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Evolutionary Game and Simulation Analysis of Food Safety Regulation under Time Delay Effect

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Abstract: This study develops a tripartite evolutionary game dynamic model with a time delay effect to analyze the interactions among food enterprise, government regulatory, and food inspection agencies in managing food safety risks. This model enables government regulatory agencies to more accurately assess and predict food safety risks, thereby implementing more effective preventative measures, ensuring the maximization of policy effectiveness and reducing food safety incidents. The results emphasize the significance of recent company performance by showing that regulatory and inspection entities' strategic decisions are significantly impacted by delay effects from food companies. This study also shows that negative self-feedback intensity drives food enterprises to develop safer products and encourages tighter government oversight. Recommendations include improving consumer reporting channels, changing government incentives and penalties, allocating resources efficiently, and advancing information technology to decrease the effects of time delays and improve food safety management. Governments can improve food safety regulation by using strategic insights from numerical simulations.

Keywords: time-delay; evolutionary game model; food safety; government regulation

MSC: 37M05; 65P40

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1. Introduction

In recent years, numerous food safety incidents have occurred worldwide, highlighting significant regulatory gaps and shortcomings in food safety management. Notable incidents include the widespread egg contamination in Europe in 2018, the salmonella-contaminated onions in the United States in 2021, and the issues with meat product storage in Brazil in 2022. These events have laid bare several critical issues, including the absence of government oversight, flaws in food testing mechanisms, and the ineffectiveness of public complaints. Despite the establishment of a series of food safety standards and regulatory measures by governments and international organizations, the problem of delays in the implementation process remains a significant challenge. For instance, the timeline from the production of contaminated products by food enterprises to their detection by testing institutions, followed by the public disclosure of test results, and finally, the government's regulatory response to these issues, illustrates the critical need for timely decision making and action in food safety governance. Therefore, the government should give significant consideration to the impact of time-lag effects during the food safety regulation process. This paper constructs a tripartite evolutionary game dynamic model with a time delay effect, aimed at its application in food safety regulation to help achieve more accurate and forward-looking decision making. At the same time, the model can aid in understanding the delayed manifestation of policy effects, providing a basis for policy formulation. This assists in adjusting the allocation of resources and the intensity of enforcement in regulatory strategies to maximize policy effectiveness, reducing unnecessary economic losses.

2. Literature Review

Food safety issues have garnered significant attention from scholars globally, leading to extensive research and significant findings in several key areas. The strict food safety legal frameworks established by governments are considered crucial for ensuring consumer safety, highlighting the importance of regulatory measures [1–3]. Research delves deep into each critical stage from farm to table, analyzing the factors influencing decision-making processes and identifying key intervention points to enhance food safety [4,5]. Moreover, the convergence and divergence of interests at different points in the food supply chain reveal the complexity of stakeholder relationships, emphasizing the necessity of adopting a balanced approach [6–9]. The significant impact of information traceability on food safety management is also recognized as an effective tool for mitigating food safety risks [10,11]. Within the broader theoretical backdrop of food safety governance, modern frameworks underscore the collaborative governance among multiple entities to effectively address food safety challenges [12,13]. Additionally, the dynamics of interaction and mutual influence among different parties within various regulatory frameworks are analyzed, highlighting the dynamic interplay in the pursuit of food safety [14–17].

Evolutionary game theory, anchored in the concept of bounded rationality from game theory, provides a robust framework for examining food safety governance. It profoundly investigates the influence of government strategies on agricultural production and the safety of the food supply chain. By developing evolutionary game models, the research unveils the effectiveness of government penalties and collaborative governance strategies in safeguarding food safety. Studies using evolutionary game models have shown the impact of government strategies on agricultural production and food supply chain safety, highlighting the effectiveness of government penalties and co-governance approaches [18,19]. Further research within this framework analyzes the interaction between government regulation and supply chain participants, assessing its effect on the food safety management system and the shift towards organic agriculture, underscoring the pivotal role of government in food safety [20–22]. The application of evolutionary game theory exposes the dynamic mechanisms of food safety regulation and the evolutionary stability of risk communication, offering fresh insights for enhancing regulatory effectiveness and improving communication efficiency. The application of evolutionary game theory exposes the dynamic mechanisms of food safety regulation and the evolutionary stability of risk communication, offering fresh insights for enhancing regulatory effectiveness and improving communication efficiency [23,24]. By establishing a tripartite evolutionary game model involving food manufacturers, the government, and consumers, Wu explored how blockchain technology influences the behavior strategies of food manufacturing enterprises and the level of social co-governance within China's social co-governance framework [25]. Additionally, analyses of quality management and information disclosure strategies in the food supply chain through evolutionary game models have further validated the critical importance of regulatory strategies and public oversight in ensuring food safety [26–28].

In evolutionary game theory, various governance entities hold differing expectations, and their choices of decision-making strategies may be subject to time-delay effects. These decision delays can significantly affect the dynamic behavior of participants within the food safety governance system. By employing delay differential equation methods to analyze the time-delay effects, Sun has quantified the temporal gap between governance actions and outcomes, aiming to assess and optimize existing regulatory strategies [29]. Further integrating evolutionary game theory, Cheng have examined the comprehensive impact of equilibrium states, system bifurcation conditions, and time-delay factors on the outcomes of evolutionary games [30]. By merging the prisoner's dilemma model, punishment mechanisms, and analysis of time-delay effects, the evolutionary dynamics of cooperation and betrayal strategies have been revealed. Wang discusses how strategy stability is influenced by punishment severity and time delays, providing a theoretical basis for promoting cooperative behaviors [31]. Moreover, the introduction of time-delayed evolutionary game models and the application of dynamic system theory have deeply analyzed the impact of delayed feedback on cooperative behaviors, common resource management, and the stability of three-strategy game models, offering new perspectives for understanding the sustainable utilization of common resources and time-delay effects [32,33].

The recent study focuses on exploring the combination of evolutionary strategies for food safety regulation under various policies or mechanisms from the perspective of evolutionary game models, without sufficiently considering the impact of time-delay effects in the food safety regulation process. Therefore, this paper innovatively integrates the tripartite evolutionary game dynamic model with time delay effect, analyzing the impact of time delay effect on the evolutionary trend of food safety regulation strategies under the government's reward and punishment mechanism and consumer feedback mechanism. The research gaps are shown in Table 1, and below are the main innovative points of the paper.

- Integrating consumer feedback and government reward and punishment systems, this paper constructs a tripartite game model involving food enterprise, food inspection agency, and government regulatory. By employing ordinary differential equations and delay differential equations, it analyzes the strategy stability of each game participant and verifies the effectiveness and consistency of the model analysis under different initial conditions.
- Combining government reward and punishment mechanisms with consumer feedback, this study investigates the influence of different mechanisms on the strategic choices of food enterprise, food inspection agency, and government regulatory, as well as the impact of each element on the rate of evolution.
- Taking into account the strategic choices of food enterprise, food inspection agency, and government regulatory, which include cross-delay and self-feedback delays, this study uses numerical analysis and simulation to explore how delay terms and self-feedback intensity affect the strategic choices of each entity and their interactions. It focuses on analyzing how internal feedback mechanisms in the system lead to adjustments in strategy decisions and how such strategic adaptive adjustments impact the system's long-term dynamics and stability.

Lastly, based on the analytical conclusions, the paper proposes strategies and recommendations for the government to improve the food safety regulatory system.

Table 1. Differences between ODEs and DDEs.

Feature	Ordinary Differential Equations (ODEs)	Delay Differential Equations (DDEs)
Definition	Equations that describe the relationship between one or more variables and their derivatives, assuming these relationships occur instantaneously.	In addition to the relationships between variables and their current derivatives, it considers the impact of the variable values at a past moment on the current state, introducing the concept of time delay.
Research Gap	- Unable to effectively handle or reflect the delay effects between decision making and implementation. - Ignores the impact of historical data on current decisions.	- Capable of simulating the delay between decisions and outcomes, more in line with real situations. - Considers the influence of historical states on current decisions and strategy changes, providing a more comprehensive analytical framework.
Limitations	Oversimplify actual problems when ignoring time delays.	More complex in terms of computation.

3. Model Establishment and Assumptions

In food safety supervision, the main participants include food enterprise, food inspection agency, and government regulatory, which are considered to be decision makers with bounded rationality. The strategic choices of these entities are dynamic and gradually evolve into optimal strategies, consistent with the basic assumptions of evolutionary game theory [34]. The logical relationship among the tripartite evolution game entities in food safety supervision constructed in this paper is shown in Figure 1.

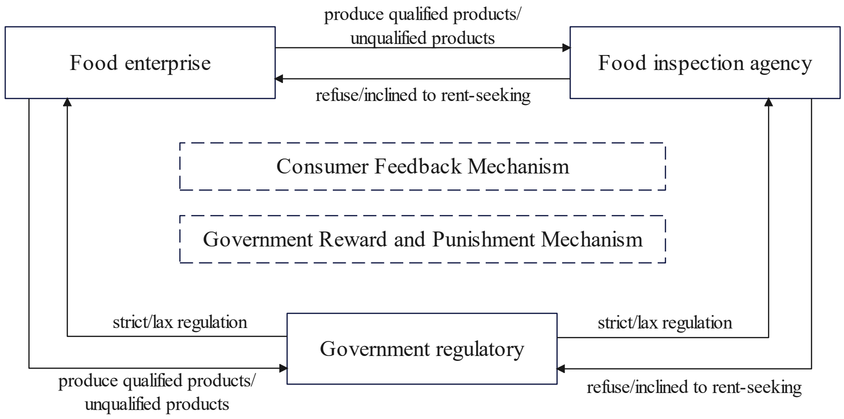

Figure 1. Logical diagram of the tripartite evolutionary game model.

