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Macroeconomic Components of the Risks to Fiscal Sustainability in Hungary

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Abstract: Introducing uncertainty under fiscal sustainability conditions for the public debt provides a framework for analyzing debt dynamics. Such methods are commonly used for fiscal projections, but our aim here is retrospective; we evaluate the sudden jump in the Hungarian public debt following the global financial crisis in 2008. Based on a traditional debt-deficit stock-flow identity combining the fiscal component (primary deficit) and the interactions among real sector components, we model the debt dynamics by a vector error correction model (VECM). Uncertainty is represented in the model by shocks that are identified in the VECM framework. Using this method for simulation starting from 2006, we found that the debt-to-GDP ratio in 2008 and after could not be ruled out by 90 percent probability. Such an event was coded in the Hungarian debt dynamics and very likely would have materialized even without the unfortunate events of the global financial crisis.

Keywords: Hungary; likelihood of public debt crisis; public debt sustainability; probabilistic approach to public debt; stochastic properties in a VECM framework simulation

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1. Introduction

The path of public debt may reflect complex changes in the macro variables and their interrelationships. When the level of public debt surpasses certain limits, sustainability may become an issue. When debt sustainability is in doubt, it imposes a strong constraint altering the behavior of macro variables and their interrelationships. Debt dynamics influence future potential performance and change the economic outlook.

This paper examines the sustainability of fiscal policy under uncertainty in the Hungarian economy for the period of 1999Q1–2011Q4[1]. First, we analyze the evolution of the Maastricht gross nominal debt-to-GDP ratio based on a commonly used decomposition of the changes in the debt-to-GDP ratio. For Hungary, a similar decomposition is extended with a probabilistic approach to analyze public debt sustainability in Ábel and Kóbor (2011), inspired by Garcia and Rigobon (2004) and Tanner and Samake (2008). The approach was based on a simple macroeconomic VAR model like the one used in Tanner and Samake (2008), but in this paper, we take this method a step further by using a VACM model reflecting the decomposition of the factors and the impact of shocks contributing to the increase in the debt-to-GDP ratio. Choosing this method is based on three considerations. First, an econometric approach may be better suited for assessing fiscal sustainability than an accounting one by combining the interactions among macroeconomic variables and thus their joint impact on debt accumulation (Tanner and Samake 2008). Such an approach was also followed by Hostland and Karam (2005), Celasun et al. (2006) for prospective assessment, and by Mendoza and Oviedo (2004) for retrospective assessment. Most of these and other assessments were done in a VAR or an SVAR framework. Changing this framework is motivated by our second consideration. As most of the macroeconomic variables are not stationary, a VAR or SVAR model could be unstable if we would use the levels of the variables. The third option would be to use the differences of the variables for the estimation, but the variables are cointegrated, so even this option would not solve the statistical stability problem of this framework.[2] To avoid these traps, we propose the use of a vector error correction model (VECM) in this paper.

Several factors may change the debt-to-GDP ratio on the short run but several of them have only a transitory effect. Debt sustainability is a long-term concept of which we can derive certain limits on debt dynamics. These limits or thresholds, however, are not static. Debt tolerance may change abruptly with changes in external economic conditions and/or the weakening of economic policy and financial soundness. This paper does not deal with these external or internal factors of debt tolerance. Reinhart and Rogoff (2009) give a detailed and robust analysis of historical changes in debt tolerance. Gilchrist and Zakrajšek (2012), Gilchrist et al. (2022) and Miranda-Agrippino and Rey (2020) analyzed how investors' changing risk appetite reflected in global financial risk during the global financial cycle may influence sovereign risk. Instead, we use a probabilistic approach and focus on the effects of the changes in the main non-fiscal macroeconomic variables, such as the exchange rate, growth rate, inflation rate, and interest rate, and for the fiscal variable we use the primary fiscal balance. There is a simple and widely used technique to attribute changes in the debt-to-GDP ratio to the changes in these fiscal and non-fiscal macroeconomic factors. Although this decomposition itself may reveal interesting historical tendencies of debt dynamics, more importantly, this decomposition offers a way to channel in information from a simple VECM based on identified shocks to allow a probabilistic assessment of debt dynamics and debt sustainability.

