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Deviation from Slow-Roll Regime in the EGB Inflationary Models with $r \sim N_e^{-1}$ †

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Abstract: We consider Einstein–Gauss–Bonnet (EGB) inflationary models using the effective potential approach. We present evolution equations in the slow-roll regime using the effective potential and the tensor-to-scalar ratio. The choice of the effective potential is related to an expression of the spectral index in terms of e-folding number N_e. The satisfaction of the slow-roll regime is mostly related to the form of the tensor-to-scalar ratio r. The case of $r \sim 1/N_e^2$ leads to a generalization of α-attractors inflationary parameters to Einstein–Gauss–Bonnet gravity with exponential effective potential. Moreover, the cosmological attractors include models with $r \sim 1/N_e$. And we check the satisfaction of the slow-roll regime during inflation for models with $r \sim 1/N_e$.

Keywords: Einstein–Gauss–Bonnet gravity; slow-roll regime; inflation

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1. Introduction

The observations data [1] allow to check different types of inflationary models due to the known values of the spectral index n_s, the amplitude A_s of scalar perturbations and the restriction to the tensor-to-scalar ratio r. The first model to historically satisfy current observation constrains [1] is the R^2 inflationary model [2–4] with:

$$n_s = 1 - \frac{2}{N_e + N_0}, \qquad r = \frac{12}{(N_e + N_0)^2} \tag{1}$$

in leading order of inverse e-folding number $1/(N_e + N_0)$, where N_0 is a constant, N_e is number of e-foldings. The generalizations of R^2 inflationary scenario [5,6] were introduced as cosmological attractors models [7–9] which lead to spectrum (1) and at the same time allow two different relations between the tensor-to-scalar ratio and e-folding number: $r \sim (N_e + N_0)^{-2}$ and $r \sim (N_e + N_0)^{-1}$. The case of $r \sim (N_e + N_0)^{-2}$ belongs to α-attractor models. The cosmological attractor models include inflationary scenarios inspired by particle physics [10–18]. Multi-fields inflationary scenarios [19–21] with scalar fields non-minimally coupled with Ricci scalar allow α-attractors approximation [22].

The Einstein–Gauss–Bonnet gravity is inspired by string theory framework as a quantum correction to general relativity [23–30]. The construction-appropriate inflationary scenarios in the Einstein–Gauss–Bonnet gravity is an actively studied problem [31–52]. The models with the Gauss–Bonnet term and the Ricci scalar multiplied by functions of the field can be considered such generalizations of the models with minimal coupling [39,40]. The appropriate inflationary scenarios can be obtained both numerically and analytically. The analytical studying of inflationary scenarios can be performed using the e-folding numbers N_e presentation[53]. The model of the Einstein–Gauss–Bonnet gravity leading to the α attractor inflationary parameters was reconstructed in [54] and studied in [55] using the presentation of inflationary scenarios in terms of the e-folding number N_e. In [34], the models inspired by chaotic inflation [56,57] with monomial potential $V \sim \phi^n$ and an inverse function before the Gauss–Bonnet term $\xi \sim \phi^{-n}$ were studied in a slow-roll regime. However, the case of $n = 2$ leads to deviation from the slow-roll regime before the end of inflation, as in the case $n = 4$, the value of the spectral index becomes sufficiently small to satisfy recently observed data [1]. Models constructed in [34] lead to the tensor-to-scalar ratio of the form $r \sim 1/N_e$. In [31], the correction to function $\xi(\phi)$ was introduced to obtain appropriate inflationary scenarios. However, a more complicate form of the tensor-to-scalar ratio r was obtained. In the present paper, models without the introduction of $V(\phi)$ and $\xi(\phi)$ leading to inflationary parameters of cosmological attractors with $r \sim (N_e + N_0)^{-1}$ are considered. In our consideration, the effective potential formulated for Einstein–Gauss–Bonnet gravity [58] applicable near de Sitter solution [59] or in slow-roll regime [31,55] is used.

