Exploring the Effect of Misinformation on Infectious Disease Transmission

Supplementary Material

A simple SI model example is presented to illustrate the LTM method and behavioural analysis of the model. Figure S1 presents the SI model, which runs from T_1 (time) to T_{20} (time). In the model, the N (total population) size is 10000 people with 9999 initial S (susceptible) and 1 initial I (infected). The SI model's basic flow and auxiliaries—R_0 (reproduction number), D (delay duration), β (per capita contact), λ (force of infection), and IR (infection rate)—are shown in Equations (1–8).

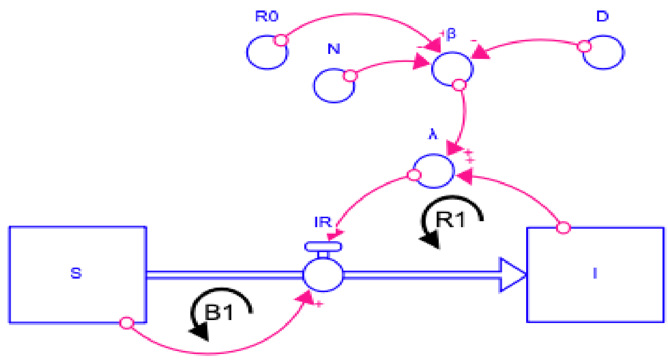

Figure S1. SI model structure.

$$d(S(t))/dt = -\lambda * S(t) \quad (1)$$
$$d(I(t))/dt = \lambda * S(t) \quad (2)$$
$$N \text{ (total population)} = 10000 \quad (3)$$
$$R_0 \text{ (reproduction number)} = 2 \quad (4)$$
$$D \text{ (delay duration)} = 2 \quad (5)$$
$$\beta \text{ (per capita contact)} = R_0/(N*D) \quad (6)$$
$$\lambda \text{ (force of infection)} = \beta * I(t) \quad (7)$$
$$IR \text{ (infection rate)} = S(t)*\lambda \quad (8)$$

The model contains two feedback loops: R_1 reinforcing feedback loop and B_1 balancing feedback loop, with the point of inflection (POI) between time T_{13} and T_{14}. Table S1 presents the details of R_1 and B_1 loop variables and links.

Table S1. Details of R1 and B1 loops.

B_1 Balancing Feedback Loop		**R_1 Reinforcing Feedback Loop**	
Number of Variables	Number of Links	Number of Variables	Number of Links
1. S 2. IR	1. S→IR 2. IR→S	1. I 2. λ 3. IR	1. λ→IR 2. IR→I 3. I→λ

The LTM method makes computations as time progresses in the model, and the first computation starts when the model is initialised. The following Equations (1–4) are used to calculate the link scores without integration, link scores with integration, loop scores, and relative loop scores, respectively [46, 47].

$$LS(S \rightarrow IR) = \{ (|\Delta_S IR/\Delta IR| \;.\; sign(\Delta_S IR/\Delta S)) , 0, \Delta IR = 0 \text{ or } \Delta S = 0 \quad (1)$$

$$\text{Inflow: } LS(i \rightarrow S) = (|\Delta i/\Delta S_t - \Delta S_{t-dt}| *1) \quad \text{Outflow: } LS(o \rightarrow S) = (|\Delta o/\Delta S_t - \Delta S_{t-dt}| *-1) \quad (2)$$

$$\text{Loop Score}(L_S) = (LS\ (u_1 \rightarrow v_1)\ .\ LS\ (u_1 \rightarrow v_1) \ldots\ldots LS\ (u_n \rightarrow v_n)) \quad (3)$$

$$\text{Relative Loop Score } L_S = (\text{Loop Score}(L_S) / \sum_{\lambda=0}^{n} |\text{Loop Score } (L_\lambda)|) \quad (4)$$

