mathematics

Article

Fuzzy Optimization Model for Decision-Making in Single Machine Construction Project Planning

Nilthon Arce Fernández [1,*], Flabio Gutiérrez Segura [2], Manuel Emilio Milla Pino [3], Jose Manuel Palomino Ojeda [4], Alfredo Lázaro Ludeña Gutiérrez [5] and River Chávez Santos [6]

1 Academic Department of Basic and Applied Sciences, Universidad Nacional de Jaén, Jaén 06800, Peru
2 Mathematics Department, Faculty of Science, Universidad Nacional de Piura, Castilla, Piura 20002, Peru; flabio@unp.edu.pe
3 Department of Civil Engineering, Faculty of Engineering, Universidad Nacional de Jaén, Jaén 06800, Peru; manuel.milla@unj.edu.pe
4 Data Science Research Institute, Universidad Nacional de Jaén, Jaén 06800, Peru; jose.palomino@est.unj.edu.pe
5 Research Office, Universidad Nacional de Jaén, Jaén 06800, Peru; alfredo.ludena@unj.edu.pe
6 Academic Department of Education, Communication Sciences and Basic Sciences, Faculty of Education and Communication Sciences, Universidad Nacional Toribio Rodríguez de Mendoza de Amazonas, Chachapoyas 01001, Peru; river.chavez@untrm.edu.pe
* Correspondence: nilthon_arce@unj.edu.pe

Abstract: Scheduling for a construction project with a limited number of machines is a critical and well-studied problem. Most studies assume that task processing times are exact; in practice, delays frequently occur, rendering the initial work plan invalid. Therefore, adaptability is crucial to the success of a project. This work introduces a fuzzy optimization model for the planning of construction projects executed simultaneously and having only one backhoe. The model assumes imprecise task processing times, represented by triangular fuzzy sets, that accept delays up to a permitted degree of tolerance. The model solution obtains a fuzzy work plan. This is a robust plan that supports incidents (delays). A method to apply the model was created. The fuzzy model can help construction companies reduce delays in the delivery of their projects and avoid excessive penalties. The model was implemented in the CPLEX solver, which can quickly obtain an optimal solution for small and medium instances. For large instances, the model must be solved with metaheuristics. This scientific contribution is important for future work since it consists of the application of fuzzy optimization in a specific area of civil engineering.

Keywords: construction project; fuzzy optimization; fuzzy sets; penalties; single-machine task scheduling; decision-making

MSC: 90C70

check for updates

Citation: Arce Fernández, N.; Gutiérrez Segura, F.; Milla Pino, M.E.; Palomino Ojeda, J.M.; Ludeña Gutiérrez, A.L.; Chávez Santos, R. Fuzzy Optimization Model for Decision-Making in Single Machine Construction Project Planning. *Mathematics* **2024**, *12*, 1088. https://doi.org/10.3390/math12071088

Academic Editor: Hsien-Chung Wu

Received: 25 February 2024
Revised: 23 March 2024
Accepted: 26 March 2024
Published: 4 April 2024

1. Introduction

The construction sector contributes directly to the national and global economies, so new and diverse technologies are implemented every day to manage projects efficiently. Managing construction projects requires the application of knowledge, skills, tools, and techniques in all programmed activities [1].

The existing uncertainty during execution is one of the fundamental limitations to delivering a construction project on time. The delay in delivery is due to a sum of delays in each of the tasks that are performed. This problem occurs because construction projects are executed in uncertain environments, which can be caused by weather conditions, site conditions, equipment conditions, late delivery of materials, worker productivity, inflation, etc. [2]. For example, in Saudi Arabia, 70% of public construction projects are delayed due to various factors that create uncertainty [3]. Peru and developing countries are no strangers to this reality; due to economic factors, small and medium construction companies lack the machinery to execute two or more construction projects simultaneously.

To plan the time and estimate the cost of a project, it is necessary to take into account the uncertainty that exists in the different phases, for example, in the processing time of the tasks. The duration and cost of tasks in a construction project are rarely known with precision, which leads to estimation errors by experts [4]. Uncertainty can be of the random or imprecise types. Randomness is modeled with probability theory; in this work, it is assumed that the task processing time of a construction project is imprecise, i.e., the desired time for the duration of the task is known, but not the exact time it takes to execute in reality. This type of uncertainty can be modeled with fuzzy logic.

To handle uncertainty in project scheduling, two approaches are used: reactive or proactive. In the reactive approach, project managers continuously review and modify plans in response to incidents. However, frequent plan revisions are not desirable from a resource planning perspective. The proactive approach is preferable, meaning a robust model that can withstand incidents and is easily adaptable. In project task scheduling, to create plans that optimize resources with a proactive approach, stochastic optimization and fuzzy optimization can be used.

In most construction projects, the processing time of tasks is considered to be exact. In real situations, it is always imprecise due to the various phenomena that generate uncertainty.

As far as is known, in the literature review, there are few works on mathematical optimization applied to the scheduling in construction projects that contemplate the imprecision in task processing time.

Senouci and Mubarak [5] proposed a multi-objective optimization model for the scheduling of construction projects under extreme weather conditions. Using applicative examples, it was verified that the model minimized the time and cost of the projects but did not consider the imprecision in task processing time.

Several works model the imprecision that occurs in the execution of projects with fuzzy logic. For example, Itoh and Ishii [6] proposed an optimization model for projects with fuzzy processing times and due dates; the model minimized the number of delayed jobs. Niu et al. [7] addressed a scheduling problem with fuzzy processing times. They modeled the uncertainty using triangular fuzzy numbers, managing to find a sequence of jobs that minimized Makespan. Knyazeva et al. [8] developed a fuzzy project scheduling problem with limited resources and deadlines. However, it does not deal with construction projects that have the following characteristics for their execution: the delivery date is fixed, a heavy penalty is applied per day of delay, a single machine is used for several projects, and the machine works in the open field and is likely to suffer more delays than a machine in a factory where there are control systems and automated processes.

In other models, uncertainty in project duration and cost is often modeled with triangular fuzzy sets. For example, Al-Zarrad and Fonseca [9] proposed a model to find time–cost trade-off alternatives while taking into account uncertainty in project time and cost. This helps to reduce the risk of projects going over budget or being delayed. Nguyen et al. [10] developed a hybrid model using fuzzy logic to address the trade-off between time, cost, and quality in construction projects. The model accounts for uncertainties in project time, cost, and quality. However, in these studies, task processing times are not considered fuzzy.

In developing countries, small and medium-sized construction companies often lack machinery, resulting in a single machine being used for several projects. Arce et al. [11] proposed an optimization model to obtain the optimal work plan for a machine, minimizing the total penalty generated by delays in the delivery of construction projects. The model was applied to the case of a backhoe loader to develop 18 tasks that correspond to four construction projects executed simultaneously. In this model, uncertainty was not taken into account.

In this work, we study the problem of minimizing the total penalty generated by delays in the delivery of construction projects executed simultaneously and operated by a single backhoe. We suppose that the distributions probability for the task processing times is unknown, that is, the problem cannot be treated with stochastic optimization. We propose a fuzzy optimization model; triangular fuzzy sets are used to model the existing imprecision in the task processing time. The efficiency of the model is analyzed by applying it to small, medium, and large instances.

2. Materials and Methods

2.1. Scheduling of Single-Machine Construction Projects

Arce et al. [11] modeled the scheduling problem for a set of construction projects, P = {Project 1, Project 2, . . ., Project n}, that are executed simultaneously with only one backhoe loader. The execution of the projects requires the completion of a set of tasks, T = {T1, T2, . . ., Tm}, each with an exact (deterministic) processing time.

Figure 1 shows the tasks for a machine in the execution of "n" projects.

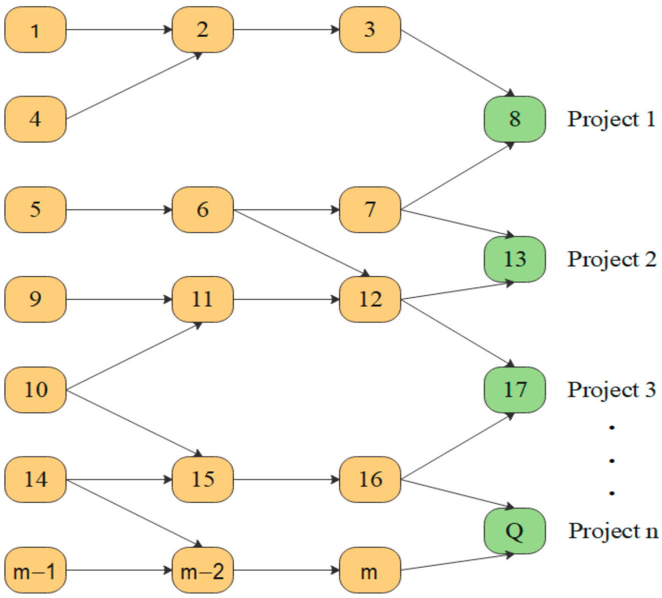

Figure 1. Tasks for the execution of "n" construction projects.

