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Integrated Profitability Evaluation for a Newsboy-Type Product in Own Brand Manufacturers

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Abstract: Effective inventory management depends on accurate estimates of product profitability to formulate ordering and manufacturing strategies. The achievable capacity index (ACI) is a simple yet efficient approach to measuring the profitability of newsboy-type products with normally distributed demand, wherein profitability is presented as the probability of achieving the target profit under the optimal ordering quantity. Unfortunately, the ACI is applicable only to retail stores with a single demand. In the current study, we addressed the issue of measuring the integrated profitability of newsboy-type products sold in multiple locations with independent demand levels, such as own-branding-and-manufacture (OBM) companies with multiple owned channels. We began by formulating profitability in accordance with multiple independent normal demands, and then developed an integrated ACI (IACI) to simplify expression. We also derived the statistical properties of the unbiased estimator to determine the true IACI in situations where demand patterns are unknown. Finally, we conducted hypothesis testing to determine whether the integrated profitability meets a stipulated minimum level. For convenience, we tabulated the critical values as a function of sample size, confidence level, the number of channels, and the stipulated minimum level. One can make decisions simply by estimating the IACI based on historical demand data from all channels and then looking up the critical value in the corresponding tables. Consequently, the proposed methods make it possible for OBM managers to address integrated profitability evaluation, which is effective in deciding the optimal timing to pull unprofitable items from the shelves by looking up generic tables. Furthermore, we also performed numerical and sensitivity analyses for a real-world case to illustrate the applicability and some managerial implications of the proposed scheme.

Keywords: inventory; achievable capacity index; newsboy problem; OBM; profitability estimation

MSC: 90B05

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1. Introduction

The newsboy problem is a classic optimization problem that originated from a boy who sells the neighborhood newsletter and decides on copies under uncertainty in weekly demand. Now, most of the literature involving to newsboy problem has extended to products with a restricted shelf-life (e.g., daily newspapers and fresh food with an expiry date) under uncertain demand. Note that some of the literature also refers to this type of product as a newsboy-type product. For newsboy-type products, there is a single ordering opportunity at the start of the selling period. If the order quantity exceeds demand after the end of the period, then the seller must bear the disposal costs of the excess inventory. If the order quantity falls short of demand, then the seller must consider the opportunity cost associated with the shortage. This type of product is also called a single-period product. The sellers of single-period products generally seek to determine the optimal order quantity for a stipulated objective function (e.g., minimizing the expected cost, maximizing the expected profit, or maximizing the probability of achieving a target profit) within a probabilistic demand framework. Note that the newsboy problem is applicable to both the manufacturing and retail industries.

Most of the literature related to inventory problems considered the practical scenarios involving marketing activities and government policies. Edalatpour et al. [1] considered that sustainable production and inventory management simultaneously take into account the economic, environmental, and social criteria when determining pricing policies or inventory levels. For example, the carbon tax and cap-and-trade regulations are environmentally friendly approaches to reducing carbon emissions, which both reduce emissions by encouraging the lowest-cost emissions reductions. Lee [2] developed an EOQ model jointly determining optimal order quantity and investment in carbon emissions reduction under the cap-and-price regulation policy. Entezaminia et al. [3] explored trading and production planning in an unreliable manufacturing system with cap-and-trade regulation. Hasan et al. [4] optimized the inventory level and technology investment under a carbon tax, cap-and-trade, and strict carbon limit regulations. Khanna and Yadav [5] developed an EOQ model for freshness and stock-dependent demand with an expiration date under the cap-and-trade mechanism. Qi et al. [6] considered cap-and-trade regulation and uncertain market demand in a risk-averse firm. Konstantaras et al. [7] proposed a continuous review production-inventory model in the green supply chain under carbon tax regulation. Karampour et al. [8] developed a bi-objective nonlinear optimization model to maximize the profit of inventory and minimize the carbon emissions of transportation, simultaneously. Zhang et al. [9] studied an inventory model involving a consignment stock policy and a cap-and-trade regulation. Hasan et al. [10] considered green technology investment and carbon regulations (i.e., carbon tax, cap-and-offset, and cap-and-trade) in their inventory model. Maity et al. [11] establish a green inventory model, in which the demand rate depends on selling price, stock, and green concern level. San-José et al. [12] considered a deteriorating product with power demand and full backlogging in a sustainable inventory model under a carbon emission tax. In recent years, most of the literature involving the newsboy problem also considers these two regulations for carbon emission in their models. Bai and Chen [13] studied a distributionally robust newsboy problem with dual sourcing under carbon tax and cap-and-trade regulations. Lee et al. [14] considered a newsboy model with a carbon emission cap-and-trade system under the assumption that policymakers determining the cap for carbon emissions also have the power to regulate prices in the carbon trading market. Qu et al. [15] considered cap-and-trade regulations in which the carbon footprint exists in the production, sales, residual disposal, and return warranty service stages. Chen et al. [16] explored the optimal joint decision problem of shipping quantity, product pricing strategies, and carbon emissions of online retailers under a new logistics mode. Zhao et al. [17] analyzed the optimal joint strategies on order quantity and sustainable technology investment, in which sustainable technology can decrease greenhouse gas emissions. Adhikary and Kar [18] considered perishable goods in a multiobjective newsboy problem which simultaneously maximizes expected profit and minimizes the total carbon emissions. Gayon and Hassoun [19] studied a newsboy problem with greenhouse gas emissions at the disposal stage regulated by a cap-and-trade policy. Other researchers have addressed the newsboy problem in virtual stores. Ma et al. [20] considered a problem with drop-shipping and resalable returns, wherein both bricks-and-clicks and online businesses were available. Ma and Jemai [21] analyzed two policies aimed at optimizing the rationing of store inventory, including a threshold policy and a fixed-portion policy. Some of the literature also used data-driven methods to address the newsboy problem, such as deep learning and machine learning. Xu et al. [22] constructed a distribution ambiguity set with the nonparametric characteristics of the true distribution based on data, and then used it to make robust decisions for the newsboy problem. Kou and Wan [23] combined absolute robust optimization with a data-driven uncertainty set to address the multi-item newsboy problems. Lin et al. [24] proposed a data-driven approximation to the theoretical risk-averse newsboy model under a value-at-risk constraint. Neghab et al. [25] presented an integrated learning and optimization method based on deep learning for the data-driven newsboy problem with observable and unobservable features. Yang et al. [26] studied a data-driven newsvendor problem, in which the replenishment decisions are obtained by mapping high-dimensional and mixed-frequency features of historical data. Tian et al. [27] investigated a data-driven newsboy problem, in which textual review data and historical data were accessible. Chen [28] considered decision-makers' risk preferences in the data-driven newsboy models.

To maximize profits, retailers must consider "what/which to order" as well as "how much to order", particularly in situations with limited shelf space. The products that make it onto the shelves are generally those with the highest estimated profitability. Thus, estimating profitability is a key task for retailers. A number of researchers have used the estimated results of the objective function to explain the market performance of newsboy-type products. Kevork [29] developed an estimator for the maximum expected profit as a representation of product profitability. They also used the statistical properties of the estimator for inference. Halkos and Kevork [30,31] further derived the interval estimation for the maximum expected profit based on normally-, exponentially-, and Rayleigh-distributed demand. Sundar et al. [32] derived the lower bound of interval estimation for the maximum expected profit to present the conservative profitability of the product. Su and Pearn [33] considered that the maximum probability of achieving a target profit can also be used to measure the market performance of the newsboy-type product. Therefore, they defined the maximum probability in the case of normally distributed demand as the profitability of a newsboy-type product. Furthermore, they also proposed the achievable capacity index (ACI), which is positively correlated with profitability, and further studied profitability estimation using ACI under unknown mean and variance parameters. Note that the simplified formulation of ACI makes it easy to derive the statistical properties of the ACI estimator, and researchers have explored the practical applicability of the ACI based on statistical inference. Su and Pearn [33] established a hypothesis test that involves selecting from among multiple homogeneous products. Su and Pearn [34] addressed the problem of determining whether the profitability of a product meets a stipulated minimum level in order to determine the optimal timing to pull unprofitable items from the shelves. To meet the conditions encountered in real-world scenarios, profitability analysis has also been used to investigate the assumptions of uncertain demand. Su et al. [35] considered the issue of data collection in situations involving multiple samples. Su et al. [36] derived the ACI and statistical properties of its estimator under the assumption that demand is highly uncertain and obeys an exponential distribution. To eliminate overestimates of profitability due to sampling error, Su et al. [37] and Su et al. [38] obtained the lower confidence bound of the ACI as an estimate of conservative profitability for newsboy-type products. Su et al. [39] also considered the issue of conservative profitability for newsboy products with exponentially distributed demand based on multiple samples.

1.1. Problem Statement and Existing Methods

Since competition in the manufacturing sector has increased, many manufacturers are dedicated to transforming themselves from original equipment manufacturers (OEM) or original design manufacturers (ODM) to own brand manufacturers (OBM). OBM refers to a type of manufacturing company that not only manufactures products, but also develops and owns the brand associated with those products. Therefore, the OBM companies are responsible for the entire product lifecycle, including design, production, branding, and marketing. The founder of ACER, President Shih, also advocates that manufacturers should further develop R&D and marketing to create more added value [40]. In an effort to safeguard brand image, common OBM firms manufacture and sell products under their own brand within their own sales channels (e.g., flagship stores, direct-selling stores, franchise stores, counters, and stipulated retailers with contracts). This model is not conducive to business expansion; however, it does allow the transfer of stock among channels to reduce the risk of overstocking or running out of stock due to uncertain demand. To the best of our knowledge, it appears that most OBM firms adjust their manufacturing and sales strategies in accordance with evaluations of profitability based on the total revenue in all channels. Note, however, that the total revenue gives no indication of the impact of operating costs, such as surpluses, shortages, disposal costs, or the cost of manufacturing. Moreover, using total revenue as a means to gauge a product's profitability might not encompass the nuances of statistical inference, due to unpredictable demand. Based on our review of the literature, the ACI index appears to be a useful tool for retailers seeking an estimate of profitability for a newsboy-type product. Unfortunately, the ACI is applicable only to retail stores with single demand; i.e., the ACI cannot be used to measure the integrated profitability of a newsboy-type product sold in multiple locations, such as OBM companies with multiple owned channels.

