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Review

Strong Interaction Dynamics and Fermi β Decay in the Nucleon and the Nucleus

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Abstract: Nuclear super-allowed β decay has been used to obtain tight limits on the value of the CKM matrix element V_{ud} that is important for unitarity tests and, therefore, for tests of the standard model. Current requirements on precision are so intense that effects formerly thought too small to matter are now considered relevant. This article is a brief review of personal efforts to include the effects of strong interactions on Fermi β decay. First, I examine the role of isospin violation in the decay of the neutron. The size of the necessary correction depends upon detailed strong-interaction dynamics. The isospin violating parts of the nucleon wave function, important at the low energy of β decay, can be constrained by data taken at much higher energies, via measurements, for example, of $ed \to e'\pi^{\pm} + X$ reactions at Jefferson Laboratory. The next point of focus is on the role of nuclear short-ranged correlations, which affect the value of the correction needed to account for isospin violation in extracting the value of V_{ud}. The net result is that effects previously considered as irrelevant are now considered relevant for both neutron and nuclear β decay.

Keywords: short-range correlations; isospin violation; charge symmetry breaking; semi-inclusive deep inelastic scattering

Citation: Miller, G.A. Strong Interaction Dynamics and Fermi β Decay in the Nucleon and the Nucleus. *Universe* **2023**, *9*, 209. https://doi.org/10.3390/universe9050209

Academic Editor: Benjamin Grinstein

Received: 28 February 2023
Revised: 11 April 2023
Accepted: 21 April 2023
Published: 27 April 2023

1. Introduction

The problem of precisely understanding β decay of nuclei has recently come into prominence because of its possibility to use as a probe of new physics beyond the Standard Model [1,2]. Nuclear β decay transitions have been used to extract the value of V_{ud}, which governs the probability that a down quark decays into an up quark, which is important for testing the unitarity of the CKM matrix [3]. At present, the most precise determination of V_{ud} is obtained from $0^+ \to 0^+$ (superallowed) decays of nuclei. The value obtained from a set of more than 200 measurements in 21 different nuclei [4] is

$$V_{ud} = 0.97373 \pm 0.00031. \tag{1}$$

The Particle Data Group [5] has essentially obtained the same central value of V_{ud} but a smaller uncertainty of ± 0.00014. Extracting such values requires that the effects of nuclear isospin-breaking (as well as other small effects) be removed from the experimental measurement. The necessary precision is remarkably high, especially because the nuclear structure theory needed to make various corrections is of very early vintage.

Concerns about the accuracy of the Towner–Hardy formalism used for nuclear structure corrections have led us to propose a different formalism [6,7] that allows the inclusion of shell model states of higher energies. Recent experimental work (see, e.g., the review [8]) showing that the effects of nucleon–nucleon short-ranged correlations can be and have been measured stimulated us [9] to revisit the subject of nuclear superallowed β decay. We found that the effects of short-ranged correlations are relevant for superallowed β decay.

The subject of neutron β decay naturally arose during the study of nuclear decays. In particular, the assumption that the quark-level β decay operator is the same as the neutron-level operator has been widely used. This assumption is correct if isospin symmetry (more precisely, charge symmetry) is upheld in the nucleon wave function. We examined this assumption in [10], with results similar to that of an earlier study [11], finding that there is a correction with the value about equal to the current uncertainty. It is also noteworthy that the parton content of nucleons is modified by charge symmetry-breaking effects [12–14]. A study of parton charge symmetry breaking is under current investigation in JLab experiment E12-09-002, in which semi-inclusive $\pi^{\pm}$ production from the deuteron is measured [15].

The outline of this article is to first review the work on the nucleon, considering Fermi β decay, and then discussing how the same charge symmetry-breaking effects in the Hamiltonian are manifest in the parton distributions, and how these cause observable effects in semi-inclusive pion production on a deuteron target. Thus, there is an interesting connection between low and high energy physics. Finally, the work on nuclear beta decay will be reviewed.