The following mixed integer linear programming model (MILP) was obtained:

$$\min Z = \sum_{p=1}^{n} (m_p \ * s_{pb}) \tag{1}$$

subject to:

$$x_i - x_j \geq t_j - M * y_{ij} \tag{2}$$

$$x_j - x_i \geq t_i - M(1 - y_{ij}) \tag{3}$$

$$x_j - x_i \geq t_i \tag{4}$$

$$x_u + t_u + s_{pa} - s_{pb} = f_p \tag{5}$$

where:

$$x_i,\ s_{pa},\ s_{pb},\ t_i \in Z_0^+$$
$$y_{ij} \in \{0, 1\}$$

where:
x_i: Start of the task i
t_i: Task processing time i

x_u: Start of the last task u of the project P
t_u: Task processing time u
s_{pa}: Variable that does not generate a penalty in the project P
s_{pb}: Variable that generates a penalty in the project P
f_p: Project delivery time P
m_p: Penalty per day of project delay P
y_{ij}: Binary variable
M: A very large integer

Where (1) minimizes the total penalty, (2) and (3) are non-interference constraints, (4) represents precedence constraints, and finally (5) represents lead-time constraints.

2.2. Fuzzy Sets

Let X be the universe of discourse; a fuzzy set $\widetilde{\mathrm{A}}$ in X is a set of ordered pairs:

$$\widetilde{\mathrm{A}} = \left\{(x, \mu_{\widetilde{\mathrm{A}}}(x)), x \in \mathrm{X}\right\} \tag{6}$$

where $\mu_{\widetilde{\mathrm{A}}} : \mathrm{X} \to [0, 1]$ is called a membership function, and $\mu_{\widetilde{\mathrm{A}}}(x)$ represents the degree to which x belongs to the set $\widetilde{\mathrm{A}}$ [12].

For our purposes, we restrict ourselves to fuzzy sets on the real line. A membership function can be triangular, trapezoidal, sigmoidal, etc.

2.3. Linear Programming with Fuzzy Resources

A linear programming (LP) model is given by:

$$\begin{array}{ll} & \max z = c * x \\ subject\ to: & \\ & Ax \leq b \\ & x \geq 0 \end{array}$$

where the parameters c (profits or costs), b (availability of resources), and A (the matrix of technological coefficients) are known numbers in exact form. However, in real problems, some of the parameters indicated may be imprecise.

If it is assumed that the imprecision of the resources is modeled with linear fuzzy sets, as in Figure 2, we are faced with an LP model with fuzzy resources, which is formulated as follows:

$$\begin{array}{ll} & \max z = c * x \\ subject\ to: & \\ & (Ax)_i \leq \widetilde{b}_i\ ;\ i = 1, 2,\ \ldots,\ m \\ & x \geq 0 \end{array} \tag{7}$$

where $c \in R^n$, A is a matrix mxn, and the symbol $\widetilde{b}_i$ indicates diffuse resources.

For each resource, i, consider a desirable amount, t_i, but the possibility that it could be extended to $t_i + d_i$ is accepted, where d_i is the maximum degree of imprecision (see Figure 2).

To solve (7), different methods have been proposed [13]. In this work, the Verdegay method is used.

2.4. Verdegay Method

Suppose that, in the model (7), the membership functions are linear (see Figure 2), with the equation as follows:

$$\mu_i(x) = \begin{cases} 1 & ;\ (Ax)_i < t_i \\ 1 - \frac{[(Ax)_i - t_i]}{d_i} & ;\ t_i \leq (Ax)_i \leq t_i + d_i \\ 0 & ;\ (Ax)_i > t_i + d_i \end{cases} \tag{8}$$

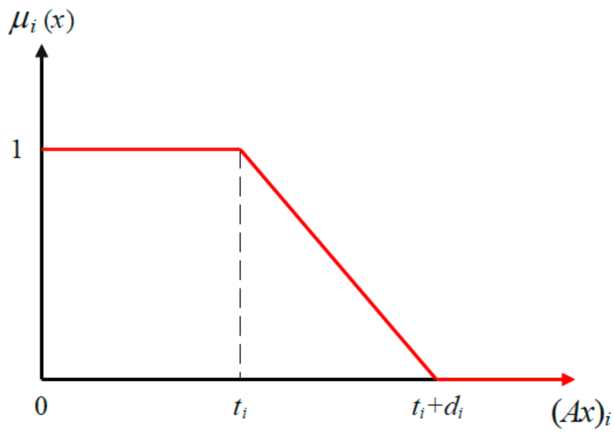

Figure 2. Degree of decision-maker satisfaction.

When $(Ax)_i < t_i$, the constraints are satisfactorily met, therefore, the degree of satisfaction is maximum $[\mu_i(x) = 1]$. When $(Ax)_i > t_i + d_i$, violation of the restrictions is not accepted, as they exceed the given tolerance. Therefore, the degree of satisfaction is zero $[\mu_i(x) = 0]$. When $t_i \le (Ax)_i \le t_i + d_i$, the violation of the constraints is accepted, and the degree of satisfaction decreases while moving away from it, t_i. It is important to note that the level of compliance with the constraints reflects the level of satisfaction of the decision-maker.

The goal is to find the optimal solution for $[\mu_i(x) = 1]$, $i = 1, 2, \ldots, m$. However, it may be acceptable to obtain an optimal solution for a value $\mu_i(x)$ greater than α_i, which is considered a minimum satisfaction level fixed a priori. This should be determined based on the nature of the problem and in interaction with the decision-maker.

According to Verdegay [14], for the linear Equation (8), problem (7) is transformed into the following parametric PL problem:

$$\begin{array}{ll} & \max z = c * x \\ \textit{subject to}: & \\ & (Ax)_i \le t_i + (1-\alpha)d_i \,, \forall i \\ & x \ge 0 \,, \alpha \in [0,\ 1] \end{array}$$

Let $\beta = 1 - \alpha$, this classic model is expressed in the following way:

$$\begin{array}{ll} & \max z = c * x \\ \textit{subject to}: & \\ & (Ax)_i \le t_i + \beta * d_i \,, \forall i \\ & x \ge 0 \,, \beta \in [0,\ 1] \end{array}$$

where β is the degree of imprecision accepted.

2.5. A Fuzzy Optimization Model for Single-Machine Construction Projects Scheduling

Considering that the imprecision in the task processing time executed by a machine in construction projects can be modeled with a fuzzy set using the deterministic model (see Section 2.1), the following fuzzy PL model is proposed.

$$\min Z^* = \sum_{p=1}^{n} (m_p * s_{pb}) \tag{9}$$

subject to:

$$x_i - x_j + M * y_{ij} \ge \widetilde{t}_j \tag{10}$$

$$x_j - x_i - M * y_{ij} \ge -M + \widetilde{t}_i \tag{11}$$

$$x_j - x_i \geq \tilde{t}_i \tag{12}$$

$$x_u + s_{pa} - s_{pb} = f_p - \tilde{t}_u \tag{13}$$

where:

$$x_i,\ s_{pa},\ s_{pb} \in R_0^+,\ t_i \in Z^+$$
$$y_{ij} \in \{0,1\}$$

Constraints (10)–(13) are fuzzy. $\tilde{t}_i$ is the resource, and i is fuzzy task processing time.

2.6. Solution of the Proposed Model

The imprecision in the processing time of the tasks performed by the machine is represented by fuzzy triangles, as shown in Figure 3.

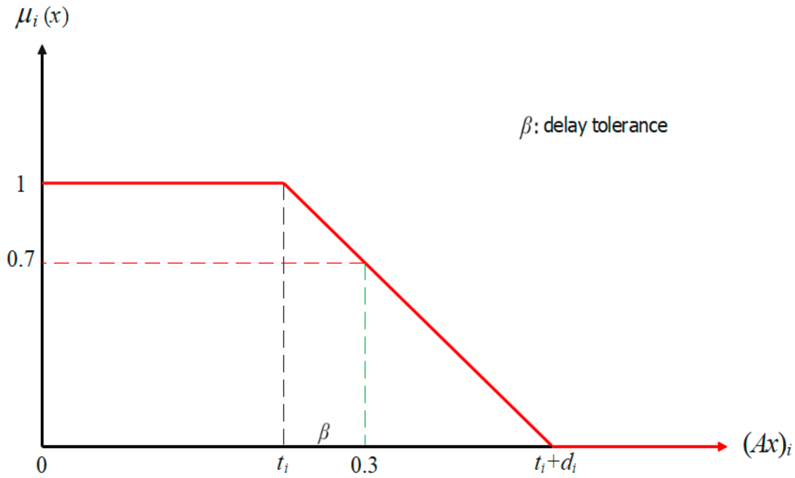

Figure 3. Imprecision in task processing time.

Where:
t_i : Task processing time i.
d_i : Maximum task tolerance i.
β : The degree of imprecision accepted by the project manager depending on the execution conditions and in coordination with the construction engineers.

