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Optimal Power Allocation for MIMO-MAC in Cognitive Radio Networks

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Abstract: This paper considers a cognitive radio (CR) network, in which the unlicensed (secondary) users (SUs) are allowed to concurrently access the spectrum allocated to the licensed (primary) users, provided that the interference of SUs with the primary users (PUs) satisfies certain constraints. It is more general and owns a stronger challenge to ensure the quality of service (QoS) of PUs, as well as to maximize the sum-rate of SUs. On the other hand, the multiple-antenna mobile user case has not been well investigated for the target problem in the open literature. We refer to this setting as multiple input multiple output multiple access channels (MIMO-MAC) in the CR network. Subject to the interference constraints of SUs and the peak power constraints of SUs, the sum-rate maximization problem is solved. To efficiently maximize the achievable sum-rate of SUs, a tight pair of upper and lower bounds, as an interval, of the optimal Lagrange multiplier is proposed. It can avoid ineffectiveness or inefficiency when the dual decomposition is used. Furthermore, a novel water-filling-like algorithm is proposed for the inner loop computation of the proposed problem. It is shown that this algorithm used in the inner loop computation can obtain the exact solution with a few finite computations, to avoid one more loop, which would be embedded in the inner loop. In addition, the proposed approach overcomes the limitation of Hermitian matrices, as optimization variables. This limitation to the optimization problem in several complex variables has not been well investigated so far. As a result, our analysis and results are solidly extended to the field of complex numbers, which are more compatible with practical communication systems.

1. Introduction

The radio spectrum is a precious resource that needs careful planning, as the currently licensed spectrum is severely underutilized [1]. Cognitive radio (CR) [2], which adapts the radio's operating characteristics to the real-time conditions, is the key technology that allows flexible, efficient and reliable spectrum utilization to be realized in wireless communications. This technology exploits the facts that the licensed spectrum is underutilized by the primary user(s) (PU), and it introduces the secondary user(s) (SU) to operating on the spectrum that is either opportunistically available or concurrently being shared by the PU and SU. The proposed paper focuses on the latter case.

Since the multiple-input multiple-output (MIMO) technology uses multiple antennas at both the transmitter and the receiver to significantly increase data throughput and link range without additional bandwidth or transmit power, it plays an important role in wireless communications today.

The works in [3–5] set up some algorithms to compute the maximum sum-rate problems for the Gaussian broadcast channel and the sum-power constrained Gaussian dual multiple-access channel. In addition, for computing the maximum weighted sum-rate for a class of the Gaussian SIMO (single-input-multiple-output) systems, [6] has also presented some algorithms to provide the max-stability policy. Those meaningful works are applied to the non-CR networks. For the SIMO-MAC in CR networks, as a special case of the MIMO-MAC in CR networks, the weighted sum-rate maximization problem has been investigated in [7] to compute the optimal power allocation. In the recent published papers [8,9], the MAC protocol identification and the joint subcarrier and bit allocation issues under CR networks were discussed, respectively. On spectrum sensing of CR, there are some meaningful works, such as [10] and the references cited therein.

In this paper, we integrate the CR networks with MIMO technology to fully ensure the quality of service (QoS) of PUs, as well as to maximize the sum-rate of SUs. Since multiple-antenna mobile users are quite common due to the efficiency of the mobile terminals, we simply term this setting as a multiple input multiple output multiple access channel (MIMO-MAC) in CR networks and confine the topic to the MIMO-MAC situation. We set up more practical mathematical models for the target problem. In addition, with the added constraints, conventional water-filling algorithms cannot be used due to a more complicated problem structure. By exploiting the structure of the sum-rate optimization problem, we propose a dual decomposition algorithm based on the water-filling principle to compute the optimal allocation distribution and the maximum sum-rate.

The proposed algorithm owns several improvements, shown below. First, to avoid ineffectively using the dual decomposition algorithm and to make the proposed algorithm more efficient, a tight pair of upper and lower bounds, as an interval for the optimal Lagrange multiplier, is proposed. Especially when the number of the users is sufficiently large, the benefits of utilizing this interval can be significant.

Second, we reduce the sum-rate problem into solving a decoupled system. Each block of the decoupled system can be solved by a highly efficient water-filling-like algorithm, due to the characteristics of both the objective function and the decoupled constraint system, with each of the blocks solved by the water-filling-like algorithm.

Moreover, the convergence of the proposed algorithm can be guaranteed through the rigorous mathematical proof presented in this paper. As a result, the proposed algorithm offers fast convergence. It is important to note that the convergence of the proposed algorithm is based on the theoretical advances from the fundamental results of the previously-mentioned acclaimed papers. In particular, the proposed approach overcomes the limitation of Hermitian matrices as independent variables. This limitation to the optimization problem in several complex variables has not been well investigated in the open literature. The existing optimization theory and methods, including the convex optimization theory and methods, deal with the problems with the optimization variables in the real n-dimensional Euclidean space (refer to [11–13]), but not in the complex n-dimensional unitary space. The target problem falls into the latter category. As a result, our analysis and results are solidly extended to the field of complex numbers, which are more compatible with practical communication systems.