The paper is organized as follows. In Section 2, the action and the evolution equations in the slow-roll regime are briefly introduced and presented in terms of e-folding numbers using the effective potential formulation. In Section 3, the slow-roll parameters are presented using the effective potential, the tensor-to-scalar ratio and nonminimal coupling function. The consideration includes the case of minimal coupling of field with Ricci scalar. The expressions of the effective potential leading to an appropriate spectral index are considered. Subsequently, the model with an exponential effective potential with $r \sim (8r_0)/(N_e + N_0)$ is studied and the breaking of the slow-roll regime due to the small value of r_0 is clearly demonstrated. In Section 4, the results of our consideration formulate the conclusion.

2. Slow-Roll Regime in EGB Gravity

In this paper, the gravity model with a scalar field is considered, nonminimally coupled with both the Ricci curvature scalar and the Gauss–Bonnet term, as described by the following action:

$$S = \int d^4x \frac{\sqrt{-g}}{2} \left[F(\phi)R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V(\phi) - \xi(\phi)\mathcal{G} \right], \tag{2}$$

where the functions $F(\phi)$, $V(\phi)$, and $\xi(\phi)$ are differentiable ones, R is the Ricci scalar and:

$$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

is the Gauss–Bonnet term. We assume that $F(\phi) > 0$ and $V(\phi) > 0$ during inflation.

The system of evolution equations in the spatially flat Friedmann–Lemaître–Robertson–Walker metric with $ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$ was obtained and considered in slow-roll approximation in [34,39]:

$$\dot{\phi}^2 \ll V,\ |\ddot{\phi}| \ll 3H|\dot{\phi}|,\ 4|\dot{\xi}|H \ll F,\ |\ddot{\xi}| \ll |\dot{\xi}|H,\ |\ddot{F}| \ll H|\dot{F}| \ll H^2F,$$

where $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor, dots denote the derivatives with respect to the cosmic time t. In [54,55], the slow-roll approximation of evolution equations in terms of e-folding number N_e was formulated using $dA/dt = -HdA/dN_e$. The obtained equations can be presented as follows:

$$H^2 \simeq \frac{W}{3}, \quad (\phi')^2 \simeq 4WV'_{eff}. \tag{3}$$

where a prime denotes the derivative with respect to N_e, $W \equiv V/F$ and the effective potential [58]:

$$V_{eff} = -\frac{F^2}{4V} + \frac{\xi}{3}. \tag{4}$$

In our consideration, we use $N_e = -\ln(a/a_e)$ following the choice of notations in Ref. [9,53–55]. Note that in many papers [21,31,34,39,50], $N = -N_e$ is used as a new independent variable for evolution equations. The slow-roll parameters as functions of N_e can be presented such as

$$\epsilon_1 = \frac{1}{2}\ln'(W), \quad \zeta_1 = -\ln'(F), \quad \delta_1 = -\frac{4W}{3F}\xi', \tag{5}$$

$$\epsilon_{i+1} = -\ln'(\epsilon_i), \qquad \zeta_{i+1} = -\ln'(\zeta_i), \qquad \delta_{i+1} = -\ln'(\delta_i). \tag{6}$$

In our consideration, we use expressions for the tensor-to-scalar ratio r and the spectral index of scalar perturbations n_s obtained in [55]:

$$r = \frac{32WV'_{eff}}{F}, \tag{7}$$

$$n_s = 1 + \frac{V''_{eff}}{V'_{eff}}. \tag{8}$$

The model of EGB gravity leading to cosmological attractor inflationary parameters with $r \sim (N_e + N_0)^{-2}$ was assumed in [54]. The model was later studied in detail and generalized to EGB gravity models with the field nonminimally coupled with Ricci scalar. In the next section, we consider EGB gravity models leading to the n_s coinciding with n_s of cosmological attractors with $r \sim (N_e + N_0)^{-1}$.