The following mixed integer linear programming model (Algorithm 1) was obtained by applying the Verdegay method (see Section 2.4) to the proposed fuzzy model:

Algorithm 1. Mixed Integer Linear Programming model

Input: Set of "T" tasks to perform a set of "P" projects
Output: Scheduling of tasks for the realization of "P" projects
For each $\beta = \{0,\ 0.1,\ 0.2,\ 0.3,\ \ldots,\ 1\}$

$$\min Z^* = \sum_{p=1}^{n} (m_p * s_{pb}) \tag{14}$$

subject to:

$$x_i - x_j + M * y_{ij} \geq t_j + \beta * d_j \tag{15}$$
$$x_j - x_i - M * y_{ij} \geq -M + t_i + \beta * d_i \tag{16}$$
$$x_j - x_i \geq t_i + \beta * d_i \tag{17}$$
$$x_u + s_{pa} - s_{pb} = f_p - (t_u + \beta * d_u) \tag{18}$$

where:

$x_i,\ s_{pa},\ s_{pb} \in R_0^+;\ t_i \in Z^+$
$y_{ij} \in \{0,1\}$
$\beta \in [0,1]$

3. Results

We implemented the model using IBM ILOG CPLEX Optimization Studio software, version student 12.4. The software was run on a personal computer equipped with an Intel (R) Core (TM) i5-8250U CPU @ 1.60 GHz 1.80 GHz and 6.00 GB RAM.

3.1. Case Study

To apply the proposed fuzzy model, we used a case study presented in [11] as a reference. However, we took into account the imprecision in the processing time of the tasks. The data used in this new case study were collected through interviews with experts selected by purposive sampling.

A construction company in the province of Jaen, Peru plans to execute four construction projects simultaneously but only has one backhoe to perform 18 tasks. The network of tasks for the backhoe was designed with non-interference, precedence, and deadline constraints (see Figure 4).

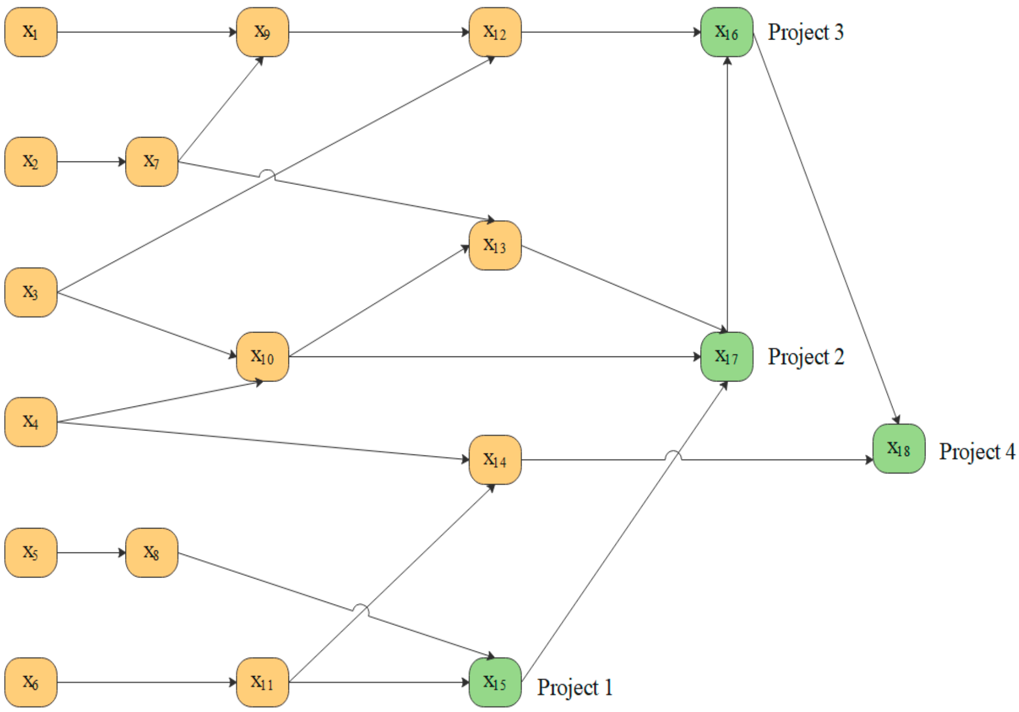

Figure 4. Machine tasks in project execution.

Additionally, we have data on the time used by the backhoe loader to execute each task. This time is imprecise, so the decision-maker would only accept up to a maximum tolerance, as shown in Table 1. The imprecision is represented using triangular fuzzy sets. For example, in Figure 5, we observe the triangular fuzzy set for task T6. The expected processing time is 15 days, but it can be delayed up to 2 days.

Table 1. Processing time and maximum tolerance for backhoe.

Task	Processing Time Days	Tolerance Days
T1	10	1
T2	17	2
T3	13	1
T4	20	2
T5	14	1

Table 1. *Cont.*

Task	Processing Time Days	Tolerance Days
T6	15	2
T7	21	2
T8	30	3
T9	25	3
T10	16	2
T11	18	2
T12	23	2
T13	12	1
T14	20	2
T15	26	3
T16	18	2
T17	14	1
T18	28	3

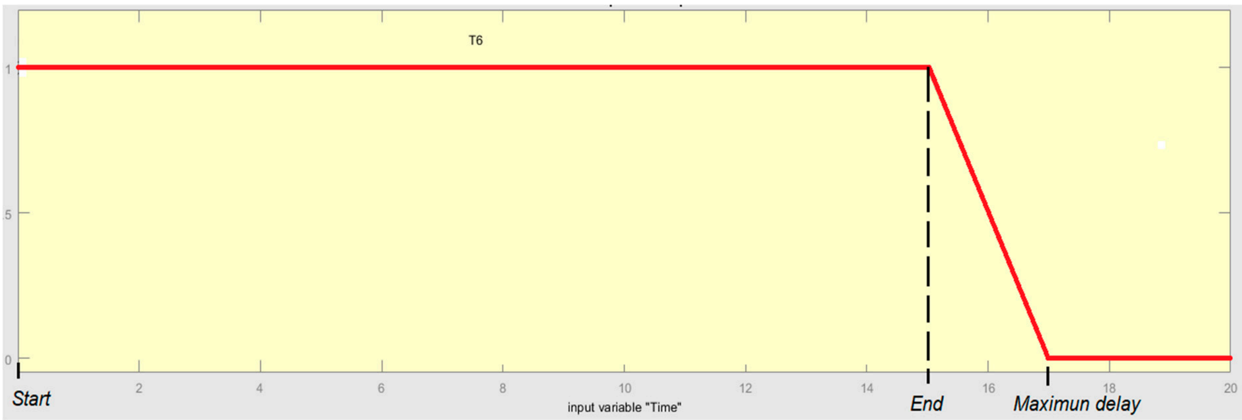

Figure 5. Imprecision in the task processing time T6.

The delivery time per construction project and the penalty per day of delay can be seen in Table 2.

Table 2. Delivery time and penalty per day of delay.

Projects	Delivery Time Days	Penalty Dollars/Day
P_1	95	1000
P_2	205	1600
P_3	280	2500
P_4	330	3000

The parametric MILP model (19) was obtained by applying the proposed model to the case study and by using the Verdegay method.

The objective function is given by the following:

$$\min Z^* = 1000s_{1b} + 1600s_{2b} + 2500s_{3b} + 3000s_{4b} \quad (19)$$

The above is subject to the following restrictions of non-interference (15) and (16), precedence (17), and delivery time (18). Table 3 presents all the non-interference restrictions for task processing, totaling 170.

Table 3. Non-interference restrictions of the parametric model.