Key notations that are used in this paper are as follows: $|A|$ and $\text{Tr}(A)$ give the determinant and the trace of a square matrix A, respectively; $A \succeq 0$ means that the matrix A is positive-semidefinite; $E[X]$ is the expectation of the random variable X; the capital symbol I for a matrix denotes the identity matrix with an appropriate size; in addition, for any complex matrix, its superscript † denotes the conjugate transpose of the matrix. Some notations or symbols are listed in Table 1.

Table 1. List of variables and abbreviations. SU, secondary user.

Variables & Abbreviations	Representations or Interpretations
k^*	water level step (highest step under water)
E	expectation on probability
K	total number of channels
P_t	upper bound for total power or sum-power
P_k	upper bound for the peak power constraint at step k
Tr	trace operation of a square matrix
x^i	signal transmitted by the i-th SU, which is a column vector
S_i	$E[x_i(x_i)^\dagger]$, which is a matrix
λ	Lagrange multiplier or dual variable

2. MIMO-MAC in a CR Network and Its Sum-Rate

For the MIMO-MAC in CR networks, assume that there is one base-station (BS) with N_r antennas, K SUs and N PUs, each of which is equipped with N_t antennas. In this section, assume that the MIMO-MAC under the CR network is described as:

$$y = \sum_{i=1}^{K} H_i x^i + \sum_{j=1}^{N} \hat{H}_j \hat{x}^j + Z, \text{ where}$$

$$H_i \in \mathbb{C}^{N_r \times N_t}, i = 1, 2, \cdots, K, \text{ and} \tag{1}$$

$$\hat{H}_j \in \mathbb{C}^{N_r \times N_t}, j = 1, 2, \cdots, N,$$

are the given fixed channel matrices of SUs and PUs, respectively. As general assumptions, the signal $x^i \in \mathbb{C}^{N_t \times 1}$ is a complex input signal vector from the i-th SU, and it is a Gaussian random vector having zero mean with independent entries; and the signal $\hat{x}^j \in \mathbb{C}^{N_t \times 1}$ is a complex input signal vector from the j-th PU, and it is a Gaussian random vector having zero mean with independent entries. Further, $Z \in \mathbb{C}^{N_r \times 1}$ is an additive Gaussian noise random vector, *i.e.*, $Z \sim \mathbb{N}(0, \sigma^2 I)$. Thus, $y \in \mathbb{C}^{N_r \times 1}$ is the received signal at the BS. Furthermore,

$$S_i \triangleq E\left[x^i \left(x^i\right)^{\dagger} \right], i = 1, 2, \cdots, K \tag{2}$$

The MIMO-MAC in the CR network is illustrated in Figure 1.

Figure 1. Cognitive radio (CR)-MIMO-MAC system model.
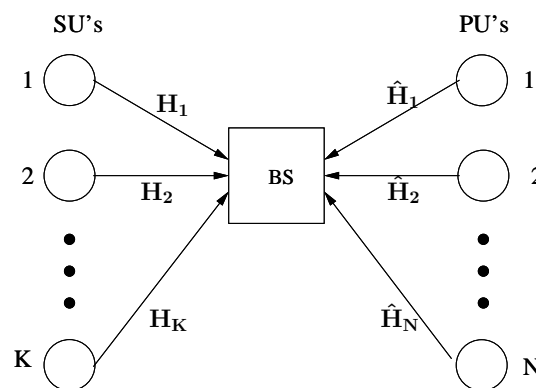
To clarify the role of the transmitted signals from the SUs to the BS (represented by the thick solid lines), with the transmitted signal from the PUs to the BS playing the role of the interference to the SUs (represented by the dashed lines), it is illustrated in Figure 2.

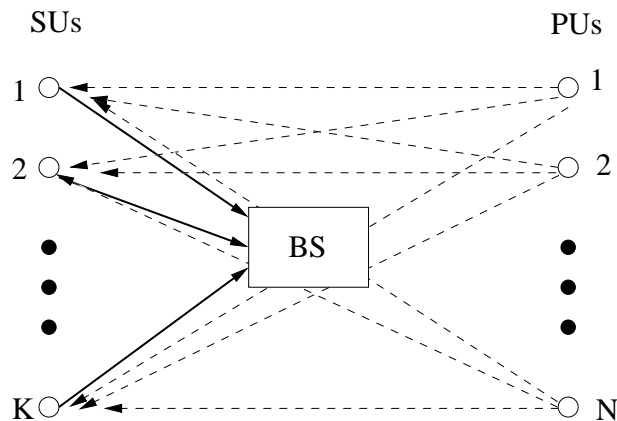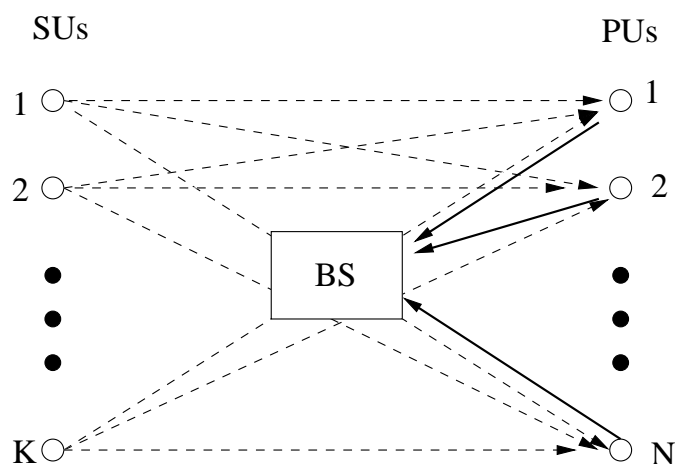**Figure 2.** SUs to BS and signals from primary users (PUs) as the interference.