2. Charge Symmetry Breaking and Fermi β Decay of the Neutron

In the Standard Model, the strength of the interaction $d \to u\,W^-$ is governed by the isospin operator acting on quarks. The standard assumption made in going from quark-based to nucleon-based calculations has been that the isospin operator acting on quarks is the same operator that changes a neutron into a proton. This assumption is valid only if the nucleon wave function is invariant under the specific isospin rotation denoted to be the charge symmetry rotation. This operator rotates an up quark into a down quark and in formal notation is a rotation, $P_{\rm cs}$ of π about the y-axis in isospin space [16,17]:

$$P_{\rm cs} = e^{i\pi T_2}, \tag{2}$$

with T_2 as the y-component of the isospin operator. Behrends and Sirlin [18] examined the accuracy of the assumption long ago. Those authors found very small corrections to the beta decay matrix element, on the order of 10^{-6}. This result was based on taking the neutron–proton mass difference to be the perturbing operator. However, this operator does not change the nucleon wave function. The work of [10] used the non-relativistic quark model to provide a more detailed estimate. An outline of that calculation is presented next.

The non-relativistic quark model [19] was used, with harmonic oscillator confinement. The Hamiltonian also includes the non-relativistic kinetic energy, the electromagnetic interaction, and the one-gluon exchange interaction. The charge symmetry-breaking Hamiltonian, H_1, originates from the up–down quark mass difference and the electromagnetic fine structure constant, α. This statement is true more generally. All charge symmetry-breaking effects are caused by the up–down quark mass difference and electromagnetic effects. These effects are manifest in a great variety of manners: isospin-violating hadronic-mass differences, isospin mixing of mesons, and isospin violation in nucleon–nucleon scattering and in nuclear structure. See the reviews, e.g., [16,17].

Well-known theorems tell us that the effects of charge symmetry-breaking on beta decay enter only at second and higher orders in H_1. Three different quark models were employed in the calculations of Crawford and Miller. Each model was constrained to yield the correct value of the neutron–proton mass difference. The differences between models arose from different estimates of the strength of the one-gluon exchange hyperfine interaction and the influence of the pion cloud on the size of the nucleon. These effects were compensated by using different values of the up and down quark masses.

The matrix element of the isospin operator τ_+ is given to second order by

$$\langle p|\tau_+|n\rangle = 1 - 2\sum_{k=1} \frac{(p|H_1|p_k)(p_k|H_1|p)}{(\bar{M} - M_k)^2}, \tag{3}$$

where the index $k \geq 1$ denotes the k'th radial excitation, $\bar{M}$ is the average of neutron (n) and proton (p) masses, the rounded brackets represent states computed with the charge symmetry conserving part of the Hamiltonian, and $|p)$ is the proton ground state. Note that the charge symmetry-breaking effects of H_1 cause the matrix element to be less than unity.

In computing the neutron–proton mass difference, the contributions of the up–down quark mass difference are partially cancelled by the charge symmetry-breaking effects of the kinetic energy, electromagnetic interaction and the one-gluon exchange interaction, leading to the small difference of 1.29 MeV [20]. However, the up–down quark mass difference does not change the nucleon wave function, but the other effects act together in concert to modify the wave function. Thus, we obtained [10] a reduction of the β decay matrix element on the order of 10^{-4}, which is about 100 times larger than the earlier estimate [18].

The schematic calculation of [18] used second-order perturbation theory in the form of $(\frac{\langle V \rangle}{\Delta E})^2$, with $\langle V \rangle$ taken to be the neutron–proton mass difference and the energy denominator ΔE set equal to the nucleon mass. The actual value of $\langle V \rangle$ must be larger than that for reasons explained above, and the energy denominator can be as small as about half the nucleon mass, as determined from the Roper resonance–nucleon mass difference. Thus, the early result was a vast underestimate. Our results of effects of about 10^{-4} (with a spread of about a factor of 5) depend on the details of the quark model such that the details of the strong interaction model, mentioned above, enter in important ways. Nevertheless, it is encouraging that these results are very similar to those of an earlier MIT bag model calculation [11]. The size of these effects is about the same as the current experimental uncertainties. Future improvements in those uncertainties would increase the importance of more accurately determining the size of these charge symmetry-violating corrections.