Non-Interference Restrictions	
$x_1 - x_2 + 5000y_{1,2} \geq 17 + 2\beta$	$x_{10} - x_5 - 5000y_{5,10} \geq -4986 + \beta$
$x_2 - x_1 - 5000y_{1,2} \geq -4990 + \beta$	$x_5 - x_{11} + 5000y_{5,11} \geq 18 + 2\beta$
$x_1 - x_3 + 5000y_{1,3} \geq 13 + \beta$	$x_{11} - x_5 - 5000y_{5,11} \geq -4986 + \beta$
$x_3 - x_1 - 5000y_{1,3} \geq -4990 + \beta$	$x_5 - x_{12} + 5000y_{5,12} \geq 23 + 2\beta$
$x_1 - x_4 + 5000y_{1,4} \geq 20 + 2\beta$	$x_{12} - x_5 - 5000y_{5,12} \geq -4986 + \beta$
$x_4 - x_1 - 5000y_{1,4} \geq -4990 + \beta$	$x_5 - x_{13} + 5000y_{5,13} \geq 12 + \beta$
$x_1 - x_5 + 5000y_{1,5} \geq 14 + \beta$	$x_{13} - x_5 - 5000y_{5,13} \geq -4986 + \beta$
$x_5 - x_1 - 5000y_{1,5} \geq -4990 + \beta$	$x_5 - x_{14} + 5000y_{5,14} \geq 20 + 2\beta$
$x_1 - x_6 + 5000y_{1,6} \geq 15 + 2\beta$	$x_{14} - x_5 - 5000y_{5,14} \geq -4986 + \beta$
$x_6 - x_1 - 5000y_{1,6} \geq -4990 + \beta$	$x_6 - x_7 + 5000y_{6,7} \geq 21 + 2\beta$
$x_1 - x_7 + 5000y_{1,7} \geq 21 + 2\beta$	$x_7 - x_6 - 5000y_{6,7} \geq -4985 + 2\beta$
$x_7 - x_1 - 5000y_{1,7} \geq -4990 + \beta$	$x_6 - x_8 + 5000y_{6,8} \geq 30 + 3\beta$
$x_1 - x_8 + 5000y_{1,8} \geq 30 + 3\beta$	$x_8 - x_6 - 5000y_{6,8} \geq -4985 + 2\beta$
$x_8 - x_1 - 5000y_{1,8} \geq -4990 + \beta$	$x_6 - x_9 + 5000y_{6,9} \geq 25 + 3\beta$
$x_1 - x_{10} + 5000y_{1,10} \geq 16 + 2\beta$	$x_9 - x_6 - 5000y_{6,9} \geq -4985 + 2\beta$
$x_{10} - x_1 - 5000y_{1,10} \geq -4990 + \beta$	$x_6 - x_{10} + 5000y_{6,10} \geq 16 + 2\beta$
$x_1 - x_{11} + 5000y_{1,11} \geq 18 + 2\beta$	$x_{10} - x_6 - 5000y_{6,10} \geq -4985 + 2\beta$
$x_{11} - x_1 - 5000y_{1,11} \geq -4990 + \beta$	$x_6 - x_{12} + 5000y_{6,12} \geq 23 + 2\beta$
$x_1 - x_{13} + 5000y_{1,13} \geq 12 + \beta$	$x_{12} - x_6 - 5000y_{6,12} \geq -4985 + 2\beta$
$x_{13} - x_1 - 5000y_{1,13} \geq -4990 + \beta$	$x_6 - x_{13} + 5000y_{6,13} \geq 12 + \beta$
$x_1 - x_{14} + 5000y_{1,14} \geq 20 + 2\beta$	$x_{13} - x_6 - 5000y_{6,13} \geq -4985 + 2\beta$
$x_{14} - x_1 - 5000y_{1,14} \geq -4990 + \beta$	$x_7 - x_8 + 5000y_{7,8} \geq 30 + 3\beta$
$x_1 - x_{15} + 5000y_{1,15} \geq 26 + 3\beta$	$x_8 - x_7 - 5000y_{7,8} \geq -4979 + 2\beta$
$x_{15} - x_1 - 5000y_{1,15} \geq -4990 + \beta$	$x_7 - x_{10} + 5000y_{7,10} \geq 16 + 2\beta$
$x_1 - x_{17} + 5000y_{1,17} \geq 14 + \beta$	$x_{10} - x_7 - 5000y_{7,10} \geq -4979 + 2\beta$
$x_{17} - x_1 - 5000y_{1,17} \geq -4990 + \beta$	$x_7 - x_{11} + 5000y_{7,11} \geq 18 + 2\beta$
$x_2 - x_3 + 5000y_{2,3} \geq 13 + \beta$	$x_{11} - x_7 - 5000y_{7,11} \geq -4979 + 2\beta$
$x_3 - x_2 - 5000y_{2,3} \geq -4983 + 2\beta$	$x_7 - x_{14} + 5000y_{7,14} \geq 20 + 2\beta$
$x_2 - x_4 + 5000y_{2,4} \geq 20 + 2\beta$	$x_{14} - x_7 - 5000y_{7,14} \geq -4979 + 2\beta$
$x_4 - x_2 - 5000y_{2,4} \geq -4983 + 2\beta$	$x_7 - x_{15} + 5000y_{7,15} \geq 26 + 3\beta$
$x_2 - x_5 + 5000y_{2,5} \geq 14 + \beta$	$x_{15} - x_7 - 5000y_{7,15} \geq -4979 + 2\beta$
$x_5 - x_2 - 5000y_{2,5} \geq -4983 + 2\beta$	$x_8 - x_9 + 5000y_{8,9} \geq 25 + 3\beta$
$x_2 - x_6 + 5000y_{2,6} \geq 15 + 2\beta$	$x_9 - x_8 - 5000y_{8,9} \geq -4970 + 3\beta$
$x_6 - x_2 - 5000y_{2,6} \geq -4983 + 2\beta$	$x_8 - x_{10} + 5000y_{8,10} \geq 16 + 2\beta$
$x_2 - x_8 + 5000y_{2,8} \geq 30 + 3\beta$	$x_{10} - x_8 - 5000y_{8,10} \geq -4970 + 3\beta$
$x_8 - x_2 - 5000y_{2,8} \geq -4983 + 2\beta$	$x_8 - x_{11} + 5000y_{8,11} \geq 18 + 2\beta$
$x_2 - x_{10} + 5000y_{2,10} \geq 16 + 2\beta$	$x_{11} - x_8 - 5000y_{8,11} \geq -4970 + 3\beta$

Table 3. *Cont.*

Non-Interference Restrictions	
$x_{10} - x_2 - 5000y_{2,10} \geq -4983 + 2\beta$	$x_8 - x_{12} + 5000y_{8,12} \geq 23 + 2\beta$
$x_2 - x_{11} + 5000y_{2,11} \geq 18 + 2\beta$	$x_{12} - x_8 - 5000y_{8,12} \geq -4970 + 3\beta$
$x_{11} - x_2 - 5000y_{2,11} \geq -4983 + 2\beta$	$x_8 - x_{13} + 5000y_{8,13} \geq 12 + \beta$
$x_2 - x_{14} + 5000y_{2,14} \geq 20 + 2\beta$	$x_{13} - x_8 - 5000y_{8,13} \geq -4970 + 3\beta$
$x_{14} - x_2 - 5000y_{2,14} \geq -4983 + 2\beta$	$x_8 - x_{14} + 5000y_{8,14} \geq 20 + 2\beta$
$x_2 - x_{15} + 5000y_{2,15} \geq 26 + 3\beta$	$x_{14} - x_8 - 5000y_{8,14} \geq -4970 + 3\beta$
$x_{15} - x_2 - 5000y_{2,15} \geq -4983 + 2\beta$	$x_9 - x_{10} + 5000y_{9,10} \geq 16 + 2\beta$
$x_3 - x_4 + 5000y_{3,4} \geq 20 + 2\beta$	$x_{10} - x_9 - 5000y_{9,10} \geq -4975 + 3\beta$
$x_4 - x_3 - 5000y_{3,4} \geq -4987 + \beta$	$x_9 - x_{11} + 5000y_{9,11} \geq 18 + 2\beta$
$x_3 - x_5 + 5000y_{3,5} \geq 14 + \beta$	$x_{11} - x_9 - 5000y_{9,11} \geq -4975 + 3\beta$
$x_5 - x_3 - 5000y_{3,5} \geq -4987 + \beta$	$x_9 - x_{13} + 5000y_{9,13} \geq 12 + \beta$
$x_3 - x_6 + 5000y_{3,6} \geq 15 + 2\beta$	$x_{13} - x_9 - 5000y_{9,13} \geq -4975 + 3\beta$
$x_6 - x_3 - 5000y_{3,6} \geq -4987 + \beta$	$x_9 - x_{14} + 5000y_{9,14} \geq 20 + 2\beta$
$x_3 - x_7 + 5000y_{3,7} \geq 21 + 2\beta$	$x_{14} - x_9 - 5000y_{9,14} \geq -4975 + 3\beta$
$x_7 - x_3 - 5000y_{3,7} \geq -4987 + \beta$	$x_9 - x_{15} + 5000y_{9,15} \geq 26 + 3\beta$
$x_3 - x_8 + 5000y_{3,8} \geq 30 + 3\beta$	$x_{15} - x_9 - 5000y_{9,15} \geq -4975 + 3\beta$
$x_8 - x_3 - 5000y_{3,8} \geq -4987 + \beta$	$x_9 - x_{17} + 5000y_{9,17} \geq 14 + \beta$
$x_3 - x_9 + 5000y_{3,9} \geq 25 + 3\beta$	$x_{17} - x_9 - 5000y_{9,17} \geq -4975 + 3\beta$
$x_9 - x_3 - 5000y_{3,9} \geq -4987 + \beta$	$x_{10} - x_{11} + 5000y_{10,11} \geq 18 + 2\beta$
$x_3 - x_{11} + 5000y_{3,11} \geq 18 + 2\beta$	$x_{11} - x_{10} - 5000y_{10,11} \geq -4984 + 2\beta$
$x_{11} - x_3 - 5000y_{3,11} \geq -4987 + \beta$	$x_{10} - x_{12} + 5000y_{10,12} \geq 23 + 2\beta$
$x_3 - x_{14} + 5000y_{3,14} \geq 20 + 2\beta$	$x_{12} - x_{10} - 5000y_{10,12} \geq -4984 + 2\beta$
$x_{14} - x_3 - 5000y_{3,14} \geq -4987 + \beta$	$x_{10} - x_{14} + 5000y_{10,14} \geq 20 + 2\beta$
$x_3 - x_{15} + 5000y_{3,15} \geq 26 + 3\beta$	$x_{14} - x_{10} - 5000y_{10,14} \geq -4984 + 2\beta$
$x_{15} - x_3 - 5000y_{3,15} \geq -4987 + \beta$	$x_{10} - x_{15} + 5000y_{10,15} \geq 26 + 3\beta$
$x_4 - x_5 + 5000y_{4,5} \geq 14 + \beta$	$x_{15} - x_{10} - 5000y_{10,15} \geq -4984 + 2\beta$
$x_5 - x_4 - 5000y_{4,5} \geq -4980 + 2\beta$	$x_{11} - x_{12} + 5000y_{11,12} \geq 23 + 2\beta$
$x_4 - x_6 + 5000y_{4,6} \geq 15 + 2\beta$	$x_{12} - x_{11} - 5000y_{11,12} \geq -4982 + 2\beta$
$x_6 - x_4 - 5000y_{4,6} \geq -4980 + 2\beta$	$x_{11} - x_{13} + 5000y_{11,13} \geq 12 + \beta$
$x_4 - x_7 + 5000y_{4,7} \geq 21 + 2\beta$	$x_{13} - x_{11} - 5000y_{11,13} \geq -4982 + 2\beta$
$x_7 - x_4 - 5000y_{4,7} \geq -4980 + 2\beta$	$x_{12} - x_{13} + 5000y_{12,13} \geq 12 + \beta$
$x_4 - x_8 + 5000y_{4,8} \geq 30 + 3\beta$	$x_{13} - x_{12} - 5000y_{12,13} \geq -4977 + 2\beta$
$x_8 - x_4 - 5000y_{4,8} \geq -4980 + 2\beta$	$x_{12} - x_{14} + 5000y_{12,14} \geq 20 + 2\beta$
$x_4 - x_9 + 5000y_{4,9} \geq 25 + 3\beta$	$x_{14} - x_{12} - 5000y_{12,14} \geq -4977 + 2\beta$
$x_9 - x_4 - 5000y_{4,9} \geq -4980 + 2\beta$	$x_{12} - x_{15} + 5000y_{12,15} \geq 26 + 3\beta$
$x_4 - x_{11} + 5000y_{4,11} \geq 18 + 2\beta$	$x_{15} - x_{12} - 5000y_{12,15} \geq -4977 + 2\beta$
$x_{11} - x_4 - 5000y_{4,11} \geq -4980 + 2\beta$	$x_{12} - x_{17} + 5000y_{12,17} \geq 14 + \beta$
$x_4 - x_{12} + 5000y_{4,12} \geq 23 + 2\beta$	$x_{17} - x_{12} - 5000y_{12,17} \geq -4977 + 2\beta$
$x_{12} - x_4 - 5000y_{4,12} \geq -4980 + 2\beta$	$x_{13} - x_{14} + 5000y_{13,14} \geq 20 + 2\beta$