To further refine the model, analyzing the stability of the strategies and equilibrium points of all parties as well as the influence relationships among various elements, the following assumptions are made:

Assumption 1. *The strategic domain for food enterprise is defined as* $\alpha = (\alpha_1, \alpha_2)$*, representing the actions of producing qualified products and producing unqualified products, respectively. Enterprises adopt strategy* α_1 *with probability* x *and* α_2 *with probability* $1 - x$*, where* $x \in [0, 1]$*. The strategy set for food inspection agency is* $\beta = (\beta_1, \beta_2)$*, signifying the choice of refusing or being inclined to rent seeking. These institutions select* β_1 *with probability* y *and* β_2 *with probability* $1 - y$*, where* $y \in [0, 1]$*. Lastly, the strategic options for government regulatory are denoted by* $\gamma = (\gamma_1, \gamma_2)$*, which correspond to strict or lenient regulation. They implement* γ_1 *with probability* z *and* γ_2 *with probability* $1 - z$*, where* $z \in [0, 1]$*.*

Assumption 2. *The revenue from product sales for food enterprise,* S_p*, exceeds the costs of producing qualified* C_{ph} *and substandard goods* C_{pl}*, with* $S_p > C_{ph} > C_{pl}$*. Additionally, producing unqualified products incurs extra costs,* C_s*, including rent-seeking and unethical practices like document fabrication and false advertising.*

Assumption 3. *Food products undergo safety evaluations, with those meeting standards allowed in the market, and non-compliant ones removed. Inspection agencies earn an income,* V_t*, from these evaluations. The cost incurred by the inspection agency in rent-seeking,* C_t*, including actions like falsifying reports, is less than their revenue from testing,* R_t*, i.e.,* $C_t < R_t$*.*

Assumption 4. *Under strict government oversight, food enterprise distributing subpar products and inspection agencies engaging in rent-seeking face monetary penalties,* F_c *and* F_t *respectively. Conversely, compliant enterprises and integrity-driven inspection agencies receive incentives,* A_c *and* A_t*. With lax regulation, neither incentives nor sanctions apply. The cost of enforcing strict regulations is* C_g*.*

Assumption 5. *Food enterprise producing unqualified products face brand reputation damage, leading to customer loss, reduced potential consumer interest, and negative feedback, economically quantified as* P_c*. Similarly, trust in local food inspection agencies declines with market penetration of unqualified products, affecting their service demand or leading to license revocation, with economic impacts represented by* P_t*.*

Assumption 6. *Manufacturing quality-compliant products results in a net social gain, A_s, enhancing public health, economic growth, and social cohesion. Conversely, inferior products increase public healthcare costs, hinder well-being and economic progress, with government bearing remediation costs, D_g. A lenient regulatory environment risks public health and may lead to censure and administrative sanctions on regulatory bodies, T_g, with the cost of strict regulation, C_g, being lower than the penalties for regulatory failure, $C_g < T_g$.*

Assumption 7. *The model incorporates delay mechanisms, suggesting that strategic decision-making is influenced by both current conditions and specific past states, represented by delay parameters τ_x, τ_y, and τ_z for food enterprise, food inspection agency, and government regulatory, respectively, as shown in Figure 2. Additionally, self-feedback intensity parameters N_α, N_β, and N_γ quantify the impact of past strategies on current recalibrations for food enterprise, food inspection agency, and government regulatory, reflecting their sensitivity to historical strategies and guiding current operations and regulatory approaches.*

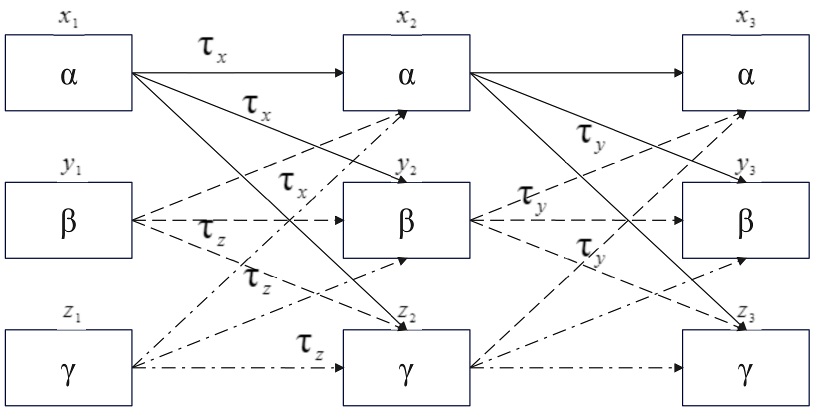

Figure 2. Conceptual diagram of time-delay effects among entities.

Based on the above assumptions, the mixed strategy game matrix of food enterprise, food inspection agency, and government regulatory is shown in Table 2.

Table 2. Mixed strategy game matrix for food enterprise, food inspection agency, and government regulatory department.

	Food Inspection Agency		**Government Regulatory**	
			Strict Regulation (z)	**Lax Regulation (1 − z)**
Food Enterprise	Qualified Products (x)	Refuse Rent-seeking (y)	$S_p - C_{ph} + A_c$, $V_c + A_t$, $-C_g - A_c - A_t + A_s$	$S_p - C_{ph}$, V_t, A_s
		Intend Rent-seeking (1 − y)	$S_p - C_{ph} + A_c$, $V_t - C_t - F_t - P_t$, $-C_g - A_c + F_t + A_s$	$S_p - C_{ph}$, $V_t - C_t - P_t$, A_s
	Produce Unqualified Products (1 − x)	Refuse Rent-seeking (y)	$-C_{pl} - F_c - C_s - P_c$, $V_t + A_t$, $-C_g + F_c - A_t$	$-C_{pl} - C_s - P_c$, V_t, 0
		Intend Rent-seeking (1 − y)	$S_p - C_{pl} - C_s - R_t - F_c - P_c$, $V_t + R_t - C_t - F_t - P_t$, $-C_g + F_c + F_t - D_g$	$S_p - C_{pl} - C_s - R_t - P_c$, $V_t + R_t - C_t - P_t$, $-D_g - T_g$

4. Model Analysis

4.1. Analysis of the Strategic Stability of Food Enterprise

The expected profits for food enterprise producing qualified or unqualified products, and its average expected profits (E_x, E_{1-x}), are, respectively, defined as follows:

$$E_x = S_p - C_{ph} + zA_c \tag{1}$$

$$E_{1-x} = S_p - C_{pl} - C_s - R_t - P_c + y(R_t - S_p) - zF_c \tag{2}$$

The ordinary differential equations (ODEs) and the delay differential equations (DDEs) for the strategy choice of food enterprise are defined as follows:

$$F(x) = x(x-1)(C_{ph} - C_{pl} - C_s - R_t - P_c + y(R_t - S_p) - z(A_c + F_c)) \tag{3}$$

$$\begin{aligned} x'(t) = x(t)(x(t)-1)(C_{ph} - C_{pl} - C_s - R_t - P_c + y(t-\tau_y)(R_t - S_p) \\ - z(t-\tau_z)(A_c + F_c) + N_\alpha x(t-\tau_x)) \end{aligned} \tag{4}$$

Corollary 1. *The probability of food enterprise producing compliant products increases as the rate at which local food testing institution refuse rent seeking and the strictness of government regulatory oversight rise.*

Proof. Consider the first-order partial derivatives of the Equation (3) with respect to y and z, and the first-order partial derivatives of the Equation (4) with respect to τ_y, τ_z: $\partial F(x)/\partial y = R_t - S_p < 0$, $\partial F(x)/\partial z = -(A_c + F_c) < 0$, $\partial x'(t)/\partial \tau_y = -y(R_t - S_p) > 0$, $\partial x'(t)/\partial \tau_z = z(A_c + F_c) > 0$. □

Implications of Corollary 1: As the probability of rent-seeking behavior decreases within inspection agencies and government regulatory bodies strengthen measures for rigorous oversight, the likelihood of food enterprises producing compliant products correspondingly increases. This suggests that to enhance food safety standards, regulatory bodies can achieve this by either intensifying governmental oversight or by enhancing the impartiality and independence of third-party inspection agencies. The implementation of such strategies not only encourages food enterprises to produce high-quality, compliant products but also contributes to the establishment of a more equitable and transparent food safety regulatory framework. By strengthening collaboration and supervision between the government and third-party entities, it becomes possible to more effectively mobilize societal engagement in food safety regulation, collectively safeguarding public health and market integrity. This integrated system of quality regulation, which combines governmental and societal forces, holds significant importance in elevating food safety standards.

According to the stability of the Equation (3) the probability of food enterprises choosing to produce compliant products is stable if and only if $F(x) = 0$ and $\frac{dF(x)}{dx} < 0$. Hence, when $(y, z) = \left(0, \frac{C_{ph} - C_{pl} - C_s - R_t - P_c}{A_c + F_c}\right)$ and $\left(\frac{C_{pl} - C_{ph} + C_s + R_t + P_c}{R_t - S_p}, 0\right)$, at these points $F(x) \equiv 0$ and $\frac{d(F(x))}{dx} \equiv 0$, making it unclear to define the optimal stable strategy for food enterprises. When $y < \frac{C_{ph} - C_{pl} - C_s - R_t - P_c - z(A_c + F_c)}{S_p - R_t}$, then $\frac{d(F(x))}{dx} < 0$, and $x = 0$ is the Evolutionarily Stable Strategy (ESS) for food enterprises. Conversely, $x = 1$ is ESS. Let $y^{(1)} = \frac{C_{ph} - C_{pl} - C_s - R_t - P_c - z(A_c + F_c)}{S_p - R_t}$. The evolutionary phase diagram for the strategies of food enterprises is shown in Figure 3.

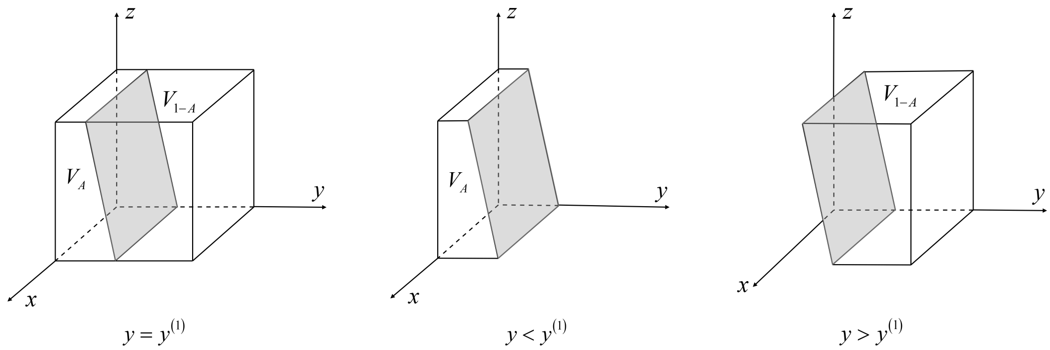

Figure 3. Phase diagram of the strategy evolution for food enterprise.