Figure 3 illustrates the case when the roles are exchanged between the SUs and PUs.

Figure 3. PUs to BS and signals from SUs as the interference.

The mathematical model of the sum-rate optimization problem for the MIMO-MAC in the CR network can be written as follows:

$$f_{mac}\left(H_1, \cdots, H_K; P_1, \cdots, P_K; P_t\right) =$$

$$\max_{\{S_k\}_{k=1}^K : S_k \succeq 0, \mathrm{Tr}(S_k) \leq P_k, \ \sum_{k=1}^K g_k \mathrm{Tr}(S_k) \leq P_t} \log\left|I + \sum_{j=1}^K H_j S_j H_j^\dagger\right|,$$

(3)

where the upper bound of the peak power constraint on the i-th SU is denoted by $P_k(>0), k = 1, \ldots, K$. Assume $P_t > 0$ and $H_i \neq 0$ for any i. Further, without loss of generality, $\sum_{i=1}^N \hat{H}_i \hat{x}^i + Z$ is assumed to have the identity covariance matrix in the model (3).

The constraint:

$$\sum_{k=1}^K g_k \mathrm{Tr}(S_k) \leq P_t$$

(4)

of the sum-rate optimization problem (3) of the MIMO-MAC in the CR network is called the sum-power constraint with gains. The gains denoted by $\{g_k\}_{k=1}^K$ are defined as follows. Let:

$$y = \sum_{j=1}^N \hat{H}_j \hat{x}^j + \left(\sum_{i=1}^K H_i x^i + Z \right) \tag{5}$$

where $\sum_{j=1}^N \hat{H}_j \hat{x}^j + Z$ is the additive interference and noise of the transmitted signal $\sum_{i=1}^K H_i x^i$, which is transmitted to the BS. To guarantee the QoS for PUs, the power of the interference and noise is less than the transmitted power by PUs. That is to say, setting up a threshold P_t limits the transmitted power by the SUs and guarantees the QoS for PUs. The ratio of the threshold P_t to the power used by PUs can be determined (for example, refer to [14]).

Its mathematical expression can be expressed as:

$$\mathrm{Tr}[E(\sum_{i=1}^K H_i x^i + Z)(\sum_{i=1}^K H_i x^i + Z)^\dagger]$$

$$= \sum_{i=1}^K \mathrm{Tr}[H_i E(x^i (x^i)^\dagger) H_i^\dagger] + E(ZZ^\dagger) \tag{6}$$

$$\leq P_t$$

i.e.,

$$\sum_{i=1}^K \mathrm{Tr}(H_i^\dagger H_i S_i) + \mathrm{Tr}(\sigma^2 I) \leq P_t. \tag{7}$$

Let g_k be the maximum eigenvalue of $H_k^\dagger H_k, \forall k$. Then, we only need:

$$\sum_{k=1}^K g_k \mathrm{Tr}(S_k) \leq P_t - N_r \sigma^2 \tag{8}$$

such that Equation (6) holds. Thus, an innovation in our model is that we can construct the parameters $g_1, \cdots, g_K$ from the channel gain matrix and is referred to as the gains of the sum-power constraint. Let:

$$P_t \Leftarrow P_t - N_r \sigma^2 \tag{9}$$

where the symbol "$\Leftarrow$" means the assignment operation, and P_t is called the upper bound of the sum-power constraint with the gains.

Further, based on the same principle, a better transmission throughput model can also be obtained. Our approach reflects the essence of the target problem more practically.

In addition, assume that $M = \mathrm{rank}([H_1, \cdots, H_K])$ and $[H_1, \cdots, H_K] \in \mathbb{C}^{N_r \times K}$. This case corresponds to the SIMO-MAC case in the CR networks.

Applying the QR decomposition, $H = QR$. Let

$$Q = [q_1, \cdots, q_M] \in \mathbb{C}^{N_r \times M} \tag{10}$$

has orthogonal and normalized column vectors. $R \in \mathbb{C}^{M \times K}$ is an upper triangle matrix with $r_{m,k}$ denoting the (m, k)-th entry of the matrix R. $Q^\dagger$ is regarded as an equalizer to the received signal by the BS.

