

Supplemental to:

“Generalized Pattern Search Algorithm for Crustal Modeling”

2. Materials and Methods

2.1 Receiver functions

In the time domain receiver functions are computed using deconvolution of vertical component seismic signal from radial component seismic signal. Under ideal conditions, receiver functions can also be determined in the frequency domain using the ratio of radial component and vertical component seismograms:

$$RF(\omega) = \frac{R(\omega)}{V(\omega)} \quad (1)$$

where $RF(\omega)$ is frequency-domain receiver function, $R(\omega)$ and $V(\omega)$ are frequency-domain radial and vertical component seismograms, respectively (Cassidy, 1992). Practically, due to noise in the data and the band-limited nature of the signal, the water-level technique is used to stabilize this spectral division (Clayton and Wiggins, 1976), and therefore, the receiver function estimate $\overline{RF}(\omega)$ given by:

$$\overline{RF}(\omega) = \frac{R(\omega)}{V(\omega)} A(\omega) \quad (2)$$

where $A(\omega)$ is the averaging function obtained using the relationship

$$A(\omega) = \frac{R(\omega)R^*(\omega)G(\omega)}{\phi(\omega)} , \quad (3)$$

with

$$G(\omega) = \exp\left(\frac{-\omega^2}{4a^2}\right), \quad (4)$$

and a is the width of the Gaussian filter used to remove high-frequency noise

$$\phi(\omega) = \max\{V(\omega)V(\omega)^*, c. \max\{V(\omega)V(\omega)^*\}\}, \quad (5)$$

c is the water-level parameter expressed as a fraction of the maximum vertical component power spectra (Ammon, 1991; Cassidy, 1992).

The time domain receiver function for H- κ analysis is obtained by applying inverse Fourier Transform to Equation 10 above:

$$\overline{rf}(t) = F^{-1}\{\overline{RF}(\omega)\} \quad (6)$$

Tangential receiver functions can also be obtained by de-convolving vertical component seismograms from tangential component seismograms. The tangential receiver functions are usually important to check if there is inhomogeneity in the crustal structure.

2.2 H-k Stacking

Receiver functions display a number of phases whose arrival times are related with discontinuities in the crust and upper mantle. The phases that are considered in crustal studies are depicted in Figure 2.4(a). These phases include the reference direct P wave, P-to-S converted phase (Ps), PpPs phase and PpSs+PsPs phase. These phases originate from an impinging plane P-wave

at the crust-mantle boundary (the Moho). Figure 2.4 (b) displays the receiver function corresponding to the crustal structure in Figure 2.4 (a).

In Figure (2.4), the times, t_1 , t_2 , and t_3 are relative arrival times of the above three phases with respect to the arrival time of the direct P wave. Relationships of the crustal thickness H , crustal velocities V_p and V_s with the different phase arrival times in the receiver function are given by the following equations (Zandt et al., 1995):

$$H = \frac{t_{Ps} - t_p}{\sqrt{\frac{1}{V_s^2} - p^2} - \sqrt{\frac{1}{V_p^2} - p^2}} \quad (7)$$

$$H = \frac{t_{PpPs} - t_p}{\sqrt{\frac{1}{V_s^2} - p^2} + \sqrt{\frac{1}{V_p^2} - p^2}} \quad (8)$$

$$H = \frac{t_{PpSs} + P_s P_s - t_p}{2 \sqrt{\frac{1}{V_s^2} - p^2}} \quad (9)$$

where $p = \frac{\sin \theta}{V_p}$ is the ray parameter of the seismic signal. Here θ is the angle of incidence between the seismic ray and a normal and V_p is the velocity of the P-wave. The relative arrival times can be given by:

$$t_1 = \left(\sqrt{\kappa^2 - V_p^2 p^2} - \sqrt{1 - V_p^2 p^2} \right) \cdot \frac{H}{V_p} \quad (10)$$

$$t_2 = \left(\sqrt{\kappa^2 - V_p^2 p^2} + \sqrt{1 - V_p^2 p^2} \right) \cdot \frac{H}{V_p} \quad (11)$$

$$t_3 = \frac{2H}{V_p} \cdot \left(\sqrt{\kappa^2 - V_p^2 p^2} \right) \quad (12)$$

H- κ stacking of receiver functions is a technique used to estimate crustal thickness H and crustal Vp-to-Vs ratio κ (Zhu and Kanamori, 2000). Using the relative times t_1 , t_2 , and t_3 as well as H and κ , we can rewrite the objective function proposed by Zhu and Kanamori (2000) as a maximization problem as follows:

Maximize

$$S(H, \kappa) = \sum_{j=1}^N w_1 r_j(t_1(H, \kappa)) + w_2 r_j(t_2(H, \kappa)) + w_3 r_j(t_3(H, \kappa)) \quad (13)$$

subject to

$$w_1 + w_2 + w_3 = 1 \quad (14)$$

where w_1 , w_2 , w_3 are weights (with $w_3=1-w_1-w_2$). The $r_j(t_i)$, $i = 1, 2, 3$, are the receiver function amplitude values at the predicted arrival times of the Ps, PpPs, and PsPs+PpSs phases, respectively, for the j^{th} receiver function, and N is the total number of receiver functions used. By performing a grid search through H and κ parameter space, the H and κ values corresponding to the maximum value of the objective function can be determined. The main hypothesis behind H- κ stacking is that the weighted sum stack will attain its maximum value when H and κ take their proper values (Zhu and Kanamori, 2000).

Following Christensen (1996) and Zandt et al. (1995), Poisson's ratio (σ) is directly related to κ , and the relations are:

$$\kappa = \frac{V_p}{V_s} = \sqrt{(1 - p^2 V_p^2) \left(2 \left(\frac{t_1}{t_2 - t_1} \right) + 1 \right)^2 + p^2 V_p^2} \quad (15)$$

$$\sigma = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)} = \frac{\kappa^2 - 2}{2(\kappa^2 - 1)} \quad (16)$$

3.2 GUI Implementation for the GPS Algorithm.

GUI implementation is important not only for examining and displaying initial parameters and final parameters, but also for demonstrating the convergence of the algorithm. A GUI has been developed using Matlab for the GPS algorithm. The GUI is very significant especially for the implementation of the GPS initial value dependence testing algorithm in this dissertation. Since the GUI clearly shows both the initial as well as final values of parameters and weights, observation of initial value dependence will be evident.

3.3 GPS Convergence Test Algorithm. The algorithm of GPS Convergence Test is performed to examine final parameters and final objective function values at least for two extreme initial values. Using the GUI that is developed in this research work, the convergence for seismic station ARBA can be shown.

3.3.1 GPS Convergence Test for Seismic Station ARBA. It is observed that, for two extreme initial values and combinations of extreme values as well as for some intermediate initial values, GPS gives similar results. Though their initial parameter values as well as their initial objective function values are different, the final parameter and objective function values are similar; i.e. the GPS results converge to the same final values. Figure 4.7 (a) and (b) show that the final values from the GPS converge to the same final or optimal values irrespective of the initial values. Therefore, in such a case we don't need to resort to applying the FPN to find the best initial values because the inversion is independent of initial values or model parameters. Figure 3 displays two extreme initial values for the region of investigation: (a) smallest and (b) highest initial parameter values.

DATA AND RESOURCES

Seismic data collected by station ARBA from the Ethiopia Broadband Seismic Experiment (Nyblade and Langston, 2002) has been used in this study for testing the implementation of the GPS algorithm. The data was collected between 2000 and 2002, and is accessible through the IRIS website: http://www.iris.edu/SeismiQuery/month_f2.html, network XI.