2. Debt Dynamics

The term debt dynamics refers to the study of the evolution of the measured debt-to-GDP ratio. Describing how the debt level has evolved in the past may provide a distinctive perspective for future actions. It helps to identify which factors dominated debt accumulation in the past and how could it be altered in the future by appropriate policy measures. Several simple and widely used methods are collected and their use is well demonstrated in Burnside (2005). Such an analysis of debt dynamics is the starting point for us to illustrate risks to public finances in Hungary.

The analysis starts from a simple budget identity expressed in local currency units, which states that the level of debt (D) today is the sum of the inherited debt plus interest (r) due on the debt stock augmented by the current primary fiscal deficit (P): $D_t = D_{t-1}(1+r) + P_t$. The economy also grew, not just the debt, and nominal GDP had increased by the rate of growth (g) and inflation (π): $GDP_t = GDP_{t-1}(1+g)(1+\pi)$. The evolution of the debt-to-GDP ratio is given by (1)

$$d_t = d_{t-1}\frac{1+r}{(1+g)(1+\pi)} + p_t \tag{1}$$

where

d is the debt-to-GDP ratio;
p is the primary fiscal deficit to GDP ratio (P/GDP);
g is the real GDP growth rate;
π is the inflation rate;
r is the effective interest rate on the debt.

The effective interest rate on debt is the ratio of effective interest payments in period t and the stock of debt, D_{t-1}, outstanding at the beginning of the year (closing of the previous year). The debt stock has a domestic D^d and a foreign D^f component. Foreign currency debt carries a foreign interest rate r^f and is exposed to exchange rate risks, while the effective interest rate on domestic debt is denoted by r^d. Introducing these distinctions in Formula (1) we arrive at Formula (2):

$$d_t = \frac{(1 + r_t^f)(1 + x)d_{t-1}^f + \left(1 + r_t^d\right)d_{t-1}^d}{(1 + g)(1 + \pi)} + p_t \quad (2)$$

where

d^f is the foreign-denominated debt to GDP ratio;
d^d is the domestic debt to GDP ratio;
x is the measure of depreciation of the domestic currency (in percentage);
r^f is the effective interest rate on the foreign component of the debt;
r^d is the effective interest rate on the domestic component of the debt.

Formula (2) summarizes the main macroeconomic factors that influence the debt dynamics, but there are many other things that may change the debt volume. These other factors may include temporary or one-off factors, such as privatization revenues used for debt repayment. Valuation or accounting changes, such as the mark-to-market valuation, which reflects changes in market sentiments and other changes, may also alter the value of the debt. We can find a useful and quite extensive list of other such factors in several places in Burnside (2005). We will show the effect of these other factors, sometimes called one-off, off-budget, or stock-flow adjustment, under the heading of "stock-flow adjustment" calculated as a residuum in the attribution exercise and will not go into further detail concerning them. An informative discussion of the motives of governments to rely on stock-flow adjustments and the characteristics of such practices in different countries is presented in Kiss (2007), Koen and Van den Noord (2005), and Weber (2012).

To separate the effects of the macroeconomic variables we rewrote Formula (2) as follows:

$$d_t - d_{t-1} = d_{t-1}\frac{wr_t^f + (1 - w)r_t^d}{(1 + g)(1 + \pi)} + d_{t-1}\frac{wx\left(1 + r_t^f\right)}{(1 + g)(1 + \pi)} - d_{t-1}\frac{g}{(1 + g)(1 + \pi)} - d_{t-1}\frac{\pi(1 + g)}{(1 + g)(1 + \pi)} + p_t \quad (3)$$

In order to arrive at an additive formula in (3), we denoted the share of foreign debt by w, where $w = \frac{d_{t-1}^f}{d_{t-1}}$. The share of domestic debt is complementary, $(1 - w)$.

There might be many other possible ways to break the formula into components, but we prefer to use this particular form in the empirical attribution exercise. The first part on the right-hand side of Equation (3) reflects the impact of interest rates, both domestic and foreign interest rates, on debt dynamics (*interest rate effect*). The second part is the *exchange rate effect*, with the simplification that we apply the same exchange rate to the stock of the foreign debt and the interest payment due on it. The third part gives the reduction of the debt-to-GDP ratio caused by the increase in GDP (*growth effect*). The next is the formula for the *inflation effect*, which covers the reduction in the debt-to-GDP ratio caused by the inflationary increase in GDP, combining both the direct inflationary effect and the cross effect of the real-growth and inflation. The last part denotes the effect of the primary deficit on the debt-to-GDP ratio (*primary deficit effect*).