Thus, the i-th SU should have the rate:

$$\log\left(1 + \frac{|r_{i,i}|^2 S_i}{\sigma^2 + \sum_{n=1}^N \hat{S}_n q_i^\dagger \hat{R}_n q_i + \sum_{j=i+1}^K |r_{i,j}|^2 S_j}\right) \tag{11}$$

where $\hat{S}_n = E\left[\hat{x}^n(\hat{x}^n)^\dagger\right]$ and $\hat{R}_n = \hat{H}_n \hat{H}_n^\dagger, n = 1, \cdots, N$. For easy processing, the third term in the denominator above has been ignored in some earlier works.

Extending the separating hyperplane theorem [11] in convex optimization theory over the field of real numbers, or the direct product of the fields of real numbers to that over the field of complex numbers, or the direct product of the fields of complex numbers, we may obtain the following proposition.

Proposition 1. *The optimization problem (3) is equivalent to the following optimization problem:*

$$\min_{\lambda \geq 0}\{\max_{\{S_k\}_{k=1}^K} \quad \log|I + \sum_{j=1}^K H_j S_j H_j^\dagger| - \lambda(\sum_{k=1}^K g_k Tr(S_k) - P_t)\} \tag{12}$$

subject to $\qquad S_k \succeq 0, Tr(S_k) \leq P_k, \forall k$

i.e., the optimal objective values of the optimization problems (3) and (12) are equal. Furthermore, with the exception of the part of the dual variable, the restriction of any optimal solution of (12) (to the part of the original variable) is the same as the optimal solution of (3). Further, the optimal solution, λ^*, to (12) is the optimal dual variable of the sum power constraint with gains in (3).

3. Algorithm ALW1

To efficiently solve the optimization problem (12), a new method is proposed as follows.

Given $\lambda \geq 0$ and the optimization problem:

$$\max_{\{S_k\}_{k=1}^K} \quad \log|I + \sum_{j=1}^K H_j S_j H_j^\dagger| - \lambda(\sum_{k=1}^K g_k\text{Tr}(S_k) - P_t) \tag{13}$$

subject to $\quad S_k \succeq 0, \text{Tr}(S_k) \leq P_k, \forall k$

an efficient iterative algorithm is proposed here, and the optimal objective function value for the problem (13) is denoted by $g(\lambda)$. It is seen that $g(\lambda)$ is a convex function over $\lambda \geq 0$, and λ is a scalar. Thus, we may use the sub-gradient algorithm or a line search to obtain the optimal solution λ^* to the problem (12). Since the range to search is important, a pair of upper and lower bounds are proposed as follows.

Proposition 2. *For the optimization problem (12), the optimal solution:*

$$\lambda^* \in [0, 1] \tag{14}$$

Proof of Proposition 2. Since the optimization problems (3) and (12) are equivalent, for given λ^*, the latter is further transformed into the following problem:

$$\max_{\{x_i\}_{i=(k-1)N_t+1}^{kN_t}} \quad \sum_{i=(k-1)N_t+1}^{kN_t} \log(1 + \lambda_i x_i) - \lambda^*(g_k \sum_{i=(k-1)N_t+1}^{kN_t} x_i - P_t) \tag{15}$$

subject to $\qquad x_i \geq 0, \text{ as } (k-1)N_t + 1 \leq i \leq kN_t;$
$\qquad \sum_{i=(k-1)N_t+1}^{kN_t} x_i \leq P_k, \text{ as } k = 1, \ldots, K,$

to find the solution to (3), where the matrix diag $\left(\lambda_{(k-1)N_t+1}, \cdots, \lambda_{kN_t}\right)$, with $\{\lambda_i\}_{i=(k-1)N_t+1}^{kN_t}$ being decreasingly ordered, is equivalent to the matrix $H_k^\dagger(I + \sum_{j \neq k} H_j S_j H_j^\dagger)^{-1} H_k$, under the meaning of the unitary similarity transformation by a unitary matrix U. We have the KKTcondition:

$$\frac{\lambda_{k^*(k)}}{1 + \lambda_{k^*(k)}x_{k^*(k)}} = \lambda^* g_k + \nu_{k^*(k)}, \tag{16}$$

where the optimal dual variable $\nu_{k^*(k)} \geq 0$ and $k^*(k)$ is the highest step under the water during the steps from $(k-1)N_t + 1$ to kN_t. From the properties of positive semi-definite matrices and the definitions of $\lambda_{k^*(k)}$ and g_k, it is seen that $\lambda_{k^*(k)} \leq g_k$. Thus,

$$\frac{\lambda_{k^*(k)}}{1 + \lambda_{k^*(k)}x_{k^*(k)}} \geq \lambda^* g_k \tag{17}$$

implies $\lambda^* \leq 1$.

Therefore, Proposition 2 is proven.

The tightness of the interval means that there exists a set of channel gains, such that its optimal Lagrange multiplier λ^* touches either of the ends of the interval. For example, as $K = 1, g_1 = 1, P_t = 2, P_1 = 1$ and $h_1 = 1$, it is seen that $\lambda^* = 0$.