3. Application

We try to reproduce the cosmological attractor inflationary parameters with $r \sim 1/(N_e + N_0)$ in EGB gravity. Accordingly, with (8) and the expression of the spectral index with the second order correction [55], we can write:

$$\frac{V''_{eff}}{V'_{eff}} = -\frac{2}{N_e + N_0} + \frac{C_2}{(N_e + N_0)^2} \tag{9}$$

The satisfaction of the equation is possible by two ways:

1. If $C_2 \neq 0$ we can suppose $V_{eff} = C_{eff}\exp\left(-\frac{C_2}{N_e+N_0}\right)$;
2. And if $C_2 = 0$, we can suppose $V_{eff} = \frac{C_{eff}}{N_e+N_0}$.

In [55], the exponential presentation of effective potential with second order tensor-to-scalar ratio $r \sim 1/(N_e + N_0)^2$ was considered. After the choice of tensor-to-scalar ratio, the form of the potential will be related to the choice of nonminimal coupling (7) by

$$W = \left(\frac{F\,r}{32V'_{eff}}\right) \tag{10}$$

At the same time, the function ξ can be presented through the effective potential and the tensor-to-scalar ratio:

$$\xi = 3\,V_{eff} + \frac{24\,V'_{eff}}{r} \tag{11}$$

The dependence of slow-roll parameters ϵ_1, ζ_1, δ_1 from $N_e + N_0$ related with the effective potential, the tensor-to-scalar ratio and the nonminimal coupling function:

$$\epsilon_1 = \frac{1}{2}\left(\frac{r'}{r} - \frac{V''_{eff}}{V'_{eff}} + \frac{F'}{F}\right) \tag{12}$$

$$\delta_1 = \left(\frac{r'}{r} - \frac{r}{8} - \frac{V''_{eff}}{V'_{eff}}\right)F = \left(2\epsilon_1 + \zeta_1 - \frac{r}{8}\right)F \tag{13}$$

$$\zeta_1 = -\frac{F'}{F} \tag{14}$$

We suppose that at the end of inflation, the nonminimal coupling function tends to 1. To simplify the analysis of only dealing with Gauss–Bonnet coupling, we consider the case of constant coupling $F = 1$. In this case, the expressions for the slow-roll parameters ϵ_1 and δ_1 can be simplified as follows:

$$\epsilon_1 = \frac{1}{2}\left(\frac{r'}{r} - \frac{V''_{eff}}{V'_{eff}}\right), \quad \delta_1 = \left(\frac{r'}{r} - \frac{r}{8} - \frac{V''_{eff}}{V'_{eff}}\right) = 2\epsilon_1 - \frac{r}{8}, \tag{15}$$

the slow-roll parameters ζ_i are absent. We assume that in the case of

$$r = \frac{8r_0}{(N_e + N_0)} \tag{16}$$

the upper values of parameter r_0 are rather small to save the slow-roll regime during inflation. In the next section, our supposition will be estimated.

The first step of the discrimination of models is checking the values first order slow-roll parameters during inflation. The second step of the discrimination is the consideration of the second order slow-roll parameters. During the analysis, one should remember the appropriate values of the tensor-to-scalar ratio. As such, we have two variants of effective potentials leading to the same spectral index in leading order of inverse e-folding number, and thus consider the corresponding effective potentials in two different subsections.

3.1. Power-Law Effective Potential

In this subsection, the power-law variant of effective potential is considered:

$$V_{eff} = \frac{C_{eff}}{(N_e + N_0)} \tag{17}$$

which leads to the exact reproduction of the spectral index without second order correction. At the same time, the supposition $r \sim (N_e + N_0)^{-2}$ leads to constant potential. The first slow-roll, $\epsilon_1 = 0$, has no end.