3. Detecting Charge Symmetry Breaking in the Nucleon

Semi-inclusive pion production from the deuteron via lepton deep inelastic scattering, $ed \rightarrow e'\pi^{\pm} + \mathrm{X}$, could be a sensitive probe of charge symmetry-breaking effects in nucleon valence parton distributions [12–14] if the experiments are sufficiently accurate and if uncertainties in fragmentation functions can be accurately handled. The basic idea is that π^+ are produced mainly by the interactions between a photon and up quarks and π^- are produced by down quarks. As a result, if charge symmetry is conserved in the various wave functions, the yields, $N_{\text{fav}}^{D\pi^{\pm}}$, obey the relation [12]

$$N_{\text{fav}}^{D\pi^+}(x,z) = 4N_{\text{fav}}^{D\pi^-}(x,z), \tag{4}$$

where x is the Bjorken variable and z is the ratio of the pion energy to the photon energy. The factor of 4 arises from the proportionality of the cross section to the square of quark charges. The charge symmetry-breaking distributions are defined by

$$\delta d(x) \equiv d^p(x) - u^n(x); \; \delta u(x) \equiv u^p(x) - d^n(x). \tag{5}$$

In addition to effects of the charge symmetry-breaking Hamiltonian, H_1, there is a kinematic effect of the quark mass difference that appears in evaluating the probability that a quark carries a light-front momentum fraction x. Calculations of these quantities are reviewed in [21], but there is disagreement on which of the effects are dominant. Benesh and Goldman [22], in contrast with other authors, find that the kinematic effects of the quark mass difference are not dominant. A direct measurement of δd and δu would shed light on various aspects of confinement as well as be related to fundamental β decay of the neutron.

The ratio [12]

$$R^D(x,z) \equiv \frac{4N^{D\pi^-}(x,z) - N^{D\pi^+}(x,z)}{N^{D\pi^+}(x,z) - N^{D\pi^-}(x,z)} \tag{6}$$

is sensitive to the charge symmetry-breaking quantity

$$R^D_{CSB}(x) = \frac{4}{3}\frac{\delta d(x) - \delta u(x)}{u^p_v(x) + d^p_v(x)}, \tag{7}$$

where the subscript v stands for valence.

Jefferson Laboratory experiment E12-09-02 has taken data necessary to obtain precise measurements of ratios of charged pion electroproduction in semi-inclusive deep inelastic scattering from a deuterium target. The preliminary results indicate that the quantity $\delta d(x) - \delta u(x)$ can be extracted from the data [15]. A future measurement at an energy-upgraded version of Jefferson Laboratory [23] could be capable of obtaining precision results because of the greater kinematic range available for testing aspects (factorization) of the reaction mechanism.

4. Nuclear Superallowed β Decay

The dominant contribution to the first-row unitarity test of the CKM matrix of the Standard Model comes from measuring the up–down quark matrix element V_{ud}. The value of V_{ud} has been extracted from nuclear beta decays by Hardy and Towner in a long series of papers displaying increasing precision culminating in [4]. Many (>20) measurements of $0^+ \to 0^+$ decays from nuclei ranging from ^{10}C to ^{74}Rb have been made with great precision. There is a only a very small uncertainty, as noted in the Introduction, because the different measurements very similar values of V_{ud}. over a wide range of nuclei.