1.2. Research Objective

The research gaps in the above-mentioned section motivated us to develop a suitable ACI for the estimation and measurement of product profitability for use by OBM companies with multiple owned channels, in which the transfer of products among channels is permitted to moderate operational losses resulting from demand uncertainty. Furthermore, it also motivated us to explore the integrated profitability evaluation for a newsboy-type product. The main contribution of evaluation results is effective for OBM managers in deciding the optimal timing to pull unprofitable items from the shelves. The objectives of this research are as follows. In Section 2, we formulated the integrated profitability for a newsboy-type product sold in multiple channels under normally distributed demand, and developed an integrated ACI (IACI) index, which has a simplified form and positively correlated with integrated profitability. Then, we derived the statistical properties of the IACI estimator for use in statistical inference. Note that a discussion of a scenario case involving multiple samples is also explored. In Section 3, based on the statistical properties of the IACI estimator, we established a hypothesis test, which involves determining whether the integrated profitability of the product meets the stipulated minimum level, i.e., integrated profitability evaluation. We also established guidelines and steps for decision-making. Section 4 illustrates the applicability of the proposed scheme in a numerical example involving profitability evaluation in an OBM company. The sensitivity analysis of the numerical example also performed to obtain managerial insights. In Section 5, we outlined the concluding remarks and future research directions.

2. Integrated Profitability

In this section, we first formulate integrated profitability for a single-period product, which is defined as the probability of achieving a target profit for all channels under optimal manufacturing quantity. Note that the time length of the single period is defined as the time length of shelf-life. Then, we develop an IACI index with a simplified form to express integrated profitability, and derive the statistical properties of its estimator. For convenience, the notation used to construct the integrated profitability is listed as follows:

p the selling price of a product;
c the manufacturing cost of a product;
c_p the net profit of a product without any cost impact, i.e., $c_p = p - c$;
c_s the shortage cost of a product;
c_d the disposal cost for a surplus product;
c_e the excess cost for a product, i.e., $c_e = c_d + c$;
h the number of channels, the set of all channels is denoted as $\{\psi_1, \psi_2, \ldots, \psi_h\}$;
d_i the demand for the channel ψ_i, where $i = 1, 2, \ldots, h$;
Q_i the order quantity for the channel ψ_i;
Q_t the total order quantity for all channels per period, i.e., $Q_t = \sum_{i=1}^{h} Q_i$;
d_t the total demand for all channels per period, i.e., $d_t = \sum_{i=1}^{h} d_i$;
k_t the stipulated target profit;
T_t the target demand.

2.1. Formulations of Integrated Profitability and IACI

This study considers an OBM company with h owned channels selling a newsboy-type product, wherein the demands of all channels are independent random variables obeying a normal distribution; i.e., $d_i \sim N(\mu_i,\ \sigma_i^2)$, where $i = 1,\ 2,\ \ldots,\ h$. Note that all products can be transferred among channels to reduce the risk of overstock and shortages. Assume that the selling price of a product (p), the manufacturing cost of a product (c), the shortage cost of a product (c_s), and the disposal cost for a surplus product (c_d) are the same and constant in all channels. At the end of a given sales period, we assume that the subsets of channels $\{\psi_1,\ \psi_2,\ \ldots,\ \psi_s\}$ and $\{\psi_{s+1},\ \psi_{s+2},\ \ldots,\ \psi_h\}$ are out of stock and meet the demand or have surplus stocks, respectively, in which s is the number of channels with shortage. Based on the above scenario, we formulate the total profit for all channels per period (denoted by Z_t) as follows:

$$Z_t = \begin{cases} p\sum_{i=1}^{h} d_i - c\sum_{i=1}^{h} Q_i - c_d\left[\sum_{i=s+1}^{h}(Q_i - d_i) - \sum_{i=1}^{s}(d_i - Q_i)\right] \ for\ \sum_{i=s+1}^{h}(Q_i - d_i) \geq \sum_{i=1}^{s}(d_i - Q_i). \\ p\sum_{i=1}^{h} Q_i - c\sum_{i=1}^{h} Q_i - c_s\left[\sum_{i=1}^{s}(d_i - Q_i) - \sum_{i=s+1}^{h}(Q_i - d_i)\right] \ for\ \sum_{i=1}^{s}(d_i - Q_i) > \sum_{i=s+1}^{h}(Q_i - d_i). \end{cases} \tag{1}$$

where Q_i is the order quantity for channel ψ_i. From Equation (1), the total profit can be divided into the overall excess case (above) and overall shortage case (below) at the end of a sales period. To simplify the formulation of Equation (1), we respectively denote $c_p = p - c$ and $c_e = c_d + c$ as the net profit and excess cost for a product, to obtain the following:

$$Z_t = \begin{cases} (c_p + c_e)d_t - c_e Q_t,\ \ for\ d_t \leq Q_t \\ (c_p + c_s)Q_t - c_s d_t,\ \ for\ d_t > Q_t \end{cases} \tag{2}$$

where $Q_t = \sum_{i=1}^{h} Q_i$ and $d_t = \sum_{i=1}^{h} d_i$ indicate the total order quantity and total demand for all channels per period, respectively. Using Equation (2), we can obtain an interval of total demand that meets the stipulated target profit (denoted by k_t), i.e., $d_t \in [LAL_t(Q_t),\ UAL_t(Q_t)]$, where $LAL_t(Q_t) = (k_t + c_e Q_t)/(c_p + c_e)$ and $UAL_t(Q_t) = [-k_t + (c_p + c_s)Q_t]/c_s$, both of which depend on the total order quantity. Since $d_t \sim N(\sum_{i=1}^{h}\mu_i,\ \sum_{i=1}^{h}\sigma_i^2)$, the probability of achieving the target profit is

$$\begin{aligned} \Pr(z_t \geq k_t) &= \Pr(LAL_t(Q_t) \leq d_t \leq UAL_t(Q_t)) \\ &= \Phi\left(\frac{UAL_t(Q_t) - \sum_{i=1}^{h}\mu_i}{\sqrt{\sum_{i=1}^{h}\sigma_i^2}}\right) - \Phi\left(\frac{LAL_t(Q_t) - \sum_{i=1}^{h}\mu_i}{\sqrt{\sum_{i=1}^{h}\sigma_i^2}}\right), \end{aligned} \tag{3}$$

where $\Phi(\cdot)$ is the cumulative distribution function of standard normal distribution. From Equation (3), it is easy to see that the probability depends on the total order quantity. According to the extreme value theorem, the optimal total order quantity maximizing the probability can be obtained by solving $\mathbf{d}\Pr(z_t \geq k_t)/\mathbf{d}Q_t = 0$, i.e.,

$$\begin{aligned} Q_t^* = T_t &+ \frac{c_s(c_p + c_e)(c_p\sum_{i=1}^{h}\mu_i - k_t)}{c_p(c_p A + 2c_e c_s)} \\ &+ \sqrt{\left[\frac{c_s(c_p + c_e)(c_p\sum_{i=1}^{h}\mu_i - k_t)}{c_p(c_p A + 2c_e c_s)}\right]^2 + \frac{2c_s^2(c_p + c_e)^2\omega\sum_{i=1}^{h}\sigma_i^2}{c_p A(c_p A + 2c_e c_s)}} > T_t, \end{aligned} \tag{4}$$

where $A = c_p + c_e + c_s > 0$, $\omega = \ln(1 + c_p A/c_s c_e) > 0$, and $T_t = k_t/c_p$ is the target demand (i.e., the minimum demand required to achieve the target profit). Note that the optimal total order quantity exceeds the target demand, $Q_t^* > T_t$, which is in line with market intuition. The sufficient condition for maximizing the probability of achieving the target profit is also shown as follows:

$$\frac{\mathbf{d}^2\Pr(z_t \geq k_t)}{\mathbf{d}Q_t^2}\Big|_{Q_t = Q_t^*} = -\frac{(c_p + c_s)\exp\left\{-\frac{1}{2}\left[\frac{UAL_t(Q_t^*) - \mu_t}{\sum_{i=1}^h \sigma_i^2}\right]^2\right\}}{\sqrt{2\pi}\left(\sum_{i=1}^h \sigma_i^2\right)^3 c_s^2(c_p+c_e)} \times\left\{\frac{[UAL_t(Q_t^*) - LAL_t(Q_t^*)](c_pA + 2c_ec_s)}{2} + \frac{c_pA\omega\sum_{i=1}^h \sigma_i^2}{UAL_t(Q_t^*) - LAL_t(Q_t^*)}\right\} < 0. \tag{5}$$

Thus, the integrated profitability for a newsboy-type product (denoted by Ω_t) can be obtained by substituting Equation (4) into Equation (3), as follows:

$$\begin{aligned}
\Omega_t &= \Pr(Z_t > k_t | Q_t = Q_t^*) \\
&= \Phi\left\{\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{2(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}} + \sqrt{\left[\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{2(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}}\right]^2 + \frac{c_pA\omega}{2(c_pA + 2c_ec_s)}}\right. \\
&\quad \left. + \frac{\omega}{\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}} + 2\sqrt{\left[\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{2(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}}\right]^2 + \frac{c_pA\omega}{2(c_pA + 2c_ec_s)}}}\right\} \\
&\quad + \Phi\left\{-\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{2(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}} - \sqrt{\left[\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{2(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}}\right]^2 + \frac{c_pA\omega}{2(c_pA+2c_ec_s)}}\right. \\
&\quad \left. + \frac{\omega}{\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}} + 2\sqrt{\left[\frac{c_pA\left(\sum_{i=1}^h \mu_i - T_t\right)}{2(c_pA + 2c_ec_s)\sqrt{\sum_{i=1}^h \sigma_i^2}}\right]^2 + \frac{c_pA\omega}{2(c_pA + 2c_ec_s)}}}\right\}.
\end{aligned} \tag{6}$$

Based on the above equation, for given values of c_p, c_e, c_s, and T_t, the integrated profitability depends on a ratio term, and we refer to it as the integrated achievable capacity index (IACI, denoted by I_A^T), i.e.,

$$I_A^T = \frac{\sum_{i=1}^h \mu_i - T_t}{\sqrt{\sum_{i=1}^h \sigma_i^2}}. \tag{7}$$

Consequently, the integrated profitability in Equation (6) can be rearranged as a function of I_A^T (denoted by $\Omega_t(I_A^T)$). Note that the numerator of I_A^T indicates the difference between the mean total demand and target demand, while the denominator is the root of total variance. To the best of our knowledge, the value of I_A^T is proportional to the integrated profitability. Taking the first-order derivative of $\Omega_t(I_A^T)$ with respect to I_A^T, we obtain

$$\begin{aligned}
\frac{\mathbf{d}\Omega_t(I_A^T)}{\mathbf{d}I_A^T} &= \frac{A_1^2 I_A^T + A_1\sqrt{\left(A_1 I_A^T\right)^2 + A_1\omega}}{\sqrt{2\pi\left[A_1\left(I_A^T\right)^2 + A_1\omega\right]}}\left\{1 + e^{\omega} + \frac{\omega(e^{\omega}-1)}{2\left[A_1 I_A^T + \sqrt{\left(A_1 I_A^T\right)^2 + A_1\omega}\right]^2}\right\} \\
&\quad \times \exp\left\{-\frac{1}{2}\left[A_1 I_A^T + \sqrt{\left(A_1 I_A^T\right)^2 + A_1\omega} + \frac{1}{2A_1 I_A^T + 2\sqrt{\left(A_1 I_A^T\right)^2 + A_1\omega}}\right]^2\right\} > 0,
\end{aligned} \tag{8}$$

where $A_1 = c_pA(2c_pA + 4c_ec_s)^{-1} > 0$. This implies that $\Omega_t(I_A^T)$ is a monotonic increasing function of I_A^T. Note that the simplicity of IACI makes it an efficient measure of integrated profitability.