Table 3. *Cont.*

Non-Interference Restrictions	
$x_4 - x_{15} + 5000y_{4,15} \geq 26 + 3\beta$	$x_{14} - x_{13} - 5000y_{13,14} \geq -4988 + \beta$
$x_{15} - x_4 - 5000y_{4,15} \geq -4980 + 2\beta$	$x_{13} - x_{15} + 5000y_{13,15} \geq 26 + 3\beta$
$x_5 - x_6 + 5000y_{5,6} \geq 15 + 2\beta$	$x_{15} - x_{13} - 5000y_{13,15} \geq -4988 + \beta$
$x_6 - x_5 - 5000y_{5,6} \geq -4986 + \beta$	$x_{14} - x_{15} + 5000y_{14,15} \geq 26 + 3\beta$
$x_5 - x_7 + 5000y_{5,7} \geq 21 + 2\beta$	$x_{15} - x_{14} - 5000y_{14,15} \geq -4980 + 2\beta$
$x_7 - x_5 - 5000y_{5,7} \geq -4986 + \beta$	$x_{14} - x_{16} + 5000y_{14,16} \geq 18 + 2\beta$
$83.x_5 - x_9 + 5000y_{5,9} \geq 25 + 3\beta$	$x_{16} - x_{14} - 5000y_{14,16} \geq -4980 + 2\beta$
$x_9 - x_5 - 5000y_{5,9} \geq -4986 + \beta$	$x_{14} - x_{17} + 5000y_{14,17} \geq 14 + \beta$
$x_5 - x_{10} + 5000y_{5,10} \geq 16 + 2\beta$	$x_{17} - x_{14} - 5000y_{14,17} \geq -4980 + 2\beta$

Table 4 presents all precedence restrictions for task processing, totaling 22.

Table 4. Parametric model precedence restrictions.

Precedence Restrictions		
$x_9 - x_1 \geq 10 + \beta$	$x_9 - x_7 \geq 21 + 2\beta$	$x_{16} - x_{12} \geq 23 + 2\beta$
$x_7 - x_2 \geq 17 + 2\beta$	$x_{13} - x_7 \geq 21 + 2\beta$	$x_{17} - x_{13} \geq 12 + \beta$
$x_{10} - x_3 \geq 13 + \beta$	$x_{15} - x_8 \geq 30 + 3\beta$	$x_{18} - x_{14} \geq 20 + 2\beta$
$x_{12} - x_3 \geq 13 + \beta$	$x_{12} - x_9 \geq 25 + 3\beta$	$x_{17} - x_{15} \geq 26 + 3\beta$
$x_{10} - x_4 \geq 20 + 2\beta$	$x_{13} - x_{10} \geq 16 + 2\beta$	$x_{18} - x_{16} \geq 18 + 2\beta$
$x_{14} - x_4 \geq 20 + 2\beta$	$x_{17} - x_{10} \geq 16 + 2\beta$	$x_{16} - x_{17} \geq 14 + \beta$
$x_8 - x_5 \geq 14 + \beta$	$x_{14} - x_{11} \geq 18 + 2\beta$	
$x_{11} - x_6 \geq 15 + 2\beta$	$x_{15} - x_{11} \geq 18 + 2\beta$	

Table 5 presents the restrictions on project delivery times.

Table 5. Parametric model delivery time restrictions.

Delivery Time Restrictions
$x_{15} + s_{1a} - s_{1b} = 69 - 3\beta$
$x_{17} + s_{2a} - s_{2b} = 191 - \beta$
$x_{16} + s_{3a} - s_{3b} = 262 - 2\beta$
$x_{18} + s_{4a} - s_{4b} = 302 - 3\beta$

Where:

$$x_i,\ s_{pa},\ s_{pb} \in R_0^+;\ t_i \in \mathbb{Z}^+;\ y_{ij} \in \{0,1\};\ \beta \in [0,1]$$

3.2. Case Study Evaluation

For the case study, 11 degrees of imprecision (degrees of delay) $\beta = \{0,\ 0.1,\ 0.2,\ 0.3,\ \ldots,\ 1\}$ generated eleven work plans for the backhoe in the construction projects (schedule).

As an illustrative example, the graphs of the three work plans corresponding to $\beta = \{0,\ 0.3,\ 1\}$ are shown in Figures 6–8.

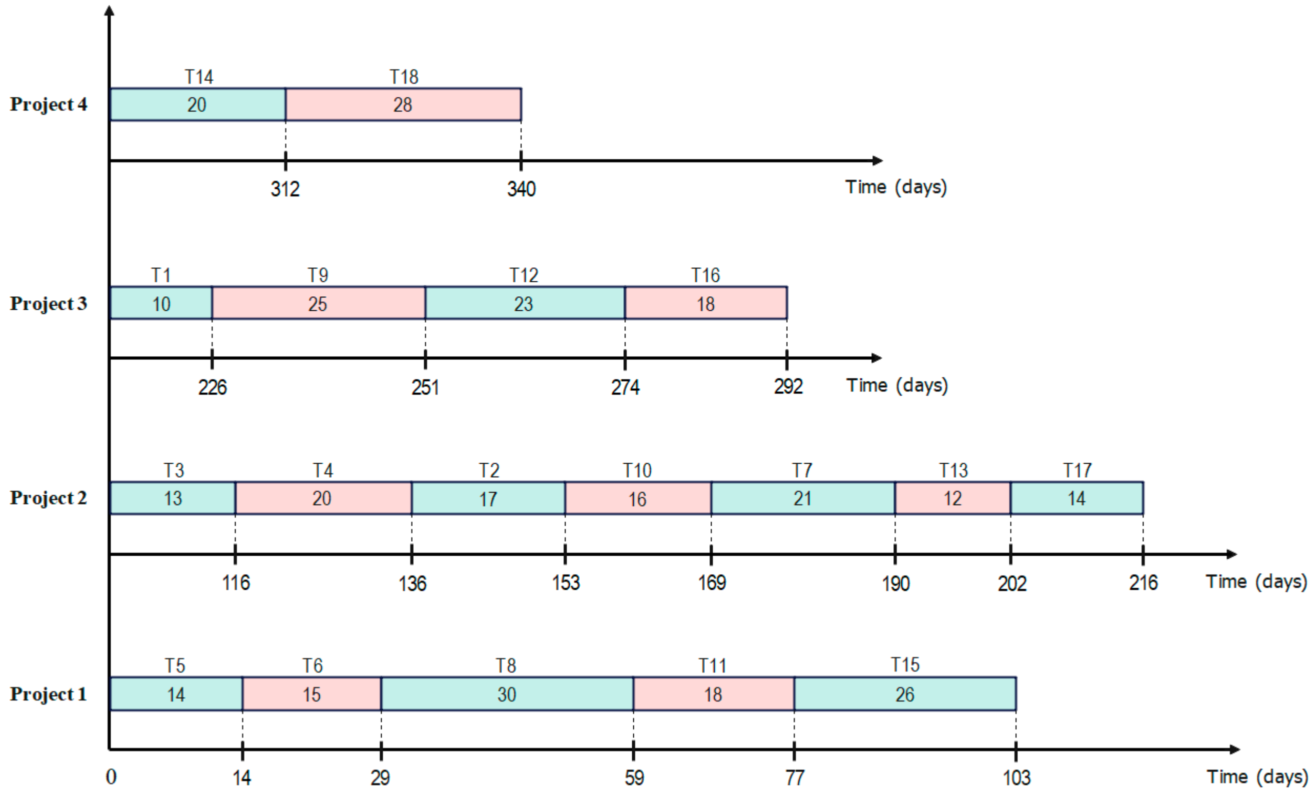

Figure 6. Work plan for $\beta = 0$ (0.0% tolerated delay).

