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Abstract: Detonation initiation is a prerequisite to normal operations of an oblique detonation engine (ODE), and initiation-assistant measures are imperative in cases of initiation failure that occur in a length-limited combustor under wide-range flight conditions. This study numerically investigates the initiation characteristics of oblique detonation waves (ODWs) in H<sub>2</sub>-fueled ODE combustors at wide-range flight Mach numbers  $Ma_f$  or flight altitudes  $H_f$ . Failures of ODW initiation are observed at both low  $Ma_f$  and high  $H_f$  if no measure is taken to assist initiation. Through analyses of the flow fields and theoretical predictions of the ignition induction length  $L_{ind}$ , the data reveal that the detonation failure at low  $Ma_f$  is raised by the significant decrease in the post-shock temperature due to insufficient shock compression, leading to a significant increase in  $L_{ind}$ . The detonation failure at high  $H_f$  is caused by the rapid decrease in the combustor inflow pressure as  $H_f$  increases, which also results in an increase in Lind. With further identifications of the key flow structures crucial to detonation initiation, an initiation-assistant concept employing a transverse  $H_2$  jet is proposed. The simulation results show that through an interaction between the incident oblique shock wave and the jet shock wave, the transverse jet helps to initiate an ODW in the combustor at a low  $Ma_{f}$ , and the initiation location is relatively fixed and determined by the jet location. At high  $H_{fr}$  a Mach reflection pattern is formed in the combustor under the effects of the transverse jet, and detonative combustion is achieved by the generated Mach stem and its reflected shock waves. The proposed concept of using transverse jets to assist detonation initiation provides a practical reference for future development of ODEs that are expected to operate under wide-range flight conditions.

**Keywords:** oblique detonation wave; initiation; transverse jet; oblique detonation engine; wide-range flight condition

# 1. Introduction

Driven by the motivation to achieve faster and more efficient atmospheric transportation, the research into and development of hypersonic aircraft have rapidly increased in recent decades [1,2], and the relevant propulsion system has become a technical bottleneck limiting further advancements [3,4]. By forming a stabilized oblique detonation wave (ODW) in the combustor to replace the traditional diffusive combustion mode, an oblique detonation engine (ODE) is a promising propulsion solution for future hypersonic flights [5,6]. As one type of extreme combustion phenomenon featuring tight coupling between a shock wave and a flame, detonation is capable of providing high efficiency in chemical-to-mechanical energy conversion and consequently increasing the propulsion performance [7,8]. The rapid heat release rate of the detonative combustion mode also implies a short combustor, which helps to reduce the massive heat losses and frictions on walls, simplify the thermal management system, and reduce the total weight of the engine [9,10]. Moreover, an ODW is expected to self-adapt to various flow perturbations by adjusting its wave angle, which benefits the stable operation of the ODE. Recently, such an oblique detonation propulsion solution using gaseous hydrogen or liquid aviation kerosene



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). as the fuel has been experimentally demonstrated in a hypersonic shock tunnel [11,12], which signifies that ODEs are anticipated to be used in the near future. However, various technical difficulties associated with ODEs, such as hypersonic mixing, premature ignition prevention, oblique detonation initiation, and stabilization, still need to be fully solved before ODEs can be implemented in practical applications.

An ODW is typically formed when a high-speed combustible mixture flows through a wedge [13]. A numerical study conducted by Li et al. [14] showed that a wedge-induced ODW generally transitions from a nonreactive oblique shock wave (OSW), which was later supported by the experimental observations obtained by Viguier et al. [15] and Lin et al. [16]. Further simulations found that the detailed OSW–ODW transition patterns can be classified into two main types: smooth with a curved shock front and abrupt with a multiwave point [17,18]. More complex flow structures within the induction zone of an ODW under certain flow conditions have been reported by Liu et al. [19] and Yang et al. [20]. In engineering applications, the wedge length needed for ODW initiation (i.e., the transition length from an OSW to an ODW) is more important and shows great sensitivities to the free-stream parameters [21], mixture reactivity [22], and wedge angle [23].

An ODE is commonly designed at one single point, whereas in practical applications, the system operates under a wide range of flight Mach numbers and flight altitudes. Under off-design conditions (for example, at low flight Mach numbers), the initiation length of ODW could be significantly increased due to the insufficient compression of the OSW [22,24]. When the ODW initiation length is much longer than the length scale of the ODE combustor, detonation initiation failure occurs, and consequently, the engine might be unable to generate thrust [25–27]. A similar detonation initiation issue could emerge when the ODE is operated at high altitudes where the atmospheric temperature does not change much but the pressure drops drastically [24,28]. Therefore, detonation initiation within a length-limited combustor under wide-range flight conditions needs to be further examined and investigated to advance the implementation of ODEs in practical applications.

Many novel methods or concepts have been proposed to control and promote ODW initiation. The direct initiation of ODWs by spherical blunt bodies has been widely studied through high-speed projectile-launching experiments, and the criticalities of the formation of stabilized ODWs have been discussed [29,30]. Fang et al. [31] numerically investigated the direct initiation of ODWs induced by blunted wedges, and a theoretical criterion based on the detonation cell sizes was proposed to predict the critical blunt radius. The effectiveness of leading-edge blunting in the initiation assistance of wedge-induced ODWs was experimentally validated by Gong et al. [32]. Other modifications to a pure wedge geometry to facilitate the rapid initiation of ODWs include the use of a step [33], dual-angle wedges [34,35], curved wedges [36,37], etc. Inspired by the formation mechanism of the abrupt OSW–ODW transition pattern, Han et al. [38] proposed introducing a small bump on the wedge surface to promote ODW initiation through shock/shock interactions. The effectiveness of such a concept was later validated numerically by Xiang et al. [39] and experimentally by Han et al. [12].

Hot jets have been widely employed in detonation-based engines [40,41]. Han et al. [42] discussed, in terms of the relationship between the inflow velocity and the detonation velocity, the formations and propagations of two types of detonations that are triggered by hot jets in supersonic premixed flows. The formation process of a detonation wave initiated by a hot transverse jet was numerically investigated by Cai et al. [43,44], and the role of a contracting flow channel formed by the hot jet on the propagation of the detonation wave was analyzed. The initiation characteristics of ODWs via a combination of a jet and a wedge were studied numerically by Yao & Lin [45], and the transformations of different combustion regimes controlled by two nondimensional parameters (i.e., the momentum flux ratio and the penetration ratio) were thus revealed.