3. Decomposition of Debt Dynamics

The decomposition described in Equation (3) of the change in gross debt is presented in Figure 1.

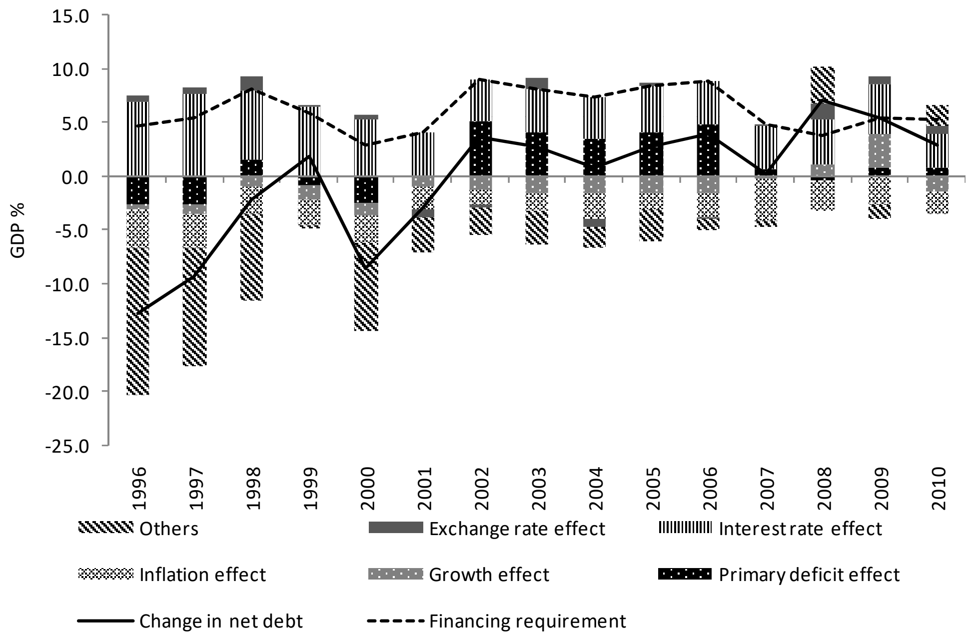

Figure 1. The components of the increase in the gross debt (decomposition based on Equation (3)). Source: Authors' calculation.

In this decomposition, the change in gross debt attributed to other factors (*stock-flow adjustment*) not shown in Equation (3) is typically negative, indicating that these factors normally reduced the level of debt in Hungary. Weber (2012) provided a cross-country perspective on such stock-flow adjustments, which may occur for a large number of reasons, including the sale or acquisition of assets and asset revaluations. He noted that even in advanced countries such practices are common and may cause significant discrepancies. In less advanced countries, "a lack of fiscal transparency may make it easier for their governments to engage in deceptive fiscal stratagems" (Weber 2012, p. 14). Koen and Van den Noord (2005) elaborating on one-off measures, creative accounting, and the three generations of deficit gimmicks explain why such adjustments can be so popular for governments pretending to reduce the debt. Kiss (2007) proposed a correction for the one-off measures to show the "true" economic impact of fiscal measures.

The stock-flow adjustment was especially dominant in the years 1996 and 1997, possibly related to privatization revenues used for debt repayment, valuation changes related to the debt swap between the central bank and the government, or other one-off factors. We have to admit that the method used for the decomposition also uses shortcuts and simplifications. The currency composition of foreign debt may easily change from one period to the other, but Formula (3) assumes only one foreign currency and one exchange rate. Similarly, interest rates may also differ not just for different currencies but also for different maturities in the same currency and different dates of the same maturity. These simplifications help to keep the formula used for the attribution tidy, but at a cost of some errors. The deviations caused by these errors show up in the residuum called other factors.

The change in debt attributed to the *interest rate effect* increased the debt-to-GDP ratio significantly, although with some fluctuation. Taking the whole period, interest rates roughly followed a downward trend, which over time moderated the debt accumulation attributed to the interest rates. Prior to 2000, the interest rate effect was accountable for about 6–7 percent of the GDP increase in debt, but after 2000, the debt accumulation attributed to the interest rates fell to around 3–4 percent of the GDP. The increase in the level of debt, however, led to an increase in the interest rate effect even with relatively stable or falling interest rates. After 2004, the interest rate effect increased to more than 4 percent of the GDP and reached 4.6 percent in 2009, before falling to 3.2 percent of the GDP in 2010.

