Supplementary Material

This material documents the variables, run specifications, arrays, optimisation results, initial properties and equations used in the SEIR model described in the article "COVID-19 case rates in the UK: modelling uncertainties as lockdown lifts". The full model, written in 'Stella Architect' software, is available on request from the authors.

1. Variables

Table S1: Summary of variables used in SEIR Model of COVID-19 pandemic in the UK

Total	Count	Including Array Elements
Variables	73	102
Stocks	11	15
Flows	12	18
Converters	50	69
Constants	25	33
Equations	37	54
Graphicals	2	2
Macro Variables	10	

2. Run Specifications

Table S2: Run specifications for SEIR Model of COVID-19 pandemic in the UK

Run Specs	
Start Time	0
Stop Time	700
DT	1/4
Fractional DT	True
Save Interval	0.25
Sim Duration	1.5
Time Units	Days
Pause Interval	0
Integration Method	Euler

3. Arrays

Table S3: Arrays used in SEIR Model of COVID-19 pandemic in the UK

Array Dimension	Indexed by	Elements
awareness	Label (2)	Unknown, Known
dose	Label (2)	One, Two
vaccine	Label (2)	PB, AZ

4. Optimisation

The Powell method was used to minimise squared error for the variable 'smoothed known new cases' against actual reported smoothed known new cases from 1st Feb 2020 to 12th July 2021. All parameters were of the same order of magnitude so no scaling adjustments were necessary.

The first optimisation was performed with results up to Feb 2021 to estimate values of contact rates, which are different for known and unknown cases, the known proportion of cases in Feb 2021 and the ratio of infectiousness of known infections to unknown infections.

Table S4: Optimisation parameters for known proportion calculation

Parameter / Value range	unconstrained infecting contact rate[unknown]	unconstrained infecting contact rate[known]	unknown infectiousness ratio	final known proportion (Feb 2021)
min_value	0.4 contacts per day	0 contacts per day	0	0
max_value	0.7 contacts per day	0.3 contacts per day	1	1
scaling	1	1	1	1

Table S5: Optimisation results for known proportion calculation

Parameter / 95% CI bounds	unconstrained infecting contact rate[unknown]	unconstrained infecting contact rate[known]	unknown infectiousness ratio	final known proportion (Feb 2021)
Lower	0.5220	0.0983	0.7657	0.2112
Value	0.5228	0.0995	0.7666	0.2126
Upper	0.5235	0.1005	0.7675	0.2132

The known proportion from Optimisation 1 was assumed to have increased from 21% in Feb 2021 to 50% in July 2021 as a result of increased testing and this was reflected in the model through a graphical function which changed over time.

A second set of optimisation runs were performed in July 2021 in order to include the additional case history accumulated between Feb 2021 and July 2021. This optimisation calculated contact rates, which are different for known and unknown cases, the ratio of infectiousness of known infections to unknown infections and the relative infectiousness of the Alpha and Delta variants. Optimisations using the same ranges for the five parameters were run for the three different immunity scenarios of 8 months, 5 months and 12 months for both recovered and vaccinated immunity.

Table S6: Optimisation parameters for final model values calculation

Parameter / Value range	unconstrained infecting contact rate[unknown]	unconstrained infecting contact rate[known]	unknown infectiousness ratio	Alpha strain relative infectivity	Delta strain relative infectivity
Lower	0.4 contacts per day	0.02 contacts per day	0.5	1.3	1.5
Value	0.7 contacts per day	0.3 contacts per day	0.9	1.7	2.5
Upper	1	1	1	1	1

Table S7: Optimisation results for 8-month immunity parameter values calculation

Parameter / 95% CI bounds	unconstrained infecting contact rate[unknown]	unconstrained infecting contact rate[known]	unknown infectiousness ratio	Alpha strain relative infectivity	Delta strain relative infectivity
Lower	0.5611	0.1407	0.7221	1.3162	1.9960
Value	0.5616	0.1418	0.7229	1.3238	2.0023
Upper	0.5623	0.1426	0.7237	1.3311	2.0096

