3.1.1. Selection of Wavelet Basic Function

Before processing the wavelet transform, scholars should obtain the wavelet basic function that extracts a result describing the signal's considerable impact, which precisely reflects the incoming signal's feature. Using the SCWT method, two types of mutation points are identified in the spatial sequence signal [1]: the first type corresponds to smooth signals without mutations, with discontinuous and mutated first derivatives; the second type corresponds to signals with sudden amplitude changes at certain positions, which lead to signal discontinuity. In the urban fringe detection process, the sudden variation of NLI values, as reflected by the mutation points on the NLI map, emerges within a distinctly sharp rise. Thus, the mutation points were described by the second kind of discontinuity [2]. The Db3 wavelet was then chosen as the wavelet basic function, acquired from the Daubechies wavelet family employed in the second type of discontinuity identification work in this study.

3.1.2. Determination of Spatial Scale in SCWT

Choosing a proper transformation scale for identifying the mutation points is critical. Notably, many mutation points will be neglected when the scale is large. If the scale is small, the result will contain significant noise. The current work employs a plot of the wavelet coefficients' variance to choose the best-fitting scale [3]. As the scale reaches the highest variance value, it achieves a considerable signal impact, which can precisely reflect the incoming signal features. Fig. 1 depicts that the curve of obtained variance indicated that the second processing scale (i.e., Scale a = 2) had the maximum variance of the coefficients between the scales of wavelet transform. Thus, the local maxima in the coefficients processed in Scale = 2 are explored by the model as the mutation points, and eventually, we mapped all the mentioned regions (of all the transect coefficients) as a map of mutation points.

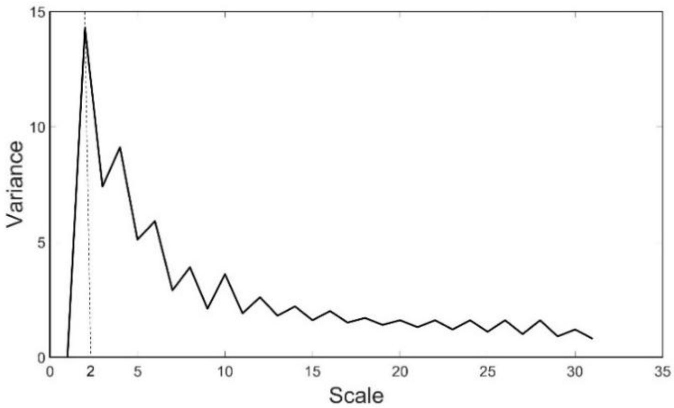

Figure S1. Wavelet transform coefficients versus the scales.

3.1.3. Elimination of "Pseudo" Mutation Point

Researchers usually label the location of mutation points by recording and mapping the "maximum module" of wavelet transform coefficients. Although the "modulus maximum" of the wavelet transform can correspond to more signal mutations, plenty of noise (i.e., "pseudo" mutation points) may be produced, making it challenging to place the mutation points more precisely [4]. As shown in Figure 2a, the wavelet coefficient curve was obtained via wavelet transform, and the wavelet coefficient's maximum and minimum values are also calculated and marked. Many sudden urbanization-level variations happen in the urban fringe, indicating a remarkable transition from one stable state to another, characterized by high wavelet coefficient values. Thus, eliminating the "pseudo" mutation point, characterized by low wavelet coefficient values, will aid in attaining accurate urban fringe detection.

To delineate the "true" mutation points within the urban fringe, the slicing threshold should derive the urban fringe areas, limited by mutations with high wavelet coefficient values with maximum continuity. The literature on delimitating different city regions generally presents a value of k-standard deviations. Suppose that we have a set consisting of the points of mutation ×1, ×2, ×3 ..., ×n, where their average $\bar{x}$ and k standard deviations are described as the following:

$$|x_i - \bar{x}| > k\sigma. \tag{S1}$$

It is assumed that the wavelet coefficient values of "pseudo" mutation points in the interval of x ± $k\sigma$ are normal, and that scores of "true" mutation points exceeding the range are abnormal (Figure 3b). The current test employed an appropriate value of two standard deviations.

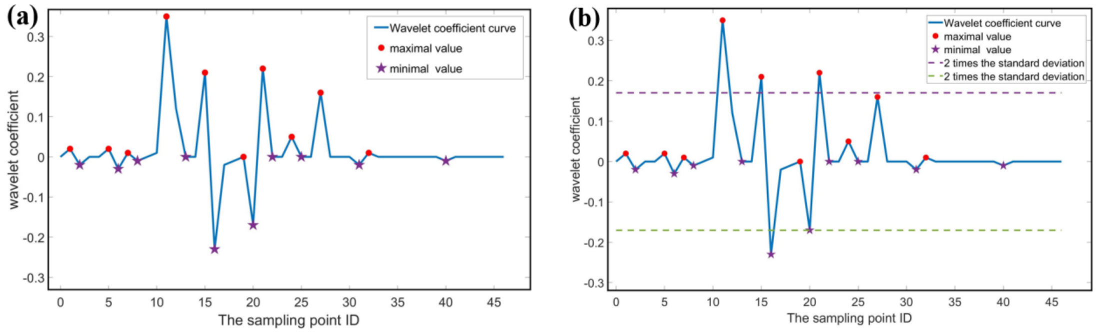

Figure S2. Elimination of "pseudo" mutation point: (**a**) "maximum module" of wavelet transform coefficients; (**b**) "pseudo" mutation points detected via k-standard deviations.

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