Let V_A be the probability of a food enterprise stably producing unqualified products, and V_{1-A} be the probability of a food enterprise stably producing qualified products. The values are obtained by integral calculation as follows:

$$\begin{aligned} V_A &= \int_0^1 \int_0^1 \frac{C_{ph} - C_{pl} - C_s - R_t - P_c + y(R_t - S_p)}{A_c + F_c} \\ &= \frac{2(C_{ph} - C_{pl} - C_s - P_c) - (R_t + S_p)}{2(A_c + F_c)} \end{aligned} \quad (5)$$

$$V_{1-A} = 1 - V_A \quad (6)$$

Corollary 2. *The probability, represented as V_{1-A}, of a food enterprise producing qualified products is positively correlated with the speculative cost of food, the rent-seeking cost, the intensity of government rewards and penalties, the consumer negative feedback, and the cost of producing unqualified products. Conversely, it is negatively correlated with the cost of producing qualified products.*

Proof. Consider the probability V_{1-A} of a food enterprise producing qualified products. Calculate the first-order partial derivative of the elements with respect to Equation (6): $\partial V_{1-A}/\partial R_t > 0$, $\partial V_{1-A}/\partial S_p > 0$, $\partial V_{1-A}/\partial C_{pl} > 0$, $\partial V_{1-A}/\partial C_s > 0$, $\partial V_{1-A}/\partial P_c > 0$, $\partial V_{1-A}/\partial A_c > 0$, $\partial V_{1-A}/\partial F_t > 0$, and $\partial V_{1-A}/\partial C_{ph} < 0$. Consequently, an increment in S_p, C_{pl}, R_t, C_s, P_c, A_c, F_t, or a decrement in C_{ph}, enhances the probability of producing qualified products. □

Implications of Corollary 2: In the food production industry, ensuring that enterprises produce qualified products is crucial for maintaining their sales revenue. To achieve this, the government needs to adopt a comprehensive strategy to promote compliant production. This includes strengthening reward and punishment mechanisms, which directly impact the cost-benefit analysis of enterprises, encouraging them to lean towards producing qualified products. Additionally, by enhancing information transparency and expanding public supervision, such as promoting the use of information technology and strengthening media publicity, the government can indirectly increase enterprises' speculative costs and enhance public attention to food safety through media networks. The implementation of these strategies helps improve food quality while promoting a healthier, more transparent market environment.

4.2. Analysis of the Strategic Stability of Food Inspection Agency

The expected profits for food inspection agencies refusing rent-seeking or inclined to rent-seeking, and their average expected profits (E_y, E_{1-y}), are, respectively, defined as follows:

$$E_y = V_t + zA_t \quad (7)$$

$$E_{1-y} = R_t - P_t - C_t + V_t - xR_t - zF_t \quad (8)$$

The ordinary differential equations and the delay differential equations for the strategy choices of food inspection agency are defined as follows:

$$F(y) = y(y-1)(R_t - P_t - C_t - xR_t - z(A_t + F_t)) \tag{9}$$

$$\begin{aligned} y'(t) = {} & y(t)(y(t)-1)\big(R_t - P_t - C_t - x(t-\tau_x)R_t \\ & - z(t-\tau_z)(A_t + F_t) + N_\beta y(t-\tau_y)\big) \end{aligned} \tag{10}$$

Corollary 3. *The probability of food inspection agency resisting rent-seeking behavior increases with enhanced supervision by government regulatory and the likelihood of enterprise producing more effective food products.*

Proof. Consider the first-order partial derivatives of the Equation (9) $F(y)$ with respect to x and z, and the first-order partial derivatives of Equation (10) $y'(t)$ with respect to τ_x, τ_z: $\partial F(y)/\partial x = -R_t < 0$, $\partial F(y)/\partial z = -z(A_t + F_t) < 0$, $\partial y'(t)/\partial \tau_x = xR_t > 0$, $\partial y'(t)/\partial \tau_z = z(A_t + F_t) > 0$. □

Implications of Corollary 3: In the food safety regulatory system, the strategic choices of food enterprises and government regulatory departments significantly influence the behavior of third-party food testing agencies. Strengthened government oversight and the inclination of food enterprises towards producing compliant products can effectively encourage third-party testing agencies to reject rent-seeking behavior. To promote the healthy development of the food industry and ensure the fairness of testing, strict regulatory policies must be implemented, and companies encouraged to adhere to safety production standards. Such synergistic strategies will foster a virtuous cycle, contributing to the enhancement of food safety standards, protecting consumer health, and improving the overall credibility and sustainability of the industry.

Based on the stability of the Equation (9), the probability of food inspection agencies choosing to refuse rent-seeking is stable if and only if $F(y) = 0$ and $\frac{d(F(y))}{dt} = 0$. Therefore, when $(x,z) = \left(0, \frac{R_t - P_t - C_t}{A_t + F_t}\right)$ and $(x,z) = \left(\frac{R_t - P_t - C_t}{R_t}, 0\right)$, at these points $F(y) \equiv 0$ and $\frac{d(F(y))}{dy} \equiv 0$, making it impossible for food inspection agencies to determine their equilibrium strategy. When $x < \frac{R_t - P_t - C_t - z(A_t + F_t)}{R_t}$, $\frac{d(F(y))}{dy} < 0$, $y = 0$ is the Evolutionarily Stable Strategy (ESS). Conversely, $y = 1$ is ESS. Let $x^{(2)} = \frac{R_t - P_t - C_t - z(A_t + F_t)}{R_t}$, the evolutionary phase diagram of strategies for food inspection agency is shown in Figure 4.

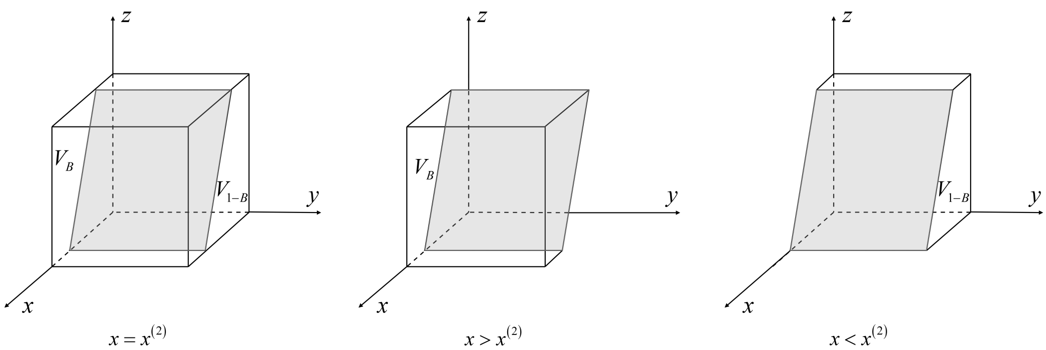

Figure 4. Phase diagram of the strategy evolution for food inspection agencies.

Let V_B be the probability of a food inspection agency stably refusing rent seeking, and V_{1-B} be the probability of a food inspection agency stably inclined to rent seeking. These probabilities are obtained through integral calculation as follows:

$$V_{1-B} = \int_0^{\frac{R_t - P_t - C_t}{R_t}} \int_0^1 \frac{R_t - P_t - C_t - xR_t}{A_t + F_c} dydx = \frac{(C_t + P_t - R_t)^2}{2R_t(A_t + F_c)} \tag{11}$$

$$V_B = 1 - V_{1-B} \tag{12}$$

Corollary 4. *The probability V_{1-B} of a food inspection agency refusing rent-seeking is positively correlated with the government's reward and punishment mechanism. Specifically, when the rent-seeking revenue exceeds the speculative cost and negative feedback, it is also positively correlated with the speculative cost and consumer negative feedback.*

Proof. According to the expression for the probability V_{1-B} of food inspection agency refusing rent seeking, calculate the first-order partial derivative of the elements with respect to Equation (11): $\partial V_{1-B}/\partial A_t > 0$, $\partial V_{1-B}/\partial F_c > 0$, when $C_t + P_t > R_t$, $\partial V_{1-B}/\partial R_t > 0$, $\partial V_{1-B}/\partial C_t < 0$, $\partial V_{1-B}/\partial P_t < 0$. □

Implications of Corollary 4: When the rent-seeking benefits of regulatory bodies are significant, the government should strengthen strict supervision over local food inspection agencies. This can be achieved by increasing rewards and penalties, expanding media disclosure, and other measures, which would help increase the probability of inspection agencies refusing rent-seeking activities.