The *exchange rate effect* on debt dynamics was much less in size than the interest rate effect, which is not surprising. The exchange rate effect also fluctuated significantly. In most years, it contributed to the increase in debt, but there were periods, such as the years 2001, 2002, 2004, and 2006, when the appreciation of the domestic currency *reduced* the debt-to-GDP ratio. In 2008, a reduction of 1.4 percent of the GDP was a result of the appreciation of the Hungarian forint. In 2009, a 0.6 percent GDP reduction, and in 2010, a 0.7 percent GDP reduction in the debt were attributed to exchange rate appreciation.

Almost every year, about a 3–4 percentage point reduction in the debt-to-GDP ratio is attributed to the nominal growth of GDP. This is the combined contribution of the *growth effect* and the *inflation effect*. The components of Equation (3) handle these effects separately, and it would not be difficult to calculate these two effects accordingly, but at this level, it would not add much detail to the picture, other than variance. When real growth was lower, inflation was normally higher, and vice versa, but the sum of these two factors deducted 3–4 percentage points from the debt-to-GDP ratio almost every year. Variance does matter though, and indeed, in the model, we will use both, real GDP growth and inflation. Their contribution to the macroeconomic risks is significant, as we will demonstrate in the second half of this paper.

The *primary deficit effect* was debt reduction in the years 1996, 1997, 1999, 2000, and 2008, when the budget recorded a primary surplus. For the whole 15-year period, however, the primary deficit was the most common occurrence with levels as high as 4–5 percent of the GDP, which increased the debt-to-GDP ratio by the same magnitude.

4. Integrating the Effects of Shocks in the Debt Dynamics

In practice, the dynamics of the debt-to-GDP ratio given in Equation (2) depend on macroeconomic variables but also on unexpected shocks affecting the economy (Chalk and Hemming 2000; Cherif and Hasanov 2012; Favero and Giavazzi 2007). There are several approaches to analyzing the impact of uncertainty on the debt dynamics written in Equation (2). A simple methodology to run stochastic debt simulations is described in Di Giovanni and Gardner (2008), which starts with a debt accumulation path similar to the one in Equation (2), and applying random shocks directly to this equation builds confidence intervals around the baseline.

A somewhat more detailed analysis of the effects of uncertainty is given in Beynet and Paviot (2012) for Hungary by using a similar approach but introducing isolated random shocks to the components (i.e., exchange rate, GDP, and interest rate) independently. Here, we also start from the same formula of debt dynamics, rearranged in the following convenient form:

$$d_t = \frac{(1+r_t^f)(1+x)D_{t-1}^f + \left(1+r_t^d\right)D_{t-1}^d}{GDP_t} + p_t \quad (4)$$

To emphasize the interrelationships among the macroeconomic variables, we depart from the isolated approach. In order to incorporate the interrelationships between the economic variables in Formula (4) of debt dynamics and to introduce shock, Tanner and Samake (2008) used a vector auto-regression approach (VAR) for a retrospective analysis of public debt in Brazil, Mexico, and Turkey. Eller and Urvová (2012) presented another interesting application for Hungary combining a VAR for non-fiscal determinants of debt and a fiscal reaction function for the fiscal determinants. We have two concerns about applying such an approach to Hungary. First, most of the variables are not stationary and a VAR could be unstable; consequently, the estimation using the levels of the variables is unsatisfactory. Instead, we could use the differences between the variables for the estimation, but the very likely possibility that the variables are cointegrated would not solve this problem.[3] To avoid these traps, we propose the use of a vector error correction model (VECM). Our basic VECM specification is

$$dY_t = \delta + FdY_{t-1} - A(BY_{t-1}) + \varepsilon_t \quad (5)$$

where the six-dimensional vector of variables is the following:

$$Y_t = \begin{bmatrix} \log(real\ GDP) \\ \log(price\ level) \\ \log(exchange\ rate) \\ R(policy\ interest\ rate) \\ \log(net\ taxes) \\ \log(government\ purchases) \end{bmatrix} \tag{6}$$

ε_t is the corresponding six-dimensional vector of zero mean residuals.