In this study, transverse jets are utilized to control initiations of ODWs in two typical ODE combustors designed for wide-range applications of flight Mach number or flight altitude and are numerically investigated. First, the general geometry of a representative

ODE combustor, the mathematical models, the numerical algorithms, and a case summary are introduced in Section 2. Then, the initiation characteristics of ODWs in the combustors without any assistance measures are analyzed for operations in a wide range of flight Mach numbers or flight altitudes in Section 3. This section also includes a thorough discussion of the crucial flow structures for detonation initiation. In Sections 4.1 and 4.2, respectively, the initiation-assistant methods using a transverse jet as well as the corresponding numerical validations for addressing the initiation failure issues at low flight Mach numbers or high flight altitudes are presented. Finally, conclusions are provided in Section 5.

# 2. Details of the Methodology

# 2.1. Geometry of an ODE Combustor

This study considers a classical two-dimensional (2-D) configuration of ODEs with external fuel injection [9], as schematically shown in Figure 1a. The inlet is two-stage and is designed to compress the incoming airflow and implement fuel/air mixing. Immediately after the inlet but before the nozzle, there is a short detonative combustor in between, with its detailed geometry illustrated in Figure 1b. The combustor enveloped within points OCDE has a flow channel of a constant height (denoted as  $DE = h_c$ ). An ODW is generated from the leading edge of the cowl wall (i.e., point C), extends downstream, and reflects over the wall on the other side. According to Zhang et al. [46], the formed ODW is easily destabilized if it is reflected before the first expansion corner of the combustor (i.e., point O); hence, it is suggested to set the reflection locus right at or downstream of the first expansion corner for more redundancy of stabilization. The actual reflection location is determined by the initiation length of the ODW, the wave angle of the ODW, the height of the combustor  $h_{c'}$  and the upstream-extending distance of the cowl's leading edge compared to the first expansion corner (denoted as  $CK = d_c$ ). Notably, the initiation length and the wave angle of the ODW depend on the free-stream parameters. Therefore, it is practical to control the reflection location of the ODW by determining  $d_c$  with a fixed  $h_c$  based on the most upstream posture of the ODW under the given operation condition range of the ODE, i.e., an ODW with a near-zero initiation length and a maximum wave angle (as shown by the red solid line in Figure 1b).



**Figure 1.** Schematics of (**a**) the classical configuration of an external-injected ODE and (**b**) the detailed geometry of the combustor.

Although the combustor channel has no significant effect on the performance of the ODW (assuming it can be successfully initiated), a considerable length (denoted as  $OE = l_c$ ) is commonly required to prevent the significant expansion effects of the nozzle on the strength of the OSW and subsequently the initiation of the ODW. Notably,  $d_c$  is nearly

proportional to  $h_c$  for given conditions. Typical and fixed values of  $h_c = 6$  cm and  $l_c = 20$  cm are used in this study, equal to those values employed by Zhang et al. [47]. Then, the value of  $d_c$  is determined by the certain operation conditions of the ODE and fixed for the combustor. Upstream of the combustor, the inlet wall has an inclined angle of  $\theta_i$ , indicating that the flow deflection angle after flowing into the combustor is  $\theta_i$  as well. To control the growth and upstream motion of the separation zone formed through the ODW/boundary-layer interaction, a simple floor bleed structure is employed at the entrance of the combustor (i.e., point O), which expels the incoming boundary layer from the inlet and reestablishes a thin boundary layer downstream [11,47,48]. Finally, a one-side expanding nozzle is located downstream of the combustor.

#### 2.2. Mathematical Models and Numerical Algorithms

Since the effectiveness of a transverse jet in assisting oblique detonation initiation is the main focus of this study, complex 3-D effects of the flow field, such as 3-D wave structures and boundary layers over the combustor's side walls, are neglected for simplicity [43–45], which is believed to have limited impact on the conclusions. Hence, the flow field in the ODE combustor is governed by the 2-D multispecies reactive Reynolds-averaged Navier-Stokes (RANS) equations. Menter's shear-stress transport (SST)  $k-\omega$  turbulence model [49] is used to model the turbulent stresses. The well-known NASA Glenn database [50] is used to model the thermodynamic properties of each species. Hydrogen is used as the fuel, and its combustion is modeled by a modified Jachimowski's reaction model [51] involving 19 reversible elementary reactions among 8 species: H<sub>2</sub>, H, O<sub>2</sub>, O, OH, HO<sub>2</sub>, H<sub>2</sub>O<sub>2</sub> and H<sub>2</sub>O and with N<sub>2</sub> serving as the bathing species. This detailed chemistry mechanism for H<sub>2</sub>/air combustion was proved to well predict the ignition delay time [52] and has been widely used in numerical simulations of unsteady shock-induced combustion phenomena [48,53,54] scramjets [55,56], ODWs [45], and ODEs [48,57]. More details regarding the mathematical modeling are available in Zhang et al. [47].

The governing equations are numerically solved using a quadrilateral-grid-based finite volume method [58], where the spatial discretization is implemented through a second-order total variation diminishing (TVD) scheme [59]. The convective flux on each cell edge is evaluated based on a nonlinear Harten–Lax–van Leer contact (HLLC) approximate Riemann solver together with the use of a minmod limiter to suppress the non-physics oscillations that easily occur in the vicinity of flow discontinuities. An implicit dual time-stepping method is adopted for integration in the time direction in time-dependent simulations. The applicability of the above numerical methods to hypersonic reactive flows, including the combustion flow fields in ODEs, has been well demonstrated in, for example, refs. [37,47,48]. In particular, a preliminary comparison of the oblique detonation flow field in an ODE combustor model between a wind-tunnel experiment and a 2-D numerical simulation using the present methods was made in the supplementary materials of Zhang et al. [47], and a good agreement was obtained.

#### 2.3. Cases

The initiations of ODWs in ODE combustors under two types of wide-range operation conditions, i.e., wide-range flight Mach number  $Ma_f$  and wide-range flight altitude  $H_f$ , are individually investigated. Two sets of inflow parameters and combustor geometric parameters are employed accordingly. First, a wide operation range of the flight Mach number from  $Ma_f = 8$  to 13 is assumed, while the flight altitude is set at  $H_f = 30$  km. To maintain the structural simplicity of the ODE under such wide-range conditions, all the compression angles of the engine are expected to be fixed, and they are typically designed at high-flight Mach numbers to ensure that the post-compression airflow temperature falls within an appropriate range for normal operations. Thus, a fixed inclined angle of  $\theta_i = 16^{\circ}$  is set to the inlet wall for the aforementioned wide-range- $Ma_f$  operation conditions.