The Hardy and Towner theory approach has successfully reproduced a vast amount of data. Despite this, the crucial importance of the super-allowed beta decay process in testing the Standard Model has mandated that the theory behind the analysis be continually re-examined. The work of Condren and Miller was only concerned with the isospin-breaking correction, known as δ_C (to be explicitly defined below).

$$|V_{ud}|^2 \propto \frac{1}{1+\delta_{\rm NS}-\delta_C}, \tag{8}$$

so that variation of $\Delta\delta_C$ would cause a variation in V_{ud} given by

$$\frac{\Delta(V_{ud}^2)}{V_{ud}^2} \approx \Delta\delta_C. \tag{9}$$

A specific example is the value of $\delta_C = 0.960(63)\%$ for the $0f_{7/2}$ orbital of ^{42}Ti [4]. A twenty percent change in that number is about 0.2%, and V_{ud} would be changed by half that, 10^{-3}, a number that is larger than the current uncertainty. The Particle Data Group [5] finds an even smaller uncertainty, and that value of $\Delta\delta_C$ would be even more significant.

Schwenk and Miller [6,7] raised questions about the Towner–Hardy approach to the isospin correction. We now explain their argument. In the absence of isospin-breaking effects, which are usually small, the beta decay matrix element would depend only on the value of the nuclear isospin. However, the requirements of extremely high precision require the careful evaluation of the corrections due to isospin-breaking effects. Therefore, one must compute the contributions from electromagnetic and charge-dependent strong interactions to the Fermi matrix element of the isospin raising operator τ_+, $M_F = V_{ud}\langle f|\tau_+|i\rangle$, between the initial and final states for superallowed β decay, $|i\rangle$ and $|f\rangle$.

Towner and Hardy [24] write the Fermi matrix element as

$$M_F \approx V_{ud}\langle f|w_+|i\rangle, \tag{10}$$

using a second quantization formulation. Here,

$$w_+ = \sum_{\alpha,\beta} a_\alpha^\dagger a_\beta I_{\alpha\beta}^{np}, \tag{11}$$

where $a_\alpha^\dagger$ creates a neutron in a single-particle, shell-model state with quantum numbers denoted as α (n,l,j,m), and a_β annihilates a proton in state β. The single-particle matrix element $I_{\alpha\beta}^{np}$ is given by

$$I^{np}_{\alpha\beta} = \delta_{\alpha,\beta} \int_0^\infty R^n_\alpha(r)\, R^p_\beta(r)\, r^2 dr \equiv \delta_{\alpha,\beta}\, r_\alpha\,, \tag{12}$$

where $R^n_\alpha(r)$ and $R^p_\beta(r)$ are the neutron and proton radial wave functions. The operator w_+ changes wave functions and therefore acts on spatial degrees of freedom. Thus, it cannot be the same as the isospin operator τ_+ that acts *only* on isospin degrees of freedom. The operator τ_+ does not change spatial wave functions. Instead, w_+ of Equation (11) is the plus component of the W-spin operator of MacDonald [25], reviewed in Ref. [26].

Note the appearance of the delta function in Equation (12). This means that the radial excitations are missing. In making this approximation, Towner and Hardy neglected radial excitations of energy $2\hbar\omega$ and higher above the relevant orbitals. This approximation truncated the necessary shell model space. Corrections to the Towner and Hardy formalism based on the collective isovector monopole state were presented in [27,28]. Work on the effects of short-ranged correlations appears in [29] that concludes, "we present a new set of isospin-mixing corrections . . ., different from the values of Towner and Hardy. A more advanced study of these corrections should be performed".

The Towner and Hardy approach that truncates the available shell-model space specifically eliminates a very important effect. This is the influence of short-ranged nucleon–nucleon correlations. Such produce components of the nuclear wave function involving nucleons in orbitals high above the given shell model space. This effect of the strong nucleon–nucleon interaction reduces the probability that a decaying nucleon is in a valence single-particle orbital and suggests that the magnitude of δ_C could be smaller than that of previous calculations.

Condren and Miller [9] argued that the influence of short-ranged correlations between nucleons should be included in calculating the beta decay matrix elements. This influence, sometimes small and absent in the work of Towner and Hardy, may cause changes in the value of δ_C that are large in the context of the required very high accuracy. This means that V_{ud} could be different than its present value. Any change would depend on future theoretical efforts to includes the effects of nucleon–nucleon correlations in a more precise manner than in [9].