2.2. Statistical Properties of IACI

In situations where the demand patterns of all channels (i.e., μ_i and σ_i^2) are unknown, historical demand data must be collected from all channels to estimate the true I_A^T. Based on the sample data set for h channels of size n (denoted as $\mathbf{x} = \{x_{ij} | i = 1, 2, \ldots, h; j = 1, 2, \ldots, n\}$), we respectively replace the unknown μ_i and σ_i^2 with their unbiased estimators $\overline{x}_i = \sum_{j=1}^{n} x_{ij}/n$ and $s_i^2 = \sum_{j=1}^{n} (x_{ij} - \overline{x}_i)^2/(n-1)$ to obtain the nature estimator of I_A^T (denoted as $\hat{I}_A^T$), as follows:

$$\hat{I}_A^T = \frac{\sum_{i=1}^{h} \overline{x}_i - T_t}{\sqrt{\sum_{i=1}^{h} s_i^2}}. \tag{9}$$

After rearranging Equation (9), we obtain the following:

$$\hat{I}_A^T = \frac{\frac{\sum_{i=1}^{h} \overline{x}_i - \sum_{i=1}^{h} \mu_i}{\sqrt{\sum_{i=1}^{h} \sigma_i^2}} + \frac{\sum_{i=1}^{h} \mu_i - T_t}{\sqrt{\sum_{i=1}^{h} \sigma_i^2}}}{\sqrt{\frac{\sum_{i=1}^{h} s_i^2}{\sum_{i=1}^{h} \sigma_i^2}}} = \frac{\frac{\sum_{i=1}^{h} \overline{x}_i - \sum_{i=1}^{h} \mu_i}{\sqrt{\sum_{i=1}^{h} \sigma_i^2}} + I_A^T}{\sqrt{\frac{\sum_{i=1}^{h} s_i^2}{\sum_{i=1}^{h} \sigma_i^2}}}. \tag{10}$$

In the above equation, the denominator cannot be presented as a chi-square random variable, due to unequal variance in demand. This means that the form of $\hat{I}_A^T$ cannot be further simplified as a common estimator. In this study, we first explored a case involving identical variance for all channels, i.e., $\sigma_1 = \sigma_2 = \ldots = \sigma_h = \sigma$. Thus, we obtain the following:

$$\begin{aligned} \hat{I}_A^T &= \frac{\frac{\sum_{i=1}^{h} \overline{x}_i - \sum_{i=1}^{h} \mu_i}{\sqrt{h\sigma^2}} + I_A^T}{\sqrt{\frac{1}{h}\sum_{i=1}^{h} \frac{s_i^2}{\sigma^2}}} = \frac{\frac{\sum_{i=1}^{h} \overline{x}_i - \sum_{i=1}^{h} \mu_i}{\sqrt{h\sigma^2/n}} + \sqrt{n} I_A^T}{\sqrt{\frac{n}{h}\sum_{i=1}^{h} \frac{s_i^2}{\sigma^2}}} \\ &= \frac{Z + \sqrt{n} I_A^T}{\sqrt{\frac{n}{h(n-1)}\sum_{i=1}^{h} \frac{(n-1)s_i^2}{\sigma^2}}} = \frac{1}{\sqrt{n}} \frac{Z + \theta}{\sqrt{\frac{\chi^2_{h(n-1)}}{h(n-1)}}} = \frac{1}{\sqrt{n}} t_{h(n-1)}(\theta), \end{aligned} \tag{11}$$

where $Z = (\sum_{i=1}^{h} \overline{x}_i - \sum_{i=1}^{h} \mu_i)(h\sigma^2/n)^{-1/2}$ is a standard normal random variable, $\chi^2_{h(n-1)} = \sum_{i=1}^{h} (n-1)s_i^2/\sigma^2$ is a chi-square random variable with $h(n-1)$ degrees of freedom, and $t_{h(n-1)}(\theta)$ is a noncentral t random variable with the noncentrality parameter $\theta = \sqrt{n} I_A^T$ and $h(n-1)$ degrees of freedom. Considering that $\hat{I}_A^T$ is a biased estimator (i.e., $E(\hat{I}_A^T) = (\sqrt{h(n-1)/2}\Gamma((h(n-1)-1)/2)/\Gamma(h(n-1)/2)) \times I_A^T \neq I_A^T$), we define $b_t = \sqrt{2/h(n-1)}\Gamma(h(n-1)/2)/\Gamma((h(n-1)-1)/2)$ as a correction factor. This allows us to obtain an unbiased estimator (denoted as $\widetilde{I}_A^T$) by multiplying b_t with $\hat{I}_A^T$ (i.e., $\widetilde{I}_A^T = b_t \hat{I}_A^T$).

2.3. Multiple Samples: Discussion

Market information related to demand can be obtained from multiple samples, rather than a single sample. For example, demand records for all channels could be collected weekly, monthly, or periodically. For multiple samples that include m groups of size n in h channels ($\mathbf{x}' = \{x'_{ilj} | i = 1, 2, \ldots, h; l = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$), the nature estimator of I_A^T based on this group of samples (denoted as $\hat{I}'^T_A$) can be defined as follows:

$$\hat{I}'^T_A = \frac{\sum_{i=1}^{h} \overline{\overline{x}}'_i - T_t}{\sqrt{\sum_{i=1}^{h} s'^2_i}}, \tag{12}$$

where $\overline{x}'_{il} = \sum_{j=1}^{n} x'_{ilj}/n$, $\overline{\overline{x}}'_{i} = \sum_{l=1}^{m} \overline{x}'_{il}/m$, $s'^2_{il} = \sum_{j=1}^{n} \left(x'_{ilj} - \overline{x}'_{il}\right)^2/(n-1)$, and $s'^2_i = \sum_{l=1}^{m} s'^2_{il}/m$. We can further derive $\hat{I}'^T_A$ as follows:

$$\begin{aligned}\hat{I}'^T_A &= \frac{\frac{\sum_{i=1}^{h}\overline{\overline{x}}'_i - \sum_{i=1}^{h}\mu_i}{\sqrt{h\sigma^2}} + I^T_A}{\sqrt{\frac{1}{h}\sum_{i=1}^{h}\frac{s'^2_i}{\sigma^2}}} = \frac{\frac{\sum_{i=1}^{h}\overline{\overline{x}}'_i - \sum_{i=1}^{h}\mu_i}{\sqrt{h\sigma^2/mn}} + \sqrt{mn}I^T_A}{\sqrt{\frac{mn}{h}\sum_{i=1}^{h}\frac{s'^2_i}{\sigma^2}}} \\ &= \frac{Z' + \sqrt{mn}I^T_A}{\sqrt{\frac{mn}{hm(n-1)}\sum_{i=1}^{h}\frac{m(n-1)s'^2_i}{\sigma^2}}} = \frac{1}{\sqrt{mn}}\frac{Z' + \sqrt{mn}I^T_A}{\sqrt{\frac{\chi^2_{hm(n-1)}}{hm(n-1)}}} = \frac{1}{\sqrt{mn}}t_{hm(n-1)}(\theta'),\end{aligned} \tag{13}$$

where $Z' = \left(\sum_{i=1}^{h}\overline{\overline{x}}'_i - \sum_{i=1}^{h}\mu_i\right)\left(h\sigma^2/mn\right)^{-1/2}$ is a standard normal random variable, $\chi^2_{hm(n-1)} = \sum_{i=1}^{h} m(n-1)s'^2_i/\sigma^2$ is a chi-square random variable with $hm(n-1)$ degrees of freedom, and $t_{hm(n-1)}(\theta)$ is a noncentral t random variable with the noncentrality parameter $\theta' = \sqrt{mn}I^T_A$ and $hm(n-1)$ degrees of freedom. Similarly, we define a correction factor, $b'_t = \sqrt{2/hm(n-1)}\Gamma(hm(n-1)/2)/\Gamma((hm(n-1)-1)/2)$. This makes it possible to obtain an unbiased estimator (denoted as $\widetilde{I}'^T_A$) after multiplying b'_t with $\hat{I}'^T_A$ (i.e., $\widetilde{I}'^T_A = b'_t\hat{I}'^T_A$).

3. Evaluating Integrated Profitability Using IACI

In addition to measuring the integrated profitability for all channels, the IACI can also be used by OBM managers to assess whether the integrated profitability meets a stipulated minimum level (denoted as $I^{T(R)}_A$); i.e., to determine if $I^T_A \geq I^{T(R)}_A$. The results of this assessment can then be used by the OBM manager to guide decision-making. For example, if $I^T_A \geq I^{T(R)}_A$, then the product is deemed profitable according to the standards set by the OBM manager, indicating that it is worth continuing to produce and sell. If the product fails to meet this standard, then the OBM managers should consider discontinuing production or reducing volumes or the number of sales points.

