

Gauss–Bonnet Inflation and the String Swampland

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Received: 22 August 2019; Accepted: 12 September 2019; Published: 15 September 2019



Abstract: The swampland criteria are generically in tension with single-field slow-roll inflation because the first swampland criterion requires small tensor-to-scalar ratio while the second swampland criterion requires either large tensor-to-scalar ratio or large scalar spectral tilt. The challenge to single-field slow-roll inflation imposed by the swampland criteria can be avoided by modifying the relationship between the tensor-to-scalar ratio and the slow-roll parameter. We show that the Gauss–Bonnet inflation with the coupling function inversely proportional to the potential overcomes the challenge by adding a constant factor in the relationship between the tensor-to-scalar ratio and the slow-roll parameter. For the Gauss–Bonnet inflation, while the swampland criteria are satisfied, the slow-roll conditions are also fulfilled, so the scalar spectral tilt and the tensor-to-scalar ratio are consistent with the observations. We use the potentials for chaotic inflation and the E-model as examples to show that the models pass all the constraints. The Gauss–Bonnet coupling seems a way out of the swampland issue for single-field inflationary models.

Keywords: inflation; Gauss–Bonnet inflation; swampland

1. Introduction

Inflation solves the flatness and horizon problems in standard cosmology [1–5], and is usually modeled by a single slow-roll scalar field which is obtained from low-energy effective field theories. In order to embed such scalar fields in a string quantum gravity theory successfully, they have to satisfy the following swampland criteria [6,7]:

- Swampland Criterion I (SCI) [8]: The scalar field excursion, normalized by the reduced Planck mass, in field space is bounded from above

$$|\Delta\phi| \leq d, \quad (1)$$

where the reduced Planck mass $M_{\text{Pl}} = 1/\sqrt{8\pi G} = 1$ and the order one constant $d \sim \mathcal{O}(1)$.

- Refined de Sitter Conjecture (SCII) [9,10]: The gradient of the field potential V with $V > 0$ should satisfy

$$\frac{|\nabla V|}{V} \geq c, \quad (2)$$

or

$$\frac{\min(\nabla_i \nabla_j V)}{V} \leq -\tilde{c}, \quad (3)$$

where the order one constants $c \sim \mathcal{O}(1)$ and $\tilde{c} \sim \mathcal{O}(1)$.

For single-field slow-roll inflation, the first two slow-roll parameters are $\epsilon_V = (V'/V)^2/2$ and $\eta_V = V''/V$, and the tensor-to-scalar ratio is $r = 16\epsilon_V$, where $V' = dV/d\phi$. In terms of the slow-roll

parameters, condition (2) becomes $\epsilon_V \geq c^2/2$ and condition (3) becomes $\eta_V \leq -\bar{c}$. Obviously, condition (2) violates the slow-roll condition and poses a threat to inflationary models by requiring a large tensor-to-scalar ratio $r \sim 8c^2$. For example, even if we chose $c = 0.1$ [11], it is still inconsistent with the observational constraint $r_{0.002} < 0.064$ [12,13] because $r \sim 0.08 > 0.064$. Condition (3) also poses a threat to single-field slow-roll inflation because it requires that $n_s = 1 + 2\eta_V - 3r/8 < 1 - 2\bar{c} \sim -\mathcal{O}(1)$, which is inconsistent with the observation. As pointed out in Reference [14], a viable way to solve this problem is by using models with the tensor-to-scalar ratio r reduced by a factor while keeping the lower bound on the field excursion $\Delta\phi$ as required by the Lyth bound [15], such as warm inflation [16]. See References [17–48] for more discussions on this issue.

In Reference [49], we find a powerful mechanism to reduce the tensor-to-scalar ratio r . With the help of the Gauss–Bonnet coupling, for any potential, the tensor-to-scalar r is reduced by a factor of $(1 - \lambda)^2$ with the order one parameter λ , so it may solve the swampland problem. Furthermore, the Gauss–Bonnet term is induced from the superstring theory, and it may solve the singularity problem of the Universe [50–54]. The speed of gravitational waves in Gauss–Bonnet inflation is usually different from the speed of light, $c_T \neq 1$. However, as discussed in [55], the effect of the Gauss–Bonnet term is negligible at low energy, and the Gauss–Bonnet model is compatible with the observational constraint $c_T \approx 1$ [56]. Furthermore, it was found that by choosing the coupling function appropriately we can keep $c_T = 1$ in Gauss–Bonnet inflation [57].