Letting $\lambda \in (0, 1]$, consider the evaluation of $g(\lambda)$. Note that the problem (13) has decoupled constraints. Therefore, the block coordinate ascend algorithm (BCAA) or the cyclic coordinate ascend algorithm (CCAA) (see [12,15]) can be used to solve the problem efficiently. The iterative algorithm works as follows. In each step, the objective function is maximized over a single matrix-valued variable S_k, while keeping all other S_ks fixed, $k = 1, \cdots, K$ and then repeating this process. Since the objective is nondecreasing with each iteration, the algorithm must converge to a fixed point. Using the fixed point theory, the fixed point is an optimal solution to the problem (13). Without loss of generality, let us consider an optimization problem with the optimization variable $S_k, k = 1$, with respect to all other S_ks being fixed, as follows:

$$\max_{\{S_1\}} \quad \log|I + \sum_{j=1}^{K} H_j S_j H_j^\dagger| - \lambda(\sum_{k=1}^{K} g_k \text{Tr}(S_k) - P_t)$$
$$\text{subject to} \quad S_1 \succeq 0, \text{Tr}(S_1) \leq P_1 \tag{18}$$

It is known that $g_i > 0$ due to $H_i \neq 0, \forall i$. Note that the single user case above with respect to the fixed other users is different from the existing ones for the MIMO-MAC cases, since it has a more complicated objective or feasible set. However, we can still exploit the water-filling-like approach to solve (18). The skeleton is stated as follows.

An optimal solution to (18) can be computed by exploiting the water-filling-like approach (refer to [16,17] and the references therein), since the problem (18) is equivalent to the following problem.

$$\max_{\{S_1\}} \quad \log|I + GS_1G^\dagger| - \lambda(g_1\text{Tr}(S_1) - P_t)$$
$$\text{subject to} \quad S_1 \succeq 0, \text{Tr}(S_1) \leq P_1 \tag{19}$$

where the mentioned $G = (I + \sum_{j \neq 1} H_j S_j H_j^\dagger)^{-\frac{1}{2}} H_1$. Similarly, the problem (19) can be equivalent to the problem (20) below,

$$\max_{\{x_i\}_{i=1}^{N_t}} \quad \sum_{i=1}^{N_t} \log(1 + \lambda_i x_i) - \lambda(g_1 \sum_{i=1}^{N_t} x_i - P_t)$$

(20)

$$\text{subject to} \quad x_i \geq 0, \forall i, \sum_{i=1}^{N_t} x_i \leq P_1$$

similarly, where the matrix diag $(\lambda_1, \cdots, \lambda_{N_t})$, with $\{\lambda_i\}$ being decreasing ordered, is equivalent to the matrix $G^\dagger G$, under the meaning of the unitary similarity transformation by a unitary matrix U. It is seen that we can use a similar method to the water-filling-like approach [18] to compute the optimal solution $\{x_i^*\}$ to the problem (20) and then obtain the optimal solution Udiag $(\lambda_1, \cdots, \lambda_{N_t}) U^\dagger$ to the problem (18).

How to compute the optimal solution $\{x_i^*\}$ to the problem (20) is concisely and uniquely proposed as follows: if $\lambda_i = 0$, then $x_i^* = 0$; if $0 < \lambda_i \leq \lambda g_1$, then $x_i^* = 0$. Further, if $\exists k_0$, such that $\lambda g_1 < \lambda_{k_0}$, $k^* \triangleq \max\{k | \lambda g_1 < \lambda_k, 1 \leq k \leq N_t\}$. Thus, under the condition mentioned above, if $\frac{k^*}{\lambda g_1} - \sum_{k=1}^{k^*} \frac{1}{\lambda_k} < P_1$, then $x_i^* = \frac{1}{\lambda g_1} - \frac{1}{\lambda_i}, \forall i : 1 \leq i \leq k^*$; and $x_i^* = 0, \forall i : k^* < i \leq N_t$. Else if $\frac{k^*}{\lambda g_1} - \sum_{k=1}^{k^*} \frac{1}{\lambda_k} \geq P_1$, then $x_i^* = \frac{P_1 + \sum_{k=1}^{k^*} \frac{1}{\lambda_k}}{k^*} - \frac{1}{\lambda_i}$, for $i = 1, \ldots, k^*$; and $x_i^* = 0$, for $i = k^* + 1, \ldots, N_t$.

For λ and the given $\left\{ S_1^{(n)}, \cdots, S_K^{(n)} \right\}$, the BCAA is used from the first matrix-valued variable to the K-th variable, and we obtain:

$$\left\{ S_1^{(n+1)} \cdots, S_K^{(n+1)} \right\}.$$

(21)

Thus, a mapping can be defined, which projects:

$$\left\{ S_1^{(n)}, \cdots, S_K^{(n)} \right\} \text{ to } \left\{ S_1^{(n+1)}, \cdots, S_K^{(n+1)} \right\}, \forall n$$

(22)

This mapping is denoted by f.

With the assumptions and the concepts introduced, a new iterative water-filling-like algorithm, ALW1, is concisely proposed as follows.