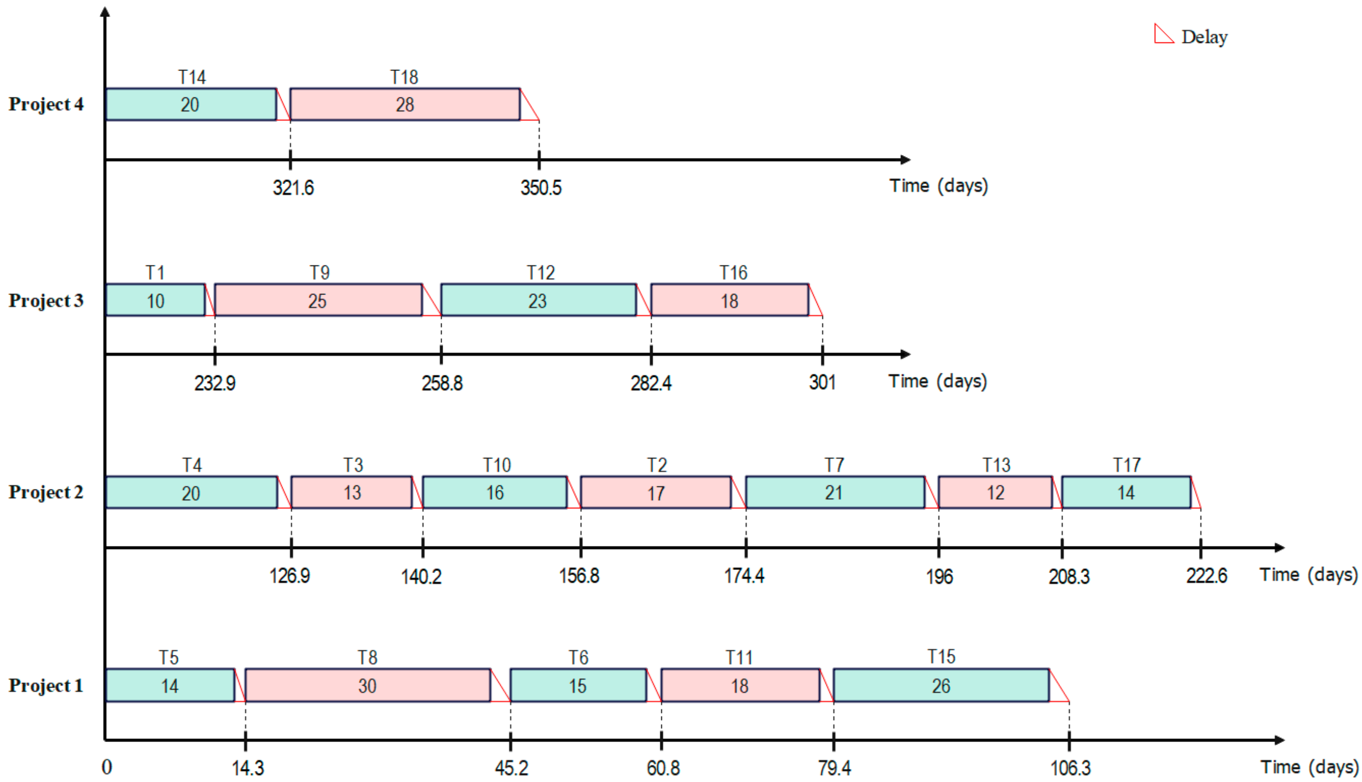

Figure 7. Work plan for $\beta = 0.3$ (30.0% tolerated delay).

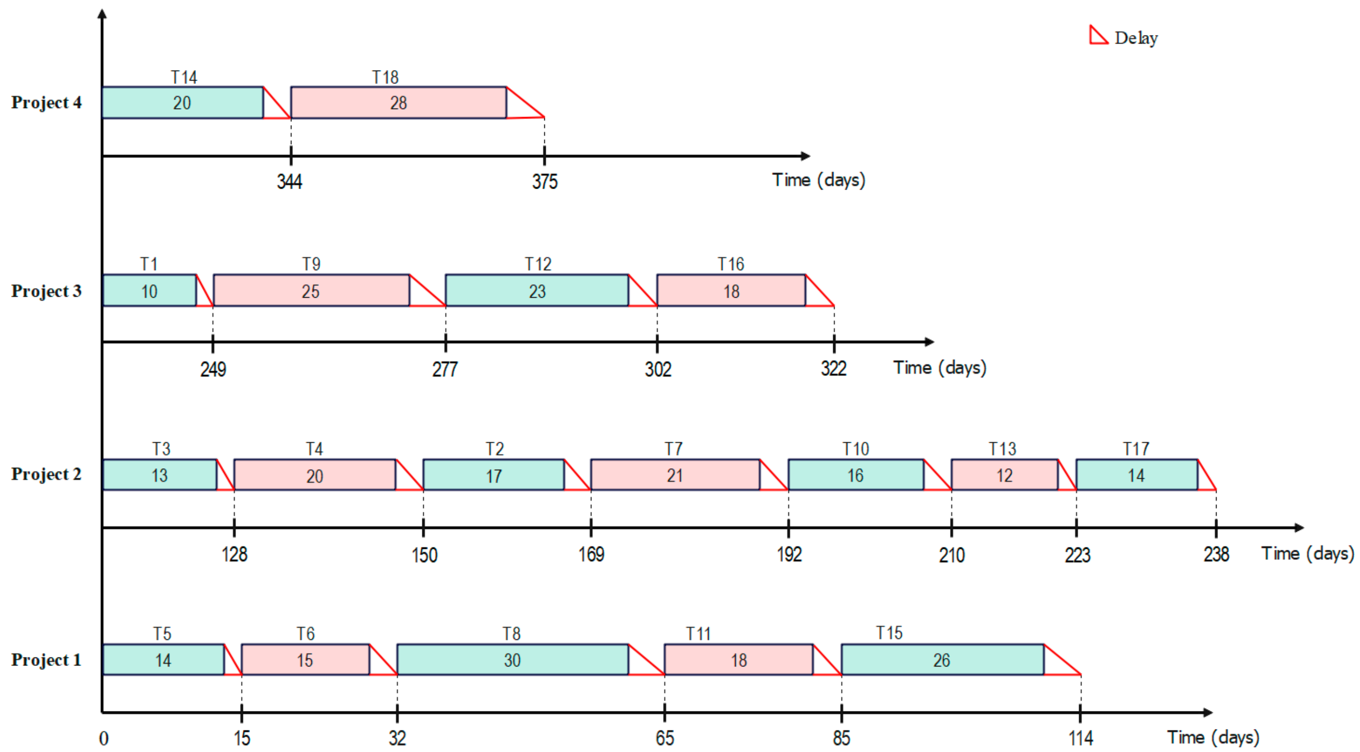

Figure 8. Work plan for $\beta = 1$ (100.0% tolerated delay).

Regarding the degree of imprecision $\beta = 0$ (see Figure 6), results show there is no tolerance for delay in the processing of tasks. For example, if task T4 of Project 2, which must be completed on day 136, is delayed, the execution of task T2 cannot begin on day 136, rendering the work plan infeasible.

For a degree of imprecision of $\beta = 0.3$ (see Figure 7), this implies that a delay of up to 30% is acceptable for all tasks. For instance, task T4 of Project 2 should end on day 126.3 (refer to Table 6), but the plan allows for a delay until day 126.9 while remaining valid.

Table 6. Detailed solution of the case study with $\beta = 0.3$ (degree of delay).