If the demand patterns of all channels (i.e., μ_i and σ^2_i) are unknown, then an evaluation of integrated profitability based on statistical inference must be performed. In the current study, we present the following statistical hypothesis test: $H_0: \ I^T_A \leq I^{T(R)}_A$ *versus* $H_A: \ I^T_A > I^{T(R)}_A$. This analysis focuses on a critical value (denoted as c_0) with stipulated Type-I error α (i.e., the chance of incorrectly judging $I^T_A \leq I^{T(R)}_A$ as $I^T_A > I^{T(R)}_A$). Based on historical demand data from all channels, the null hypothesis is rejected if the observed value of the statistic $\widetilde{I}^T_A$ exceeds the critical value (i.e., estimate $\widetilde{I}^T_A$ falls into the rejection region, $[c_0, \infty)$). This implies that the integrated profitability meets the stipulated minimum level with a confidence level of $(1-\alpha) \times 100\%$. Since $\widetilde{I}^T_A$ is distributed as $b_t n^{-1/2} t_{h(n-1)}(\theta)$, the value of c_0 can be obtained by solving the following equation:

$$\begin{aligned}\alpha &= P\left\{\widetilde{I}^T_A \geq c_0 \middle| I^T_A = I^{T(R)}_A\right\} \\ &= P\left\{\frac{b_t}{\sqrt{n}}t_{h(n-1)}(\theta) \geq c_0 \middle| I^T_A = I^{T(R)}_A\right\} = P\left\{t_{h(n-1)}(\theta) \geq \frac{\sqrt{n}}{b_t}c_0 \middle| I^T_A = I^{T(R)}_A\right\}.\end{aligned} \tag{14}$$

For the sake of simplicity, we denote $qt_{h(n-1),\,\alpha}(\sqrt{n}I^{T(R)}_A)$ as the upper α quantile of a noncentral t distribution with the noncentrality parameter $\sqrt{n}I^{T(R)}_A$ and $h(n-1)$ degrees of freedom, while satisfying $\alpha = P\left\{t_{h(n-1)}(\sqrt{n}I^{T(R)}_A) \geq qt_{h(n-1),\,\alpha}(\sqrt{n}I^{T(R)}_A)\right\}$. Thus, Equation (14) becomes

$$qt_{h(n-1),\,\alpha}(\sqrt{n}I^{T(R)}_A) = \frac{\sqrt{n}}{b_t}c_0. \tag{15}$$

After rearranging Equation (15), we obtain a closed form of the critical value as follows:

$$c_0 = b_t \frac{qt_{h(n-1),\,\alpha}\left(\sqrt{n} I_A^{T(R)}\right)}{\sqrt{n}}. \tag{16}$$

Tables 1–9 tabulate the critical values for various sample sizes and stipulated minimum levels as $\alpha = 0.1,\ 0.05,\ 0.01$ and $h = 5,\ 10,\ 15$. Note that these tables are applicable to any decision-making situation involving the evaluation of integrated profitability in OBM firms, simply by calculating the value of $\widetilde{I}_A^T$ and comparing it with the critical value in the corresponding table. For example, consider an estimate $\widetilde{I}_A^T = 1.5$, which is based on a sample data set involving $h = 10$ channels of size $n = 100$. If the stipulated minimum profit is $I_A^{T(R)} = 1.4$ and the confidence level is 95%, then the corresponding critical value ($c_0 = 1.5737$) can be found in Table 5. Since $\widetilde{I}_A^T = 1.5 < c_0 = 1.5737$, the null hypothesis is accepted, which means that the integrated profitability fails to meet the stipulated minimum profit with a 95% confidence level. Figure 1 also plots the three-dimensional critical value graphs of sample sizes and stipulated minimum profit with $\alpha = 0.1,\ 0.05,\ 0.01$ and $h = 5,\ 10,\ 15$. From Tables 1–9 and Figure 1, we derived the following observations:

(1) If the values of $I_A^{T(R)}$, h, and α were fixed, then the critical value would decrease with an increase in sample size n. This means that an increase in sample size would widen the rejection region of the test (i.e., a higher likelihood of $\widetilde{I}_A^T$ falling into the rejection region). Note, however, that increasing the sample size would not necessarily increase the probability of rejecting H_0, due to the fact that estimate $\widetilde{I}_A^T$ could change.

(2) If the values of n, h, and α were fixed, then the critical value would increase with an increase in the stipulated minimum profit $I_A^{T(R)}$. This means that increasing the stipulated minimum profit would reduce the width of the rejection region and thereby reduce the likelihood of rejecting the null hypothesis. This is in line with intuition and statistical inference.

Table 1. Critical values for various values of n and $I_A^{T(R)}$ as $h = 5$ and $\alpha = 0.1$.

n \ $I_A^{T(R)}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
10	1.3977	1.4982	1.6444	1.7503	1.8566	1.9632	2.0702	2.1774	2.2849	2.3928	2.5009
20	1.2901	1.3913	1.5113	1.6152	1.7192	1.8235	1.9281	2.0328	2.1377	2.2428	2.3481
30	1.2396	1.341	1.4533	1.5564	1.6596	1.7630	1.8665	1.9703	2.0742	2.1783	2.2825
40	1.2088	1.3102	1.4190	1.5215	1.6243	1.7272	1.8302	1.9334	2.0367	2.1402	2.2438
50	1.1875	1.2888	1.3956	1.4979	1.6003	1.7029	1.8056	1.9084	2.0113	2.1144	2.2176
60	1.1716	1.2729	1.3784	1.4805	1.5827	1.6850	1.7874	1.8900	1.9926	2.0954	2.1983
70	1.1592	1.2604	1.3651	1.4670	1.5690	1.6711	1.7734	1.8757	1.9782	2.0807	2.1834
80	1.1491	1.2503	1.3543	1.4561	1.5580	1.6599	1.7620	1.8642	1.9665	2.0689	2.1714
90	1.1408	1.2419	1.3454	1.4471	1.5488	1.6507	1.7527	1.8547	1.9569	2.0591	2.1615
100	1.1337	1.2348	1.3379	1.4395	1.5411	1.6429	1.7448	1.8467	1.9488	2.0509	2.1531
110	1.1275	1.2286	1.3315	1.4329	1.5345	1.6362	1.7380	1.8398	1.9418	2.0438	2.1459
120	1.1222	1.2232	1.3258	1.4272	1.5288	1.6304	1.7321	1.8338	1.9357	2.0376	2.1396
130	1.1174	1.2185	1.3209	1.4222	1.5237	1.6252	1.7268	1.8285	1.9303	2.0322	2.1341
140	1.1132	1.2142	1.3164	1.4177	1.5191	1.6206	1.7222	1.8238	1.9255	2.0273	2.1292
150	1.1094	1.2104	1.3125	1.4137	1.5151	1.6165	1.7180	1.8196	1.9212	2.0230	2.1248
160	1.1060	1.2069	1.3089	1.4101	1.5114	1.6128	1.7142	1.8158	1.9174	2.0190	2.1208
170	1.1028	1.2037	1.3056	1.4068	1.5081	1.6094	1.7108	1.8123	1.9138	2.0154	2.1171
180	1.1000	1.2009	1.3026	1.4038	1.5050	1.6063	1.7077	1.8091	1.9106	2.0122	2.1138
190	1.0973	1.1982	1.2999	1.4010	1.5022	1.6034	1.7048	1.8062	1.9076	2.0092	2.1107
200	1.0949	1.1957	1.2973	1.3984	1.4996	1.6008	1.7021	1.8035	1.9049	2.0064	2.1079

Table 2. Critical values for various values of n and $I_A^{T(R)}$ as $h = 10$ and $\alpha = 0.1$.

n \ $I_A^{T(R)}$	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2.0**
10	1.4016	1.5018	1.6251	1.7281	1.8314	1.9349	2.0385	2.1423	2.2463	2.3505	2.4548
20	1.2883	1.3890	1.4991	1.6011	1.7032	1.8055	1.9078	2.0103	2.1129	2.2157	2.3185
30	1.2368	1.3375	1.4438	1.5454	1.6470	1.7488	1.8507	1.9527	2.0547	2.1569	2.2592
40	1.2058	1.3065	1.4109	1.5123	1.6137	1.7152	1.8168	1.9185	2.0202	2.1221	2.2240
50	1.1844	1.2851	1.3885	1.4897	1.5910	1.6923	1.7937	1.8952	1.9968	2.0984	2.2001
60	1.1686	1.2692	1.3720	1.4731	1.5742	1.6754	1.7767	1.8781	1.9795	2.0810	2.1825
70	1.1562	1.2569	1.3592	1.4602	1.5612	1.6624	1.7635	1.8648	1.9661	2.0675	2.1689
80	1.1462	1.2468	1.3489	1.4498	1.5508	1.6518	1.7529	1.8541	1.9553	2.0566	2.1579
90	1.1380	1.2385	1.3403	1.4412	1.5421	1.6431	1.7441	1.8452	1.9464	2.0476	2.1488
100	1.1309	1.2315	1.3331	1.4339	1.5348	1.6357	1.7367	1.8377	1.9388	2.0399	2.1411
110	1.1249	1.2254	1.3269	1.4277	1.5285	1.6294	1.7303	1.8313	1.9323	2.0334	2.1345
120	1.1196	1.2201	1.3215	1.4222	1.5230	1.6238	1.7247	1.8257	1.9267	2.0277	2.1288
130	1.1149	1.2155	1.3167	1.4174	1.5182	1.6190	1.7198	1.8207	1.9217	2.0227	2.1237
140	1.1108	1.2113	1.3124	1.4131	1.5138	1.6146	1.7154	1.8163	1.9172	2.0182	2.1192
150	1.1071	1.2075	1.3086	1.4093	1.5100	1.6107	1.7115	1.8123	1.9132	2.0141	2.1151
160	1.1037	1.2041	1.3052	1.4058	1.5065	1.6072	1.7080	1.8088	1.9096	2.0105	2.1114
170	1.1006	1.2011	1.3020	1.4026	1.5033	1.6040	1.7047	1.8055	1.9063	2.0072	2.1081
180	1.0978	1.1982	1.2991	1.3997	1.5004	1.6010	1.7018	1.8025	1.9033	2.0042	2.1050
190	1.0952	1.1956	1.2965	1.3971	1.4977	1.5983	1.6990	1.7998	1.9006	2.0014	2.1022
200	1.0928	1.1932	1.2940	1.3946	1.4952	1.5958	1.6965	1.7972	1.8980	1.9988	2.0996

Table 3. Critical values for various values of n and $I_A^{T(R)}$ as $h = 15$ and $\alpha = 0.1$.