Algorithm ALW1:

(1) Given $\varepsilon > 0$, initialize:

$$\left\{ S_1^{(0)} = 0, \cdots, S_K^{(0)} = 0 \right\}, \lambda_{\min} \text{ and } \lambda_{\max}$$

(23)

(2) Set $\lambda = (\lambda_{\min} + \lambda_{\max})/2$.

(3) Compute:

$$\left\{ S_k^{(n+1)} \right\}_{k=1}^K = f \left(\left\{ S_k^{(n)} \right\}_{k=1}^K \right)$$

(24)

Then, $n <= n + 1$. Repeat Procedure (3) mentioned until the optimal solution $\{S_k^*\}_{k=1}^K$ to the problem (13) is reached.

(4) If $\sum_{k=1}^K g_k \text{Tr}(S_k^*) - P_t > 0$, then $\lambda_{\min}$ is assigned by λ;

if $\sum_{k=1}^K g_k \text{Tr}(S_k^*) - P_t < 0$, then $\lambda_{\max}$ is assigned by λ;

if $\sum_{k=1}^K g_k \text{Tr}(S_k^*) - P_t = 0$, stop.

(5) If $|\lambda_{\min} - \lambda_{\max}| \leq \varepsilon$, stop. Otherwise, go to Step (2)

Note that the initial $\lambda_{\min}$ is chosen as zero, and $\lambda_{\max}$ is chosen as one, respectively. If the initial values $\lambda_{\min} \geq 0$ and $\lambda_{\max} \geq 0$ are chosen as two points outside of the available range of the λ^*, there exists an example to account for the fact that algorithms via the dual decomposition principle cannot find any optimal solution. In Step (4) of ALW1, it is not difficult to see that $P_t - \sum_{k=1}^{K} g_k \text{Tr}(S_k^*)$ is a subgradient of the function $g(\lambda)$. This is because:

$$
\begin{aligned}
g(\lambda_1) = & \ \max_{\{S_k\}_{k=1}^K} \{\log |I + \sum_{j=1}^{K} H_j S_j H_j^\dagger| - \\
& \ \lambda_1 (\sum_{k=1}^{K} g_k \text{Tr}(S_k) - P_t) \} \\
\geq & \ \log |I + \sum_{j=1}^{K} H_j S_j^* H_j^\dagger| - \\
& \ \lambda_1 (\sum_{k=1}^{K} g_k \text{Tr}(S_k^*) - P_t) \\
\geq & \ g(\lambda) + (\lambda_1 - \lambda)(P_t - \sum_{k=1}^{K} g_k \text{Tr}(S_k^*))
\end{aligned}
\tag{25}
$$

where $\{S_j^*\}$ is the optimal solution to (3).

According to the definition of the subgradient, $P_t - \sum_{k=1}^{K} g_k \text{Tr}(S_k^*)$ is a subgradient of the function $g(\lambda)$, although $g(\lambda)$ cannot be guaranteed to be differential. Thus, we can follow the subgradient algorithm to search better λ in the Step (4) mentioned.

Example 1. $K = 2, P_1 = P_2 = P_t = 2$ and $H_1 = H_2 = 1$, the problem (13) is instanced as follows.

$$
\begin{aligned}
\max_{\{S_k\}_{k=1}^2} & \quad \log(1 + \sum_{j=1}^{2} S_j) - \lambda(\sum_{k=1}^{2} S_k - 2) \\
\text{subject to} & \quad S_k \geq 0, S_k \leq 2, k = 1, 2
\end{aligned}
\tag{26}
$$

Let the initials $\lambda_{\min} = 6$ *and* $\lambda_{\max} = 8$. *Dual decomposition algorithms cannot be used to find any optimal solution to this sum-rate maximization problem.*

Therefore, from Example 1, it is easily seen that without proper initial values of $\lambda_{\min}$ and $\lambda_{\max}$, it might result in the ineffectiveness and/or the inefficiency of a class of primal-dual algorithms with the subgradient approach.

4. Convergence of Algorithm ALW1

First, we will prove that for the fixed λ, the iterative water-filling can compute an optimal solution to the optimization problem (18).

Proposition 3. *Via the proposed water-filling-like algorithm with a fixed λ, an optimal solution can be found to the optimization problem (18).*

This proposition can be proven by the KKT condition [12] of the optimization problem (18) and a similar method in [18].

Second, convergence for the algorithm ALW1 is discussed as follows.

Theorem 1. *For the optimal power allocation problem (3), ALW1 is convergent, following Proposition 3.*

Proof. Due to construction of the tight interval or bounds for the optimal Lagrange multiplier λ^*, the convexity of both the optimization problems (12) and $g(\lambda)$ and characteristic of the sub-gradient method, the Lagrange multiplier obtained by the iterations of the outer loop computation can approximate to λ^* only when the inner loop can guarantee convergence. It is seen that the cyclic coordinate ascent algorithm [15] is used by the inner loop computation, and it is convergent if the solved problem is a convex optimization problem with a compact Cartesian direct product set as the feasible set and a continuously differential objective function for each of (15). The problem (3) can guarantee the conditions mentioned above. Therefore, ALW1 is convergent. □

5. Performance Results and Comparison

In this section, some numerical examples to illustrate the effectiveness of the proposed algorithm are presented. Since there is no existing method to compute the maximum sum-rate of the MIMO MAC in the CR networks, we made some development to the iterative water-filling-like algorithm in [6] to solve the target problem for comparison purposes. This algorithm is referred to as Algorithm AW in this paper.