The supposition (16) leads to the following model:

$$V = -\frac{r_0\,(N_e + N_0)}{4C_{eff}}, \quad \xi = \frac{3C_{eff}\,(r_0 - 1)}{r_0\,(N_e + N_0)}, \quad \xi' = -\frac{3C_{eff}\,(r_0 - 1)}{r_0\,(N_e + N_0)^2} \tag{18}$$

and slow-roll parameters:

$$\epsilon_1 = \frac{1}{2\,(N_e + N_0)}, \quad \epsilon_2 = \frac{1}{(N_e + N_0)} \tag{19}$$

$$\delta_1 = -\frac{r_0 - 1}{(N_e + N_0)}, \quad \delta_2 = \epsilon_2 \tag{20}$$

We suppose that the exit from inflation is defined at $N_0 = 1/2$ at which $\epsilon_1(N_e = 0) = 1$. At the $N_0 = 1/2$, the second slow-roll parameters ϵ_2 and δ_2 reach 2 , thus the slow roll regime is infracted during inflation and the slow-roll approach is not applicable to the considered model.

3.2. Exponential Effective Potential

The case of an exponential potential and $r \sim (N_e + N_0)^{-2}$ was considered in [54,59]. Now, we consider the case of the exponential potential and $r \sim (N + N_0)^{-1}$:

$$V_{eff} = C_{eff} \exp\left(-\frac{C_2}{N_e + N_0}\right). \tag{21}$$

The choice of effective potential (21) and tensor-to-scalar ratio (16) leads to the following model in terms of e-folding number:

$$V = \frac{r_0\,(N_e+N_0)}{4C_{eff}C_2\left(\exp\left(-\frac{C_2}{N_e}\right)\right)}, \quad \xi = \frac{3\,C_{eff}\exp\left(-\frac{C_2}{N_e+N_0}\right)(r_0\,(N_e+N_0)+C_2)}{r_0(N_e+N_0)} \tag{22}$$

Which leads to the following slow-roll parameters:

$$\epsilon_1 = \frac{1}{2(N_e+N_0)} - \frac{C_2}{2(N_e+N_0)^2}, \quad \epsilon_2 = \frac{-(N_e+N_0)+2\,C_2}{(N_e+N_0)(-(N_e+N_0)+C_2)} \tag{23}$$

$$\delta_1 = -\frac{r_0-1}{N_e+N_0} - \frac{C_2}{(N_e+N_0)^2}, \quad \delta_2 = \frac{(r_0-1)(N_e+N_0)+2\,C_2}{(N_e+N_0)((r_0-1)(N_e+N_0)+C_2)} \tag{24}$$

Solving equation $\epsilon_1(N_e=0)=1$, we obtain constant C_2:

$$C_2 = -(2\,N_0-1)N_0 \tag{25}$$

The substitution of (25) to expressions for slow-roll parameters leads to:

$$\epsilon_2 = \frac{4\,{N_0}^2+N_e-N_0}{(N_e+N_0)\left(2\,{N_0}^2+N_e\right)}, \tag{26}$$

$$\delta_1 = -\frac{r_0\,(N_e+N_0)-2\,{N_0}^2-N_e}{(N_e+N_0)^2}, \tag{27}$$

$$\delta_2 = \frac{(r_0-1)(N_e+N_0)-4\,{N_0}^2+2\,N_0}{(N_e+N_0)\left((r_0-1)(N_e+N_0)-2\,{N_0}^2+N_0\right)} \tag{28}$$

At the end of inflation, this slow-roll parameters can be reduced to:

$$\epsilon_2 = \frac{4\,N_0-1}{2{N_0}^2} \tag{29}$$

$$\delta_1 = 2-\frac{r_0}{N_0} \tag{30}$$

$$\delta_2 = \frac{-r_0-1+4\,N_0}{N_0\,(-r_0+2\,N_0)} \tag{31}$$

Evidently, to save the slow-roll regime we should have $-1<\delta_1<1$, considering the biggest r_0 and smallest N_0. The slow-roll parameter ϵ_2 reaches the value 1 at $N_e=0$ if $N_0 = 1+\frac{\sqrt{2}}{2}$ and after that, the slow-roll parameter ϵ_2 grows and becomes bigger then 1. Thus, we suppose that $N_0 = 1+\frac{\sqrt{2}}{2}$ is the smallest appropriate value for the sum N_e+N_0 during slow-roll regime.