n \ $I_A^{T(R)}$	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2.0**
10	1.4028	1.5030	1.6185	1.7206	1.8228	1.9251	2.0276	2.1302	2.2329	2.3358	2.4388
20	1.2878	1.3882	1.4950	1.5963	1.6977	1.7993	1.9009	2.0026	2.1044	2.2062	2.3082
30	1.2359	1.3364	1.4406	1.5416	1.6427	1.7439	1.8452	1.9466	2.0480	2.1495	2.2510
40	1.2047	1.3052	1.4082	1.5091	1.6100	1.7111	1.8122	1.9133	2.0145	2.1158	2.2171
50	1.1834	1.2838	1.3861	1.4869	1.5878	1.6887	1.7896	1.8907	1.9917	2.0928	2.1940
60	1.1675	1.2680	1.3699	1.4706	1.5713	1.6722	1.7730	1.8740	1.9749	2.0759	2.1770
70	1.1552	1.2556	1.3572	1.4579	1.5586	1.6593	1.7601	1.8610	1.9619	2.0628	2.1638
80	1.1453	1.2457	1.3470	1.4477	1.5483	1.6490	1.7498	1.8506	1.9514	2.0523	2.1532
90	1.1370	1.2374	1.3386	1.4392	1.5398	1.6405	1.7412	1.8419	1.9427	2.0435	2.1444
100	1.1300	1.2304	1.3315	1.4320	1.5326	1.6332	1.7339	1.8346	1.9353	2.0361	2.1369
110	1.1240	1.2244	1.3253	1.4259	1.5264	1.6270	1.7276	1.8283	1.9290	2.0298	2.1305
120	1.1187	1.2191	1.3200	1.4205	1.5210	1.6216	1.7222	1.8228	1.9235	2.0242	2.1250
130	1.1141	1.2145	1.3153	1.4158	1.5163	1.6168	1.7174	1.8180	1.9186	2.0193	2.1200
140	1.1100	1.2103	1.3111	1.4115	1.5120	1.6126	1.7131	1.8137	1.9143	2.0150	2.1157
150	1.1063	1.2066	1.3073	1.4077	1.5082	1.6087	1.7093	1.8098	1.9104	2.0111	2.1117
160	1.1029	1.2032	1.3039	1.4043	1.5048	1.6053	1.7058	1.8063	1.9069	2.0075	2.1082
170	1.0998	1.2001	1.3008	1.4012	1.5016	1.6021	1.7026	1.8032	1.9037	2.0043	2.1049
180	1.0970	1.1973	1.2979	1.3983	1.4988	1.5992	1.6997	1.8002	1.9008	2.0014	2.1020
190	1.0944	1.1947	1.2953	1.3957	1.4961	1.5966	1.6971	1.7976	1.8981	1.9986	2.0992
200	1.0921	1.1924	1.2929	1.3933	1.4937	1.5941	1.6946	1.7951	1.8956	1.9961	2.0967

Table 4. Critical values for various values of n and $I_A^{T(R)}$ as $h = 5$ and $\alpha = 0.05$.

n \ $I_A^{T(R)}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
10	1.5177	1.6190	1.7856	1.8942	2.0033	2.1127	2.2226	2.3329	2.4435	2.5545	2.6659
20	1.3762	1.4781	1.6062	1.7116	1.8173	1.9233	2.0295	2.1360	2.2428	2.3498	2.4570
30	1.3103	1.4123	1.5293	1.6335	1.7379	1.8426	1.9475	2.0526	2.1579	2.2634	2.3690
40	1.2701	1.3721	1.4841	1.5876	1.6913	1.7953	1.8994	2.0037	2.1082	2.2128	2.3177
50	1.2424	1.3442	1.4535	1.5565	1.6598	1.7633	1.8669	1.9707	2.0747	2.1788	2.2830
60	1.2217	1.3235	1.4309	1.5337	1.6367	1.7398	1.8431	1.9465	2.0501	2.1538	2.2576
70	1.2056	1.3073	1.4135	1.5161	1.6188	1.7216	1.8246	1.9278	2.0311	2.1345	2.2380
80	1.1925	1.2941	1.3995	1.5019	1.6044	1.7070	1.8098	1.9128	2.0158	2.1190	2.2223
90	1.1816	1.2832	1.3879	1.4901	1.5925	1.6950	1.7976	1.9004	2.0032	2.1062	2.2093
100	1.1724	1.2739	1.3781	1.4802	1.5825	1.6848	1.7873	1.8899	1.9926	2.0954	2.1983
110	1.1645	1.2659	1.3697	1.4717	1.5739	1.6761	1.7784	1.8809	1.9835	2.0861	2.1889
120	1.1575	1.2590	1.3624	1.4643	1.5663	1.6685	1.7707	1.8731	1.9755	2.0781	2.1807
130	1.1514	1.2528	1.3560	1.4578	1.5597	1.6618	1.7639	1.8662	1.9685	2.0710	2.1735
140	1.1459	1.2473	1.3502	1.4520	1.5538	1.6558	1.7579	1.8600	1.9623	2.0646	2.1671
150	1.1410	1.2423	1.3451	1.4468	1.5486	1.6505	1.7524	1.8545	1.9567	2.0590	2.1613
160	1.1366	1.2378	1.3404	1.4421	1.5438	1.6456	1.7475	1.8495	1.9516	2.0538	2.1561
170	1.1325	1.2337	1.3362	1.4378	1.5394	1.6412	1.7431	1.8450	1.9471	2.0492	2.1514
180	1.1288	1.2300	1.3323	1.4339	1.5355	1.6372	1.7390	1.8409	1.9429	2.0449	2.1470
190	1.1254	1.2265	1.3288	1.4302	1.5318	1.6335	1.7352	1.8371	1.9390	2.0410	2.1431
200	1.1222	1.2233	1.3255	1.4269	1.5284	1.6301	1.7318	1.8336	1.9354	2.0374	2.1394

Table 5. Critical values for various values of n and $I_A^{T(R)}$ as $h = 10$ and $\alpha = 0.05$.

n \ $I_A^{T(R)}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
10	1.5190	1.6197	1.7531	1.8575	1.9622	2.0671	2.1722	2.2776	2.3833	2.4891	2.5951
20	1.3720	1.4730	1.5872	1.6900	1.7929	1.8961	1.9993	2.1027	2.2063	2.3101	2.4139
30	1.3053	1.4064	1.5150	1.6172	1.7195	1.8219	1.9244	2.0271	2.1299	2.2329	2.3359
40	1.2651	1.3661	1.4722	1.5741	1.6760	1.7781	1.8802	1.9825	2.0849	2.1874	2.2899
50	1.2375	1.3385	1.4432	1.5448	1.6465	1.7483	1.8502	1.9522	2.0543	2.1565	2.2588
60	1.2171	1.3180	1.4218	1.5232	1.6248	1.7264	1.8281	1.9299	2.0318	2.1338	2.2359
70	1.2011	1.3020	1.4052	1.5065	1.6079	1.7094	1.8110	1.9126	2.0144	2.1162	2.2181
80	1.1882	1.2891	1.3918	1.4930	1.5944	1.6957	1.7972	1.8987	2.0004	2.1021	2.2038
90	1.1776	1.2783	1.3808	1.4819	1.5831	1.6844	1.7858	1.8873	1.9888	2.0904	2.1920
100	1.1685	1.2693	1.3714	1.4725	1.5737	1.6749	1.7762	1.8776	1.9790	2.0805	2.1821
110	1.1607	1.2614	1.3634	1.4644	1.5655	1.6667	1.7679	1.8692	1.9706	2.0720	2.1735
120	1.1539	1.2546	1.3564	1.4574	1.5584	1.6595	1.7607	1.8619	1.9632	2.0646	2.1660
130	1.1479	1.2486	1.3502	1.4512	1.5522	1.6532	1.7543	1.8555	1.9568	2.0581	2.1595
140	1.1425	1.2432	1.3447	1.4456	1.5466	1.6476	1.7487	1.8498	1.9510	2.0523	2.1536
150	1.1377	1.2384	1.3398	1.4407	1.5416	1.6426	1.7436	1.8447	1.9459	2.0471	2.1483
160	1.1333	1.2340	1.3353	1.4362	1.5371	1.6380	1.7390	1.8401	1.9412	2.0424	2.1436
170	1.1294	1.2300	1.3313	1.4321	1.5329	1.6339	1.7348	1.8359	1.9370	2.0381	2.1393
180	1.1257	1.2263	1.3275	1.4283	1.5292	1.6301	1.7310	1.8320	1.9331	2.0342	2.1353
190	1.1224	1.2230	1.3241	1.4249	1.5257	1.6266	1.7275	1.8285	1.9295	2.0306	2.1317
200	1.1193	1.2199	1.3210	1.4217	1.5225	1.6234	1.7243	1.8252	1.9262	2.0272	2.1283

Table 6. Critical values for various values of n and $I_A^{T(R)}$ as $h = 15$ and $\alpha = 0.05$.

n \ $I_A^{T(R)}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
10	1.5194	1.6199	1.7422	1.8451	1.9483	2.0516	2.1551	2.2588	2.3626	2.4666	2.5707
20	1.3706	1.4713	1.5808	1.6827	1.7847	1.8868	1.9890	2.0913	2.1937	2.2963	2.3989
30	1.3037	1.4044	1.5101	1.6116	1.7132	1.8148	1.9165	2.0184	2.1203	2.2223	2.3244
40	1.2635	1.3641	1.4682	1.5695	1.6708	1.7721	1.8736	1.9752	2.0768	2.1785	2.2803
50	1.2359	1.3365	1.4397	1.5408	1.6419	1.7431	1.8444	1.9458	2.0472	2.1487	2.2503
60	1.2155	1.3161	1.4187	1.5196	1.6207	1.7218	1.8229	1.9242	2.0255	2.1268	2.2282
70	1.1996	1.3002	1.4023	1.5032	1.6042	1.7052	1.8063	1.9074	2.0086	2.1098	2.2111
80	1.1868	1.2874	1.3892	1.4900	1.5909	1.6919	1.7929	1.8939	1.9950	2.0962	2.1974
90	1.1762	1.2767	1.3783	1.4791	1.5799	1.6808	1.7817	1.8827	1.9838	2.0849	2.1860
100	1.1672	1.2677	1.3691	1.4699	1.5706	1.6715	1.7724	1.8733	1.9743	2.0753	2.1764
110	1.1594	1.2599	1.3612	1.4619	1.5627	1.6635	1.7643	1.8652	1.9661	2.0671	2.1681
120	1.1527	1.2531	1.3543	1.4550	1.5557	1.6565	1.7573	1.8581	1.9590	2.0599	2.1609
130	1.1467	1.2471	1.3483	1.4489	1.5496	1.6503	1.7511	1.8519	1.9527	2.0536	2.1545
140	1.1414	1.2418	1.3428	1.4434	1.5441	1.6448	1.7455	1.8463	1.9471	2.0480	2.1489
150	1.1366	1.2370	1.3380	1.4386	1.5392	1.6399	1.7406	1.8413	1.9421	2.0429	2.1438
160	1.1322	1.2327	1.3336	1.4341	1.5348	1.6354	1.7361	1.8368	1.9376	2.0384	2.1392
170	1.1283	1.2287	1.3296	1.4301	1.5307	1.6313	1.7320	1.8327	1.9334	2.0342	2.1350
180	1.1247	1.2251	1.3259	1.4264	1.5270	1.6276	1.7283	1.8289	1.9297	2.0304	2.1312
190	1.1214	1.2218	1.3225	1.4231	1.5236	1.6242	1.7248	1.8255	1.9262	2.0269	2.1277
200	1.1183	1.2187	1.3194	1.4199	1.5205	1.6210	1.7217	1.8223	1.9230	2.0237	2.1244