The abovementioned Equation (1) calculates the link score (without integration). In the SI model, links without the integration process are "S→IR", "λ→IR", and "I→λ". Table S2 includes all the necessary steps to calculate links' scores for T_2. In the link "S→IR", IR is a flow defined by the equation IR=f(S*λ). The first term in Equation (1), "$|\Delta_S IR/\Delta IR|$", measures the contribution of S to IR by reporting the proportion of the change in IR that originated from S, where the partial change in IR occurred. To calculate the link score magnitude for the link "S→IR", first, determine "ΔIR", which is the actual change in IR (0.9996). Next, determine the partial change in IR for S over the interval, represented with the symbol "$\Delta_S IR$", by substituting the previous value of λ (0.0001) and the current value of S (9998) into the equation for IR. Then, take the computed value of IR using those values (0.9998) and subtract from it the previous value of IR (0.9999) to yield "$\Delta_S IR$" (-0.0001). To complete the calculation of the link score magnitude, divide "$\Delta_S IR$" (-0.0001) by "ΔIR" (0.9996) to obtain the result (0.0001) of the change in IR, which is caused by the change in S. The second term in Equation (1), "$sign(\Delta_S IR/\Delta S)$", measures link polarity. The sign function extracts the sign for the link. To complete the link score calculation, multiply link score magnitude by link polarity. Repeat the same steps for other link scores, "λ→IR" and "I→λ".

Table S2. Demonstration of link scores for S→IR, λ→IR, and I→λ by using Equation (1). Calculations are based on the following equations: IR = S*λ and λ = I*β.

Variable Link	**Time T_1**	**Time T_2**	**Variable Change**	**Partial Change IR**	**Link Score Magnitude**	**Link Polarity**	**Link Score**
S **S→IR**	9999	9998	ΔS = -1	$\Delta_S IR$ = -0.0001	$\|\Delta_S IR/\Delta IR\|$ = 0.0001	$sign(\Delta_S IR/\Delta S)$ = +1	$\| \Delta_S IR/\Delta IR\|.$ $sign(\Delta_S IR/\Delta S)$ = **0.0001**
λ **λ→IR**	0.0001	0.0002	$\Delta\lambda$ = 0.0001	$\Delta_\lambda IR$ = 0.9999	$\|\Delta_\lambda IR/\Delta IR\|$ = 1.0003	$sign(\Delta_\lambda IR/\Delta\lambda)$ = +1	$\| \Delta_\lambda IR/\Delta IR\|.$ $sign(\Delta_\lambda IR/\Delta\lambda)$ = **1.0003**
IR= S*λ	0.9999	1.9995	ΔIR = 0.9996				
I **I→λ**	1	2	ΔI = 1	$\Delta_I\lambda$ = 0.0001	$\|\Delta_I\lambda/\Delta\lambda\|$ = 1	$sign(\Delta_I\lambda/\Delta\lambda)$ = +1	$\|\Delta_I\lambda/\Delta\lambda\|.$ $sign(\Delta_I\lambda/\Delta\lambda)$ = **1**
β	0.0001	0.0001					
λ= β*I	0.0001	0.0002	$\Delta\lambda$ = 0.0001				

The abovementioned Equation (2) calculates the link score with integration, where "i" is the inflow, "o" is the outflow, "S" is the stock, and "t" represents time. "Δi" and "Δo" represent the first-order partial changes in stock S for the flow. "ΔS_t - ΔS_{t-dt}" is the change in the net flow, which is the second-order change in stock S. The terms "$|\Delta i/\Delta S_t - \Delta S_{t-dt}|$" and "$|\Delta o/\Delta S_t - \Delta S_{t-dt}|$" measure the contribution of flow to the stock by reporting the proportion of the change in the stock. To complete the calculation, multiply the inflow result by "1" and multiply the outflow result by "-1". In the SI model, the links that contain the integration process are "IR→S" and "IR→I". Table S3 includes all the necessary steps to calculate the link scores for T_2.

Table S3. Demonstration of link scores for IR→S and IR→I by using Equation (2).

Variable Link	**Time T_1**	**Time T_2**	**Variable Change**	**Link Score Magnitude**
i = non	0	0	Δi = 0	$\|\Delta i/\Delta S_t - \Delta S_{t-dt}\| * 1 = 0$
o = IR **IR→S**	0.9999	1.9995	Δo = 0.9996	$\|\Delta o/\Delta S_t - \Delta S_{t-dt}\| * -1 = \mathbf{-1}$
S = ∫ (in – out) Initial = 9999	9999	9998	$\Delta S_t - \Delta S_{t-dt}$ = 0.9996	
i = IR **IR→I**	0.9999	1.9995	Δi = 0.9996	$\|\Delta i/\Delta I_t - \Delta I_{t-dt}\| * 1 = \mathbf{1}$
o = non **IR→I**	0	0	Δo = 0	$\|\Delta o/\Delta I_t - \Delta I_{t-dt}\| * -1 = 0$
I = ∫ (in – out) Initial = 1	1	2	$\Delta I_t - \Delta I_{t-dt}$ = 0.9996	

Equations (1) and (2) both have the same concept to calculate the partial change, i.e., Δi and Δo are the same concepts as ΔIR or Δλ, and sign () is replaced with "-1" or "+1". In the SI model, both S and I stocks have a single flow. "IR→S" and "IR→I" link scores are nonzero; therefore, the links between the flow and the stock are active. The S and I stocks will change throughout the simulation. Table S4 contains B_1 and R_1 loop scores, as well as relative loop scores for T_2, T_8, T_{10}, T_{13}, T_{14}, T_{16}, and T_{20}.