The value of constant r_0 included slow-roll parameters related to the restriction of the tensor-to-scalar ratio:

$$r = \frac{8r_0}{N_b+N_0} = 0.065\cdot k, \quad \text{where} \quad 0<k<1, \quad N_b \quad \text{is a start point of inflation} \tag{32}$$

and parameter r_0 can be expressed as

$$r_0 = \frac{0.065\cdot k\cdot(N_b+N_0)}{8} = 0.008125\,k\,(N_b+N_0). \tag{33}$$

The start point of inflation N_b is related to the appropriate value of the spectral index:

$$n_s = 1 - \frac{2}{N_b + N_0} - \frac{(2N_0 - 1)N_0}{(N_b + N_0)^2}. \tag{34}$$

From here, we obtain an equation for $(N_b + N_0)$:

$$\frac{-2\,{N_0}^2 + N_0 - 2\,(N_b + N_0)}{(N_b + N_0)^2} = n_s - 1 \tag{35}$$

having two solutions:

$$N_b = \frac{-1 + \sqrt{-2\,{N_0}^2 n_s + 2\,{N_0}^2 + N_0\,n_s - N_0 + 1}}{n_s - 1} - N_0 \tag{36}$$

$$N_b = -\frac{1 + \sqrt{-2\,{N_0}^2 n_s + 2\,{N_0}^2 + N_0\,n_s - N_0 + 1}}{n_s - 1} - N_0 \tag{37}$$

but only the second solution allows to reproduce $55 < N_b < 65$. To obtain r_0, we substitute (37) into (33). After that we substitute the obtained r_0 into (30):

$$\delta_1 = 2 + \Delta\delta_2, \quad \Delta\delta_2 = \frac{0.008125\,k\left(1 + \sqrt{-2\,{N_0}^2 n_s + 2\,{N_0}^2 + N_0\,n_s - N_0 + 1}\right)}{(n_s - 1)N_0}. \tag{38}$$

To save slow-roll regime, the minimal condition $-3 < \Delta\delta_2 < -1$ should be checked. Since $0 < k < 1$, we would like to analyze $\Delta\delta_2/k$ using the following inequality:

$$\frac{\Delta\delta_2}{k} = -\frac{0.008125\left(1 + \sqrt{2\,{N_0}^2(1 - n_s) - N_0(1 - n_s) + 1}\right)}{(1 - n_s)N_0} < -1. \tag{39}$$

From here, we can suppose:

$$\frac{0.008125\left(1 + \sqrt{2\,{N_0}^2(1 - n_s) - N_0(1 - n_s) + 1}\right)}{(n_s - 1)N_0} = -l, \quad \text{where} \quad l > 1. \tag{40}$$

The solution of (40) can be presented in the form:

$$n_s = 1 - \frac{0.00013203125}{l^2} - \frac{0.01625}{lN_0} + \frac{0.000066015625}{l^2 N_0}. \tag{41}$$

which can be approximated:

$$n_s \approx 1 - \frac{0.01625}{lN_0} \tag{42}$$

due to $l > 1$, $N_0 > 1$ and orders of numerical values of numbers included to (41). From here, it is evident that the increase in l and N_0 leads to the increase in n_s. We substitute the minimal values of parameters $l = 1$, $N_0 = 1 + \frac{\sqrt{2}}{2}$ to expression (41) and obtain $n_s \approx 0.99039$ as the minimal values of model spectral index. The need value for the spectral index can be only be reached at $l < 1$, leading to the breaking of the slow-roll regime.

