satisfying $0 < \delta < \kappa$ such that*

$$\lim_{n \to \infty} P\,(N_n \le \delta \log m(n)) = 0 \tag{5}$$

Proof. According to Theorem 5 in [5], we note that when $p > \frac{5}{6}$ for large n. Taking the limits as $n \to \infty$, the inequality (16) in [5] derives the condition $0 < \delta < \kappa$.

Similarly, by dividing $\mathbb{A}$ into rectangular slabs of sides $\sqrt{2}l(n)\,(\kappa \log m(n) + \epsilon_n) \times \sqrt{A}$ in the vertical direction, we can show that there exists $\delta \log m(n)$ top-to-bottom crossing paths in each vertical slab. Therefore, by exploiting the union bound, we can get that there exists $\Omega\,(m(n))$ left-to-right and top-to-bottom crossing paths (*i.e.*, highways) in the area of $\mathbb{A}$ *w.h.p.* □

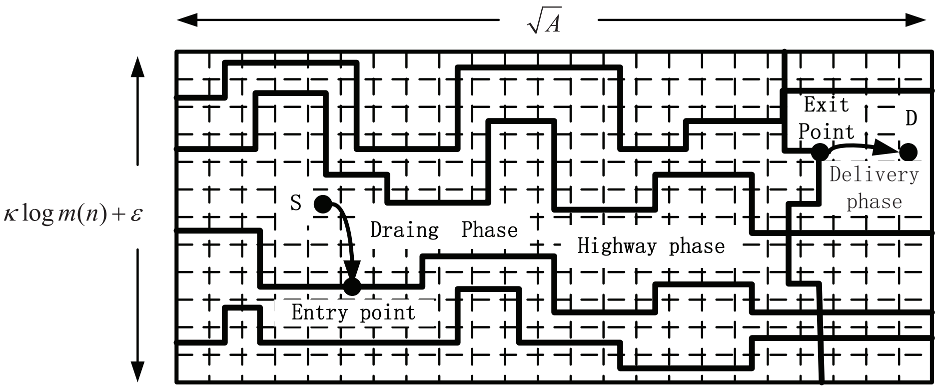


Figure 3. There are at least $\delta \log m(n)$ disjoint highways in each slab. And the three phase routing scheme is illustrated.

3.2. Routing Protocol

We now elaborate on the detailed operation in each phase of routing scheme. As shown in Figure 3, the routing scheme involves three phases: *Draining phase, Highway phase, and Delivery phase*.

(1) *Draining phase:* In the draining phase, source node s drops packets to an entry point on the nearest horizontal crossing highway.
(2) *Highway phase:* In the highway phase, packets are first moved along horizontal highway, and then along the vertical highway until they arrive at an exit point that is close to the destination node D.
(3) *Delivery phase:* In this phase, packets are delivered to the destination node D from the exit point located on the highway.

3.3. The Rate for Transporting a Packet

To achieve the rate of transmit a packet, we use the Time Division Multiplex (TDM) strategy. The idea of the TDM strategy is that when a node transmits along a path, other nodes which are far away can simultaneously transmit without causing excessive interference, as shown in Figure 4.

The following Theorem introduces the fact that the rate can be obtained *w.h.p.* on the path simultaneously. The theorem is stated in slightly more general terms considering nodes at distance d in the edge percolation grid, where d is not the Euclidean distance but the number of d squares away.

Theorem 3. *In each square, for any integer $d > 0$, the rate that source-destination pair can be obtained is*

$$R(d) = \begin{cases} \Omega\left(\left(\frac{d}{\sqrt{\zeta p(n)}}\right)^{-\alpha-2}\right) & \text{if } dl(n) = \Omega(1) \\ \Omega\left(d^{-2}\right) & \text{if } dl(n) = O(1) \end{cases} \tag{6}$$

Proof. As depicted in Figure 4, we partition the network into squares and divide the time frame into k^2 successive slots, where $k = 2(d+1)$. Then, the disjoint set of squares of s_i can transmit simultaneously.

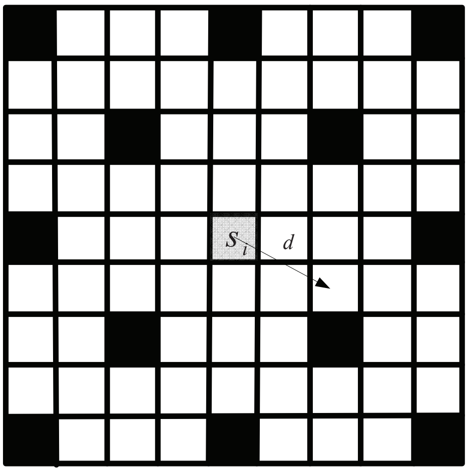


Figure 4. The time division multiplex (TDM) strategy and the case $d = 3$. Gray squares can transmit simultaneously. Notice that around each grey square there is a "silent" region of squares that are not allowed to transmit in the given time slot.