Example 2. *The performance of Algorithm ALW1, compared with Algorithm AW is presented in Figures 4 to 9 for different numbers of users, K, where $N_r = 8$. Random data are generated for the channel gain vectors and the number of PUs. The numbers, denoted by K, of SUs are eight up to 158, respectively, and the number of PUs is 258. The sum-power constraints with the gains are $P_t = 8$, and the peak constraints are chosen at random.*

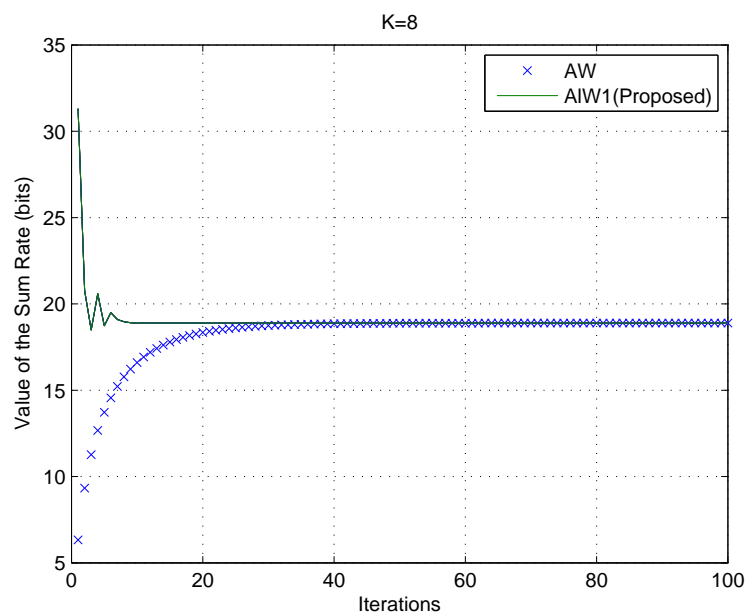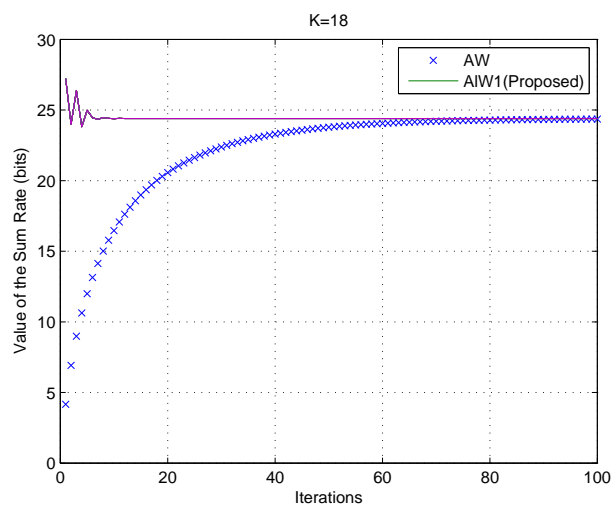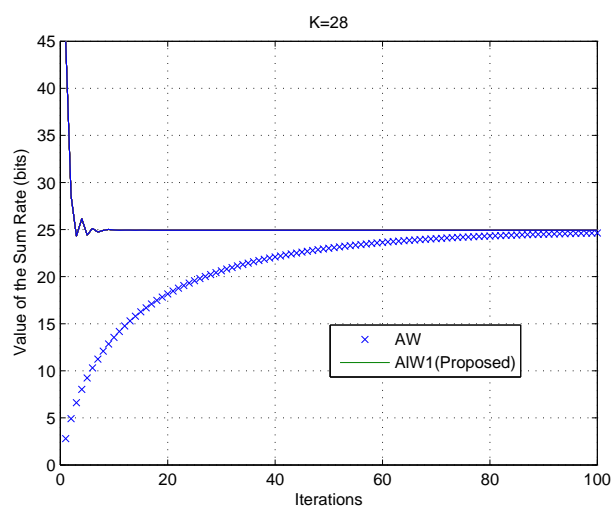**Figure 4.** Algorithm ALW1compared with Algorithm AW, as K = 8.
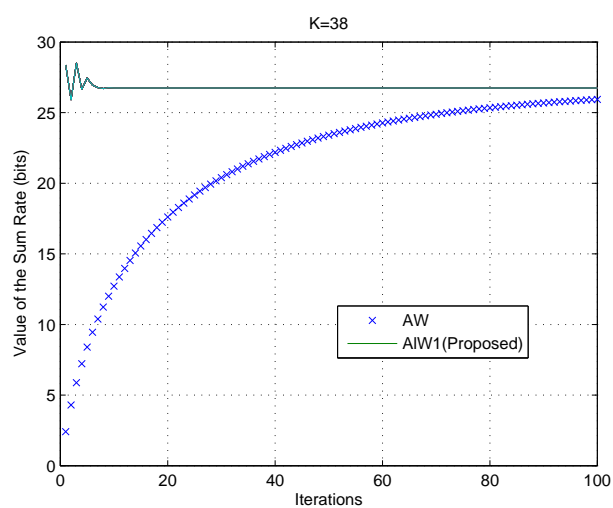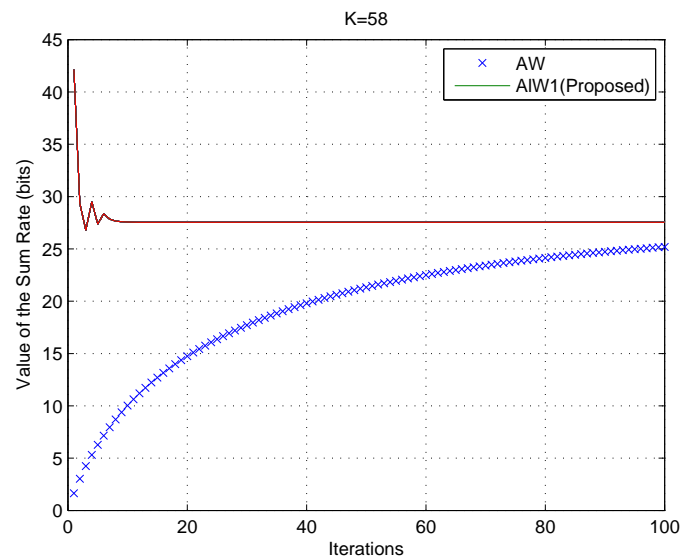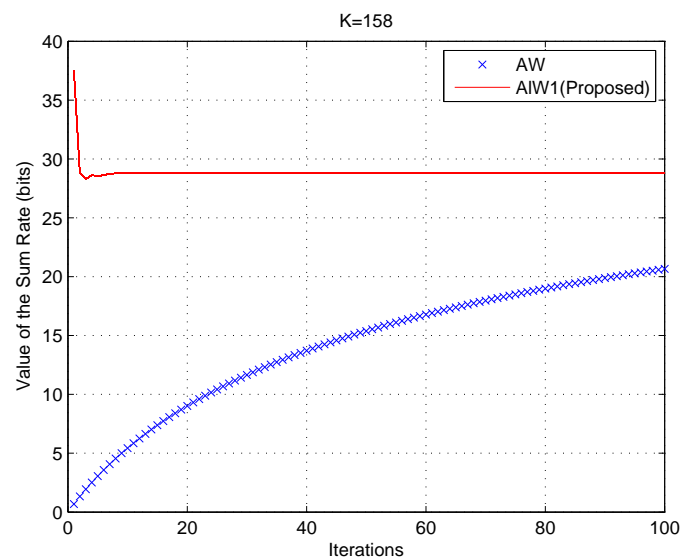
Figure 5. Algorithm ALW1 compared with Algorithm AW, as K = 18.