4.3. Analysis of the Strategic Stability of Government Regulatory

The expected benefits for government regulatory departments under strict or lenient regulation, as well as their average expected benefits (E_z, E_{1-z}), are, respectively, defined as follows:

$$\begin{aligned} E_z = &-C_g + F_c + F_t - D_g + x(D_g + A_s - A_c) \\ &+ y(D_g - F_t - A_t) - xyD_g \end{aligned} \tag{13}$$

$$E_{1-z} = -D_g - T_g + x(A_s + D_g + T_g) + y(D_g + T_g) - xy(D_g + T_g) \tag{14}$$

The ordinary differential equations and the time-delay differential equations for the strategy selection of government regulatory agencies are as follows:

$$F(z) = z(z-1)\left(C_g - F_c - F_t - T_g + x(A_c + F_c + T_g) + y(A_t + F_t + T_g) - xyT_g\right) \tag{15}$$

$$\begin{aligned} z'(t) = &z(t)(z(t)-1)\big(C_g - F_c - F_t - T_g + x(t-\tau_x)(A_c + F_c + T_g) \\ &+ y(t-\tau_y)(A_t + F_t + T_g) - x(t-\tau_x)y(t-\tau_y)T_g + N_\gamma z(t-\tau_z)\big) \end{aligned} \tag{16}$$

Corollary 5. *The strict supervision rate of government regulatory departments decreases as the compliance rate of food enterprises or the refusal rate of rent-seeking by food inspection agency increases. The impact of time-delay effects from food enterprises and food testing institutions on government regulatory departments varies over time.*

Proof. Consider the first-order partial derivatives of the Equation (15) $F(z)$ with respect to x and y, and Equation (16) $z'(t)$ with respect to τ_x, τ_y: $\partial F(z)/\partial x = A_c + F_c + T_g - yT_g > 0$, $\partial F(z)/\partial y = A_t + F_t + T_g - xT_g > 0$, $\partial z'(t)/\partial \tau_x = -x(A_c + F_c + T_g) + xy(t-\tau_y)T_g$, $\partial z'(t)/\partial \tau_y = -y(A_t + F_t + T_g) + x(t-\tau_x)yT_g$. □

Implications of Corollary 5: In the food safety regulatory system, the intensified supervision by government regulatory agencies and the strategic choice of food enterprises to increase the proportion of qualified products have a significant impact on the decision making of third-party food testing institutions. Specifically, strengthening government regulatory efforts and encouraging companies to increase the output of high-quality products can effectively motivate testing institutions to avoid rent-seeking behavior. Therefore, to promote the healthy development of the food industry and ensure the fairness of testing, the key lies in the government implementing strict regulatory policies and incentivizing companies to adhere to food safety standards. This not only safeguards consumer health but also promotes the long-term stable development of the food industry.

Based on the stability of Equation (15), the probability of government regulatory departments opting for strict supervision is stable if and only if $F(z) = 0$ and $\frac{d(F(z))}{dz} < 0$. Hence, when $(x, y) = \left(0, \frac{F_c+F_t+T_g-C_g}{A_t+F_t+T_g}\right)$ and $(x, y) = \left(\frac{F_c+F_t+T_g-C_g}{A_c+F_c+T_g}, 0\right)$, at these points $F(z) \equiv 0$ and $\frac{d(F(z))}{dz} \equiv 0$, making it unclear for government regulatory departments to define their equilibrium strategy. When $x > \frac{(F_c+F_t+T_g-C_g-y(A_t+F_t+T_g))}{A_c+F_c+T_g-yT_g}$, then $\frac{d(F(z))}{dz} < 0$, $z = 0$ is the Evolutionarily Stable Strategy (ESS). Conversely, $z = 1$ is ESS. Let $y^{(3)} = \frac{F_c+F_t+T_g-C_g-x(A_c+F_c+T_g)}{A_t+F_t+T_g-xT_g}$, the evolutionary phase diagram for the strategies of government regulatory is shown in Figure 5.

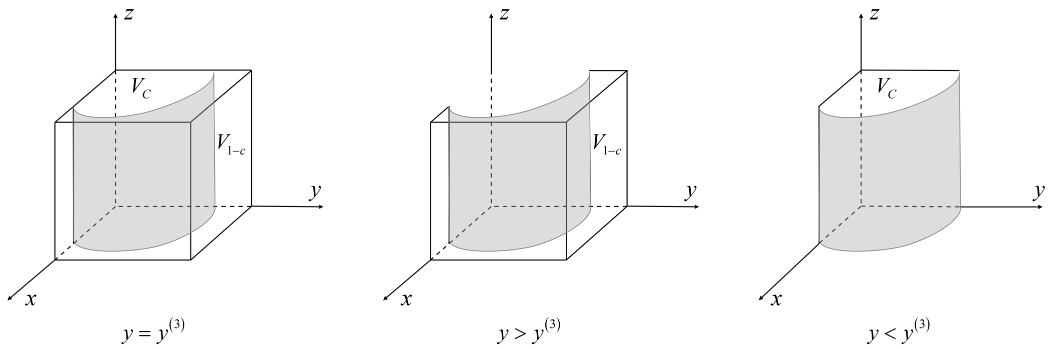

Figure 5. Phase diagram of the strategy evolution for government regulatory.

Let V_C be the probability of the government regulatory agencies maintaining strict regulation stably, and V_{1-C} be the probability of maintaining lenient regulation stably. The values are obtained through integral calculation.

$$V_C = \int_0^1 \int_0^1 \frac{F_c + F_t - C_g + T_g - x(A_c + F_c + T_g)}{A_c + F_t + T_g - xT_g} dxdz \\ \approx \frac{1}{2} - \frac{2A_c + 2C_g - F_c - F_t}{A_c + F_c + T_g} \tag{17}$$

$$V_{1-C} = 1 - V_C \tag{18}$$

Corollary 6. *The probability of strict regulation by government regulatory agencies, V_C, is positively correlated with the intensity of punishment, and negatively correlated with the intensity of rewards and the cost of regulation. When the intensity of rewards and the cost of regulation are high, the probability of strict regulation is positively correlated with higher-level administrative penalties.*

Proof. According to the expression for the probability V_C of strict regulation by government regulatory agencies, calculate the first-order partial derivative of the elements with respect to Equation (17): $\partial V_C/\partial F_t > 0$, $\partial V_C/\partial F_c > 0$, $\partial V_C/\partial C_g > 0$, $\partial V_C/\partial A_c < 0$. When $A_c + C_g > F_c + F_t/2$, $\partial V_C/\partial T_g > 0$. □

Implications of Corollary 6: The government regulatory departments, by increasing fines on food enterprises and local food testing institutions while reducing incentive measures, help strengthen the supervision of food safety. This not only increases the likelihood of third-party food testing institutions refusing inappropriate exchanges of interests, effectively preventing unqualified food from entering the market, but also imposes more severe administrative penalties on regulatory departments, prompting them to fulfill their food safety supervision duties more strictly and responsibly.

4.4. Stability Analysis of Equilibrium Points in a Tripartite Evolutionary Game System

This part explores the impact of different strategic parameters on the long-term behavior of the system and potential equilibrium points without considering historical states or time-delay effects. This aims to precisely describe the behavioral patterns of the system under simplified conditions, and also provides a theoretical basis and reference for subsequently introducing time-delay parameters.

The ODEs (3), (9) and (15) for the strategic choices of food enterprise, local food inspection agency, and government regulatory are as follows:

$$\begin{cases} \dfrac{dx}{dt} = x(x-1)(C_{ph} - C_{pl} - C_s - R_t - P_c + y(R_t - S_p) - z(A_c + F_c)), \\ \dfrac{dy}{dt} = y(y-1)(R_t - P_t - C_t - xR_t - z(A_t + F_t)), \\ \dfrac{dz}{dt} = z(z-1)(C_g - F_c - F_t - T_g + x(A_c + F_c + T_g) + y(A_t + F_t + T_g) - xyT_g) \end{cases}$$

Given $F(x) = 0$, $F(y) = 0$, $F(z) = 0$ and $x, y, z \in [0,1]$, the equilibrium points of the system can be determined as follows: $E_1(0,0,0)$, $E_2(0,0,1)$, $E_3(0,1,0)$, $E_4(1,0,0)$, $E_5(1,1,0)$, $E_6(1,0,1)$, $E_7(0,1,1)$, $E_8(1,1,1)$, $E_9(0,(F_c + F_t + T_g - C_g)/(A_t + F_t + T_g),(R_t - P_t - C_t)/(F_t + A_t)$, $E_{10}(F_c + F_t + T_g - C_g)/(A_c + F_c + T_g), 0, (C_{ph} - C_{pl} - C_s - R_t - P_c)/(A_c + F_c$, $E_{11}(R_t - P_t - C_t)/(R_t), (C_{pl} + C_s + R_t + P_c - C_{ph})/(R_t - S_p), 0)$, $E_{12}(R_t - P_t - C_t - A_t - F_t)/R_t, (C_{pl} + C_s + P_c + R_t + A_c + F_c - C_{ph})/(R_t - S_p), 1)$.

Given $F(x) = dx/dt$, $F(y) = dy/dt$, $F(z) = dz/dt$, the Jacobian matrix of the three-player evolutionary game system is obtained as follows:

$$J = \begin{bmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} & \frac{\partial F(x)}{\partial z} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} & \frac{\partial F(y)}{\partial z} \\ \frac{\partial F(z)}{\partial x} & \frac{\partial F(z)}{\partial y} & \frac{\partial F(z)}{\partial z} \end{bmatrix}$$

Calculate the first-order partial derivatives of $F(x)$, $F(y)$ and $F(z)$ with respect to x, y, and z: $\partial F(x)/\partial x = (2x-1)(C_{ph} - C_{pl} - C_s - R_t - P_c + y(R_t - S_p) - z(A_c + F_c))$, $\partial F(x)/\partial y = x(x-1)(R_t - S_p)$, $\partial F(x)/\partial z = -x(x-1)(A_c + F_c)$, $\partial F(y)/\partial x = -y(y-1)R_t$, $\partial F(y)/\partial y = (2y-1)(R_t - P_t - C_t - xR_t - z(A_t + F_t))$, $\partial F(y)/\partial z = -y(y-1)(A_t + F_t)$, $\partial F(z)/\partial x = z(z-1)(A_c + F_c + T_g - yT_g)$, $\partial F(z)/\partial y = z(z-1)(A_t + F_t + T_g - xT_g)$, $\partial F(z)/\partial z = (2z-1)(C_g - F_c - F_t - T_g + x(A_c + F_c + T_g) + y(A_t + F_t + T_g) - xyT_g)$.

In dynamics and control theory, the stability of a system at an equilibrium point is assessed using the eigenvalues of the Jacobian matrix. If all eigenvalues have negative real parts, the system will naturally return to equilibrium after a small disturbance, indicating asymptotic stability. If any eigenvalue has a positive real part, the equilibrium is unstable, and small disturbances can cause significant deviation. When all eigenvalues have non-positive real parts but one is zero, the system is critically stable, and stability cannot be judged by eigenvalues alone. The stability of the numerical equilibrium points E_1 to E_{12} is analyzed using the eigenvalues derived from the characteristic polynomial, as detailed in Table 3.