Table S4. Demonstration of loop scores and relative loop scores for B_1 and R_1.

Links	**T_2**	**T_8**	**T_{10}**	**T_{13}**	**T_{14}**	**T_{16}**	**T_{20}**
S→IR	0.0001	0.0129	0.0539	0.7020	5.3429	1.2529	1.000
IR→S	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
B1 Loop Score	**-0.0001**	**-0.0129**	**-0.0539**	**-0.7020**	**-5.3429**	**-1.2529**	**-1.000**
B1 Relative Loop Score	**-0.0099**	**-1.2719**	**-4.9913**	**-33.609**	**-55.9233**	**-96.2257**	**-100**
λ→IR	1.0003	1.0065	1.0273	1.3868	4.2110	0.0491	$1.695e^{-23}$
IR→I	1.000	1.000	1.000	1.000	1.000	1.000	1.000
I→λ	1.000	1.000	1.000	1.000	1.000	1.001	1.000

R1 Loop Score	**1.0003**	**1.0065**	**1.0273**	**1.3868**	**4.2110**	**0.0491**	**1.695e^{-23}**
R1 Relative Loop Score	**99.9900**	**98.7476**	**95.0280**	**66.3902**	**44.0766**	**3.7742**	**1.695e^{-21}**

Link scores can be multiplied together along a path and maintain the interpretation of the impact of the starting variable on the ultimate target. When that target is the variable itself, as it will be for a feedback loop, the score along that path, created by multiplying all of the link scores, is called the loop score [45]. Equation (3) calculates the loop score by multiplying all link scores in a loop from the first one, $u_1 \rightarrow v_1$ to the last, $u_n \rightarrow v_n$, where "n" is the number of links in the loop "L_S". Therefore, v_n is the same as u_1. An odd number of negative links gives a negative loop, and an even number gives a positive loop. Equation (4) relativises or normalises the loop scores to obtain percentages. To normalise the loop scores, divide the loop score by the sum of the loop scores of all loops in the cycle partition [46]. The sign of a relative loop score represents the polarity of the feedback loop. The sign for the R_1 loop is positive, and the B_1 loop is negative. By comparing relative loop scores, we determine which loops contribute the most to the behaviour [46].

Figures S2 and S3 show the LTM simulation for R_1 and B_1 loops during the runs from T_1 to T_{20}, and the model's dt (time step) is 1. The line plot shows that the relative loop scores change throughout the simulation. The R_1 relative loop score is positive, and the B_1 relative loop score is negative. The stacked area plot dimensions are divided from 0% to 100%, with 50% considered a point of the loop's equilibrium (balance). The R_1 loop in red colour covers around an average relative loop score of 66%, and the B_1 loop in blue colour covers around an average relative loop score of 33% during the selected runs from Time T_1 to T_{20}.

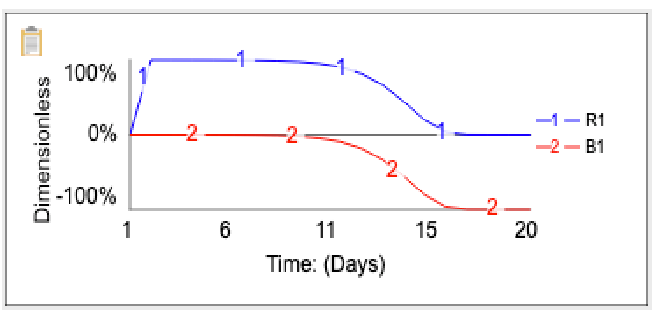

Figure S2. Relative scores of R_1 and B_1 loops in line plot**.**

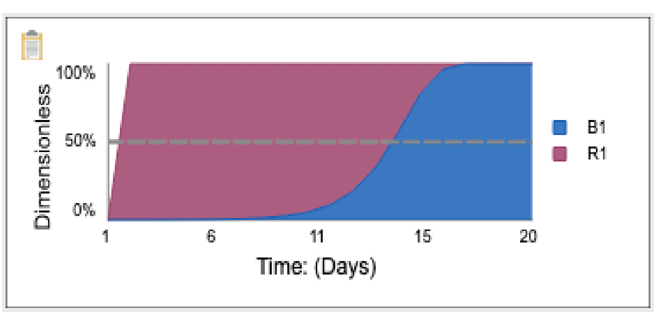

Figure S3. Relative scores of R_1 and B_1 loops in stacked-area plot.