Work Plan	Task Start (x_i)	Expected Processing Time (t_i)	Task Delay (r_i)	Final Processing Time (t_i + r_i)	Delivery Time (f_p)	Project Delay (s_{pb})	Penalty (m_p)
T5	0	14	0.3	14.3			
T8	14.3	30	0.9	30.9			
T6	45.2	15	0.6	15.6			
T11	60.8	18	0.6	18.6			
T15	79.4	26	0.9	26.9	95	11.3	$1000
T4	106.3	20	0.6	20.6			
T3	126.9	13	0.3	13.3			
T10	140.2	16	0.6	16.6			
T2	156.8	17	0.6	17.6			
T7	174.4	21	0.6	21.6			
T13	196	12	0.3	12.3			
T17	208.3	14	0.3	14.3	205	17.6	$1600
T1	222.6	10	0.3	10.3			
T9	232.9	25	0.9	25.9			
T12	258.8	23	0.6	23.6			
T16	282.4	18	0.6	18.6	280	21	$2500
T14	301	20	0.6	20.6			
T18	321.6	28	0.9	28.9	330	20.5	$3000
Optimum Penalty						$153,460	

For $\beta = 1$ (see Figure 8), results show that up to the maximum delay tolerance can be accepted in all tasks, e.g., task T4 of Project 2 must finish on day 148, but the plan accepts a delay up to day 150, and the work plan would still be valid.

The T4 task for $\beta = 0$ does not accept any delay, while for $\beta = 0.3$, it accepts a delay of up to 0.6 of a day, and for $\beta = 1$, it accepts a delay of up to 2 days. The same is true for the other tasks, i.e., as greater tolerance for delays is accepted, the work plan becomes more robust but loses optimality.

On the other hand, for $\beta = 0$, $\beta = 0.3$ y $\beta = 1$, the maximum completion times of the work plans in the execution of the four projects are 340, 350.5, and 375 days respectively, (see Figures 6–8); that is to say, there is a gain in robustness, but more time is used in the work plan.

For the analysis of the minimization of the total penalty generated by the delays in the delivery of the four construction projects, any of the degrees β of delay can be used; as an example, we take $\beta = 0.3$. Upon observing Table 6, it is evident that the construction projects were delivered with delays of 11.3, 17.6, 21, and 20.5 days, respectively, resulting in a total penalty of $153,460. In addition, the work plan implemented for the backhoe was confirmed, which minimized the total penalty generated by the delays in the delivery of the four construction projects.

Table 7 shows the values of the objective function (optimal penalty) for each degree of delay. For example, for $\beta = 0$, $\beta = 0.3$ y $\beta = 1$, the optimum penalties are s/85,600, s/153,460, and s/311,800, respectively, i.e., the greater the degree of delay that is tolerated, the higher the penalty is.

Table 7. Tolerated degree of delay in tasks and the optimum penalty.

β	Z^* (Dollars)
0	85,600
0.1	108,220
0.2	130,840
0.3	153,460
0.4	176,080
0.5	198,700
0.6	221,320
0.7	243,940
0.8	266,560
0.9	289,180
1	311,800

In order to verify the fuzzy model robustness, incidences were simulated in the task processing time (see Table 8). With these incidences, the final work plan was obtained (see Table 9).

Table 8. Incidents (delays) in task processing time.

Task	Delay Hours	Delay Days
T5	6	0.25
T8	8	0.33
T6	0	0.00
T11	10	0.42
T15	7	0.29
T4	12	0.50
T3	4	0.17
T10	11	0.46
T2	13	0.54
T7	0	0.00
T13	6	0.25

Table 8. *Cont.*

Task	Delay Hours	Delay Days
T17	5	0.21
T1	0	0.00
T9	8	0.33
T12	14	0.58
T16	8	0.33
T14	0	0.00
T18	12	0.50

Table 9. Final work plan.

Final Work Plan	Task Start (x_i)	Processing Time (t_i)	Delivery Time (f_p)	Project Delay (s_{pb})	Penalty (m_p)
T5	0	14.25			
T8	14.3	30.33			
T6	45.2	15.00			
T11	60.8	18.42			
T15	79.4	26.29	95	10.69	$1000
T4	106.3	20.50			
T3	126.9	13.17			
T10	140.2	16.46			
T2	156.8	17.54			
T7	174.4	21.00			
T13	196	12.25			
T17	208.3	14.21	205	17.51	$1600
T1	222.6	10.00			
T9	232.9	25.33			
T12	258.8	23.58			
T16	282.4	18.33	280	20.73	$2500
T14	301	20.00			
T18	321.6	28.50	330	20.1	$3000
Optimum Penalty				$150,831	

Figure 9 shows that the final work plan (red line) is part of the fuzzy work plan (black line).

3.3. Application of Fuzzy Model

To apply the fuzzy model in construction projects, it is suggested to follow these steps:

Step 1: Managers should review their portfolios of pending construction projects.

Step 2: Experts in the execution of construction projects must create a table indicating, for each task, the start time, processing time, and maximum tolerance time for the machine to complete the task. The experts should also indicate the degree of imprecision in the delays (β), which can be tolerated for all tasks.

Step 3: With the data from Step 2, the rectangular triangular fuzzy set representing the maximum delay to be tolerated must be designed for each task.

Step 4: Enter the parameters required by the model: for each task, enter the start time and the fuzzy set representing the processing time, the degree of delay tolerated for all tasks, the delivery time for each project, and the penalty for each day of delay.

Step 5: Solve the MILP auxiliary model using a linear programming solver. The decision variables obtained are x_i (start of task i).

Step 6: With the parameters and decision variables, the fuzzy work plan for the machine is elaborated.

Step 7: With the incidents (delays) within the tolerances for each task, the final work plan for the machine must be elaborated.

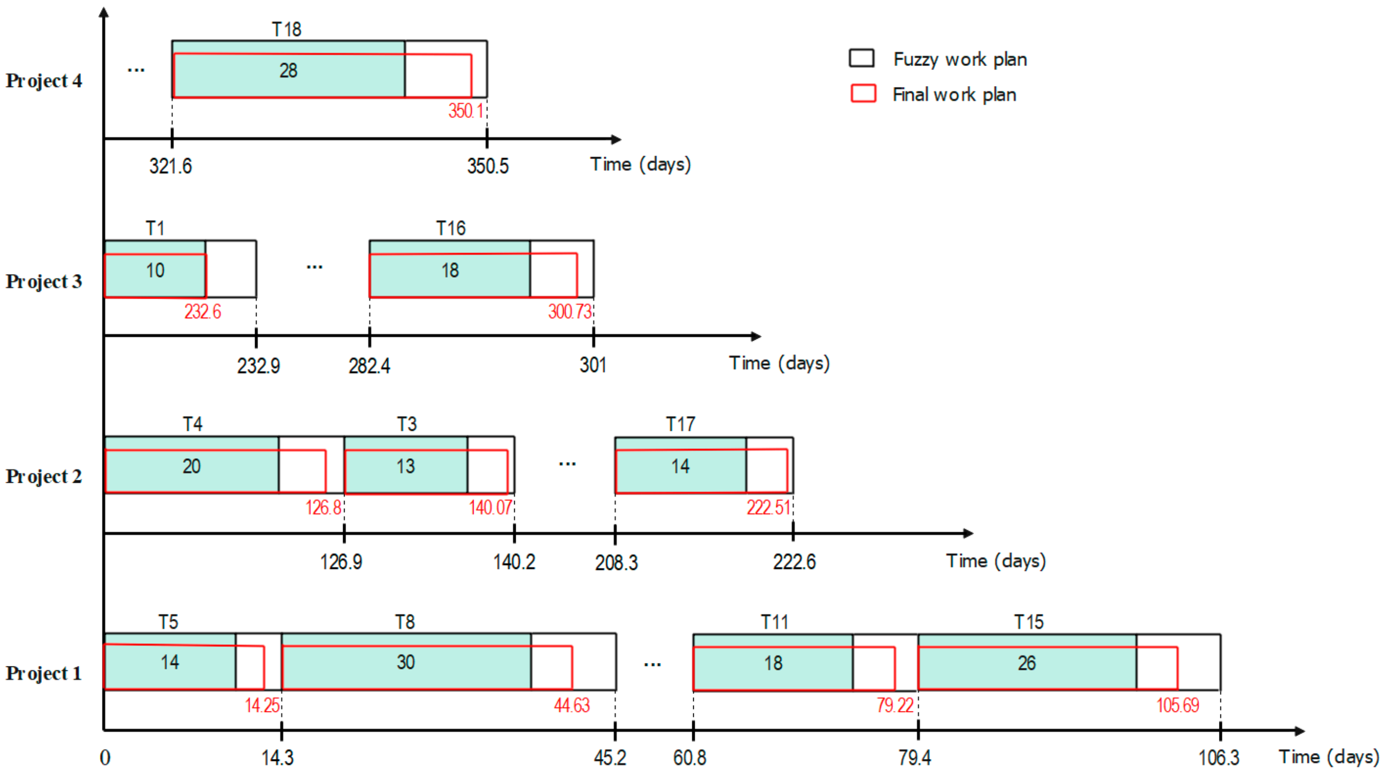

Figure 9. Final work plan included in the fuzzy work plan (See Table 9).

3.4. Fuzzy Model Efficiency

To analyze efficiency, the model was applied to four case studies with a degree of tolerance in delay of $\beta = 0.3$. The results are shown in Table 10.