Table S8: Optimisation results for 5-month immunity parameter values calculation

Parameter / 95% CI bounds	unconstrained infecting contact rate[unknown]	unconstrained infecting contact rate[known]	unknown infectiousness ratio	Alpha strain relative infectivity	Delta strain relative infectivity
Lower	0.5541	0.1072	0.7657	1.3284	1.9208
Value	0.5544	0.1079	0.7663	1.3318	1.9332
Upper	0.5550	0.1086	0.7670	1.3370	1.9367

Table S9: Optimisation results for 12-month immunity parameter values calculation

Parameter / 95% CI bounds	unconstrained infecting contact rate[unknown]	unconstrained infecting contact rate[known]	unknown infectiousness ratio	Alpha strain relative infectivity	Delta strain relative infectivity
Lower	0.5250	0.1663	0.7468	1.3017	2.0480
Value	0.5253	0.1671	0.7474	1.3030	2.0569
Upper	0.5259	0.1677	0.7481	1.3170	2.0644

5. Equations and variable values used in model

Table S10: Variables with initial values in base case of SEIR Model of COVID-19 pandemic

Variable	Equation	Properties	Units	Annotation
cumulative_deaths(t)	cumulative_deaths(t - dt) + (deaths) * dt	INIT cumulative_deaths = 0	Person	NON-NEGATIVE
incubating(t)	incubating(t - dt) + (new_infecting + recovered_infecting - becoming_infectious) * dt	INIT incubating = 30	Person	NON-NEGATIVE
known_infected_stage_2(t)	known_infected_stage_2(t - dt) + (becoming_known - recovering - deaths) * dt	INIT known_infected_stage_2 = 2	Person	NON-NEGATIVE
new_susceptible_unadjusted(t)	new_susceptible_unadjusted(t - dt) + (- new_infecting) * dt	INIT new_susceptible_unadjusted = 67886004	Person	NON-NEGATIVE
recovered_known_immune(t)	recovered_known_immune(t - dt) + (recovering - known_becoming_susceptible) * dt	INIT recovered_known_immune = 0	Person	NON-NEGATIVE
recovered_susceptible(t)	recovered_susceptible(t - dt) + (known_becoming_susceptible + unknown_becoming_susceptible - recovered_infecting) * dt	INIT recovered_susceptible = 0	Person	NON-NEGATIVE
recovered_unknown_immune(t)	recovered_unknown_immune(t - dt) + (becoming_uninfectious - unknown_becoming_susceptible) * dt	INIT recovered_unknown_immune = 0	Person	NON-NEGATIVE
unknown_infected_stage_1(t)	unknown_infected_stage_1(t - dt) + (becoming_infectious - becoming_known - unknown_evolving_into_stage_2) * dt	INIT unknown_infected_stage_1 = 60	Person	NON-NEGATIVE
unknown_infected_stage_2(t)	unknown_infected_stage_2(t - dt) + (unknown_evolving_into_stage_2 - becoming_uninfectious) * dt	INIT unknown_infected_stage_2 = 120	Person	NON-NEGATIVE
vaccinations_by_type_and_dose[vaccine, dose](t)	vaccinations_by_type_and_dose[vaccine, dose](t - dt) + (vaccinating[vaccine, dose] - protection_loss[vaccine, dose]) * dt	INIT vaccinations_by_type_and_dose[vaccine, dose] = 0	Dose	NON-NEGATIVE
vaccine_stock[PB](t)	vaccine_stock[PB](t - dt) + (- vaccinating[PB, One] - vaccinating[PB, Two]) * dt	INIT vaccine_stock[PB] = 50000000	Dose	NON-NEGATIVE
vaccine_stock[AZ](t)	vaccine_stock[AZ](t - dt) + (- vaccinating[AZ, One] - vaccinating[AZ, Two]) * dt	INIT vaccine_stock[AZ] = 150000000		
becoming_infectious	incubating/incubation_duration		Person/Day	UNIFLOW