Generally, the predictions of the inflation, n_s and r , are calculated under the slow-roll conditions $\epsilon_V \ll 1$ and $|\eta_V| \leq 1$. If the SCII criterion (2) or (3) is satisfied, then the slow-roll condition will be violated, and the predictions n_s and r may be unreliable. However, in the case with Gauss–Bonnet coupling, the slow-roll condition is $(1 - \lambda)(V'/V)^2 \ll 1$, so even the second swampland criterion (2) is satisfied—as long as $1 - \lambda \ll 1$, the model still satisfies the slow-roll condition, so the slow-roll results are applicable. In this paper, we show that with the help of the Gauss–Bonnet coupling, some inflationary models satisfy not only the swampland criteria but also the observational constraints.

This paper is organized as follows. In Section 2, we give a brief introduction to the Gauss–Bonnet inflation, and point out the reason why it is easy to satisfy the swampland criteria. In Section 3, we use the power-law potential and the E-model to show that all the constraints can be satisfied. We conclude the paper in Section 4.

2. The Gauss–Bonnet Inflation

The action for Gauss–Bonnet inflation is [58–66]

$$S = \frac{1}{2} \int \sqrt{-g} d^4x \left[R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) - \zeta(\phi) R_{GB}^2 \right], \tag{4}$$

where $R_{GB}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss–Bonnet term which is a pure topological term in four dimensions, and $\zeta(\phi)$ is the Gauss–Bonnet coupling function. For a more detailed discussion on Gauss–Bonnet inflation, please refer to [67] and references therein. In this paper, we use [49,68]

$$\zeta(\phi) = \frac{3\lambda}{4V(\phi) + \Lambda_0}, \tag{5}$$

where $0 < \lambda < 1$. The parameter $\Lambda_0 \ll (10^{16}\text{Gev})^4$ added here is to avoid the reheating problem of Gauss–Bonnet inflation [68], and it can be ignored during inflation, so in this paper we neglect the effect of Λ_0 . In terms of the horizon flow slow-roll parameters [69], the slow-roll conditions are

$$\epsilon_1 = -\frac{\dot{H}}{H^2} \ll 1, \quad \epsilon_2 = \frac{\dot{\epsilon}_1}{H\epsilon_1} \ll 1, \tag{6}$$

the e -folding number N at the horizon exit before the end of the inflation is

$$N = \int_{\phi_e}^{\phi} \frac{1}{\sqrt{2\epsilon_1}} d\phi, \tag{7}$$

the scalar spectral tilt n_s and the tensor-to-scalar ratio r are [49]

$$n_s - 1 = -2\epsilon_1 - \epsilon_2, \tag{8}$$

$$r = 16(1 - \lambda)\epsilon_1. \tag{9}$$

In terms of the potential, the slow-roll parameters are expressed as [70]

$$\epsilon_1 = \frac{1 - \lambda}{2} \left(\frac{V'}{V} \right)^2, \tag{10}$$

$$\epsilon_2 = -2(1 - \lambda) \left[\frac{V''}{V} - \left(\frac{V'}{V} \right)^2 \right]. \tag{11}$$

Due to the factor $1 - \lambda$ in Equations (10) and (11), even if the gradient of the potential is consistent with \mathcal{SCII} , the slow-roll conditions (6) can still be satisfied as long as λ is close to 1, so the slow-roll results (8) and (9) are applicable, and the e -folding number N can reach $N = 60$ easily. Substituting Equation (10) into Equation (9), we get

$$r = 8(1 - \lambda)^2 \left(\frac{V'}{V} \right)^2. \tag{12}$$

From Equation (12), we see that while condition (2) is satisfied, the tensor-to-scalar ratio r can still be very small, as long as $1 - \lambda$ is small enough. In particular, if we combine the observational constraint $r_{0.002} < 0.064$ with the condition (2), we get

$$1 - \lambda < \frac{0.09}{c}. \tag{13}$$

Now we discuss the first swampland criterion \mathcal{SCI} for the field excursion. The Lyth bound tells us that [15,49]

$$\Delta\phi > \Delta N \sqrt{\frac{r}{8}} = (1 - \lambda) \Delta N \frac{|V'|}{V}. \tag{14}$$

Without the Gauss–Bonnet term, i.e., $\lambda = 0$, if \mathcal{SCII} is satisfied, then it is impossible to satisfy \mathcal{SCI} for single-field slow-roll inflation with $\Delta N \sim 60$. With the help the Gauss–Bonnet term, it is very easy to satisfy both \mathcal{SCI} and \mathcal{SCII} conditions, as long as $1 - \lambda$ is small enough. Combining Equations (1) and (2), we obtain the constraint

$$1 - \lambda < \frac{d}{c\Delta N}. \tag{15}$$

In summary, with the help of the Gauss–Bonnet term, as long as the order one parameter λ satisfies Equations (13) and (15), the two swampland criteria \mathcal{SCI} and \mathcal{SCII} are satisfied, and the tensor-to-scalar ratio r is also consistent with the observations [12]. From Equation (8), we see that the parameter λ have no effect on the scalar spectral tilt n_s , so the constraint on n_s can also be satisfied.