Conditions:

1: $A_c + F_c + P_c + C_s + R_t < C_{ph} - C_{pl}$, $A_t + F_t + P_t < R_t - C_t$.

2: $C_s + P_c + R_t > C_{ph} - C_{pl}$, $A_t + P_t < R_t$.

3: $C_t + P_t > R_t$, $C_s + P_c + R_t < C_{ph} - C_{pl}$.

4: $C_s + P_c + R_t > C_{ph} - C_{pl}$, $C_t + P_t > R_t$.

When the conditions 1 $A_c + F_c + P_c + C_s + R_t < C_{ph} - C_{pl}$ and $A_t + F_t + P_t < R_t - C_t$ are satisfied, the replicator dynamic system has two asymptotically stable points: $E_2(0,0,1)$ and $E_5(1,1,0)$. Preliminarily, it is judged that $E_1(0,0,0)$, $E_3(0,1,0)$, $E_4(1,0,0)$, $E_7(0,1,1)$, $E_8(1,1,1)$, and $E_6(1,0,1)$ have at least one positive real root, whereas all roots of $E_5(1,1,0)$ are negative. When Condition 1 $A_c + F_c + P_c + C_s + R_t < C_{ph} - C_{pl}$ and $A_t + F_t + P_t < R_t - C_t$ is satisfied, all roots of $E_2(0,0,1)$ are negative. When Condition 2 $C_s + P_c + R_t >$

$C_{ph} - C_{pl}$ and $C_t + P_t < R_t$ is met, tr$(E_9) > 0$, making E_9 an unstable point. When Condition 3 $C_t + P_t > R_t$ and $C_s + P_c + R_t < C_{ph} - C_{pl}$ is met, tr$(E_{10}) > 0$, making E_{10} an unstable point. When Condition 4 $C_s + P_c + R_t > C_{ph} - C_{pl}$ and $C_t + P_t > R_t$ is met, tr$(E_{11}) > 0$, making E_{11} an unstable point.

Table 3. Stability Analysis of Equilibrium Points.

Equilibrium	Eigenvalue	Local Stability	Condition
$E_1(0,0,0)$	$\lambda_1, \lambda_2, \lambda_3 = F_c - C_g + F_t + T_g > 0$	Unstable	-
$E_2(0,0,1)$	$\lambda_1 = A_c - C_{ph} + C_{pl} + C_s + F_c + P_c + R_t$, $\lambda_2 = A_t + C_t + P_t + F_t - R_t$, $\lambda_3 = C_g - F_c - F_t - T_g$	ESS	1
$E_3(0,1,0)$	$\lambda_1 = C_{pl} - C_{ph} + C_s + P_c + S_p > 0, \lambda_2, \lambda_3$	Unstable	-
$E_4(1,0,0)$	$\lambda_1, \lambda_2 = C_t + P_t, \lambda_3$	Unstable	-
$E_5(1,1,0)$	$\lambda_1 = C_{ph} - C_{pl} - C_s - P_c - S_p$, $\lambda_2 = -C_t - P_t$, $\lambda_3 = -A_c - A_t - C_g$	ESS	-
$E_6(1,0,1)$	$\lambda_1, \lambda_2 = A_t + C_t + F_t + P_t > 0, \lambda_3$	Unstable	-
$E_7(0,1,1)$	$\lambda_1 = A_c - C_{ph} + C_{pl} + C_s + P_c + F_c + S_p > 0, \lambda_2, \lambda_3$	Unstable	-
$E_8(1,1,1)$	$\lambda_1, \lambda_2, \lambda_3 = A_c + A_t + C_g > 0$	Unstable	-
E_9	$\lambda_1 + \lambda_2 + \lambda_3 > 0$	Unstable	2
E_{10}	$\lambda_1 + \lambda_2 + \lambda_3 > 0$	Unstable	3
E_{11}	$\lambda_1 + \lambda_2 + \lambda_3 > 0$	Unstable	4
E_{12}	$\lambda_1, \lambda_2, \lambda_3$	Uncertain	-

Combining ODEs (3), (9) and (15), delve into the DDEs (4), (10) and (16) under certain conditions. Using numerical methods, analyze the impact of delay terms and various factors on the system's stability and dynamic characteristics.

The time-delay differential equation set for the strategy choices of food enterprise, food inspection agency, and government regulatory is as follows:

$$\begin{cases} x'(t) = x(t)(x(t)-1)\big(C_{ph} - C_{pl} - C_s - R_t - P_c + y(t-\tau_y)(R_t - S_p) \\ \qquad - z(t-\tau_z)(A_c + F_c) + N_\alpha x(t-\tau_x)\big) \\ y'(t) = y(t)(y(t)-1)\big(R_t - P_t - C_t - x(t-\tau_x)R_t - z(t-\tau_z)(A_t + F_t) + N_\beta y(t-\tau_y)\big) \\ z'(t) = z(t)(z(t)-1)\big(C_g - F_c - F_t - T_g + x(t-\tau_x)(A_c + F_c + T_g) \\ \qquad + y(t-\tau_y)(A_t + F_t + T_g) - x(t-\tau_x)y(t-\tau_y)T_g + N_\gamma z(t-\tau_z)\big) \end{cases}$$

5. Numerical Analysis and Simulation

In the theory of dynamical systems, the introduction of time-delay factors can significantly alter the dynamic characteristics of the system. Based on recent years' food safety supervision data from a province in China, summarize Set 1. Set 1: $S_p = 20$, $C_{ph} = 15$, $C_{pl} = 8$, $C_s = 1$, $R_t = 7$, $C_t = 2$, $A_t = 2$, $F_t = 2$, $A_c = 2$, $F_c = 2$, $P_t = 2$, $P_c = 2$, $C_g = 2$, $T_g = 5$, satisfying the condition $A_c + F_c + P_c + C_s + R_t > C_{ph} - C_{pl}$.

5.1. Numerical Analysis of Cross-Delay Parameter

Let $x_{\text{lag}} = x(t-\tau_x)$, $y_{\text{lag}} = y(t-\tau_y)$, $z_{\text{lag}} = z(t-\tau_z)$. To analyze the impact of changes in x_{lag} on the evolutionary game process and outcomes, let $y_{lag} = z_{lag} = 1$ and assign different values to x_{lag} such as $x_{lag} = [0.1, 0.5, 1, 2]$. Numerical analysis is shown in Figure 6. Next, let $x_{lag} = z_{lag} = 1$ and assign different values to y_{lag} as $y_{lag} = [0.1, 0.5, 1, 2]$. Numerical analysis is shown in Figure 7. Finally, let $x_{lag} = y_{lag} = 1$ and assign different values to z_{lag} as $z_{lag} = [0.1, 0.5, 1, 2]$. Numerical analysis is shown in Figure 8.

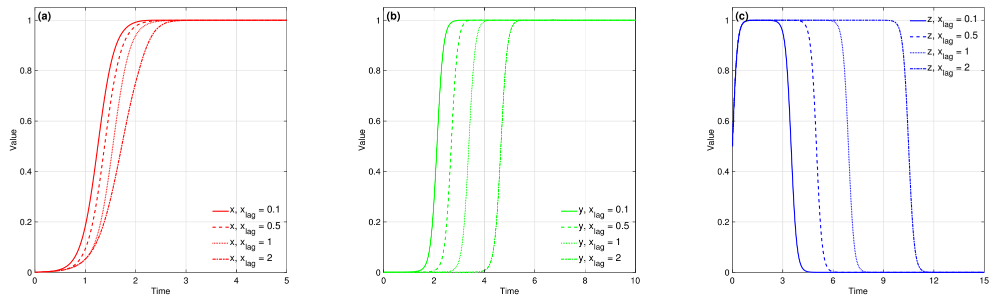

Figure 6. Influence of x_{lag} on system stability and dynamic characteristics. (**a**) the impact of x_{lag} on the evolutionary process of food enterprise, (**b**) the impact of x_{lag} on the evolutionary process of food inspection agency, (**c**) the impact of x_{lag} on the evolutionary process of government regulatory.

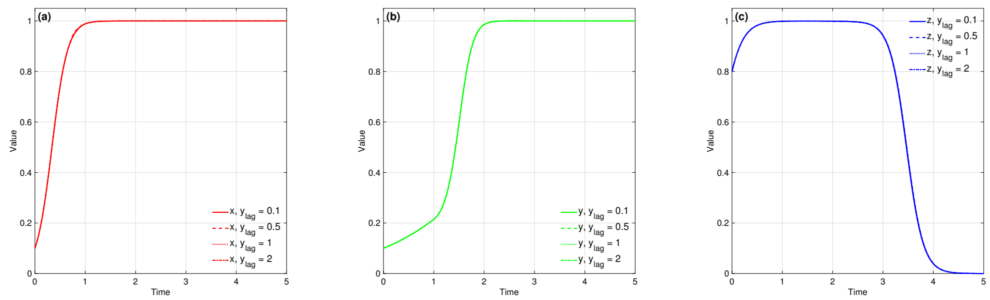

Figure 7. Influence of y_{lag} on system stability and dynamic characteristics. (**a**) the impact of y_{lag} on the evolutionary process of food enterprise, (**b**) the impact of y_{lag} on the evolutionary process of food inspection agency, (**c**) the impact of y_{lag} on the evolutionary process of government regulatory.

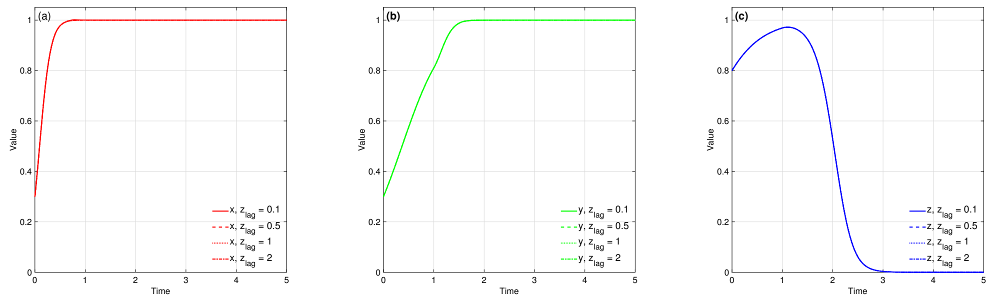

Figure 8. Influence of z_{lag} on system stability and dynamic characteristics. (**a**) the impact of z_{lag} on the evolutionary process of food enterprise, (**b**) the impact of z_{lag} on the evolutionary process of food inspection agency, (**c**) the impact of z_{lag} on the evolutionary process of government regulatory.