Figure 6. Algorithm ALW1 compared with Algorithm AW, as K = 28.

Figure 7. Algorithm ALW1 compared with Algorithm AW, as K = 38.

Figure 8. Algorithm ALW1 compared with Algorithm AW, as K = 58.

Figure 9. Algorithm ALW1 compared with Algorithm AW, as K =158.

For Figures 4 to 9, the solid curves and the cross markers represent the results of our proposed algorithm ALW1 and the algorithm AW, respectively. These results show that our proposed algorithm ALW1 exhibits faster convergence, even when the number of SUs is large. On the other hand, not only does ALW1 guarantee efficient convergence, but it also has a lower computational complexity. Each iteration of ALW1 scales linearly with K; the computation complexity [19] of the inner loop is at most $cK \times O(\log(1/\epsilon_1))$, where c denotes the number of the inner loop iterations and ϵ_1 denotes the error tolerance for computing S_k. The outer loop undergoes $O(\log(1/\epsilon_2))$ iterations to satisfy the error tolerance ϵ_2. Compared with the complexity $O(K^{3.5} \log(1/\epsilon_3))$ of the interior point algorithm, the complexity of ALW1 is significantly reduced.

6. Conclusions

In this paper, a network with MIMO-MAC and CR users is investigated. The proposed algorithm ALW1 solves the problem of the sum rate maximization and its optimal input distribution efficiently. It is an extension from the problem of the sum-rate for MIMO-MAC to that of the sum-rate for the MIMO-MAC in the CR network by an iterative water-filling-like algorithm via dual decomposition. The water-filling-like approach used by the proposed algorithm is different from the forms reported in the open literature. It provides a set of exact solutions with less computation. At the same time, we have obtained the strict convergence of the iterative water-filling-like algorithm. The proposed theory and numerical examples have shown the efficiency of the proposed algorithm. We determined the upper and lower bounds of the optimal Lagrange multiplier used at the outer loop to greatly improve the algorithm convergence rate, too.

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