Unambiguous evidence for the existence of nucleon–nucleon short-ranged correlations has been found in many recent experiments and calculations [8,30–50] in the time since Towner and Hardy started their calculations. The important effects of nucleon–nucleon short-ranged correlations, predicted long ago, have finally been measured. These correlations involve the excitations of nucleons to virtual intermediate states of high energy. The result is that radial excitations are now an important part of nuclear wave functions.

Spectroscopic factors, essentially the occupation probability of a single-particle, shell-model orbital, provide information regarding the quantitative effects of short-ranged correlations. As reviewed in [8], electron–nucleus scattering experiments, mainly $(e, e'p)$, typically observe only about 60–70% of the number of protons expected from shell-model considerations. This reduction of spectroscopic factors was widely observed over both small and large nuclear targets for experiments performed at relatively low-momentum transfer for both valence nucleon knockout using the $(e, e'p)$ reaction [51] and also pickup using the $(d,^3 He)$ reaction [52]. The missing strength of 30–40% shows that both long-ranged correlations and short-range correlations exist in nuclei. Theoretical analyses [53–56] made detailed evaluations finding that it is necessary to include the effects of both long- and short-range correlations to reproduce the values of experimentally measured spectroscopic factors. Condren and Miller argued that, in analogy with the $(e, e'p)$ and $(d,^3 He)$ reactions, superallowed beta decay measurements are influenced by the short-ranged correlations that reduce the spectroscopic strength.

Therefore, Condren and Miller [9] re-examined the calculations of superallowed beta decay rates with the goal of including the effects of short-ranged correlations absent in the Towner and Hardy formalism. That work is briefly described here.

In the simplest nuclear shell model nucleons in single-particle orbitals bound by a one-body, mean-field potential. In that case, the β decay matrix element is a simple overlap between the neutron and proton wave functions. If the Hamiltonian commutes with all of the components of the nuclear isospin operator, the spatial overlap would be unity, and the matrix element would be exactly determined by an isospin Clebsch–Gordan coefficient. However, the non-commuting interactions, such as Coulomb interaction (which is the most important for nuclei) and the diverse nuclear effects of the mass difference of up and down quarks, reduce the overlap from that obtained from isospin conservation. This leads to a non-zero value of the isospin correction known as δ_C. However, the mean field that binds the nucleons to single-particle orbitals is only a first approximation. Further modification of the value of the matrix element is required. The mean field arises from the average of two (or more)-body interactions, but residual two (or more)-nucleon effects remain that cause both long- and also short-ranged correlations.

The fundamental theory for the Fermi interaction of proton beta decay involves the isospin operator τ_+, and the Fermi matrix element is then given by $M_F = \langle f|\tau_+|i\rangle$, $|i\rangle$ and $|f\rangle$, the exact initial and final eigenstates of the full Hamiltonian $H = H_0 + V_C$ with energy E_i and E_f, respectively, and V_C denotes the sum of all interactions that do not commute with the vector isospin operator.

Condren and Miller extended the formalism of [6,7] by first developing an effective β-decay one-body operator to account for the dominant isospin-violating effect. The matrix element of this operator was then evaluated in a strongly-correlated system.

Their procedure is described here. Consider single-particle proton p and neutron n orbitals denoted by $|v, p\rangle$ and $|v, n\rangle$, in which the index v denotes the space-spin $(nljm_j)$ quantum numbers. The valence state $|v(p)\rangle$ represents a proton in a shell-model orbital of quantum numbers n, l, j, represented by the symbol v for valence. Only valence orbitals that correspond to average positions on the outer edge of the nucleus undergo beta decay. The orbitals $|v(p, n)\rangle$ are an eigenstate of a Hamiltonian, $h = h_0 + U_C(p)$, with an isospin-violating (mainly Coulomb) potential, $U_C(p)$, that acts only on protons. The eigenkets of h_0 are denoted with rounded brackets and those of h with the usual Dirac notation. Then, $|v, n\rangle = |v, n) \rightarrow |v)$ because the only difference between p and n arises from the operator $U(c)$.