The inflow parameters such as temperature  $T_i$ , pressure  $p_i'$  and velocity magnitude  $V_i$  before entering the ODE combustor for each flight Mach number are estimated through two

shock-compressions of two 8° ramps in the inlet, which are summarized in Table 1. The equivalent ratio of the inflows is then assumed to be unity. Furthermore,  $d_c$  is set to 73 mm, and then an ideal ODW at  $Ma_f = 8$  would reflect right downstream of point O. In Table 1, Cases 1–6 represent scenarios without any initiation-assistant measures, while Cases 7–10 represent scenarios employing a transverse jet to assist with detonation initiation. More details regarding the transverse-jet-assisted method are provided in Section 4.1.1.

Cases No.	$Ma_f$	$H_{f}$ (km)	<i>T<sub>i</sub></i> (K)	$p_i$ (kPa)	<i>V<sub>i</sub></i> (m/s)	$\theta_i$	<i>d</i> <sub>c</sub> (mm)	<i>h</i> <sub>c</sub> (mm)	$\Delta \phi_{ m jet}$	p <sub>0, jet</sub> (kPa)	J
1	13	30	795.4	38.25	3775	$16^{\circ}$	73	60	-	_	-
2	12	30	731.0	32.17	3480	$16^{\circ}$	73	60	-	-	-
3	11	30	670.0	26.74	3184	$16^{\circ}$	73	60	-	-	-
4	10	30	612.4	21.94	2889	$16^{\circ}$	73	60	-	-	-
5	9	30	558.5	17.75	2593	$16^{\circ}$	73	60	-	-	-
6	8	30	508.2	14.15	2296	$16^{\circ}$	73	60	-	-	-
7	11	30	670.0	26.74	3184	$16^{\circ}$	73	60	0.05	878.5	0.267
8	10	30	612.4	21.94	2889	$16^{\circ}$	73	60	0.05	715.3	0.309
9	9	30	558.5	17.75	2593	$16^{\circ}$	73	60	0.05	569.7	0.363
10	8	30	508.2	14.15	2296	$16^{\circ}$	73	60	0.075	663.0	0.656

**Table 1.** Case setups in investigations under wide-range-*Ma<sub>f</sub>* conditions.

Another batch of simulations considers wide-range- $H_f$  operation conditions. In these conditions, a similar combustor to that presented previously is considered, but with the inclined angle of the inlet wall changed to  $\theta_i = 25^\circ$ . A flight condition of  $Ma_f = 9$  and  $H_f$  = 30 km is set as the baseline case. Assuming that the free-stream airflow is compressed by two 12.5°-ramps in the inlet, the inflow parameters before the combustor for this baseline case are  $T_{ref} = 855.1$  K,  $p_{ref} = 43.96$  kPa and  $V_{ref} = 2466$  m/s. In this ODE combustor,  $d_c$  is equal to 50 mm, and the design reflection point of the ODW is located 60 mm downstream of point O. Notably, an increase in the flight altitude would result in a near-exponential decrease in atmospheric pressure, but the variation in atmospheric temperature remains relatively small (typically ranging from 210 K to 290 K). For instance, when the flight altitude increases from 20 km to 40 km, the atmospheric pressure decreases from 5529 Pa to 287 Pa, which is approximately 1/20 of the former, whereas the atmospheric temperature only increases from 217 K to 250 K (by approximately 15%). Similarly, in the case of an ascent from 20 km to 50 km, the atmospheric pressure of the latter altitude even decreases to approximately 1/70 of the former altitude, while the atmospheric temperature increases by approximately 25% from 217 K to 271 K.

Clearly, the major effects of different flight altitudes on ODW initiations come from the near-exponential variation in the combustor's inflow pressure because the pressure and temperature ratios are correlated with each other during the inlet compression process, and the inflow temperature before the combustor should be limited for premature ignition prevention (e.g., below approximately 900 K for hydrogen-fueled ODEs). In this case, we investigate this problem more simply by varying the inflow pressure  $p_i$  before the combustor with reference to the baseline case but keeping other flow parameters such as inflow temperature and velocity unchanged (i.e.,  $T_i = T_{ref}$  and  $V_i = V_{ref}$ ). Specifically, the inflow pressure in the simulated cases, as summarized in Table 2, decreases from  $p_i = p_{ref}$ to 0.2  $p_{ref}$ , 0.1  $p_{ref}$ , 0.05  $p_{ref}$ , and finally, 0.025  $p_{ref}$ . The corresponding flight altitude in terms of atmospheric pressure increases from approximately  $H_f = 30$  km to 41 km, 47 km, 52 km, and finally, 57 km. In Table 2, Cases 11–15 represent scenarios without employing any initiation-assistant measure, whereas Case 16 serves as a control counterpart to Case 15 to validate the effectiveness of using a transverse jet in ODW initiation (as is discussed further in Section 4.2).

Cases No.	$Ma_f$	H <sub>f</sub> (km)	<i>Т</i> <sub><i>i</i></sub> (К)	p <sub>i</sub> (kPa)	<i>V<sub>i</sub></i> (m/s)	$\theta_i$	$d_c$ (mm)	<i>h</i> <sub>c</sub> (mm)	$\Delta \phi_{ m jet}$	p <sub>0,jet</sub> (kPa)	J
11	9	30	855.1	43.96	2466	$25^{\circ}$	50	60	-	_	_
12	9	41	855.1	8.79	2466	$25^{\circ}$	50	60	-	_	-
13	9	47	855.1	4.40	2466	$25^{\circ}$	50	60	-	_	-
14	9	52	855.1	2.20	2466	$25^{\circ}$	50	60	-	_	_
15	9	57	855.1	1.10	2466	$25^{\circ}$	50	60	-	_	-
16	9	57	855.1	1.10	2466	$25^{\circ}$	50	60	0.2	42.50	1.73

**Table 2.** Case setups in investigations under wide-range- $H_f$  conditions.

In all cases, a grid resolution of 0.1 mm  $\times$  0.1 mm at maximum is used in the core flow regions. Near the walls, the grids are clustered in the normal direction with a minimum height of 2 µm. This grid resolution setup is similar to that used in the authors' previous study [47] and was proved suitable for capturing the key characteristics of ODW initiation.