In the next section, we will use two inflationary models, the power-law potential and the E-model, as examples to support the above discussion.

3. The Models

In the following, we consider two inflationary models, the chaotic inflation and the E-model. We show that with the help of the Gauss–Bonnet term, the two swampland criteria \mathcal{SCI} and \mathcal{SCII} are satisfied for both models. Additionally, the models also satisfy the observational constraints [12],

$$n_s = 0.9649 \pm 0.0042, \quad r_{0.002} < 0.064. \tag{16}$$

3.1. The Power-Law Potential

For the chaotic inflation with the power-law potential [71]

$$V = V_0 \phi^p, \tag{17}$$

the excursion of the inflaton is

$$\Delta\phi = \sqrt{2(1-\lambda)p} \left(\sqrt{N+\tilde{n}} - \sqrt{\tilde{n}} \right), \tag{18}$$

where N is the remaining number of e -folds before the end of inflation, and

$$\tilde{n} = \begin{cases} p/4, & 0 < p < 2, \\ (p-1)/2, & p \geq 2. \end{cases} \tag{19}$$

The scalar spectral tilt n_s and the tensor-to-scalar ratio r are

$$n_s - 1 = -\frac{p+2}{2(N+\tilde{n})}, \tag{20}$$

$$r = \frac{4(1-\lambda)p}{N+\tilde{n}}. \tag{21}$$

From Equation (20), we see that n_s is independent on λ . The slow-roll parameter $\eta_V > 0$ if $p > 1$; and $\eta_V < 0$ if $p < 1$, so the condition (3) cannot be satisfied for the chaotic model with $p > 1$. If we choose $p = 2$ and $N = 60$, the scalar spectral tilt is $n_s = 0.9669$, which is consistent with the observations (16). Although condition (3) is violated, condition (2) can be satisfied. By varying the value of λ , the values of the gradient of the potential V'/V , inflaton excursion $\Delta\phi$, and tensor-to-scalar ratio r are shown in Figure 1.

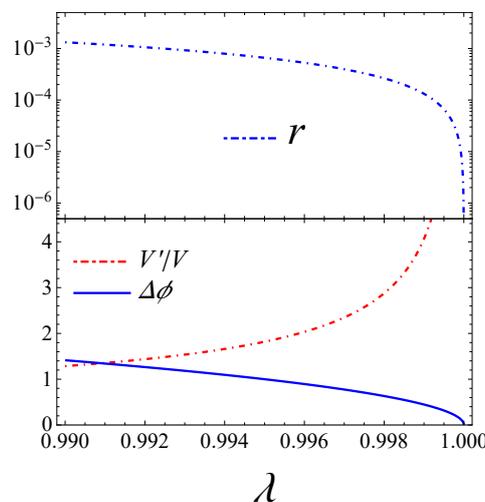


Figure 1. The dependence on λ for the power-law potential with $p = 2$. The upper panel shows the tensor-to-scalar r . The lower panel shows the gradient of the potential V'/V and the field excursion $\Delta\phi$.

As the value of $1 - \lambda$ becomes smaller and smaller, r and $\Delta\phi$ become smaller and smaller too, but V'/V will become larger and larger. If $1 - \lambda$ satisfies the conditions (13) and (15), the swampland criteria (1) and (2) as well as the observational constraints (16) are satisfied. For example, if we choose $1 - \lambda = 2 \times 10^{-3}$, we get

$$n_s = 0.9669, \quad r = 2.6 \times 10^{-4}, \tag{22}$$

$$\Delta\phi = 0.63, \quad \frac{V'}{V} = 2.9. \tag{23}$$

The predictions (22) are consistent with the observations (16), and the swampland criteria SCI (1) and $SCII$ (2) are satisfied. It is interesting to note that a large value of n_s is accommodated when neutrino properties are more consistently taken into account [72].

3.2. The E-Model

For the E-model [73,74]

$$V = V_0 \left[1 - \exp \left(-\sqrt{\frac{2}{3\alpha}} \phi \right) \right]^{2n}, \tag{24}$$

the excursion of the inflaton for $n = 1$ is

$$\Delta\phi = -\frac{1}{\sqrt{6\alpha}} [3\alpha(\tilde{N} + X - 1) + 4(1 - \lambda)N], \tag{25}$$

where

$$\tilde{N} = 1 + W_{-1} \left[-X \exp \left(-X - \frac{4(1 - \lambda)N}{3\alpha} \right) \right], \tag{26}$$

$X = 2\sqrt{1 - \lambda}/(\sqrt{3\alpha}) + 1$, and the function W_{-1} is the lower branch of the Lambert W function. The scalar spectral tilt n_s and the tensor-to-scalar ratio r are [75]

$$n_s = 1 + \frac{8(1 - \lambda)}{3\alpha\tilde{N}} - \frac{16(1 - \lambda)}{3\alpha\tilde{N}^2}, \tag{27}$$

$$r = \frac{64(1 - \lambda)^2}{3\alpha\tilde{N}^2}. \tag{28}$$

If we choose $\alpha = 1 - \lambda$ and $N = 60$, we get $n_s = 0.9678$, which is consistent with the observation. Varying λ , the values of V''/V , V'/V , $\Delta\phi$, and r are shown in Figure 2.