The use of Wigner–Brillouin perturbation theory in U_C yields the expression

$$|v, p\rangle = \sqrt{Z_C}|v, p) + \frac{1}{E_v - \Lambda_v h_0 \Lambda_v}\Lambda_v U_C|v, p\rangle \tag{13}$$

with $Z_C = 1 - \langle v, p|U_C \frac{1}{(E_v - \Lambda_v h_0 \Lambda_v)^2} U_C|v, p\rangle$, and the projection operator Λ_v removes the eigenvectors that have the eigenvalues of $h_0 : \ (v, (n, p)|\Lambda_v = 0$.

The single-particle super-allowed beta decay matrix element, $M_{\rm sp} \equiv (v, n|\tau_+|v, p\rangle$ is given by:

$$M_{\rm sp} = V_{ud}\sqrt{Z_C}. \tag{14}$$

Evaluating $\sqrt{Z_C}$ to second-order in U_C yields

$$M_{\rm sp} \approx V_{ud}\left(1 - \frac{1}{2}(v|U_C \frac{1}{(E_v - \Lambda_v h_0 \Lambda_v)^2}\Lambda_v U_C|v)\right). \tag{15}$$

Once again, as is well-known, we see that the isospin-violating corrections are of second order. The dominant isospin correction of Towner and Hardy, δ_C, is twice the second term of the parenthesized expression.

Next, we turn to nuclear super-allowed β-decay. The one-body Coulomb-correction operator appearing in Equation (15) is defined as $\widehat{\mathcal{O}}_C(v) \equiv U_C \frac{1}{(E_v - \Lambda_v h_0 \Lambda_v)^2}\Lambda_v U_C$. Consider, a situation in which the initial nucleus i consisting of a proton in a valence orbital v outside an isospin-0 core state of A nucleons beta decays to a neutron outside the same state, f.

This simplification is made so as to exhibit the main features of the calculation. The core of the state f is taken as that of the state i, so that overlap of the cores does not influence the β decay matrix element. Corrections are of a negligible $\mathcal{O}(1/A)$.

The dominant Coulomb correction is obtained by computing the expectation value of the operator $\widehat{\mathcal{O}}_C(v)$ that yields $\delta_{C0}(v)$. In coordinate space, this is given by the expression:

$$\delta_{C0}(v) = \int d^3r d^3r' \phi_v^*(\mathbf{r}) \mathcal{O}_C(\mathbf{r}, \mathbf{r}') \phi_v(\mathbf{r}'). \tag{16}$$

Spin indices are suppressed to improve the clarity.

Condren and Miller tested the validity of Equation (16) by comparing the numerical results with those of exact single-particle calculations. Their result $\delta_{C0} = 0.267\%$ agrees with the result for the ^{46}V $f_{7/2}$ state appearing in Table I in [24].

Equation (16) results from using only the simple single-particle state obtained by mean-field approximation to the nuclear wave function. The valence proton or neutron interacts strongly with the core nucleons. This causes both long- and short-ranged correlations that are not included in the mean field. Concentrating on short-ranged aspects, the two-nucleon wave function is given by

$$|v, \alpha\rangle = \sqrt{Z_S(v, \alpha)} |v, \alpha) + Q \frac{G}{e} |v, \alpha), \tag{17}$$

with α denoting one of the occupied orbitals of the core state and $|v, \alpha\rangle$ is a two-nucleon state. The operator G (the two-nucleon scattering amplitude operator, T-matrix, evaluated at negative energy and modified by Pauli-blocking effects) is the anti-symmetrized reaction matrix operator summing ladder diagrams involving two-nucleon interactions. The term Z_S maintains the normalization of the two-nucleon wave function, e represents an energy denominator, and iterations of the potential that correct the state $|i_0)$ are schematically included in the factor $Q\frac{G}{e}$. The Hermitian projection operator Q obeys $Q|v, \alpha) = 0$, and is constructed so that the effects of long-ranged correlations are excluded and that only the short-ranged correlations are included in the correction studied here.