We use quarterly data for the period of 1999Q1–2011Q4[4]. The variables for GDP, net taxes, and government purchases are seasonally adjusted because these variables normally follow a marked quarterly pattern each year. Some of these variables represent those in (3), but there are discrepancies. First, for the sake of identification of the shocks, we use two variables, net taxes, and government purchases, instead of the primary deficit, to describe the fiscal impact, following the practice proposed by Blanchard and Perotti (2002). Net taxes are revenues minus cash transfers and personal expenditures. The shocks related to net taxes have no direct instantaneous impact on GDP, but they influence incomes and consumption and indirectly, other components of GDP. Government purchases consisting of purchases of goods and services, in-kind transfers, and transfers from the European Union, however, have a direct and instantaneous impact on GDP. These are explicit components of the balance sheet of the use of GDP.

The other departure concerns the foreign and domestic effective interest rates on debt, r_t^f and r_t^d; these are missing from the VECM. There are two reasons to model the effect of these interest rates separately. First, we assume that macro variables would rather react to the announced central bank policy rate (R) than the not known effective rates used in (4), which are calculated ex-post. Secondly, we still wanted to capture the effects of these rates on the debt dynamics and modeled them separately using the following specifications:

$$r_t^d = c^d + \beta_1^d R_t + \beta_2^d r_{t-1}^d \tag{7}$$

Equation (6) assumes that the effective interest rate on domestic debt changes gradually in response to changes in the policy rate (R).

$$r_t^f = c^f + \beta_1^f SPREAD_t + \beta_2^f r_{t-1}^f \tag{8}$$

The effective interest rate on the foreign component of debt responds with some inertia to the changes in the country risk premium denoted by SPREAD in Equation (9).

$$SPREAD_t = c^S + \beta_1^S \log(HUFEUR_t) + \beta_2^S R_t + \beta_3^S SPREAD_{t-1} \tag{9}$$

Equation (9) shows that the variable SPREAD in (8) reflects changes in the HUF/EUR exchange rate with some inertia. Equations (7)–(9) give a very rudimentary characterization of the money markets, but among several other alternatives that we have tried, only this specification gave meaningful results that could be somehow linked to the practice of the Hungarian central bank (Magyar Nemzeti Bank). Magyar Nemzeti Bank targets inflation; considering the path of the policy rate, due considerations are taken regarding the implications for the exchange rate and the risk premium. With these, we covered all of the variables in (4). The next step is to calculate the shocks ε in (5).

5. Identification of the Shocks

We estimate the VECM and use its residuals for the stochastic simulation. We use a decomposition of the variance–covariance matrix of the residuals $\hat{\Omega}$. The first step is a Choleski decomposition of the variance–covariance matrix, but we need such a decomposition that transforms the stochastic shocks to economically meaningful definitions. In fact, we are looking for decomposition with the structural restrictions given in Table 1. Using the decomposition of the form $\hat{\Omega} = SS'$ and the economically interpreted and identified shocks of $u_t \sim N(0,1)$, we calculate a transformed shock $\varepsilon_t = Su_t$ that then enters the VECM in Equation (5).

Table 1. The matrix of identifying restrictions (S).

	Risk Premium Shock PREM	**Monetary Policy Shock MP**	**Demand Shock DEM**	**Supply Shock SUPP**	**Fiscal Revenue Shock TAX**	**Fiscal Spending Shock GC**
Real GDP	0	0	+	+	0	+
Price level	0	0	+	−		
Policy rate (R)	+	+				
Exchange rate (HUFEUR)	+	−				
Government purchases			0	0		+
Net taxes	0	0	+	+	+	

Sign restrictions are denoted by + or −, and zeros (0) indicate exclusion (no direct impact).

Here, we identified six macroeconomic shocks, i.e., the risk premium shock (PREM), monetary policy shock (MP), demand shock (DEM), supply shock (SUPP), fiscal revenue shock (TAX), and fiscal spending shock (GC). A well-established identification method of structural shocks is given in Mountford and Uhlig (2009) based on sign restrictions of the impulse responses of variables that are expected to react to economically meaningful policy-related shocks[5]. The sign restrictions given in Table 1 reflect a similar approach.

The identifying restrictions in the first row of the matrix reflect that unexpected movements in GDP can be due to unexpected movements in demand, unexpected supply shocks, unexpected changes in fiscal spending, or other unexpected shocks. We also assume that other shocks, such as the change in risk premium, monetary policy, or taxes (fiscal revenues), have no instantaneous effect on the GDP, indicated by zeros (0) in the matrix. GDP adjustment to these shocks may take place with lags. Similarly, the second row in the matrix of the identifying restrictions indicates that a demand shock increases, while a supply shock decreases the price level.