From Figure 6, it can be seen that the smaller the x_{lag}, the shorter the time it takes for x, y, and z to reach a stable state. This indicates that the smaller the lag effect of food companies, or the faster the response to policies, the quicker the food companies, inspection agencies, and government regulatory departments can achieve a relatively stable state. Figures 7 and 8 show that the sizes of y_{lag} and z_{lag} have almost no effect on the evolution speed of x, y, and z. This suggests that the delay in decision making by the inspection agencies and government regulatory departments has little impact on the overall food safety regulatory system. Government regulatory strategies must be highly sensitive to changes in the behavior of food enterprises to ensure the timeliness of food safety and quality control measures. Meanwhile, the food safety regulatory system demonstrates high resilience to the time-lag effects on local food inspection agencies and government regulatory departments. Therefore, the following suggestions can be made:

- Government regulatory departments can utilize advanced information technology and big data analytics to monitor the production, circulation, and sales of food enterprises in real time, promptly identifying potential risks and non-compliant behaviors.
- Enhance the information sharing and coordinated cooperation among various regulatory departments within the government as well as between the government and food enterprises, consumers, to form a pattern of joint regulatory oversight.
- Develop and improve a rapid response mechanism for food safety issues, ensuring that once risks or non-compliance are detected, measures can be quickly taken to minimize losses and impact.

Furthermore, based on Set 1, analyze the evolutionarily stable strategy combinations considering cross time-delays. Set 2: $S_p = 20$, $C_{ph} = 16$, $C_{pl} = 2$, $C_s = 2$, $R_t = 6$, $C_t = 2$, $A_t = 1$, $F_t = 1$, $A_c = 2$, $A_c = 2$, $F_c = 2$, $P_c = 2$, $P_t = 1$, $C_g = 2$, $T_g = 5$, meeting condition 1. The evolutionary outcomes over time starting from different initial strategy combinations for Set 1 and 2 are shown in Figures 9 and 10. Different colors indicate convergence to different points, where red lines signify convergence to $E_2(0,0,1)$ and blue lines signify convergence to $E_5(1,1,0)$.

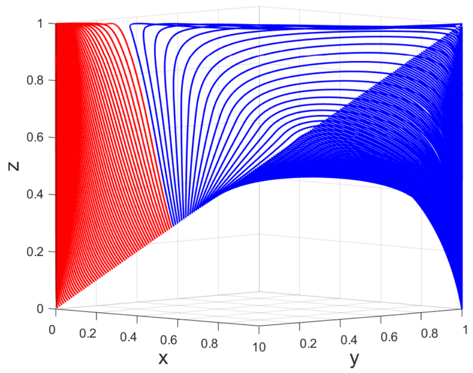

Figure 9. Set 1 evolution result.

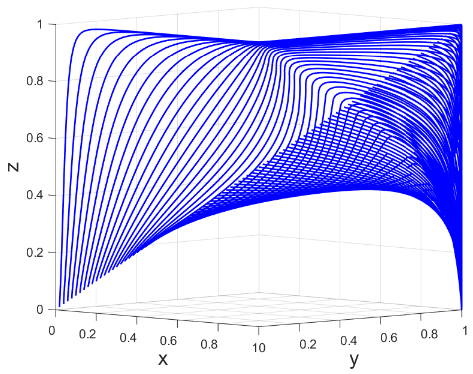

Figure 10. Set 2 evolution result.

From Figures 9 and 10, in the case of only cross delays, participants' strategic choices are influenced by the past strategic choices of others. However, these delay effects do not alter the stable strategic combinations and their stability conditions within the evolutionary game system. This indicates that, despite delays between decision making and action, the stability of the equilibrium state ultimately reached by the system remains unaffected. Therefore, this provides a theoretical foundation for the analysis and design of dynamic systems involving multi-party interactions and decision-making delays.

5.2. Numerical Analysis of Self-Feedback Delay Parameter

Building upon Set 1, this analysis examines the effects of the self-feedback time-delay intensity factors N_α, N_β, and N_γ on the evolutionary game process and its outcomes. By setting $N_\beta = N_\gamma = 0$ and varying N_α within the range of $N_\alpha = [-2, -1, 1, 5]$, the numerical analysis is illustrated in Figure 11a. Similarly, with $N_\alpha = N_\gamma = 0$ and varying N_β within $N_\beta = [-2, -1, 1, 5]$, the analysis is shown in Figure 11b. Finally, setting $N_\alpha = N_\beta = 0$ and varying N_γ within $N_\gamma = [-2, -1, 1, 5]$, the numerical analysis is presented in Figure 11c.

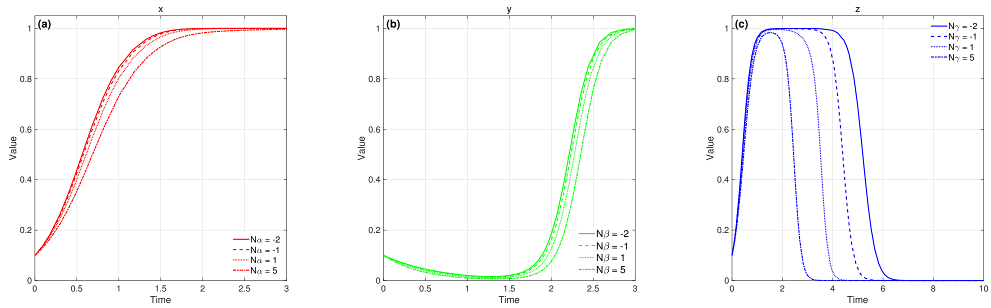

Figure 11. Influence of self-feedback intensity N_α, N_β, and N_γ. (**a**) the impact of N_α on the evolutionary process of food enterprise, (**b**) the impact of N_β on the evolutionary process of food inspection agency, (**c**) the impact of N_γ on the evolutionary process of government regulatory.

From Figure 11, it is evident that the lower the self-feedback intensities N_α and N_β, the less time it takes for x and y to reach a stable state within the same time. And it is also observed that the greater the self-feedback intensity N_γ, the less time it takes for z to reach $z = 1$. This implies that negative feedback mechanisms are more beneficial for the compliance behavior of food companies and inspection agencies. For food companies, negative feedback occurs when the production of non-compliant products leads to brand image damage, customer loss, and possible legal consequences, which encourages companies to produce compliant products to avoid negative outcomes. For food inspection agencies, the tendency to seek rent damages consumer and government trust, which could result in the long-term loss of inspection business or license revocation, generating negative feedback that prompts inspection agencies to operate legally. Therefore, the following suggestions can be proposed:

- Establish a blacklist system for enterprises that have violated regulations multiple times, listing them on the blacklist and making it public as a warning.
- Encourage the participation of food industry associations, professional institutions, and other third parties in food safety supervision. Utilizing their expertise and technical means, they can provide support and supplementation for government regulation, reducing the impact of time lag effects.

5.3. Simulation Analysis

To verify the effectiveness of the evolutionary stability analysis, the impact of S_p, $C_{ph} - C_{pl}$, C_s, C_t, F_c, F_t, A_c, A_t, P_c, P_t, C_g, and T_g on the evolutionary game process was analyzed based on Set 1.

Simulation 1. *The parameter S_p was assigned distinct values $S_p = [15, 30, 45, 60]$, and corresponding simulations were executed. The outcomes of these simulations are depicted in Figure 12. Subsequently, the parameter C_p, representing the differential between C_{ph} and C_{pl}, was assigned the values $C_p = [0, 6, 9, 12]$, followed by additional simulations. The results of these simulations are illustrated in Figure 13.*

From Figures 12 and 13, it can be seen that as sales revenue increased and cost disparity reduces, the probability of food companies choosing to produce qualified products increases, the probability of inspection agencies choosing to seek rent decreases, and the probability of government regulatory departments choosing lenient supervision increases. Therefore, the following suggestions can be proposed:

- The government can motivate enterprises to produce and sell compliant products by offering tax reductions, subsidies, and preferential procurement policies.
- Encourage food companies to adopt new technologies to improve product quality and supply chain management, while also establishing industry cooperation platforms to facilitate information exchange and technology sharing among supply chain segments.

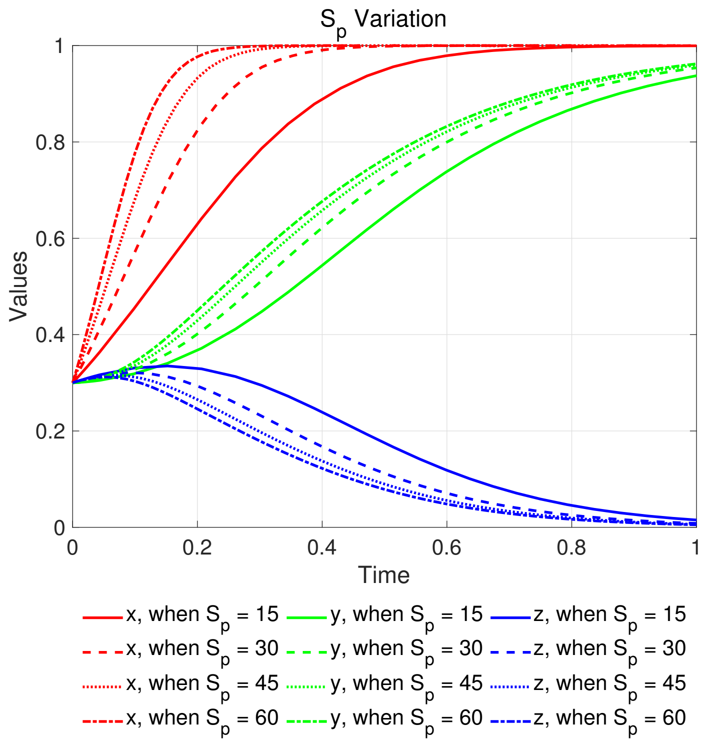

Figure 12. Influence of sales revenue S_p.