At the same time, the appropriate values of the spectral index lead to the deviation of a slow-roll regime via the parameter δ_1. Let us present this fact explicitly. The slow-roll parameter δ_1 includes the constant r_0 which is related with the value of $N_b + N_0$. The second solution (37) to equation (34) can be presented in the form:

$$(N_b + N_0) = -\frac{1 + \sqrt{\left(N_0 - 2\,{N_0}^2\right)(n_s - 1) + 1}}{n_s - 1} \tag{43}$$

The substitution $N_0 = 1 + \frac{\sqrt{2}}{2}$ to this equation leads to:

$$(N_b + N_0) = -\frac{2 + \sqrt{-(6\sqrt{2} + 8)\,(n_s - 1) + 4}}{2(n_s - 1)} \tag{44}$$

Using (44), the parameter r_0 is obtained, included in tensor-to-scalar ratio:

$$r_0 = -\frac{0.0040625\,k\left(2 + \sqrt{-(6\,\sqrt{2} + 8)\,(n_s - 1) + 4}\right)}{(n_s - 1)} \tag{45}$$

and it is substituted for slow-roll parameter δ_1:

$$\delta_1 = 2 + \frac{0.0040625\,k\left(2 + \sqrt{-(6\,\sqrt{2} + 8)\,(n_s - 1) + 4}\right)}{(n_s - 1)\left(1 + 1/\sqrt{2}\right)} \tag{46}$$

The $(n_s - 1)$ is less than zero and to obtain the smallest value of δ_1, we assume $k = 1$. The consideration of (46) in the case of $k = 1$ leads to $\delta_1 > 1$ at appropriate values of the spectral index. Let us present minimal values of δ_1 at key values of n_s:

1. if $n_s = 0.961$ then $\delta_1 \geq 1.7465$,
2. if $n_s = 0.965$ then $\delta_1 \geq 1.7186$,
3. if $n_s = 0.969$ then $\delta_1 \geq 1.6834$.

The saving of appropriate values of spectral index $n_s = 0.965 \pm 0.04$ leads to the divination of δ_1 from the slow-roll regime during inflation.

Thus, the reconstruction of a minimally coupled model in EGB gravity leading to inflationary parameters of the cosmological attractor with $r \sim (N_e + N_0)^{-1}$ during the slow-roll regime is impossible.

4. Conclusions

The introduction of effective potential and the tensor-to-scalar ratio allows to reproduce the expression for ξ in the case of minimal coupling between the field and the Ricci scalar. In the case of nonminimal coupling, the reproduction of ξ will be related to the form of the coupling function. The question of satisfaction of slow-roll regime is rather important for analytically formulated models due to the effective potential approach [55,58] and to have the positive square of sound speed in Einstein–Gauss–Bonnet gravity models [54]. The reheating after inflation is rather a popular problem [60–63]. However, the reheating is a rather open question in the Einstein–Gauss–Bonnet gravity [41]. And the deviation from can coincide with start of prereheating processes [64,65]. Therefore, we try to check satisfaction of the slow-roll regime during inflation.

In the case of $r = 8r_0/(N_e + N_0)^2$ and the exponential effective potential, the slow-roll regime can be satisfied during inflation [55]. However, the slow-roll parameter δ_1 related with the e-folding number derivative of the function before the Gauss–Bonnet term in the initial action ξ' leads to the deviation from the slow-roll regime for the models with the same effective potential and the following tensor-to-scalar ratio $r = 8r_0/(N_e + N_0)$. We demonstrate this using the representation of the function ξ thought the effective potential and the tensor-to-scalar ratio. To satisfy the restriction of observable data [1], we restrict the value of parameter r_0 considering the tensor-to-scalar ratio r in the beginning of inflation. In the models with $r \sim (N_e + N_0)^{-2}$, the upper value of r_0 is bigger then in models with $r \sim (N_e + N_0)^{-1}$. As a result, the value of $r \sim 8r_0(N_e + N_0)^{-1}$ is rather small to satisfy the condition $|\delta_1| = (2\epsilon_1 - r/8)F < 1$ due to the supposition of F limits to 1 in the end of inflation $N_e = 0$. We plan to generalize our consideration for the case of nontrivial nonminimal coupling. The detailed analysis of this situation was made for $F = 1$ and the effective potential of the exponential form.

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