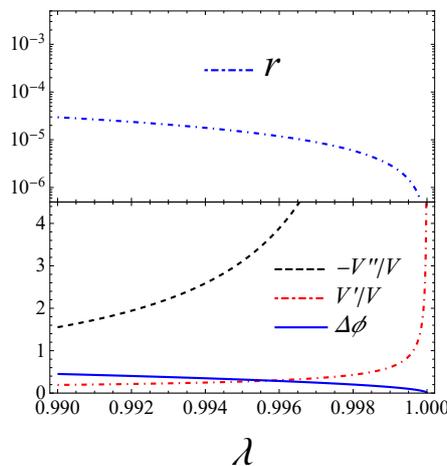


Figure 2. The dependence on λ for the the E-model with $\alpha = 1 - \lambda$. The upper panel shows the tensor-to-scalar r . The lower panel shows the gradients of the potential V''/V and V'/V , and the field excursion $\Delta\phi$.

Similar to the chaotic inflation, as the value of $1 - \lambda$ becomes smaller and smaller, r and $\Delta\phi$ become smaller and smaller and $-V''/V$ and V'/V become larger and larger. When $1 - \lambda$ satisfies the conditions (13) and (15), all the swampland criteria as well as the observational constraints (16) are satisfied. If we choose $1 - \lambda = 2 \times 10^{-3}$, we get

$$n_s = 0.9678, \quad r = 5.9 \times 10^{-6}, \quad (29)$$

$$\Delta\phi = 0.20, \quad \frac{V''}{V} = -7.8. \quad (30)$$

The predictions (29) are consistent with the observations (16), and the swampland criteria SCI (1) and $SCII$ (2) are satisfied. Further more, if we choose $1 - \lambda = 10^{-4}$, we get

$$n_s = 0.9678, \quad r = 3.0 \times 10^{-7}, \quad (31)$$

$$\Delta\phi = 0.045, \quad \frac{V'}{V} = 1.9, \quad \frac{V''}{V} = -155.3. \quad (32)$$

The field excursion $\Delta\phi$ and the gradients of the potential V'/V and V''/V all satisfy the swampland criteria (1), (2), and (3).

4. Conclusions

The two swampland criteria pose a threat on single-field slow-roll inflation. With the help of the Gauss–Bonnet coupling, the relationship between r and V'/V is described by Equation (12), i.e., r is reduced by a factor of $(1 - \lambda)^2$ compared with the result in standard single-field slow-roll inflation. Due to the reduction in r , the first swampland criterion is easily satisfied by requiring r to be small. On the other hand, it is easy to satisfy the second swampland criterion by requiring $1 - \lambda$ to be small and keeping r small.

For the chaotic inflation with $p = 2$, if we take $1 - \lambda = 5 \times 10^{-5}$, we get $n_s = 0.9669$, $r = 6.6 \times 10^{-6}$, $V'/V = 18.2$, and $\Delta\phi = 0.1$. Therefore, for the chaotic inflation with $p = 2$ and $\lambda > 0.99995$, the model satisfies not only the observational constraints, but also the swampland criteria (1) and (2). If $p < 1$, although condition (3) can be satisfied, the model is inconsistent with the observation at the 1σ confidence level. For the E-model, if we choose $1 - \lambda = 10^{-4}$, we get $n_s = 0.9678$, $r = 3.0 \times 10^{-7}$, $V'/V = 1.9$, $V''/V = -155.3$, and $\Delta\phi = 0.045$. The model satisfies not only the observational constraints, but also the swampland criteria (1), (2), and (3). In conclusion, the Gauss–Bonnet inflation with the condition (5) satisfies not only the observational constraints, but also the swampland criteria. Therefore, the Gauss–Bonnet coupling seems a way out of the swampland issue for single-field inflationary models.

Author Contributions: Conceptualization, Y.G.; investigation, Z.Y.; data curation, Z.Y.; writing—original draft preparation, Z.Y.; writing—review and editing, Y.G.; supervision, Y.G.; funding acquisition, Y.G.

Funding: This research was funded in part by the National Natural Science Foundation of China under Grant Nos. 11875136 and 11475065 and the Major Program of the National Natural Science Foundation of China under Grant No. 11690021.

Conflicts of Interest: The authors declare no conflict of interest.

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