Table 7. Critical values for various values of n and $I_A^{T(R)}$ as $h = 5$ and $\alpha = 0.01$.

n \ $I_A^{T(R)}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
10	1.7474	1.8504	2.0683	2.1828	2.2979	2.4137	2.5300	2.6469	2.7643	2.8822	3.0006
20	1.5407	1.6442	1.7913	1.9000	2.0091	2.1186	2.2284	2.3387	2.4493	2.5603	2.6715
30	1.4450	1.5483	1.6762	1.7828	1.8897	1.9970	2.1046	2.2124	2.3206	2.4290	2.5377
40	1.3868	1.4900	1.6093	1.7148	1.8205	1.9266	2.0329	2.1395	2.2464	2.3534	2.4607
50	1.3467	1.4497	1.5643	1.6691	1.7741	1.8794	1.9849	2.0907	2.1966	2.3028	2.4092
60	1.3169	1.4197	1.5314	1.6357	1.7402	1.8449	1.9499	2.0550	2.1604	2.2660	2.3717
70	1.2936	1.3963	1.5060	1.6099	1.7140	1.8183	1.9229	2.0276	2.1325	2.2376	2.3429
80	1.2748	1.3773	1.4856	1.5892	1.6930	1.7970	1.9012	2.0056	2.1102	2.2149	2.3198
90	1.2592	1.3616	1.4688	1.5722	1.6758	1.7795	1.8834	1.9875	2.0917	2.1962	2.3007
100	1.2459	1.3482	1.4547	1.5578	1.6612	1.7647	1.8684	1.9722	2.0763	2.1804	2.2847
110	1.2345	1.3367	1.4425	1.5455	1.6487	1.7520	1.8555	1.9592	2.0630	2.1669	2.2710
120	1.2245	1.3267	1.4320	1.5348	1.6378	1.7410	1.8443	1.9478	2.0514	2.1552	2.2591
130	1.2157	1.3178	1.4227	1.5254	1.6283	1.7313	1.8345	1.9378	2.0413	2.1449	2.2486
140	1.2079	1.3099	1.4144	1.5170	1.6198	1.7227	1.8257	1.9289	2.0322	2.1357	2.2393
150	1.2008	1.3028	1.4070	1.5095	1.6121	1.7149	1.8179	1.9209	2.0241	2.1275	2.2309
160	1.1945	1.2963	1.4003	1.5027	1.6053	1.7080	1.8108	1.9138	2.0168	2.1200	2.2234
170	1.1886	1.2905	1.3942	1.4965	1.5990	1.7016	1.8043	1.9072	2.0102	2.1133	2.2165
180	1.1833	1.2851	1.3886	1.4909	1.5933	1.6958	1.7985	1.9012	2.0041	2.1071	2.2103
190	1.1784	1.2802	1.3835	1.4857	1.5880	1.6905	1.7930	1.8957	1.9986	2.1015	2.2045
200	1.1739	1.2756	1.3788	1.4809	1.5832	1.6855	1.7881	1.8907	1.9934	2.0963	2.1992

Table 8. Critical values for various values of n and $I_A^{T(R)}$ as $h = 10$ and $\alpha = 0.01$.

n \ $\boldsymbol{I_A^{T(R)}}$	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2.0**
10	1.7417	1.8432	2.0017	2.1090	2.2168	2.3249	2.4333	2.5420	2.6511	2.7605	2.8702
20	1.5305	1.6323	1.7559	1.8604	1.9651	2.0700	2.1751	2.2805	2.3861	2.4919	2.5979
30	1.4349	1.5366	1.6507	1.7541	1.8577	1.9614	2.0654	2.1695	2.2738	2.3783	2.4830
40	1.3774	1.4790	1.5888	1.6916	1.7946	1.8977	2.0010	2.1045	2.2081	2.3119	2.4158
50	1.3379	1.4394	1.5468	1.6493	1.7519	1.8547	1.9576	2.0606	2.1637	2.2670	2.3705
60	1.3087	1.4101	1.5160	1.6182	1.7206	1.8231	1.9257	2.0284	2.1312	2.2342	2.3373
70	1.2859	1.3872	1.4922	1.5942	1.6963	1.7986	1.9010	2.0035	2.1061	2.2088	2.3116
80	1.2675	1.3688	1.4730	1.5749	1.6769	1.7789	1.8811	1.9834	2.0859	2.1884	2.2910
90	1.2522	1.3535	1.4572	1.5589	1.6608	1.7627	1.8648	1.9669	2.0692	2.1715	2.2740
100	1.2393	1.3405	1.4438	1.5454	1.6472	1.7490	1.8509	1.9530	2.0551	2.1573	2.2596
110	1.2282	1.3293	1.4323	1.5339	1.6355	1.7372	1.8391	1.9410	2.0430	2.1451	2.2473
120	1.2185	1.3196	1.4223	1.5238	1.6253	1.7270	1.8287	1.9306	2.0325	2.1345	2.2366
130	1.2099	1.3110	1.4135	1.5149	1.6164	1.7180	1.8196	1.9214	2.0232	2.1251	2.2271
140	1.2023	1.3033	1.4056	1.5070	1.6084	1.7099	1.8115	1.9132	2.0150	2.1168	2.2187
150	1.1954	1.2964	1.3986	1.4999	1.6013	1.7027	1.8043	1.9059	2.0076	2.1094	2.2112
160	1.1892	1.2902	1.3922	1.4935	1.5948	1.6962	1.7977	1.8993	2.0009	2.1026	2.2044
170	1.1836	1.2845	1.3864	1.4876	1.5889	1.6903	1.7917	1.8932	1.9948	2.0965	2.1982
180	1.1784	1.2793	1.3811	1.4823	1.5835	1.6849	1.7862	1.8877	1.9893	2.0909	2.1925
190	1.1736	1.2745	1.3762	1.4774	1.5786	1.6799	1.7812	1.8826	1.9841	2.0857	2.1873
200	1.1692	1.2701	1.3717	1.4728	1.5740	1.6753	1.7766	1.8780	1.9794	2.0809	2.1825

Table 9. Critical values for various values of n and $I_A^{T(R)}$ as $h = 15$ and $\alpha = 0.01$.

n \ $\boldsymbol{I_A^{T(R)}}$	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2.0**
10	1.7397	1.8407	1.9796	2.0846	2.1898	2.2952	2.4009	2.5068	2.6130	2.7194	2.8260
20	1.5271	1.6283	1.7441	1.8471	1.9502	2.0535	2.1570	2.2607	2.3645	2.4684	2.5725
30	1.4315	1.5327	1.6421	1.7444	1.8468	1.9493	2.0520	2.1548	2.2578	2.3608	2.4640
40	1.3742	1.4753	1.5818	1.6837	1.7858	1.8879	1.9901	2.0925	2.1950	2.2975	2.4002
50	1.3350	1.4360	1.5409	1.6426	1.7444	1.8462	1.9482	2.0502	2.1524	2.2546	2.3570
60	1.3059	1.4069	1.5108	1.6123	1.7139	1.8156	1.9173	2.0192	2.1211	2.2232	2.3253
70	1.2833	1.3842	1.4875	1.5889	1.6903	1.7918	1.8935	1.9951	2.0969	2.1988	2.3007
80	1.2650	1.3659	1.4688	1.5700	1.6713	1.7728	1.8742	1.9758	2.0775	2.1792	2.2810
90	1.2499	1.3507	1.4532	1.5544	1.6556	1.7570	1.8584	1.9598	2.0614	2.1630	2.2646
100	1.2371	1.3379	1.4401	1.5412	1.6424	1.7436	1.8449	1.9463	2.0478	2.1493	2.2509
110	1.2261	1.3268	1.4288	1.5299	1.6310	1.7322	1.8334	1.9347	2.0361	2.1375	2.2390
120	1.2165	1.3172	1.4190	1.5200	1.6211	1.7222	1.8234	1.9246	2.0259	2.1273	2.2287
130	1.2080	1.3087	1.4104	1.5113	1.6123	1.7134	1.8145	1.9157	2.0170	2.1183	2.2196
140	1.2004	1.3011	1.4027	1.5036	1.6045	1.7056	1.8066	1.9078	2.0090	2.1103	2.2116
150	1.1936	1.2943	1.3957	1.4966	1.5975	1.6985	1.7996	1.9007	2.0018	2.1030	2.2043
160	1.1875	1.2881	1.3895	1.4903	1.5912	1.6922	1.7932	1.8942	1.9954	2.0965	2.1978
170	1.1819	1.2825	1.3838	1.4846	1.5855	1.6864	1.7874	1.8884	1.9895	2.0906	2.1918
180	1.1767	1.2774	1.3786	1.4794	1.5802	1.6811	1.7820	1.8830	1.9841	2.0852	2.1863
190	1.1720	1.2726	1.3738	1.4745	1.5754	1.6762	1.7771	1.8781	1.9791	2.0802	2.1813
200	1.1677	1.2682	1.3693	1.4701	1.5709	1.6717	1.7726	1.8736	1.9746	2.0756	2.1767