#### 3. Initiation Characteristics of ODWs without Assistant Measures

# 3.1. ODW Initiation Characteristics in the Combustor at Different Ma<sub>f</sub>

The flow fields of the combustor at  $Ma_f = 13$ , 12, 11, 10, 9, and 8 without employing any initiation-assistant measures (i.e., Cases 1 to 6 in Table 1) are depicted in Figure 2 as the temperature and H<sub>2</sub> mole fraction contours superimposed with pressure isolines. At  $Ma_f = 13$  and 12 (as shown in Figure 2a,b), the rapid combustion of the inflowing H<sub>2</sub>/air mixture is induced at the high temperature (>1300 K) behind the OSW formed from the cowl's leading edge, and thus, an ODW is initiated from the OSW at a rather short distance (defined as the OSW–ODW transition length  $L_{tran'}$  as illustrated in Figure 2a,b). On the body side, the ODW reflects downstream of the leading edge of the upper wall of the combustor, which induces the formation of a large-scale separation zone. Overall, the combustion of the inflowing mixture is mainly completed through the ODW in these two high  $Ma_f$  cases, resulting in a predominant percentage of the oblique detonative combustion mode in the combustor.

When the flight Mach number decreases to  $Ma_f = 11$  (as shown in Figure 2c), the postshock temperature of the OSW decreases to a relatively lower value of approximately 1180 K, leading to an apparent increase in the ignition delay time and, subsequently, the OSW–ODW transition length. As a result, the percentage of the oblique detonative combustion mode reduces significantly to approximately 50%, although an ODW is successfully initiated in the combustor at this  $Ma_f$ . The remaining fuel of the mixture is burned behind the OSW with a slower combustion rate, which is considered a deflagrative combustion mode.

When the flight Mach number further decreases to  $Ma_f = 10$  (as shown in Figure 2d), the post-shock temperature of the OSW decreases to approximately 1050 K, and the corresponding ignition delay time and the OSW–ODW transition length are so large that the formed OSW fails to directly transition to an ODW within the given combustor length. In this case, a certain percentage of the oblique detonative combustion mode still occurs in the combustor through the reflected shock waves (RfSWs) in the regular interaction between the OSW and the leading-edge separated shock wave (SSW).

At a lower-flight Mach number of  $Ma_f = 9$  (as shown in Figure 2e), the post-OSW temperature is merely approximately 930 K, and similarly, no initiation of an ODW from the OSW occurs. Furthermore, no detonative combustion is induced by possible alternate flow structures in the combustor such as the RfSWs in the case of  $Ma_f = 10$ . Only some mild combustion emerges near the walls in the latter part of the combustor, but the mixture flows into the nozzle and undergoes an expansion soon. Obviously, the combustion efficiency in this case is very low, which might be insufficient to generate enough thrust for normal operations. At  $Ma_f = 8$ , the lower design limit of the ODE, no combustion is observed in the combustor (as shown in Figure 2f), implying a complete failure of the engine.



Figure 2. Cont.



**Figure 2.** The flow fields in the ODE combustor at different flight Mach numbers without any initiation-assistant measures. (a)  $Ma_f = 13$  (Case 1). (b)  $Ma_f = 12$  (Case 2). (c)  $Ma_f = 11$  (Case 3). (d)  $Ma_f = 10$  (Case 4). (e)  $Ma_f = 9$  (Case 5). (f)  $Ma_f = 8$  (Case 6).

Figure 3 depicts the variation in the theoretical ignition induction length  $L_{ind}$  of an ODW as the flight Mach number decreases from  $Ma_f = 13$  to 8.  $L_{ind}$  is estimated as a product of the ignition delay time  $\tau_{ign}$  and the post-shock velocity of the OSW  $V_s$ , i.e.,  $L_{ind} = \tau_{ign} \cdot V_s$ , where the ignition delay time  $\tau_{ign}$  is calculated from the post-shock temperature and pressure of the OSW based on a constant combustion assumption. The OSW–ODW transition lengths  $L_{tran}$  evaluated in the simulated cases are also presented in Figure 3 for comparison. As shown, the simulated  $L_{tran}$  agrees well with the theoretically predicted  $L_{ind}$  in the cases of  $Ma_f = 11$ , 12, and 13. At these high flight Mach numbers, both  $L_{ind}$  and  $L_{tran}$  are on the order of centimeters, which facilitates the transition of the formed OSW into an ODW within a short distance and maintains a significantly high percentage of the oblique detonative combustion mode within the length-limited ODE combustor.



**Figure 3.** The theoretical ignition induction length  $L_{ind}$  and the simulated OSW-ODW transition length  $L_{tran}$  as functions of the flight Mach number  $Ma_f$ .

However, when the flight Mach number decreases to a relatively low value, for example,  $Ma_f = 9$  or 8,  $L_{ind}$  increases significantly and can reach a length to the order of meters, which is so excessively large that a direct transition from the OSW to an ODW within the length-limited combustor is impossible. As a result, the percentage of the detonative combustion mode may be very low, detonative combustion may not occur, or no combustion may occur in the combustor at all. To ensure the stable operations of the ODE within its design range (i.e.,  $8 \le Ma_f \le 13$ ), certain initiation-assistant measures are necessary to facilitate the rapid initiation of an ODW within the length-limited combustor at low flight Mach numbers, e.g.,  $Ma_f = 8$ , 9, 10 and even 11.

## 3.2. ODW Initiation Characteristics in the Combustor at Different $H_f$

Note again that the variation in flight altitude considered here is simply implemented by changing the combustor's inflow pressure as described in Section 2.3. Figure 4 shows the numerical shadowgraphs and the H<sub>2</sub> mole fraction contours (superimposed with pressure isolines) of the flow fields in the combustor without any initiation-assistant measures at different inflow pressures  $p_i$ . As indicated in Figure 4a, where  $p_i = p_{ref}$  (the baseline case with  $Ma_f = 9$  and  $H_f = 30$  km), an ODW is quickly initiated, with a relatively short transition length, from the OSW on the cowl side of the combustor. On the body side, a large leadingedge separation is induced, and an SSW is formed, which is also an oblique detonation in this configuration. As a result of the interaction between the ODW and the SSW, a Mach reflection pattern occurs, and a Mach stem (MS) is generated in the middle height of the combustor. As shown, the MS is a detonation wave near-normally inclined to the incoming H<sub>2</sub>/air mixture, which actually corresponds to the strong solution of an oblique detonation [11,12]. Therefore, the detonative combustion of the premixed combustible gas occurs in the form of an ODW, an SSW, and an MS in the combustor in this case.



Figure 4. Cont.