Table 10. Cases evaluation results.

	Projects	**Tasks**	**Computer Processing Time (s)**	**Optimum**	**Target Value (Dollars)**
Case 1	2	8	2.48	yes	204.30
Case 2	3	15	3.20	yes	17,340.00
Case 3	4	18	37.01	yes	153,460.00
Case 4	4	27	29,700.00	------	------

For Cases 1, 2, and 3, it was observed that, when the number of tasks increased, the computer processing time (computational cost) increased; when adding Case 4, it was observed that the growth of the computational cost was exponential, i.e., it is a highly complex problem (see Figure 9).

It was also observed that, for the first three cases, the optimal solution was obtained. However, for Case 4, the model did not obtain any solution in a time of 8.25 h (29,700.00 s).

Figure 10 illustrates the exponential growth of the computational cost, which was caused by an increase in tasks in the studied cases.

For small and medium instances (up to approximately 25 tasks), the optimal solution was obtained; for large instances, no solution was found.

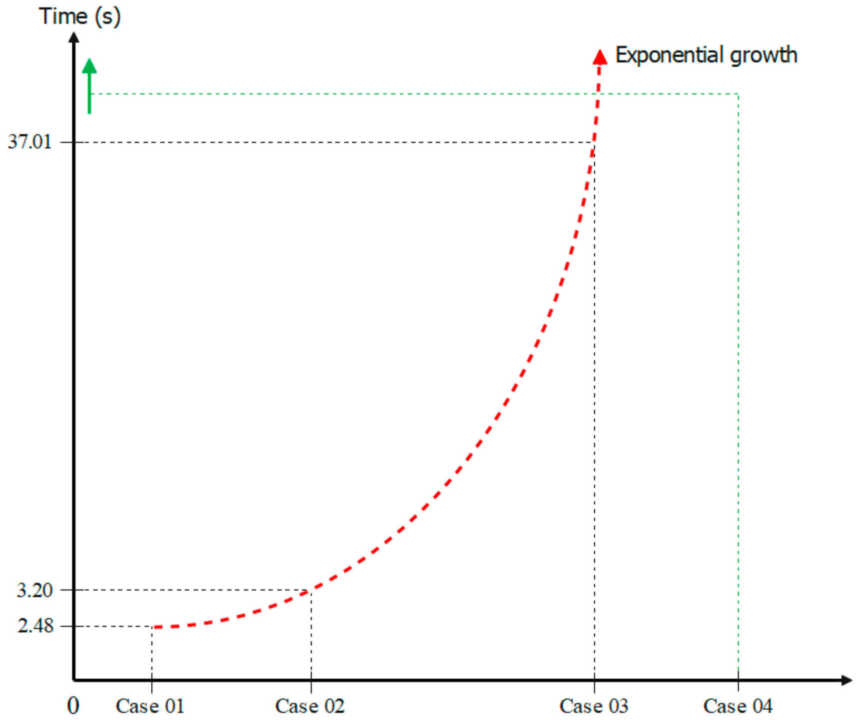

Figure 10. Computer processing time trend.

4. Discussion

The planning of construction projects requires tools that assist project managers in decision-making. Scheduling tasks for a construction project with a limited number of machines is a critical and well-studied problem. However, most studies assume that task processing times are exact. In practice, delays frequently occur, rendering the initial work plan invalid. Therefore, adaptability is crucial for successfully developing a project.

A fuzzy optimization model has been proposed for the planning of construction projects executed simultaneously with only one backhoe and considering non-interference, precedence, and lead-time constraints. The model assumes imprecise task processing times, represented by fuzzy sets with triangular membership functions, that accept delays up to a permitted degree of tolerance. The optimization tool CPLEX was used for the implementation. For previously known input data from a set of tasks, the model solution obtained a fuzzy work plan; this is a robust plan that supports incidents (delays) in task processing time. A method to apply the model was created.

The results obtained showed that the procedure can help construction project managers in their decision-making from a proactive approach, since it has, at its disposal, adaptable and optimized work plans concerning delays in the delivery of projects with the characteristic that, even with greater degrees of delay of the tasks, the model grants a completion time that supports the delays.

The results also indicate that the fuzzy optimization model can help construction companies reduce delays in the delivery of their projects and avoid excessive penalties. The scientific contribution provided by the research is very important for future work since it consists of the application of fuzzy optimization in a specific area of civil engineering.

To analyze efficiency, the model was applied to four case studies. In the first three studies, which involved instances with small and medium numbers (up to 25 tasks), the optimal solution was obtained in less than 37.1 s. However, in the fourth case, which involved a larger number of tasks, no solution was obtained within 8.25 h. When the number of tasks increases, the computational cost grows exponentially. This means that the problem is highly complex.

Finally, as a result of the research, possibilities for future research are opened up:

(i) Extend the proposed model to deal with construction projects that contemplate imprecision with multiple machines.
(ii) Use metaheuristics to solve the fuzzy optimization model when the number of tasks is large.

Author Contributions: Conceptualization, N.A.F. and F.G.S.; methodology, N.A.F. and F.G.S.; software, N.A.F. and R.C.S.; validation, M.E.M.P., J.M.P.O. and A.L.L.G.; investigation, N.A.F. and F.G.S.; data curation, M.E.M.P., J.M.P.O. and A.L.L.G.; writing—original draft preparation, N.A.F. and F.G.S.; writing—review and editing, J.M.P.O., R.C.S. and M.E.M.P. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: All the data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Bettemir, H.; Birgönül, M.T. Network Analysis Algorithm for the Solution of Discrete Time-Cost Trade-off Problem. *KSCE J. Civ. Eng.* **2017**, *21*, 1047–1058. [CrossRef]
2. Eshtehardian, E.; Afshar, A.; Abbasnia, R. Fuzzy-Based MOGA Approach to Stochastic Time-Cost Trade-off Problem. *Autom. Constr.* **2009**, *18*, 692–701. [CrossRef]
3. Alzara, M.; Kashiwagi, J.; Kashiwagi, D.; Al-Tassan, A. Using PIPS to Minimize Causes of Delay in Saudi Arabian Construction Projects: University Case Study. *Procedia Eng.* **2016**, *145*, 932–939. [CrossRef]
4. Zheng, D.X.M.; Ng, S.T. Stochastic Time-Cost Optimization Model Incorporating Fuzzy Sets Theory and Nonreplaceable Front. *J. Constr. Eng. Manag.* **2005**, *131*, 176–186. [CrossRef]
5. Senouci, A.B.; Mubarak, S.A. Multiobjective Optimization Model for Scheduling of Construction Projects under Extreme Weather. *J. Civ. Eng. Manag.* **2016**, *22*, 373–381. [CrossRef]
6. Itoh, T.; Ishii, H. Fuzzy Due-Date Scheduling Problem with Fuzzy Processing Time. *Int. Trans. Oper. Res.* **1999**, *6*, 639–647. [CrossRef]
7. Niu, Q.; Jiao, B.; Gu, X. Particle Swarm Optimization Combined with Genetic Operators for Job Shop Scheduling Problem with Fuzzy Processing Time. *Appl. Math. Comput.* **2008**, *205*, 148–158. [CrossRef]
8. Knyazeva, M.; Bozhenyuk, A.; Rozenberg, I. Resource-Constrained Project Scheduling Approach Under Fuzzy Conditions. *Procedia Comput. Sci.* **2015**, *77*, 56–64. [CrossRef]
9. Al-Zarrad, M.A.; Fonseca, D. A New Model to Improve Project Time-Cost Trade-off in Uncertain Environments. In *Contem-Porary Issues and Research in Operations Management*; University of Alabama: Tuscaloosa, AL, USA, 2018; Volume 6, pp. 96–112.
10. Nguyen, D.-T.; Le-Hoai, L.; Tarigan, P.B.; Tran, D.-H. Tradeoff Time Cost Quality in Repetitive Construction Project Using Fuzzy Logic Approach and Symbiotic Organism Search Algorithm. *Alex. Eng. J.* **2021**, *61*, 1499–1518. [CrossRef]
11. Fernandez, N.A.; Segura, F.G.; Pino, M.E.M.; Chavil, W.J.Q.; Ruiz, A.D.; Sánchez, J.L.C. Optimization Model that Minimizes the Penalty Caused by Delayed Delivery of Construction Projects. *Int. J. Prof. Bus. Rev.* **2023**, *8*, e02553. [CrossRef]
12. Lai, Y.-J.; Hwang, C.-L. *Fuzzy Mathematical Programming*; Springer: Berlín/Heidelberg, Germany, 1992; Volume 394.
13. Gutiérrez, F.A. Un Modelo de Optimización Difusa para el Problema de Atraque de Barcos. Ph.D. Thesis, Universidad Nacional de Trujillo, Trujillo, Peru, 2017. Available online: http://dspace.unitru.edu.pe/handle/UNITRU/8931 (accessed on 24 February 2024).
14. Verdegay, J. Fuzzy Mathematical Programming. In *Fuzzy Information and Decision Processes*; University of Granada: Granada, Spain, 1982; pp. 231–237.