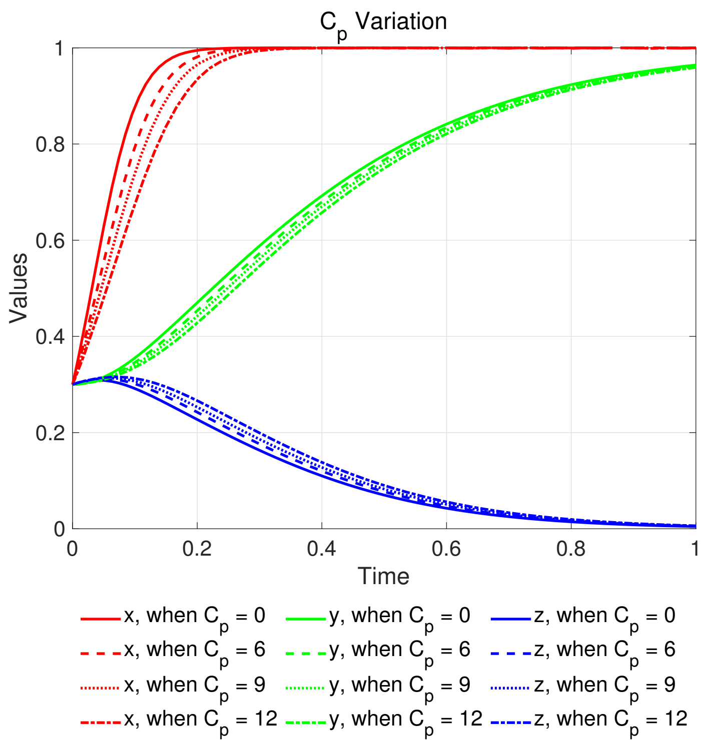

Figure 13. Influence of cost difference C_p.

Simulation 2. *Different values were assigned to* C_g*, specifically* $C_g = [0, 5, 10, 15]$*, and simulations were conducted. The results of these simulations are shown in Figure 14. Then, different values were assigned to* T_g*, specifically* $T_g = [0, 6, 12, 18]$*, and simulations were conducted. The results of these simulations are shown in Figure 15.*

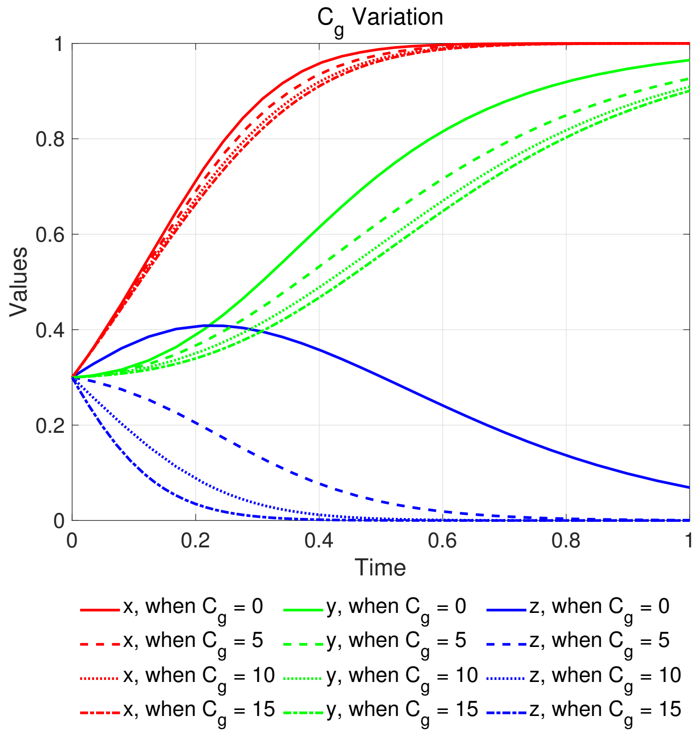

Figure 14. Influence of government regulation cost C_g.

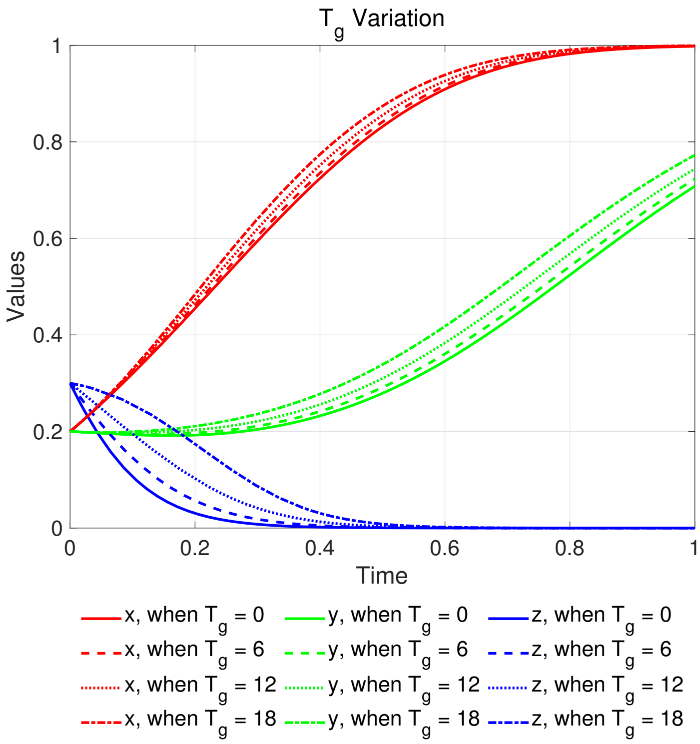

Figure 15. Influence of higher administrative punishment T_g.

From Figures 14 and 15, it can be seen that as the cost of government regulation increases and the decrease in higher-level administrative penalties, the probability of food enterprises choosing to produce substandard products increases, the probability of inspection agencies opting for rent-seeking increases, and the probability of strict regulation by government regulatory departments decreases. Therefore, the following suggestions can be made:

- The government can establish a digital supervision platform that utilizes big data and artificial intelligence for segmented assessment and prediction, in order to optimize the allocation of regulatory resources.
- Higher-level governments can encourage grassroots governments to cooperate in cross-regional regulation, implementing joint supervision of food enterprises operating across regions.
- Implement a government performance evaluation system to measure the performance of government departments at all levels based on specific indicators.

Simulation 3. *To verify the impact of consumer feedback mechanisms on the evolutionary game process, different values were assigned to P_t and P_c, specifically $P_t = [0, 2, 4, 6]$ and $P_c = [0, 2, 4, 6]$. The results of these simulations are shown in Figures 16 and 17, respectively.*

From Figures 16 and 17, with the increase of negative feedback from consumers towards food companies and inspection agencies, the probability of food companies choosing to produce qualified products increases, the likelihood of inspection agencies opting for rent-seeking behaviors decreases, and the probability of government regulatory departments choosing lenient supervision increases. Therefore, the following suggestions can be made:

- The government can enhance public awareness of food safety and food inspection through education and promotional activities, thereby strengthening consumers' self-protection abilities.
- It is also possible to provide consumers with convenient channels for easily reporting issues related to food quality and safety, such as setting up online platforms and hotline services.

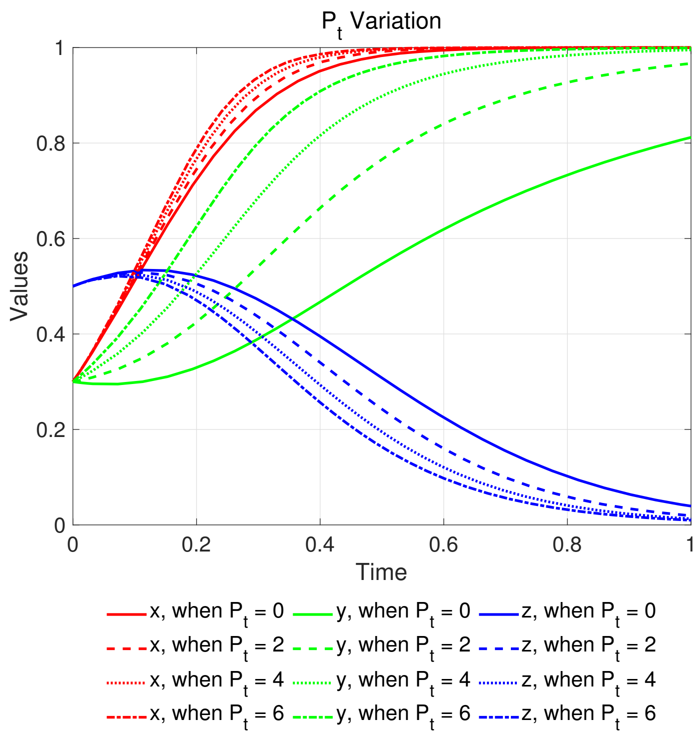

Figure 16. Influence of consumers' feedback on testing agency P_t.

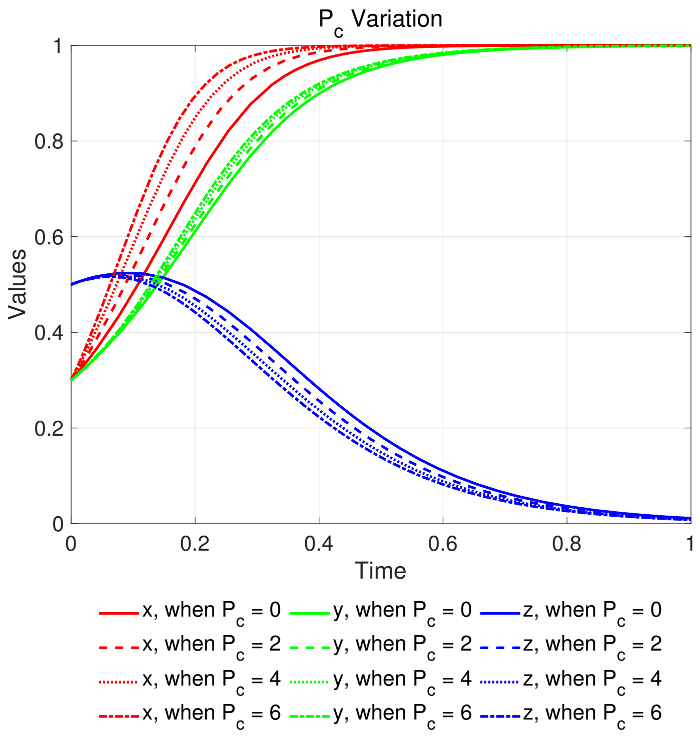

Figure 17. Influence of consumers' feedback on food enterprise P_c.