**Figure 4.** The flow fields in the ODE combustor at different inflow pressures without any initiationassistant measures. (**a**)  $p_i = p_{ref}$  (Case 11:  $H_f = 30$  km). (**b**)  $p_i = 0.2 p_{ref}$  (Case 12:  $H_f = 41$  km). (**c**)  $p_i = 0.1 p_{ref}$  (Case 13:  $H_f = 47$  km). (**d**)  $p_i = 0.05 p_{ref}$  (Case 14:  $H_f = 52$  km). (**e**)  $p_i = 0.025 p_{ref}$  (Case 15:  $H_f = 57$  km).

At  $p_i = 0.2 p_{ref}$  (Case 12 in Figure 4b, analogous to  $H_f = 41$  km), a similar flow pattern with the formation of an MS occurs in the combustor, but the combustion behavior differs from that observed at  $p_i = p_{ref}$  (Figure 4a, Case 11). On the cowl side of the combustor, the OSW fails to transition into an ODW within a short distance in this case because the  $L_{\text{ind}}$  behind the OSW significantly increases due to the low inflow pressure (as shown in Figure 5). Part of the mixture that passes through the OSW exhibits decoupled combustion downstream, while another part of the mixture exhibits some detonative combustion through the lower RfSW of the Mach reflection pattern. On the body side of the combustor, the formed SSW does not induce significant combustion of the mixture in this case; instead, the corresponding mixture undergoes detonative combustion through the upper RfSW of the Mach reflection pattern. In this middle height of the combustor, due to the higher pressure and temperature behind the MS and the resulting shorter  $L_{ind}$  (see Figure 5), the MS again manifests itself as a detonation wave, which dominates the detonative combustion mode in the combustor. In Figure 5, the theoretical ignition induction length of the MS is estimated based on the post-shock flow parameters of a normal shock wave (NSW), a proper approximation of the nature of a strong wave of the MS. Compared to the MS formed in Figure 4a, the length of the MS also slightly decreases in Figure 4b due to the reduced strength of the Mach reflection (between the OSW and the SSW) in the present lower inflow pressure case.

For a lower inflow pressure of  $p_i = 0.1 p_{ref}$  (Case 13, analogous to  $H_f = 47$  km) shown in Figure 4c, detonative combustion of the inflowing mixture still occurs in the combustor through the formed Mach reflection flow pattern, but the length of the MS decreases significantly. Moreover, the OSW no longer directly induces combustion of the mixture over a short distance due to the significant increase in the ignition induction length  $L_{ind}$ (see Figure 5); instead, the mixture undergoes detonative combustion through the RfSW of the Mach reflection and a small SSW generated by their interaction with the wall boundary layer. In both cases ( $p_i = 0.2 p_{ref}$  and 0.1  $p_{ref}$ ), although the reduction in the combustor's inflow pressure due to the increase in flight altitude results in a significant decrease in  $L_{\text{ind}}$  behind the OSW, the combustor still maintains a great percentage of the detonative combustion mode under the combined effects of the MS and the corresponding RfSWs.



**Figure 5.** The theoretical ignition induction length  $L_{ind}$  as a function of the inflow pressure  $p_i$ , calculated from the post-shock flow parameters of an OSW and an NSW.

When the combustor's inflow pressure further decreases to  $p_i = 0.05 p_{ref}$  (Case 14, analogous to  $H_f = 52$  km), as shown in Figure 4d, the length of the MS diminishes, and the reflection pattern between the OSW and the SSW appears as a regular pattern. The ignition induction length behind the OSW significantly increases to approximately  $L_{ind} \approx 0.15$  m (see Figure 5), and no combustion is observed behind the OSW. No combustion occurs behind the SSW, and the interaction between the OSW and the SSW does not induce any combustion either. The inflowing mixture only undergoes a decoupled combustion induced by a secondary reflected shock wave (2nd-RfSW) near the cowl wall of the combustor. Overall, no detonative combustion occurs within the present length-limited combustor under this low inflow pressure condition.

As the combustor's inflow pressure finally decreases to  $p_i = 0.025 p_{ref}$  (Case 15, analogous to  $H_f = 57$  km), the ignition induction length behind the OSW significantly increases to approximately  $L_{ind} \approx 0.36$  m (see Figure 5), which exceeds the length of the combustor ( $l_c = 0.2$  m or  $l_c + d_c = 0.25$  m). As a result, no combustion occurs behind the OSW, as shown in Figure 4e. The formed SSW is extremely weak in this case, failing to induce any combustion. The only regions where combustion occurs are the large leading-edge separation zone on the body wall and its wake. However, these regions are not within the main flow channel, implying that the majority of the inflowing H<sub>2</sub>/air mixture does not burn out before it expands in the nozzle and that the ODE may fail to generate enough thrust in this case. Therefore, certain initiation-assistant measures are required to ensure the normal operations of the ODE under low inflow pressure conditions, i.e., at high flight altitudes.

## 4. ODW Initiation Assisted by a Transverse Jet

## 4.1. ODW Initiation at Low Ma<sub>f</sub> Assisted by a Transverse Jet

## 4.1.1. The Transverse-Jet-Assisted Initiation Approach for Wide-Range-Ma<sub>f</sub> Applications

According to the analyses in Section 3.1, the main reason for the failure of ODW initiation in the combustor at a low flight Mach number is the excessively low temperature behind the OSW due to insufficient shock compressions, which can subsequently lead to an excessive  $L_{ind}$ . The occurrence of detonation is largely delayed in spatial aspects, or even there is an absence of combustion in some cases. Therefore, one important way to facilitate ODW initiation at low flight Mach numbers is to increase the post-shock temperature or to

generate some locally high-temperature regions (known as hot spots) without changing the key geometric configuration of the combustor.

At this point, the abrupt initiation structure of a wedge-induced ODW [17,18] can be learned. As shown in Figure 6a, after the compression of the OSW, combustion of the premixed combustible gas begins downstream of the OSW at a certain distance (i.e.,  $L_{ind}$ ). A series of compression waves (also called combustion waves) is generated within the induction zone through the heat release of combustion, propagates downstream, and spatially converges in the meantime. These convergent compression waves interact with the OSW either in a direct way or by forming a weak shock wave to interact with the OSW, leading to the initiation of the ODW.



**Figure 6.** Initiation structures of a wedge-induced ODW: (**a**) abrupted OSW–ODW transition pattern and (**b**) initiation assisted by a transverse jet.