For a specific square s_i, a node in square s_i transmits toward a destination node located in a square at distance d away. At the same time slot, there are four closest squares located at Euclidean distance at least $l(n)(d+1)$ from the receiver, the next eight closest squares are at least $l(n)(3d+3)$ Euclidean distance, and so on. By extending the sum of the interference of the network area, we can calculate the upper bound of the interference at the receiver as

$$I(d,n) \le \sum_{j=1}^{\infty} 4j \cdot P(l(n)(2j-1)(d+1))^{-\alpha} \le P(l(n)(d+1))^{-\alpha} \times \sum_{j=1}^{\infty} 4j(2j-1)^{-\alpha} \tag{7}$$

where P is the transmission power, and we notice that this sum is converged if the path loss exponent $\alpha > 2$. Thus, we can get that $I(d,n) = O\left((dl(n))^{-\alpha}\right)$.

Next we will give a lower bound of the signal received from the transmitter. According to the interference mode, we notice that the distance between transmitter and receiver is at most $\sqrt{2}l(n)(d+1)$. Hence, the lower bound of the signal $S(d)$ at the receiver is

$$S(d,n) \ge P\left(\sqrt{2}(d+1)l(n)\right)^{-\alpha} \tag{8}$$

Next, By General Physical Model [4], combining the interference and the receive signal, we can achieve transmission rate at receiver located d squares away is

$$R(d) = \lim_{n\to\infty} \log\left(1 + \frac{S(d,n)}{N_0 + I(d,n)}\right) = \begin{cases} \Omega\left((dl(n))^{-\alpha}\right) & \text{if } dl(n) = \Omega(1) \\ \Omega(1) & \text{if } dl(n) = O(1) \end{cases} \tag{9}$$

This means that there is a threshold on the rate, which is related to d and square length $l(n)$.

In addition, based on the TDM strategy we adopted, there are $k^2 = 4(d+1)^2$ time slots in our TDM strategy. Thus, the actual rate available needs to be divided by k^2. Correspondingly,

$$R(d) = \begin{cases} \Omega\left(l^{-\alpha}(n)d^{-\alpha-2}\right) & \text{if } dl(n) = \Omega(1) \\ \Omega\left(d^{-2}\right) & \text{if } dl(n) = O(1) \end{cases} \tag{10}$$

□

Equation (10) acts as the groundwork to compute the rate in this work. Since node in this paper is distributed with general node density and nodes are possibly selfish, it makes it more complicated than previous works. In particular, we partition the square dynamically to ensure there exists an AN in each square *w.h.p.* The rate between transmitter and receiver is associated with the node density ζ and probability of altruist $p(n)$.

According to Theorem 3, we can derive the achievable rate in each phase. Comparing the achievable rate of each phase, we get the rate bottleneck.

(1) *Draining phase:* Since the network is divided into squares with side length of $l(n) = c_0\sqrt{\frac{1}{\zeta p(n)}}$, we partition the network area into slices with dimension $\sqrt{A} \times l(n)$. Similar to Lemma 2 in [5], we bound the number of nodes N_s in each slice uniformly, which bounds the number of nodes accessing a crossing highway. Note that there are a total of $\frac{\sqrt{A}}{\sqrt{2}l(n)}$ slices, each node in the i-th slice transmits directly to an entry point located on the i-th crossing highway, as shown in Figure 2.

Lemma 4. *Let N_s denote the number of nodes in each slice. Then,*

$$\lim_{n\to\infty} P\left(N_s \le 2\sqrt{2n/p(n)},\ \forall s\right) = 1 \tag{11}$$

Proof. The proof follows from the Chernoff and Union bound. □

The next lemma illustrates the achievable rate in the draining phase.

Lemma 5. *The transmitter inside each square can achieve a rate to an entry node on the highway of*

$$R^{Dr} = \begin{cases} \Omega\left(p(n)\left(\frac{\log^2 np(n)}{\sqrt{\zeta p(n)}}\right)^{\alpha}\right) & \textit{if } l(n) = \Omega(\log np(n)) \\ \Omega\left(\frac{p(n)}{\log^2 np(n)}\right) & \textit{if } l(n) = O(\log np(n)) \end{cases} \tag{12}$$