Simulation 4. *To verify the impact of government reward and punishment mechanisms on the evolutionary game process, the values for F_t and F_c were set to $F_t = [0,4,8,12]$ and $F_c = [0,4,8,12]$, respectively. The results of these simulations are presented in Figure 18. Additionally, A_t and A_c were assigned values of $A_t = [4,6,8,12]$ and $A_c = [0,4,8,12]$, with the outcomes of these simulations depicted in Figure 19.*

From Figures 18 and 19, it is evident that as the level of government incentives increases, the probability of food companies choosing to produce qualified products also increases, as does the probability of inspection agencies refusing to engage in rent-seeking behavior. Conversely, the probability of strict regulation by government regulatory agencies is decreasing. From Figures 3 and 4, it can be seen that with the increase in government penalties, the likelihood of food companies opting to produce qualified products rises, the tendency of inspection agencies to seek rents decreases, and the likelihood of strict government regulatory oversight increases. Hence, it can be deduced that excessively high reward intensity may lead to counterproductive effects on the ability of government regulatory departments to perform their supervisory duties. Similarly, while excessively high penalties can effectively ensure the compliance of food enterprises and inspection departments, they may result in higher governance costs for the government. Therefore, several suggestions can be proposed:

- The government can implement differentiated reward and punishment plans, designing specific schemes for enterprises of different sizes and types, to achieve the rationalization of resource allocation.
- It is also possible to establish clear and quantifiable reward and punishment standards, rewarding or punishing enterprises and inspection agencies based on their performance, to achieve the fair distribution of resources.

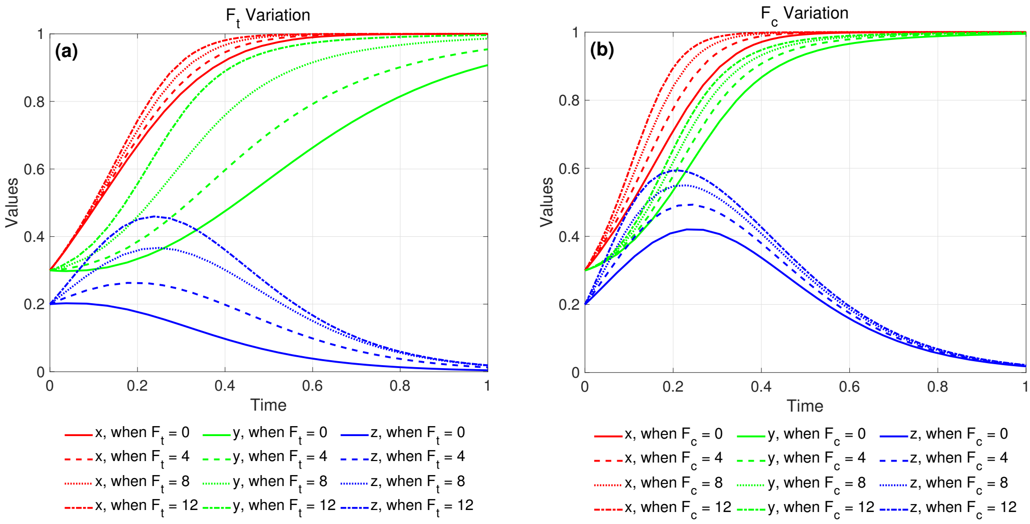

Figure 18. Influence of government punishment mechanisms F_t, F_c. (**a**) the impact of the severity of punishment for food inspection agency on the food safety regulatory system, (**b**) the impact of the severity of punishment for food enterprise on the food safety regulatory system.

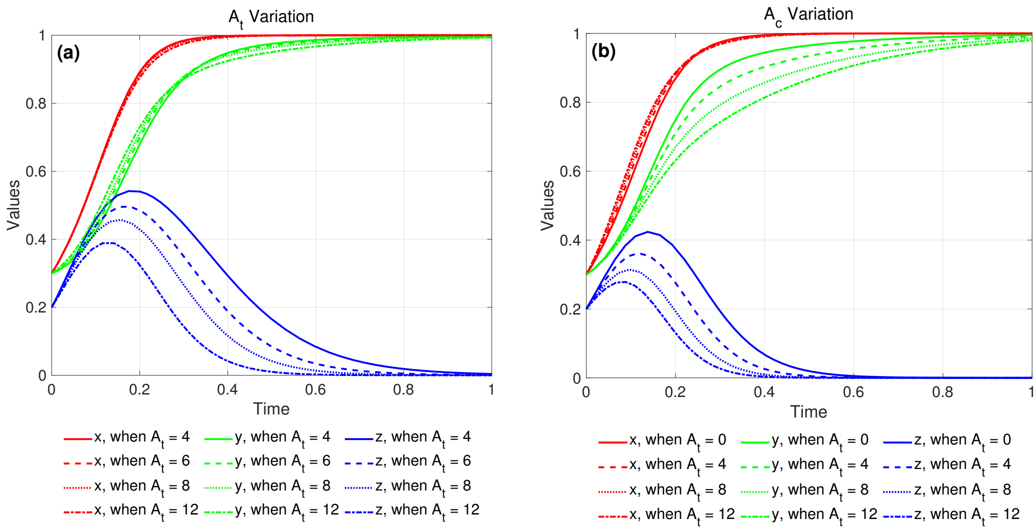

Figure 19. Influence of government incentive mechanisms A_t, A_c. (**a**) the impact of the intensity of rewards for food inspection agency on the food safety regulatory system, (**b**) the impact of the intensity of rewards for food enterprise on the food safety regulatory system.

6. Main Conclusions and Implication

This article constructs a tripartite game model of food safety supervision based on the concept of social co-governance. It innovatively introduces the time-delay effect to explore its impact on the decision making of various entities and the evolutionary process of the system. Furthermore, it incorporates different regulatory mechanisms to study their effects on food safety supervision. Compared to evolutionary game theory, considering the time-delay effect allows for a more accurate simulation of the game phenomena in the food safety supervision process, providing an effective method for understanding the decision-making choices of various entities. The research indicates the following:

- The initial states of the entities only affect the time required to reach the equilibrium point, not the outcome of the final equilibrium. $E_2(0,0,1)$ and $E_5(1,1,0)$ are the evolutionary equilibrium points of this model. As regulators, there is a preference for point $E_5(1,1,0)$, while efforts are made to avoid the occurrence of situations like point $E_2(0,0,1)$.

- The insensitivity of inspection agency and regulatory departments to the decisions of food enterprise can lead to the phenomenon of time lag, which easily results in food safety issues.
- Food enterprise, inspection agency, and government regulatory, as the main influencers in social co-governance, have their decision-making delay effects dependent on the stability of their own regulatory structures. A well-established regulatory structure is conducive to reducing food safety risks.
- Consumers, as part of social co-governance, should have their supervisory feedback roles actively utilized.
- Government regulatory departments can macro-manage the evolutionary outcome of the food safety regulatory system by adopting a series of measures.

Through numerical simulations, it offers decision-making guidance for government regulatory. These agencies can promote compliance among food enterprise and inspection agency through a series of macro-control measures, thereby alleviating the pressure on regulatory bodies in food safety management and achieving better resource allocation. At the same time, it also provides a basis for risk warning judgments for government regulatory:

- When food inspection agency and government regulatory have low sensitivity to the decisions of food enterprise, it may lead to government policies and regulations becoming outdated, thereby easily triggering food safety issues.
- When food enterprise and inspection agency themselves lack proper management, it can lead to positive feedback behaviors between the enterprises and inspection agencies. This may result in non-compliant behaviors, increasing the risk of food safety issues.
- When the speculative profit from producing substandard products is high, the probability of food enterprise producing substandard products increases, and there is a greater likelihood of rent-seeking behavior occurring between them and food inspection agencies.
- When the cost of rent-seeking for food inspection agency is low, the probability of rent-seeking behaviors between food enterprise and inspection agency increases, which is not conducive to government regulatory maintaining long-term social stability and order.

This study is based on a summary of food safety supervision data from a certain province in China in recent years. The establishment of this model conforms to the general laws of the market economy, which means that the model and conclusions are also applicable to other countries.

However, there are some certain limitations that need to be considered. In the first place, the various participating entities may adopt a diverse range of strategies, rather than simple binary strategy combinations. Second, the fundamental assumption of this study is that all entities are decision-makers with bounded rationality, ignoring the impact of irrational behaviors on the food safety regulation system. Lastly, in the study of social co-governance theory, consumers are an indispensable link, and their influence is not limited to providing negative feedback. Therefore, three further key areas have emerged from this paper for future research:

- Consumers, as an important link in social co-governance, regarding them as participants in the game as well, can better understand how consumers' behavior affects the decision-making choices of all parties involved.
- In the context of the Internet era, the Internet, as a new medium for information transmission, provides a new platform for food safety regulation. Its impact on the decision-making choices of food enterprises, inspection agencies, and government regulatory departments is worth exploring. The role played by the Internet as this medium merits deep reflection.
- In real life, the decision-making choices of all parties are not always absolutely rational. Considering the influence of irrational actors can help reduce the food safety risk index.

So constructing a four-party game model based on the "Internet+" context that considers time-delay effects to explore the interactions among food enterprise, food inspection agency, government regulatory, and consumer will be our next research.

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