Inspired by such an abrupt mechanism in the natural initiation of an ODW, we propose introducing a transverse jet from the wedge wall (as shown in Figure 6b), which generates a jet shock wave (JSW) within the induction zone. This JSW, replacing the spontaneous compression waves in natural initiation, interacts with the OSW, which is expected to be able to initiate an ODW in advance. With a suitable strength of the transverse jet, such a transverse-jet-assisted measure is expected to be able to control the initiation location of the ODW by adjusting the jet location, especially at low-flight Mach numbers.

The embedding of the transverse jet into the ODE combustor is shown schematically in Figure 7. For the wide-range- $Ma_f$  applications of the ODE, the jet is set on the cowl wall of the combustor and at a distance of  $L_{iet} = 20$  mm downstream from the leading edge, and the width of the transverse jet is set as  $W_{jet} = 1$  mm. For the jet medium, the fuel, i.e., H<sub>2</sub> gas, is chosen based on the following considerations. First, H<sub>2</sub> itself serves as the main consumable carried by aircraft in the fuel tank; hence, utilizing  $H_2$  gas as the jet medium avoids additional structural complexities, such as additional storage devices. Second,  $H_2$  gas has a low density and a high speed of sound. At the same mass flow rate and injection Mach number, the H<sub>2</sub> jet has a higher momentum and a greater penetration depth and produces a stronger JSW than most other gases. Third, the mixing process in the inlet is nonideal in practical ODEs, and the inflowing mixture into the combustor is nonuniform in terms of the equivalence ratio and is always fuel-lean near the walls [48]. In this case, the injected  $H_2$  near the combustor wall can serve as additional fuel to further mix with the unburned air and combust in that region, thereby potentially producing more thrust. However, the mixing process is neglected in the present study; instead, a uniformly premixed stoichiometric  $H_2/air$  mixture is employed for simplicity.

Moreover, the H<sub>2</sub> jet Mach number is set to one, and its total temperature is set at 300 K. Then, the mass flow rate of the H<sub>2</sub> jet can be determined based on its total pressure, and vice versa. The mass flow rate of the H<sub>2</sub> jet is characterized using a dimensionless parameter  $\Delta \phi_{jet}$ , which is defined as the absolute increment in the total equivalence ratio inside the combustor due to the H<sub>2</sub> jet. For different flight Mach numbers, the employed values of  $\Delta \phi_{jet}$ , the corresponding total pressures  $p_{0,jet}$  of the H<sub>2</sub> jet, and the corresponding jet-to-crossflow momentum ratio  $J = \rho_{jet}V_{jet}^2/\rho_sV_s^2$  are summarized in Table 1 (Cases 7–10). Here,  $\rho_{jet}$  and  $V_{jet}$  are the density and velocity magnitude of the H<sub>2</sub> jet, respectively, and  $\rho_s$ 

is the post-shock density of the OSW. As can be seen in Table 1, the required jet-to-crossflow momentum ratio increases when the flight Mach number decreases due to greater difficulty with detonation initiation at a lower flight Mach number.



Figure 7. Schematic of the ODE combustor with a transverse  $H_2$  jet for a wide-range- $Ma_f$  application.

4.1.2. Numerical Validations of Transverse-Jet-Assisted ODW Initiation at Low Ma<sub>f</sub>

Figure 8 illustrates the combustion flow fields in the combustor at  $Ma_f = 11$ , 10, 9, and 8 when a transverse H<sub>2</sub> jet is utilized on the cowl wall as described in Section 4.1.1. For comparison, the simulated OSW–ODW transition lengths  $L_{tran}$  with the transverse H<sub>2</sub> jet employed are also summarized in Figure 3. As shown, the transverse H<sub>2</sub> jet helps to initiate an ODW from the OSW in advance at  $Ma_f = 11$ , with the simulated  $L_{tran}$  reducing from approximately 0.1 m to 0.02 m, which is also demonstrated in a comparison between Figures 2c and 8a. At  $Ma_f = 10$ , although detonative combustion occurs, the OSW fails to transition to an ODW within the length-limited combustor without the jet, as described in Section 3.1. Now, with the transverse H<sub>2</sub> jet employed, the OSW transitions to an ODW rapidly with a simulated  $L_{tran}$  of approximately 0.02 m (see Figure 8b), which is reduced significantly (by an order of magnitude) compared to the theoretical  $L_{ind}$  predicted without the jet (as shown in Figure 3).

At  $Ma_f = 9$ , the no detonation initiation case without the jet, an ODW is successfully initiated from the OSW with a simulated  $L_{tran} \approx 0.02$  m by employing the transverse H<sub>2</sub> jet (as shown in Figure 8c), which facilitates oblique detonative combustion in the combustor. As indicated in Figure 3, the simulated  $L_{tran}$  with the jet at this  $Ma_f = 9$  is reduced by approximately two orders of magnitude compared to the corresponding theoretical  $L_{ind}$ predicted without the jet. At  $Ma_f = 8$ , the case without combustion in the core flow when there is no jet, an ODW is also successfully formed with the effects of the jet, and hence, the main mode occurring in the combustor is again the oblique detonative combustion (as shown in Figure 8d). Moreover, Figure 3 reveals that the reduction in the simulated  $L_{tran}$ using the jet from the theoretical  $L_{ind}$  without the jet is more than four orders of magnitude at this  $Ma_f = 8$ , and the simulated  $L_{tran}$  ( $\approx 0.02$  m) is ultimately within the length scales of the combustor ( $l_c = 0.2$  m or  $l_c + d_c = 0.273$  m).

In summary, with the effects of the transverse H<sub>2</sub> jet, an ODW is successfully initiated within the combustor at all low-flight Mach numbers from 8 to 11, indicating the occurrence of oblique detonative combustion for the majority of the inflowing combustible mixture into the combustor. In other words, the employment of a transverse jet in the combustor makes the engine capable of operating normally in the whole designed range of flight Mach numbers, i.e.,  $Ma_f = 8$  to 13. Furthermore, with the use of the transverse H<sub>2</sub> jet, the transition from the OSW to an ODW occurs at a relatively fixed position at different flight Mach numbers, which is approximately at the designed injection location of the jet, i.e.,  $L_{tran} \approx L_{jet} = 0.02$  m. In other words, the initiation location of an ODW in the length-limited ODE combustor can be easily controlled by the injection location of the transverse jet, which is of practical value.



**Figure 8.** The flow fields in the ODE combustor with a transverse H<sub>2</sub> jet. (a)  $Ma_f = 11$  and  $\Delta \phi_{jet} = 0.05$ . (b)  $Ma_f = 10$  and  $\Delta \phi_{jet} = 0.05$ . (c)  $Ma_f = 9$  and  $\Delta \phi_{jet} = 0.05$ . (d)  $Ma_f = 8$  and  $\Delta \phi_{jet} = 0.075$ .