Proof. Similar to [5], the area $\mathbb{A}$ is divided into rectangular slabs of dimensions $\sqrt{A} \times \sqrt{2}l(n)\,(\kappa \log m(n) + \epsilon_n)$, where $m(n)$ is defined in (4) and κ is chosen such that there are at least $\lceil \delta \log m(n) \rceil$ crossing paths in each slab. The crossing paths are denoted as $1, \cdots, N_n$. In order to balance the load across the highways, we slice the each slab into $\delta \log m(n)$ smaller strips, each of dimensions $\sqrt{A} \times \sqrt{2}\omega l(n)$, where ω is a constant and chosen appropriately. Note that each crossing highway may not be fully contained in its corresponding strip, but it may deviate from it. Once the source nodes are mapped to crossing paths, we choose the entry points for each source as follows: The entry point is chosen from only these open squares containing one AN. The transmitter drains the information to the entry point directly, and each transmitter finds its highway within the same slab. Hence, the distance between transmitter and entry point is never larger than $\kappa l(n) \log m(n) + \sqrt{2}l(n)$. To compute the rate that node can transport to the entry point on the highway, let $d = \kappa l(n) \log m(n) + \sqrt{2}l(n)$ and apply the Theorem 3. We can obtain that a node can communicate to its entry point at rate

$$\begin{aligned} R^{Dr} = R\left(\kappa l(n) \log m(n) + \sqrt{2}l(n)\right) &= R\left(\kappa l(n) \log\left(\sqrt{np(n)}\right) + \sqrt{2}l(n)\right) \\ &= \begin{cases} \Omega\left(\left(\frac{\log^2 np(n)}{\sqrt{\zeta p(n)}}\right)^{\alpha}\right) & \text{if } l(n) = \Omega(\log np(n)) \\ \Omega\left(1/\log^2 np(n)\right) & \text{if } l(n) = O(\log np(n)) \end{cases} \end{aligned} \tag{13}$$

□

Next, we derive the achievable rate on the highway phase.

Lemma 6. *The information along the highways can achieve a per-node rate of R^H w.h.p., where R^H is*

$$R^H = \begin{cases} \Omega\left(\frac{1}{\sqrt{n}} p^{\frac{1}{2}}(n)(\zeta p(n))^{\alpha}\right) & \textit{if } l(n) = \Omega(1) \\ \Omega\left(\frac{1}{\sqrt{n}} p^{\frac{1}{2}}(n)\right) & \textit{if } l(n) = O(1) \end{cases} \tag{14}$$

Proof. The information transported on the highway is forwarded by multi-hop routing. Under the pairwise coding and decoding, the transporting is performed along the horizontal highways first, until it reaches the crossing with the target vertical highway. Then, the same is performed along the vertical highways until it reaches the appropriate exit point for delivery.

According to Lemma 2, a node on the horizontal highway must relay for at most $2l(n)\sqrt{A}$ nodes, and the maximal distance between two hops is $\sqrt{2}l(n)$. Using Theorem 3, we can conclude that an achievable rate along the horizontal highways is

$$R^{Hh} = \begin{cases} \Omega\left(l^{-\alpha}(n)\left(\frac{p(n)}{n}\right)^{1/2}\right) & \text{if } l(n) = \Omega(1) \\ \Omega\left(\left(\frac{p(n)}{n}\right)^{1/2}\right) & \text{if } l(n) = O(1) \end{cases} \tag{15}$$

Similarly, the rate on vertical highways is

$$R^{Hv} = \begin{cases} \Omega\left(l^{-\alpha}(n)\left(\frac{p(n)}{n}\right)^{1/2}\right) & \text{if } l(n) = \Omega(1) \\ \Omega\left(\left(\frac{p(n)}{n}\right)^{1/2}\right) & \text{if } l(n) = O(1) \end{cases} \tag{16}$$

Combining Equations (15) and (16), we finish the proof of Lemma 6. □

The following lemma illustrates the achievable rate of the delivery phase.

Lemma 7. *The receiver can attain a rate of R^{Dl} from an exit point on the highway, where R^{Dl} is*

$$R^{Dl} = \begin{cases} \Omega\left(p(n)\left(\frac{\log^2 np(n)}{\sqrt{\zeta p(n)}}\right)^{\alpha}\right) & \textit{if } l(n) = \Omega(\log np(n)) \\ \Omega\left(\frac{p(n)}{\log^2 np(n)}\right) & \textit{if } l(n) = O(\log np(n)) \end{cases} \tag{17}$$

Proof. The delivery phase is an opposite process to the draining phase, while the transmission is from highways to the destination. □

Combining the Lemma 5, Lemma 6 and Lemma 7, we can derive the rate bottleneck of SWANETs

Theorem 8. *Comparing the achievable rate of each phase, we get the rate of per-node in SWANETs with general nodes density is*