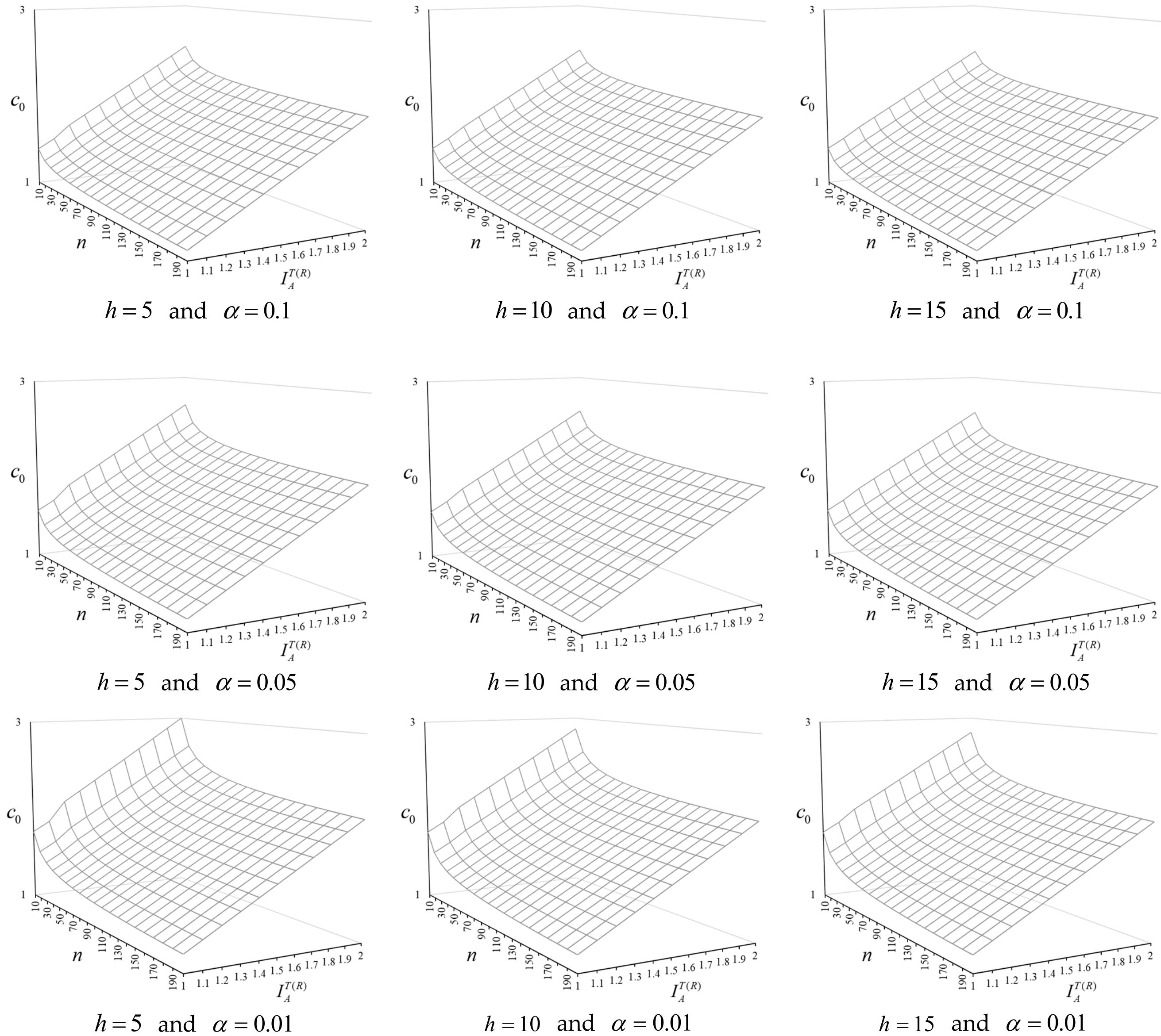

Figure 1. Three-dimensional graphs of critical value for various h and α.

If the values of $I_A^{T(R)}$, n, and h were fixed, then the critical value would increase with a decrease in Type-I error α. This means that increasing the confidence level would reduce the width of the rejection region and in so doing, decrease the probability of rejecting the null hypothesis. This is also in line with intuition and statistical inference.

4. Application Example

In this section, the applicability of IACI was demonstrated using a case study involving an OBM firm. A numerical example of this case is provided to illustrate the decision-making process. Sensitivity analysis based on the numerical example was used to obtain managerial insights.

4.1. OBM Firm

In the following, we consider an OBM firm engaged in the manufacture and sale of bedding, including mattresses, mattress protectors, pillows, bedframes, and cushions. In an effort to preserve its brand image, the products are sold only in flagship stores and at their own counters in department stores and shopping malls. This allows the transfer of stock among channels to minimize the loss of sales due to demand uncertainty. The bedding is made of cotton covers, polyester interlining, and viscoelastic polyurethane foam (so-called memory foam or low-resistance polyurethane foam). Because most consumers are sensitive to the perceived cleanliness of bedding products, managers must be highly cognizant of the appearance of the bedding and packaging. The color of foam changes from white to yellow under exposure to ultraviolet light and cloth covers are susceptible to mildew growth, particularly under imperfect storage conditions. These kinds of imperfections in no way impede the function of the product; however, they reduce the visual appeal for customers. Thus, any firm that hopes to preserve its reputation must set a shelf life for stock. Removing expired stock from shelves incurs the original costs, as well as additional costs for shipping and disposal. Note that providing discounts for expired or defective products would work against the brand image. At the same time, product shortages incur opportunity costs, as well as costs pertaining to shipping, discounts, free giveaways, and compensation. Clearly, the bedding in this model can be considered a newsboy-type product.

4.2. Numerical Example

In the following, we explore the sales strategies for the OBM bedding firm described above. The rapid development of information-and-communications technology has prompted many firms to focus on e-commerce. Note that in terms of pricing, discounts, and promotions, the operating models for virtual stores differ considerably from those for physical stores. Generally, firms must differentiate products as online-exclusive or physical-exclusive to avoid complaints due to differences in pricing or service. The company addressed in the current study transfers unprofitable products from physical stores to virtual stores, thereby necessitating an evaluation of integrated profitability in all channels.

In the following, we consider a pillow product that has for many years remained profitable, despite the introduction of new products. In this example, the OBM manager is considering listing this pillow as an online-exclusive item to increase profitability by generating more media exposure. The manager stipulates a minimum level as the criterion for decision-making. Essentially, the pillow will be listed as an online-exclusive item if its integrated profitability in all channels does not exceed the stipulated level. We use the integrated profitability evaluation based on the IACI index to decide how to handle the pillow. First, of all, the demand parameters for all channels are unknown; therefore, the manager must collect historical demand data from all channels to estimate the IACI. Note that demand data do not include the promotion activities during festivals or anniversaries. Table 10 presents the historical demand data of the classic pillow for $h = 10$ representative channels through $n = 30$ periods, as well as the results of Kolmogorov–Smirnov (K-S) and Anderson–Darling (A-D) normality tests using R software (version 1.1.419). Note that the p-values of the two tests exceeded 0.05 for all channels, indicating that the data posed no significant violation of normal distribution.

The parameter values of the considered product are as follows:

- selling price: $p = 3500$;
- manufacturing cost: $c = 2000$;
- net profit per pillow: $c_p = p - c = 1500$;
- total target profit for all channels per cycle: $k_t = 380,000$;
- total target demand for all channels per period: $T_t = k_t / c_p = 253.33$;
- shortage cost per pillow: $c_s = 250$;
- disposal cost per pillow: $c_d = 200$;
- excess cost per pillow: $c_e = c_d + c = 2200$;
- stipulated minimum level for IACI: $I_A^{T(R)} = 1.5$. The corresponding integrated profitability is $\Omega_t(1.5) = 0.88$.

Table 10. Thirty sale periods of demand from 10 channels.

Period	Channel									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	44	31	37	25	20	24	17	32	22	28
2	46	36	41	22	23	13	17	35	22	26
3	36	36	34	22	24	21	19	32	19	20
4	40	40	40	24	23	24	17	34	23	18
5	40	28	37	26	24	15	19	44	22	23
6	42	39	42	23	21	12	21	29	18	22
7	37	38	35	28	25	22	17	31	19	23
8	48	30	43	22	25	17	19	37	23	26
9	41	37	38	24	20	27	17	35	23	21
10	46	38	41	28	24	8	17	30	21	24
11	35	38	38	27	21	13	19	30	21	18
12	44	45	39	26	19	18	14	26	23	15
13	37	37	38	27	20	24	20	31	24	21
14	41	39	35	32	25	17	20	30	19	22
15	39	34	38	19	19	19	20	28	20	22
16	45	37	41	29	25	25	15	29	21	20
17	38	38	42	24	26	21	17	35	17	21
18	39	30	37	27	22	19	20	30	20	21
19	45	34	40	25	28	26	18	28	24	22
20	42	33	42	25	18	20	21	29	19	26
21	37	31	38	27	24	14	18	31	22	26
22	44	30	36	23	23	19	18	29	22	18
23	39	37	37	23	24	24	16	33	23	22
24	39	33	47	24	28	13	20	17	22	25
25	42	35	42	29	18	19	13	34	20	16
26	35	26	38	27	24	19	19	23	23	17
27	38	31	39	30	14	15	19	36	22	20
28	41	40	33	23	24	19	17	34	19	16
29	40	35	41	24	18	8	16	38	20	19
30	36	37	43	25	26	26	16	32	20	23
$\overline{x}_i$	40.533	35.100	39.067	25.333	22.500	18.700	17.867	31.400	21.100	21.367
s_i^2	12.464	17.334	9.444	7.816	10.603	26.838	3.913	23.421	3.403	11.137
K-S	0.90	0.58	0.64	0.81	0.30	0.75	0.51	0.57	0.24	0.96
A-D	0.47	0.20	0.42	0.47	0.08	0.36	0.10	0.10	0.05	0.55

Note that the historical data and parameter values pertaining to selling price and related costs have been slightly modified to protect company information. Based on historical demand data from Table 10, we first calculate the total sample mean $\sum_{i=1}^{h} \overline{x}_i = 272.967$, total sample variance $\sum_{i=1}^{h} s_i^2 = 126.374$, and estimate of the nature IACI $\hat{I}_A^T = 1.7468$. Since the correction factor for $n = 30$ and $h = 10$ is $b_t = 0.9974$, the estimate of the unbiased IACI is $\widetilde{I}_A^T = b_t \hat{I}_A^T = 0.9974 \times 1.7468 = 1.7423$. For the method of point estimation, since the estimated result of integrated profitability in all channels exceeds the stipulated level (i.e., $\widetilde{I}_A^T = 1.7423 > I_A^{T(R)} = 1.5$), the product should not be classified as an online-exclusive item. However, this result does not take into account the effect of sampling error. If the confidence level were set at 95%, the critical value for $h = 10$, $n = 30$, and $I_A^{T(R)} = 1.5$ can be obtained by looking for Table 5, i.e., $c_0 = 1.8219$. The result of the test is $\widetilde{I}_A^T = 1.7423 < c_0 = 1.8219$, such that the null hypothesis is accepted, which indicates that the physical-exclusive pillow should be recategorized as an online-exclusive item.