becoming_known	unknown_infected_stage_1*known_proportion/disease_duration_stage_1	OUTFLOW PRIORITY: 1	Person/Day	UNIFLOW
becoming_uninfectious	unknown_infected_stage_2/unknown_disease_duration_stage_2		Person/Day	UNIFLOW
deaths	MIN(case_fatality_rate, 1)*known_infected_stage_2/time_from_known_till_death	OUTFLOW PRIORITY: 2	Person/Day	UNIFLOW
known_becoming_susceptible	recovered_known_immune*(1-average_immunity_protection)/average_immunity_duration		Person/Day	UNIFLOW
new_infecting	fraction_new_susceptible*(((unknown_infected_stage_1+unknown_infected_stage_2)*controlled_infecting_contact_rate[unknown]) +(known_infected_stage_2*controlled_infecting_contact_rate[known]))		Person/Day	UNIFLOW
protection_loss[vaccine, dose]	vaccinations_by_type_and_dose/average_immunity_duration		Dose/Days	UNIFLOW
recovered_infecting	fraction_recovered_susceptible*(((unknown_infected_stage_1+unknown_infected_stage_2)*controlled_infecting_contact_rate[unknown]) +(known_infected_stage_2*controlled_infecting_contact_rate[known]))		Person/Day	UNIFLOW
recovering	(1-MIN(case_fatality_rate, 1))*known_infected_stage_2/known_disease_duration_stage_2	OUTFLOW PRIORITY: 1	Person/Day	UNIFLOW
unknown_becoming_susceptible	recovered_unknown_immune*(1-average_immunity_protection)/average_immunity_duration		Person/Day	UNIFLOW
unknown_evolving_into_stage_2	unknown_infected_stage_1*(1-known_proportion)/disease_duration_stage_1	OUTFLOW PRIORITY: 2	Person/Day	UNIFLOW
vaccinating[PB, One]	IF TIME>vaccination_start_date[PB, Two] THEN vaccination_daily_rate[PB]/2 ELSE IF TIME > vaccination_start_date[PB,One] THEN vaccination_daily_rate[PB] ELSE 0		Dose/Days	UNIFLOW
vaccinating[PB, Two]	IF TIME>vaccination_start_date[PB, Two] THEN vaccination_daily_rate[PB]/2 ELSE 0			

vaccinating[AZ, One]	IF TIME>vaccination_start_date[AZ, Two] THEN vaccination_daily_rate[AZ]/2 ELSE IF TIME > vaccination_start_date[AZ,One] THEN vaccination_daily_rate[AZ] ELSE 0		
vaccinating[AZ, Two]	IF TIME>vaccination_start_date[AZ, Two] THEN vaccination_daily_rate[AZ]/2 ELSE 0		
adjusted_infectivity	IF TIME > delta_strain_start_date THEN delta_strain_relative_infectivity ELSE IF TIME > alpha_strain_start_date THEN alpha_strain_relative_infectivity ELSE 1	Dmnl	
alpha_strain_relative_infectivity	1.32379831198934	Days	
alpha_strain_start_date	310	Dmnl	
average_immunity_duration	243	Dmnl	
average_immunity_protection	0.7	Person/Day	
case_fatality_rate	0	1/Day	
cases_to_invoke_intervention	5000		
controlled_infecting_contact_rate[unknown]	unknown_infectiousness_ratio *adjusted_infectivity *unconstrained_infecting_contact_rate[unknown]* ((100-lockdown%_modified)/100)	Person	
controlled_infecting_contact_rate[known]	unconstrained_infecting_contact_rate[known]*adjusted_infectivity* ((100-(lockdown%_modified/5))/100)	Day	
cumulative_cases	cumulative_deaths + known_infected_stage_2 + recovered_known_immune + recovered_susceptible	Dmnl	SUMMING CONVERTER
delta_strain_relative_infectivity	2	Dmnl	
delta_strain_start_date	438	Dmnl	