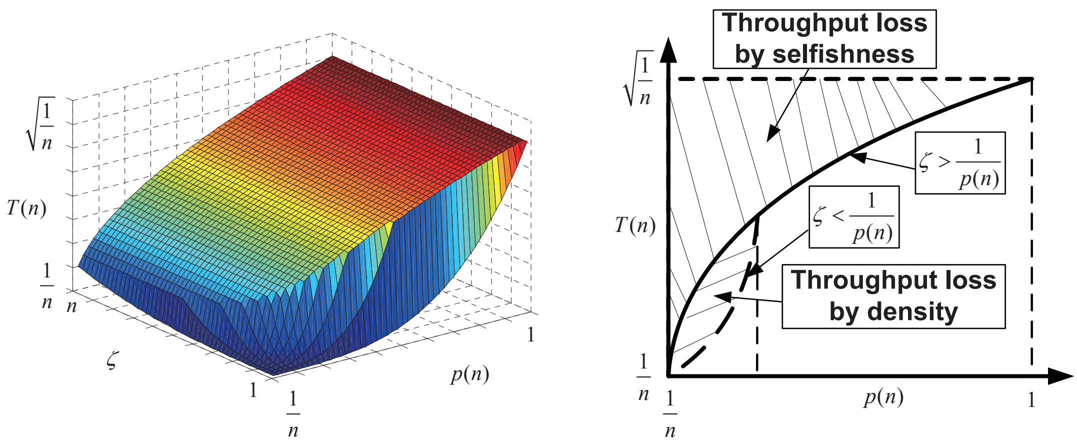
$$T(n) = \begin{cases} \Omega\left(\frac{1}{\sqrt{n}} p^{\frac{1}{2}}(n)(\zeta p(n))^{\alpha}\right) & \textit{if } l(n) = \Omega(1) \\ \Omega\left(\frac{1}{\sqrt{n}} p^{\frac{1}{2}}(n)\right) & \textit{if } l(n) = O(1) \end{cases} \tag{18}$$

4. Discussion

In this section, we will discuss the results we obtained. By comparing our results with previous literature, we get the price of selfishness and the impact of node density on the throughput capacity of WANETs.

Firstly, our results can unify previous works [4,5], when we set $p(n) = 1$ and $\zeta = 1$ or $\zeta = n$.

Secondly, As shown in Figure 5, Figure 5a depicts the relation of throughput capacity with the probability $p(n)$ and node density ζ. From Figure 5a, we find that selfish nodes can severely impact the achievable per-node rate. In particular, when the square length $l(n) = \Omega(1)$, a degradation factor of $p^{\frac{1}{2}}(n)(\zeta p(n))^{\alpha}$ emerged, while for the case of $l(n) = O(1)$, the degradation factor is $p^{\frac{1}{2}}(n)$. In addition, node density has a trivial impact on the throughput. If the node density $\zeta > \frac{1}{p(n)}$, there is no impact on the throughput, while for the case of $\zeta > \frac{1}{p(n)}$, the throughput loss caused by node density is ζ^{α}. The reason for the performance degradation is that, due to the selfish nodes, all communications operate on partial nodes, and the selfish nodes cannot be utilized. Thus, it is hard for a node to find a relay node. With the extent of selfish nodes increasing, a transmitter needs to enlarge its transmission radius to find a relay node. From [4], we know that the network capacity is decreased with increasing transmission radius, since the number of simultaneous transmissions is decreased. We can enhance the transmission power to offset the impact of node density, which also provides sufficient conditions to guarantee the connectivity of the network [20]. In addition, we also give a comparison with previous works in Figure 5b. Figure 5b demonstrates that there exists a throughput loss caused by selfish nodes and node density. This insightful result is quite important because it provides valuable insight on the desirable operating point that balances selfish nodes and node density with throughput. We need to increase $p(n)$ in order to get more altruistic nodes, but as a node itself, it is quite the opposite. Hence, for future work, we will use Game Theory to solve the benefit between nodes and network performance.



(**a**) (**b**)

Figure 5. (**a**) shows the asymptotic throughput capacity of WANETs under the impact of selfish nodes and node density. (**b**) is a section of (a); We note that there exists a threshold for node density ζ. Comparing with previous literature, we can notice the throughput loss caused by selfish nodes or node density intuitively. The scales of the axes are in terms of the order in n.

5. Conclusions

This paper introduces a modeling framework for SWANETs under general node density. We consider various scope of selfish behavior for each node. Moreover, different node density is considered. A percolation model is adopted to construct a series of highway systems to connect the transmission. The computation of throughput capacity is conducted under the model which gives a more realistic description of SWANETs. The result reveals that, although a selfish node can save resources for the node itself, it degrades the network performance significantly.

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