## 4.2. ODW Initiation at a High $H_f$ Assisted by a Transverse Jet

### 4.2.1. The Transverse-Jet-Assisted Initiation Approach for Wide-Range- $H_f$ Applications

The analyses in Section 3.2 reveal that the combustor's inflow pressure  $p_i$  decreases near-exponentially as the flight altitude  $H_f$  increases, and consequently, the predicted ignition delay time  $\tau_{ign}$  and the ignition induction length  $L_{ind}$  behind the OSW increase near-exponentially as well. The large  $L_{ind}$  prevents the OSW–ODW transition in a short distance and indicates a failure of ODW initiation in the length-limited ODE combustor. In this case, an MS generated through the shock/shock interaction between the OSW and the SSW within the space-confined combustor serves as a crucial factor in triggering detonative combustion in the combustor, as shown in Figure 4b (Case 12) and Figure 4c (Case 13). With the strong compression of the MS,  $L_{ind}$  can be reduced by approximately two orders of magnitude compared to that with the compression of an OSW (as shown in Figure 5). This reduction in  $L_{ind}$  enables combustion of the mixture to occur over a shorter distance, which, in turn, sustains the presence of the MS, resulting in a tight coupling between the shock wave and the flame, i.e., detonative combustion. However, when the flight altitude further increases and the inflow pressure further decreases, the SSW may become weak enough that the interaction between the OSW and the SSW fails to induce the formation of a Mach reflection pattern, and hence, detonative combustion (as shown in Figure 4c) or even normal combustion (as shown in Figure 4d) does not ultimately occur.

Therefore, an intuitional idea to achieve detonative combustion at high flight altitudes or low inflow pressures is to increase the strength of the SSW using some assistant measures, and a transverse jet could be the way to achieve this. That is, a transverse jet is introduced within the leading-edge separation zone on the body wall to produce a strong JSW, which interacts with the OSW to reactivate a Mach reflection pattern in the combustor, as schematically shown in Figure 9. Then, detonative combustion can be achieved through the MS and its RfSWs in a similar way as shown in Figure 4c,d. Notably, due to the low pressure, density, and velocity within the leading-edge separation zone, a not-very-strong jet is expected to penetrate deep enough to form a suitable JSW. Specifically, the jet is located at a distance of  $L_{\text{iet}} = 10 \text{ mm}$  downstream from the combustor's leading edge on the body wall, and the width of the jet is  $W_{jet} = 2$  mm. The fuel, i.e., the H<sub>2</sub> gas, is again chosen as the jet medium, following similar considerations as in Section 4.1.1. The injection Mach number of the jet is set to unity (sonic) with a total temperature of 300 K. The injection total pressure is determined by the mass flow rate, and the latter is nondimensionally expressed by  $\Delta \phi_{\text{jet}}$ , which is equal to 0.2 in this case. The corresponding jet-to-crossflow momentum ratio  $J = \rho_{\text{jet}} V_{\text{iet}}^2 / \rho_i V_i^2$  is equal to 1.73, where  $\rho_i$  is the inflow density. Notably, the inflow is assumed as the crossflow in the evaluation of J in the configuration of Figure 9.



**Figure 9.** Schematic of the ODE combustor with a transverse  $H_2$  jet for a wide-range- $H_f$  application.

4.2.2. Numerical Validations of Transverse-Jet-Assisted Detonation Initiation at High H<sub>f</sub>

To validate the effectiveness of the proposed transverse H<sub>2</sub> jet at high flight altitudes or low inflow pressures, the worst case of  $p_i = 0.025 p_{ref}$  ( $H_f = 57$  km) is taken as an example, i.e., Case 16 in Table 2. Figure 10 is a series of numerical shadowgraphs of the flow field in the combustor at different time instances after the introduction of the transverse H<sub>2</sub> jet, which depicts the process of detonation initiation induced by the jet. The two streamlines emitted from the two ends of the injection slot and the H<sub>2</sub> mole fraction isolines of 0.99 are also provided in the numerical shadowgraphs, which synergistically represent the shape and the flow path of the H<sub>2</sub> jet. The initial flow field in the combustor is a steady one without the presence of the jet, as shown in Figure 4e (Case 15) in Section 3.2, and the transverse H<sub>2</sub> jet begins to inject from the designed slot at t = 0. As indicated in Figure 10a, a JSW is generated near the body-wall leading edge of the combustor when the transverse H<sub>2</sub> jet is injected into the separation zone at t = 0.01 ms. At t = 0.07 ms (as shown in Figure 10b), the JSW is almost established and interacts with the incident OSW, forming a regular reflection flow pattern. At this moment, in addition to the formation of the aforementioned JSW, a weaker secondary jet shock wave (2nd-JSW) is generated within the jet.



**Figure 10.** The establishment process of the flow field (numerical shadowgraph) in the ODE combustor with a transverse H<sub>2</sub> jet ( $p_i = 0.025 p_{ref}$ , Case 16).

At t = 0.15 ms, the RfSW of the incident OSW interacts with the boundary layer on the cowl wall and induces a small-scale flow separation, forming typical flow separation structures including a separation zone, a separated shock, a reflected shock, and a reattached shock wave, as shown in Figure 10c. Then, the formed 2nd-JSW strengthens and moves upstream gradually, and finally, at t = 0.17 ms, it merges with and enhances the JSW, as shown in Figure 10d. With the strengthening of the JSW, the RfSW of the incident OSW also strengthens correspondingly, leading to an intensified boundary layer separation on the cowl wall of the combustor, as shown in Figure 10e. Subsequently, under the complex interaction of various RfSWs, the regular reflection between the OSW and the JSW transitions to a Mach reflection pattern at t = 0.35 ms, and an MS is formed, as shown in Figure 10f. The formed MS grows and moves upstream gradually (Figure 10g). Ultimately, the MS reaches a steady state, as shown in Figure 10h. During this process, the size of the leading-edge separation zone on the body wall of the combustor reduces gradually and eventually reaches a steady state.