Equation (17) is a first step to include the effects of short-ranged correlations. Defining $\Omega \equiv Q\frac{G}{e}$, $Z_S(v, \alpha) = 1 - (v, \alpha|\Omega^\dagger \Omega|v, \alpha)$. Then,

$$\delta_C(v) = Z_S(v)(v|\widehat{\mathcal{O}}_C(v)|v) + \sum_\alpha (v, \alpha|\Omega^\dagger \widehat{\mathcal{O}}_C(v) \Omega|v, \alpha). \tag{18}$$

with $Z_S(v) \equiv 1 - \sum_\alpha (v, \alpha|\Omega^\dagger \Omega|v, \alpha) \equiv 1 - \kappa(v)$. The factor $Z_S(v)$ is the occupation probability, known always to be <1. Terms of first order in Ω vanish, as noted above.

As a starting point, consider that the existing literature reviewed in [9] indicates that $Z_S(v) \approx 0.8$ for many states v. There is specific dependence on the state, nucleus, and interactions that somewhat changes the value, but the value of $Z_S(v)$ is never unity. The number 0.8 comes from many experimental measurements and theoretical calculations discussed above. If one neglects the second term of Equation (18), the result is that the leading Coulomb correction of Towner and Hardy is multiplied by the factor Z_S. This would potentially cause a very substantial reduction in terms of present high-precision requirements. However, one must include the effects of the second term of Equation (18).

Condren and Miller studied the effects of the second-order terms in G of Equation (18) for the ^{46}V $f_{7/2}$ state appearing in Table I in [24]. They used a simplified approach so that their results are estimates that were schematic and inconclusive. The size of the second-order terms depended on the detailed dependence on the two-nucleon separation distance. The second-order terms could be negligible or could completely compensate for the reduction caused by having $Z_S < 1$.

Thus, computations of the isospin correction were found to be very sensitive to the effects of short-ranged correlations. The size of the isospin correction depends on specific details of the model of the short-range physics. The influence could be an increase, decrease or no change. The correct evaluation of this effect can properly be assessed with the necessary precision by doing detailed calculations using different models consistent with experimentally measured spectroscopic factors. Tests of the unitarity of the CKM matrix demand very high accuracy in the nuclear structure physics. Performing detailed state-of-art nuclear calculations of superallowed β decay that avoid unnecessary truncations of the shell-model space is therefore a high priority for nuclear theorists.

5. Discussion

This paper is concerned with how strong interactions influence the necessary corrections to calculations of β decay matrix elements caused by the effects of isospin violating interactions. Although of the order of electromagnetic interactions, strong interaction physics is needed to make accurate calculations.

Differences in the details of the strong interaction influence the evaluation of the neutron decay matrix element. The size of the charge symmetry-breaking effects is about the same as the current uncertainties. Future improvements in those uncertainties would make the importance of these corrections more relevant. Oddly enough, uncertainties in the low-energy beta decay matrix element can be constrained by higher-energy measurements of the $ed \rightarrow e'\pi^{\pm} + \mathrm{X}$ reactions at Jefferson Laboratory.

This paper also discusses the role of nuclear short-ranged correlations, which affect the value of the correction needed to account for isospin violation in the extraction of the value of V_{ud} from nuclear β decay. The effects of short-range correlations have been unambiguously measured and could have a significant effect on the extraction of V_{ud}. The net result is that effects previously considered as irrelevant are now considered relevant. It is noteworthy that acquiring a sufficiently detailed understanding of beta decay will ultimately depend on knowledge of the interplay between strong electromagnetic and weak forces.

Funding: This research was partially supported by the U.S. Department of Energy Office of Science, Office of Nuclear Physics, under Award No. DE-FG02-97ER-41014.

Acknowledgments: I thank the students Levi Condren and Jacob Crawford for their contributions to the papers that are reviewed here.

Conflicts of Interest: The author declares no conflict of interest.

Abbreviation

The following abbreviation is used in this manuscript:

CKM Cabibbo–Kobashi–Maskawa

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