Reading the matrix vertically determines the character of each individual shock. A risk premium shock leads to changes in the policy rate and alters the exchange rate. An increase in the risk premium results in exchange rate (HUFEUR) depreciation and an increase in the policy rate (R). A monetary policy shock has a direct impact on the policy rate (R) and on the HUFEUR exchange rate, increasing the policy rate and contributing to an appreciation. A shock that has an impact that has a positive correlation with the GDP, the price level, and net taxes is called a demand shock. A demand shock may also have an unspecified influence on exchange rate movements and the policy rate, but not on government purchases. The impact of a supply shock is very similar to the impact of a demand shock, except that a supply shock reduces the price level, while the demand shock increases it. A fiscal revenue shock (tax increase) is identified as a shock that leads to an increase in net taxes but has no contemporaneous impact on real GDP. A fiscal spending shock shows up in the increase in government purchases and an increase in real GDP. Its effect on other variables is unrestricted (unspecified).

6. Impulse Responses

We checked the intuition on the effects of different shocks on the macroeconomic variables by looking at the impulse responses, which may also confirm the nature of the identified shocks. The estimated model was used for simulations, and impulse responses for 1000 replications were considered[6]. The size of the shocks in impulse response functions was one standard deviation.

The impulse responses of variables to the identified shock indicate that the increased market risks reflected in *a sudden jump in the country's risk premium* lead to an increase in inflation, provoke a temporary increase in the policy interest rate, and are reflected in a transitory devaluation of the currency. The effect on GDP and the fiscal variables is insignificant.

A monetary policy shock (increase in the policy interest rate) has a predominantly negative impact on GDP and results in a reduction in inflation. The monetary policy shock represents a jump in the policy rate, but because of subsequent macroeconomic adjustments, the policy rate eventually returns to its original level. The monetary policy shock goes along with a transitory appreciation of the exchange rate. The impact on government purchases is insignificant, while there is a significant loss in net revenues reflecting the decline in GDP.

The impulse responses to a demand shock are again in line with intuition. A sudden jump in demand leads to a significant increase in both the GDP and inflation and evokes an increase in the policy rate. As a result of the GDP increase, fiscal revenues (net taxes) increase with some lags, and government purchases also increase with a similar lag.

The impulse response to a positive supply shock is reflected in a decline in inflation, an appreciation of the exchange rate, and an increase in government purchases.

A fiscal revenue shock (tax increase) apparently would reduce GDP and increase inflation, but these responses are rather weak or insignificant. Similarly, a policy rate increase and a depreciation of the currency would be in line with intuition, as well as the reduction in government purchases and an increase in net taxes, but these impulse responses are also not significant.

A sudden jump in fiscal spending, however, has quite marked impulse responses. GDP increases significantly, and the exchange rate appreciates. The policy rate would be reduced, reflecting a temporary decline in inflation, but soon it would react with a permanent increase.

7. Ex-Ante Scenarios for the Public Debt Path in the VECM Framework

The VECM allows us to consider counterfactual experiments about the impact of uncertainty on debt dynamics. The components of the change in debt can be attributed to a range of macroeconomic factors, by using Formula (3). This method gave a useful description of the historical public debt path and its dynamics. In this section, we will take a step further on this path discussing *ex-ante* debt path scenarios based on various *assumptions* about the future development of these macroeconomic components (interest rates, GDP growth, inflation, exchange rate, and primary deficit). We pick the end of 2006 as the starting date and calculate the expected path for the debt-to-GDP ratio forward in quarterly steps ahead based on the hypothetical scenarios set for the macroeconomic components. The scenarios in this exercise are based on assumed expectations, but the path of each variable reflects the model-based stochastic properties of the economy described by these main macroeconomic variables. The purpose of this analysis is to illustrate how one could have estimated the expected debt path under some "optimistic" and "realistic" assumptions regarding the future path of the main macroeconomic variables.

The model is used to generate a hypothetical debt path that can be compared with the *ex-post* historical evolution (gross debt denoted by the solid line in Figures 2 and 3). The logic of constructing the model projection follows the attributions given in Equations (2) or (3). For comparison, we also calculated an ex-post evolution of the debt using the actual values for these factors; the gross debt estimated in this way is denoted by the dashed line in Figures 2 and 3.