The POI is easily detectable from the relative loop scores between the two loops. Figure S4 illustrates that the relative loop scores change throughout the simulation. At Time T_2, the R_1 relative loop score is nearly 99.9%, and the B_1 relative loop score is very low, i.e., about -0.01%. Then, the two loops shift in dominance, or inflection occurs between time T_{13} and T_{14}, as we calculated, shown in the above Table S4. At the end of the simulation, the R_1 relative loop score is less than 0.001%, and the B_1 relative loop score is close to -100%. The following plot illustrates that the selected run time, T_1 to T_{20}, completely covers the behaviour of both the stocks and the loops.

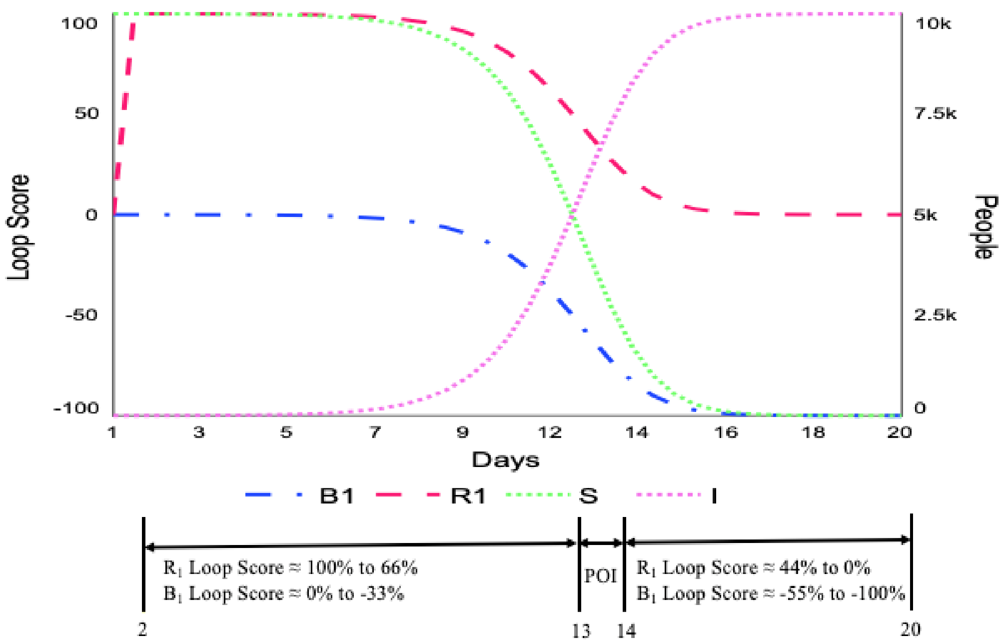

Figure S4. The R_1 and B_1 relative loop scores against S and I stocks.

The analysis of the relative loop scores at a particular point in time identifies the loops that dominate the model behaviour [46]. The above Figure S4 shows that the relative loop score magnitude for both R_1 and B_1 loops passes through the threshold for dominance at the point of inflection between T_{13} and T_{14}. The LTM analysis of the SI model identifies the specific links "S→IR" and "λ→IR" in B_1 and R_1 loops that explain the shifts in the feedback loop dominance between T_{13} and T_{14}. The "S→IR" and "λ→IR" link scores are the magnitude of R_1 and B_1 loops because IR is the only variable in R_1 and B_1 loops, which is dependent upon two dynamic variables: S and λ. The remaining "IR→S", "IR→I", and "I→λ" link scores are low (≤1) because the target variables of the three links only have a single dynamic variable. Initially, the R_1 loop score is very high, and then, between T_{13} and T_{14}, the B_1 loop score starts to increase. "A loop (or set of loops) is dominant if the loop(s) describe at least 50% of the observed change in behaviour across all stocks in the model over the selected period" [46]. Therefore, over the selected period, the R_1 loop is dominant because it covers more than 50% of the area during the selected run from T_1 and T_{20}.