Figure 11 depicts the contours of temperature and  $H_2$  mole fraction (superimposed by pressure isolines) of the ultimately stable flow field in the combustor at t = 1 ms. After introducing the transverse  $H_2$  jet, the simulated  $L_{ind}$  is significantly reduced under the influence of the formed MS and its RfSWs, and rapid combustion of the combustible mixture flowing into the combustor occurs, with the combustion products reaching a temperature exceeding 2500 K, implying a stable detonative combustion mode in the combustor. This effect is further confirmed by examining the distributions of various flow parameters along a streamline passing through the MS, as shown in Figure 12. With a rapid increase in pressure,  $H_2$  is rapidly consumed. In other words, the flame is tightly coupled with the shock front, and hence, the formed MS is an overdriven detonation wave. This example demonstrates that with the use of a transverse  $H_2$  jet into the leading-edge separation zone on the body wall, an MS is formed through the interaction between the JSW and the OSW, and stable detonative combustion of the inflowing mixture occurs in the combustor under the effects of the MS and its RfSWs, which allows the ODE to be operated normally even under low-pressure conditions at a high flight altitude.



**Figure 11.** The stable flow field in the ODE combustor with a transverse H<sub>2</sub> jet at t = 1 ms ( $p_i$  = 0.025  $p_{ref}$ , Case 16).



**Figure 12.** The flow parameter distributions along the streamline that crosses the middle point of the MS at t = 1 ms ( $p_i = 0.025 p_{ref}$ , Case 16).

#### 5. Conclusions

Initiation of an ODW in the combustor is a crucial process during the successful operation of an ODE. An ODE designed to be operated over wide ranges of flight Mach numbers and altitudes typically faces detonation initiation issues under certain off-design conditions, which indeed require effective initiation-assistant measures. In this study, numerical simulations were conducted to investigate the initiation characteristics of two H<sub>2</sub>-fueled ODE combustors designed for wide-range flight Mach numbers  $Ma_f$  and flight altitudes  $H_f$  by solving the 2-D multispecies reactive RANS equations closed by the SST k- $\omega$  turbulence model and the modified Jachimowski's H<sub>2</sub>/air combustion model.

For an ODE combustor designed for a range of  $Ma_f = 8$  to 13, detonative combustion does not occur in the combustor at the low  $Ma_f$  of 8 and 9 without any initiation-assistant measures, and detonative combustion occurs with a low percentage at  $Ma_f = 10$  and 11. With a theoretical analysis, the results show that the failure of detonation in the combustor is caused by the insufficient compression of the OSW at low  $Ma_f$ , leading to a significant decrease in the post-shock temperature and a great increase in  $L_{ind}$ . Based on the abrupt OSW–ODW transition pattern, a transverse H<sub>2</sub> jet is introduced within the induction zone to generate a JSW to initiate an ODW through a shock/shock interaction. The simulation results show that, assisted by the transverse jet, ODW initiation is successful at all low  $Ma_f = 8$  to 11, and the initiation location is relatively fixed and determined by the jet location.

For the ODE combustor designed for a range of  $H_f = 30$  km to 57 km, failure of detonation is identified at  $H_f \ge 52$  km without any initiation-assistant measures. This detonation failure mainly occurs because of the rapid decrease in the combustor's inflow pressure at high  $H_f$ , which results in a significant increase in  $L_{ind}$ . The formation of an MS is beneficial to the occurrence of detonative combustion at a moderately high  $H_f$  where the OSW–ODW transition fails. Therefore, a transverse  $H_2$  jet is introduced within the leading-edge separation zone on the body wall of the combustor at very high  $H_f$ , which generates a JSW to enhance the SSW and to make its interaction with the OSW transition into a Mach reflection pattern. Finally, detonative combustion is expected to be achieved through the formed MS and its RfSW. Considering the highest altitude  $H_f = 57$  km as an example, the effectiveness of the transverse jet in detonation initiation is validated by numerical simulation.

This study demonstrates the concept of using a transverse jet to assist detonation initiation within a length-limited ODE combustor at off-design conditions for ODEs operating at wide-range  $Ma_f$  and  $H_f$ , which is of practical value to the future development of ODEs. Although two jet configurations are proposed to solve the initiation issues encountered at low  $Ma_f$  or high  $H_f$ , a combination of these two jet configurations is expected to work for a wide-range ODE with a specific flight envelope where  $Ma_f$  and  $H_f$  vary accordingly. Furthermore, numerical simulations with imperfect inflow conditions such as mixing nonuniformity could be studied in the future to consider the effectiveness of the transverse jet under more practical conditions and to carry out necessary optimizations.

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# Nomenclature

List of Symbols

$d_c$	Upstream-extending distance of cowl's leading edge (m).
$h_c$	Height of combustor (m).
$H_{f}$	Flight altitude (m).
Ĵ	Jet-to-crossflow momentum ratio.
l.	Length of combustor (m).

*L*<sub>ind</sub> Ignition induction length (m).

$L_{jet}$	Distance of jet downstream from wall's leading edge (m).
L <sub>tran</sub>	Oblique shock wave to oblique detonation wave transition length (m).
Ma <sub>f</sub>	Flight Mach number.
$p_{0,\text{iet}}$	Total pressure of jet (Pa).
$p_i$	Inflow pressure (Pa).
$p_{\rm ref}$	Reference inflow pressure in baseline case (Pa).
$T_i$	Inflow temperature (K).
$T_{\rm ref}$	Reference inflow temperature in baseline case (K).
$V_i$	Inflow velocity magnitude (m/s).
$V_{\text{iet}}$	Velocity magnitude of jet (m/s).
V <sub>ref</sub>	Reference inflow velocity magnitude in baseline case (m/s).
$V_{\rm s}$	Post-shock velocity magnitude (m/s).
W <sub>iet</sub>	Width of transverse jet (m).
$\Delta \phi_{\rm iet}$	Increment in total equivalence ratio by fuel jet.
$\theta_i$	Inclined angle of inlet wall (°).
$ ho_i$	Inflow density $(kg/m^3)$ .
$\rho_{\rm iet}$	Density of jet $(kg/m^3)$ .
$\rho_{\rm s}$	Post-shock density (kg/m <sup>3</sup> ).
$ au_{ m ign}$	Ignition delay time (s).
Abbreviation	
2-D	Two-dimensional.
2nd-JSW	Secondary jet shock wave.
2nd-RfSW	Secondary reflected shock wave.
3-D	Three-dimensional.
HLLC	Harten–Lax–van Leer contact.
JSW	Jet shock wave.
MS	Mach stem.
NSW	Normal shock wave.
ODE	Oblique detonation engine.
ODW	Oblique detonation wave.
OSW	Oblique shock wave.
RANS	Reynolds-averaged Navier-Stokes.
RaSW	Reattachment shock wave.
RfSW	Reflected shock wave.
SST	Shear-stress transport.
SSW	Separated shock wave.
TVD	Total variation diminishing.

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