4.3. Sensitivity Analysis

The decision-making process based on an assessment of integrated profitability involves a comparison of $\widetilde{I}_A^T$ and c_0 values. Note, however, that observations (1) and (4) in Section 3 reveal that increasing the stipulated minimum profit $I_A^{T(R)}$ and/or decreasing the

Type-I error α would increase the value of c_0. As shown in Equation (9), the value of $\widetilde{I}_A^T$ increases with a decrease in the target profit k_t, which means that changing the stipulated minimum level, Type-I error, or target profit could alter the direction of the decision reached through an assessment of integrated profitability. We performed sensitivity analysis on the above numerical example to elucidate the effects of changes to parameters $I_A^{T(R)}$, α, and k_t on the test results. Figures 2–4 plot the curves for c_0 and $\widetilde{I}_A^T$ as functions of $I_A^{T(R)}$, α, and k_t. As shown in Figure 2, changing the stipulated minimum profit would not affect the value of $\widetilde{I}_A^T$, and decreasing the stipulated minimum profit by more than 5.18% would result in the rejection of the null hypothesis. As shown in Figure 3, changing the Type-I error would not affect the value of $\widetilde{I}_A^T$, and increasing the Type-I error by more than 112% would result in the rejection of the null hypothesis. In other words, decreasing the confidence level to below 89.4% would reverse the decision on the product category. As shown in Figure 4, changing the target profit would not affect the value of c_0, and decreasing the target profit by more than 35.6% would result in the rejection of the null hypothesis. Note that an excessive reduction of the stipulated profit (i.e., $I_A^{T(R)} = 1.5$ to $I_A^{T(R)} = 1.422$), confidence level (i.e., 95% to 89.4%), or target profit (i.e., $k_t = 380,000$ to $k_t = 378,647$) could lead to erroneous decisions resulting from an overestimate of profitability. Clearly, further analysis of risk is warranted.

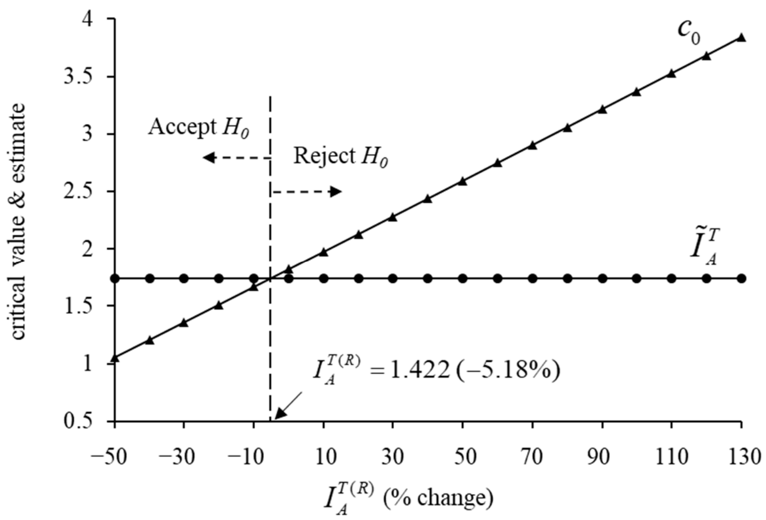

Figure 2. Curves of c_0 and $\widetilde{I}_A^T$ for changing $I_A^{T(R)}$.

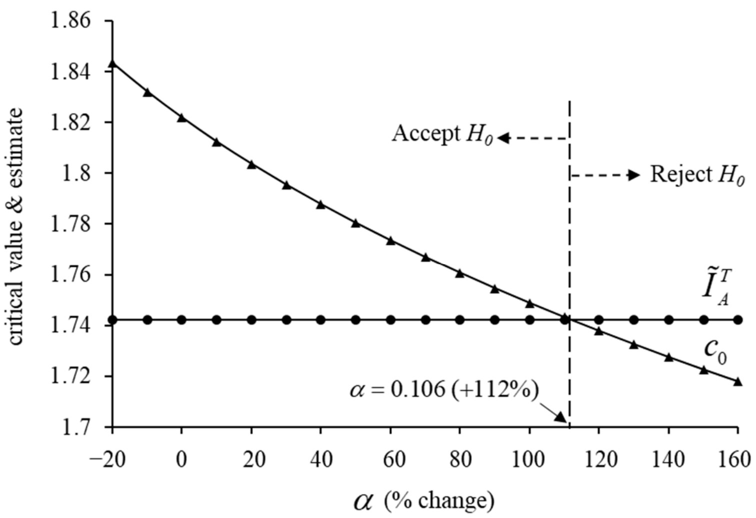

Figure 3. Curves of c_0 and $\widetilde{I}_A^T$ for changing α.

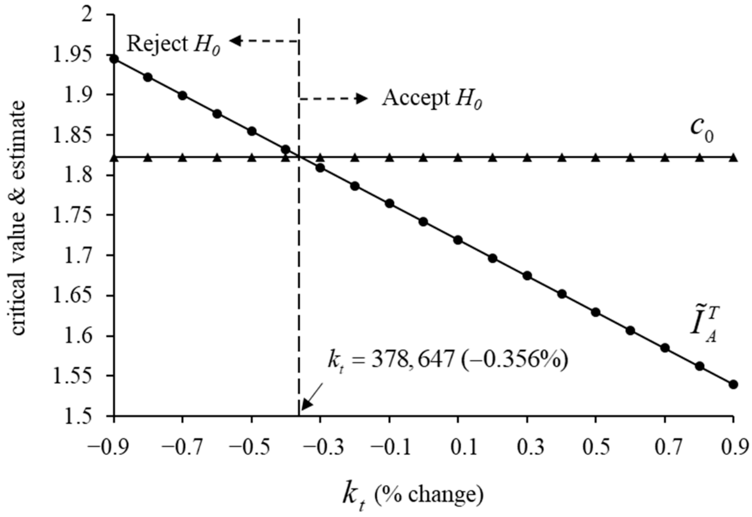

Figure 4. Curves of c_0 and $\widetilde{I}_A^T$ for changing k_t.

We determined that changing the stipulated minimum profit, Type-I error, or the target profit could alter the direction of the decision; however, there are situations in which adjusting more than one parameter would be beneficial. We first consider the effects of adjusting $I_A^{T(R)}$ and α, as in most real-world situations the target profit is fixed. Figure 5 illustrates the difference value (i.e., $\widetilde{I}_A^T - c_0$) as functions of $I_A^{T(R)}$ and α. The black curves indicate that the null hypothesis is not rejected because $\widetilde{I}_A^T < c_0$. This means that to categorize the product as online-exclusive would require that both parameters $I_A^{T(R)}$ and α fall within the gray area. In this example, if the manager is unwilling to categorize the product as online-exclusive, confidence level (i.e., α) must be increased and the stipulated minimum profit (i.e., $I_A^{T(R)}$) must be increased such that two parameters fall into the white area.

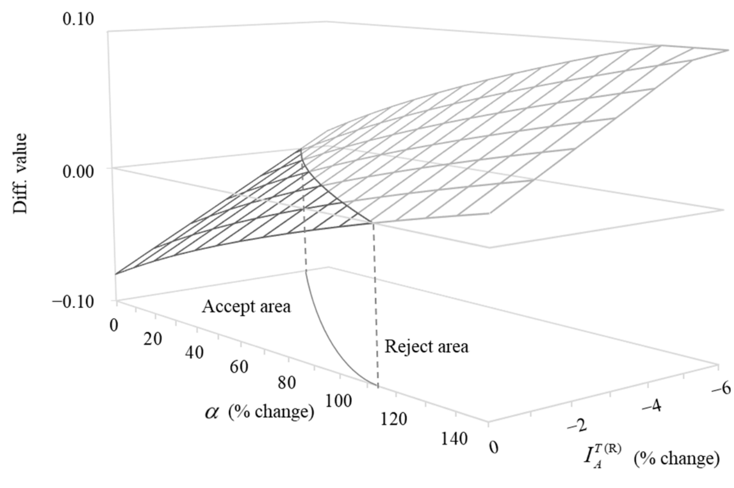

Figure 5. Difference of $\widetilde{I}_A^T$ and c_0 for changing k_t and α.

5. Conclusions

This paper addresses the issue of integrated profitability evaluation in an OBM company engaged in the manufacture and sale of newsboy-type products exclusively through owned channels. Based on the assumption of normally distributed demand in all channels, we formulated the concept of integrated profitability and developed an IACI index by which to measure it. Historical demand data from all channels was used to formulate an unbiased estimator for IACI and its sampling distribution. Data collection in situations involving multiple samplings is also discussed. Based on the statistical properties of the unbiased IACI estimator, we established a hypothesis test by which to determine whether the integrated profitability meets the stipulated minimum profit. For convenience, we tabulated the critical values as a function of sample size, confidence level, the number of channels, and stipulated minimum profit. This makes it possible for OBM managers to make decisions simply by estimating the IACI based on historical demand data from all channels, and then looking up the critical value in the corresponding table. Note that the proposed tables are universally applicable to any case involving the integrated profitability evaluation problem in an OBM company. The applicability of the proposed method was demonstrated in a practical case involving sales models (online-exclusive and physical-exclusive) in a bedding company. Numerical analysis and sensitivity analysis of this case revealed that decisions pertaining to integrated profitability evaluation could be reversed simply by adjusting the stipulated minimum profit, Type-I error, or target profit. We also derived the parameter boundary values that correspond to a decision reversal. Finally, we explored the effects of simultaneously adjusting $I_A^{T(R)}$ and α on decision-making. The findings of this study can provide OBM managers with a new measurement tool for the profitability of newsboy-type products. Since the tool can be also conducive to performing statistical inferences, the OBM managers can make logical and scientific decisions on marketing and sales strategies, rather than simply observing revenue data. Furthermore, we also provide guidelines and steps to affect the direction of decisions so that the OBM managers can easily grasp the key points in decision-making management. Therefore, we can recommend that OBM managers use the proposed method to replace the traditional measurement and evaluation modes.

We recommend the following directions for future research:

(1) In particular environments, the demand data may be collected from multiple samples rather than a single sample. Therefore, we can consider the data collection involving multiple samples, in which the sample size of each group is equal or unequal.
(2) In the current study, the statistical properties of $\widetilde{I}_A^T$ were derived under the assumption that the demand variance was the same in all channels. We recommend further research without this assumption to enhance generalizability.
(3) The use of IACI is limited to situations where demand obeys normal distribution. To enhance the generalizability of the IACI, the sample data can be made to resemble a normal distribution by applying Box–Cox and Yeo–Johnson transformations.
(4) In this study, the demands of each channel are assumed to be mutually exclusive. However, this assumption cannot be fully satisfied in the real-world scenario. To make our method more applicable in the real world, we can develop the IACI index involving the correlation factors among channels.
(5) To gain further managerial insights, it should be possible to establish a feasible loss function for the adjustable parameters (i.e., $I_A^{T(R)}$ and α). In cases where the manager desires to change decisions based on integrated profitability, it should be possible to formulate an optimal model aimed at loss minimization involving a combination of $I_A^{T(R)}$ and α as a decision variable.

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