disease_duration_stage_1	2	Dmnl
effect_on_unknown_proportion_by_vaccine_type_and_dose[vaccine, dose]	vaccinations_by_type_and_dose[vaccine,dose]*known_to_unknown_shift_by_type_and_dose[vaccine,dose]/total_pop	Dmnl
fraction_new_susceptible	((new_susceptible_unadjusted)*(1-vaccine_immune_fraction))/total_pop	Dmnl
fraction_recovered_susceptible	(recovered_susceptible* (1-vaccine_immune_fraction))/total_pop	Days
highest_known_proportion	0.5	Person
highlights_model_sensitivities	NAN	Dmnl
highlights_values_varied_in_scenarios	NAN	Dmnl
incubation_duration	3.5	Day
indicated_immune_population_by_dose[vaccine, dose]	vaccinations_by_type_and_dose* vaccine_transmisison_protection_by_dose	Person/Day
Intervention_switch	0	Dmnl
invervention_%	25	Person/Dose
known_disease_duration_stage_2	8	
known_new_cases	DELAY(becoming_known, 4)	
known_proportion	highest_known_proportion*known_proportion_graphical_trend*(1-SUM(effect_on_unknown_proportion_by_vaccine_type_and_dose[*,*]))	
known_to_unknown_shift_by_type_and_dose[PB, One]	0	Days
known_to_unknown_shift_by_type_and_dose[PB, Two]	0	Dmnl
known_to_unknown_shift_by_ty	0	Dmnl

pe_and_dose[AZ, One]		
known_to_unknown_shift_by_type_and_dose[AZ, Two]	0	Person/Day
lockdown_delay	7	Dmnl
lockdown%_modified	IF Intervention_switch=0 OR TIME<=Time_actuals_end THEN lockdown_%_planned_and_historical ELSE IF HISTORY(smoothed_known_new_cases, TIME-lockdown_delay)<cases_to_invoke_intervention THEN lockdown_%_planned_and_historical ELSE MIN(83.6, lockdown_%_planned_and_historical+invervention_%)	Days
max_immune_fraction	0.7	Dmnl
new_recorded_deaths	DELAY(deaths, 3)	
Rt[unknown]	controlled_infecting_contact_rate[unknown]*total_fraction_susceptible* (disease_duration_stage_1 +unknown_disease_duration_stage_2)	Dmnl
Rt[known]	controlled_infecting_contact_rate[known]*total_fraction_susceptible* known_disease_duration_stage_2	Days
Rt_composite	Rt[known]*known_proportion+Rt[unknown]*(1-known_proportion)	Person/Day
second_dose_offset	84	Person/Day
smoothed_known_new_cases	SMTH1(known_new_cases, 7)	Days
smoothed_new_deaths	SMTH1(new_recorded_deaths, 7)	Days
Time_actuals_end	449	Dmnl
time_from_known_till_death	11.0171561262915	Person
total_fraction_susceptible	fraction_recovered_susceptible+fraction_new_susceptible	Person/Day

total_indicated_immune_population	SUM(indicated_immune_population_by_dose[*,*])	Person	
total_new_cases	known_new_cases+unknown_evolving_into_stage_2	Person	
total_pop	incubating + recovered_susceptible + recovered_unknown_immune + recovered_known_immune + unknown_infected_stage_1 + new_susceptible_unadjusted + known_infected_stage_2 + unknown_infected_stage_2	Dose	SUMMING CONVERTER
total_susceptible	new_susceptible_unadjusted + recovered_susceptible	1/Day	SUMMING CONVERTER
total_vaccinations	SUM(vaccinations_by_type_and_dose[*,*])		
unconstrained_infecting_contact_rate[unknown]	0.561566726696367	Day	
unconstrained_infecting_contact_rate[known]	0.141751735131141	Dmnl	
unknown_disease_duration_stage_2	5	Dose/Days	
unknown_infectiousness_ratio	0.722918698809817		
vaccination_daily_rate[PB]	130000	Days	
vaccination_daily_rate[AZ]	380000		
vaccination_start_date[PB, One]	324+21		
vaccination_start_date[PB, Two]	324+7+second_dose_offset		
vaccination_start_date[AZ, One]	339+21	Dmnl	
vaccination_start_date[AZ, Two]	339+7+second_dose_offset	Person/Dose	
vaccine_immune_fraction	MIN(max_immune_fraction, total_indicated_immune_population/total_pop)		
vaccine_transmisison_protection_by_dose[PB, One]	0.65		
vaccine_transmisison_protection_by_dose[PB, Two]	0.05		

vaccine_transmission_protection_by_dose[AZ, One]	0.65
vaccine_transmission_protection_by_dose[AZ, Two]	0.05