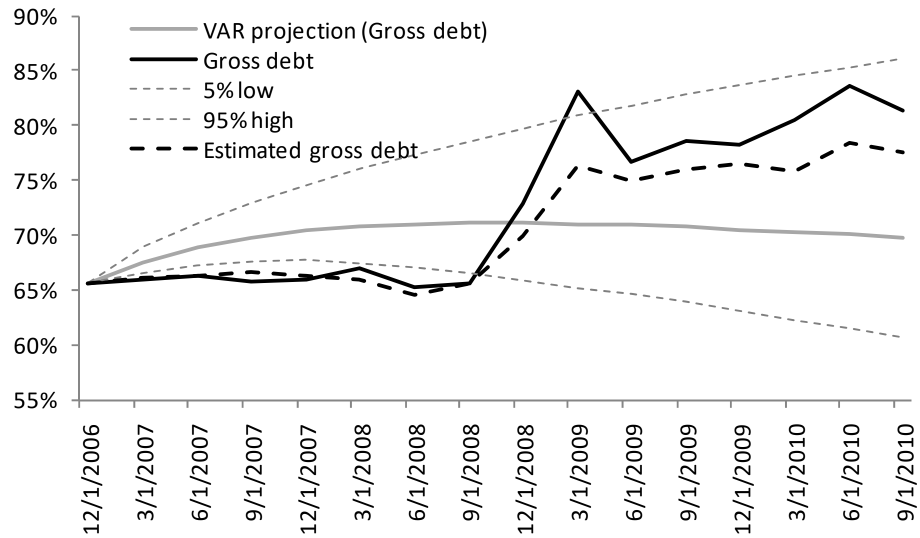

Figure 2. Hypothetical debt path estimation, starting from the end of 2006 (historical path, path estimation by realized macroeconomic factors, and path forecast based on *optimistic* macroeconomic assumptions). Note: gross nominal debt, calculated according to the Maastricht criteria.

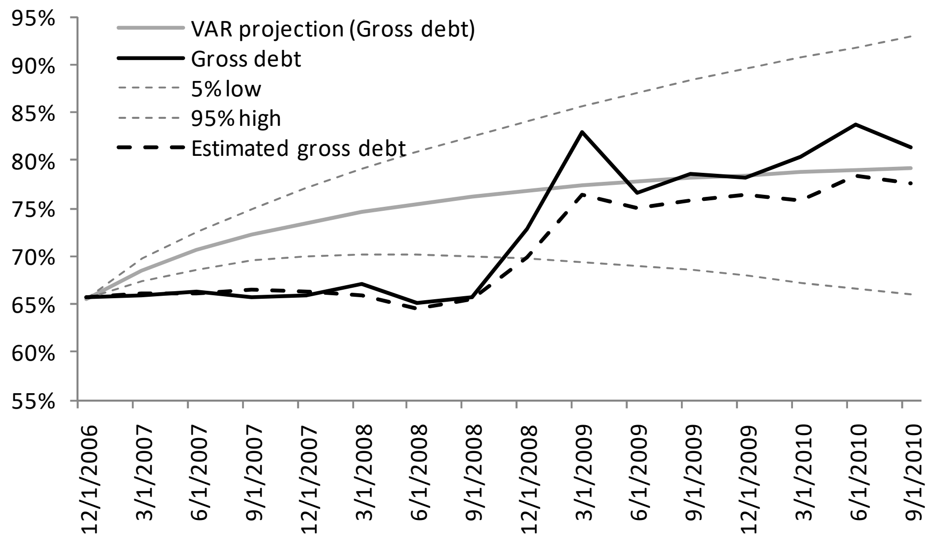

Figure 3. Hypothetical debt path estimation, starting from the end of 2006. (Historical path, *ex-post* path estimation by realized macroeconomic factors, and *ex-ante* path forecast based on *pessimistic* macroeconomic assumptions). Note: gross nominal debt, calculated according to the Maastricht criteria.

The ex-post estimated dashed line tracks the factual debt path closely, indicating that one-off factors did not divert the debt significantly in this period. However, it does not help us to better understand the stochastic characteristics (risks) inherent in the dynamics of the economic factors driving the debt path. To get a better understanding of the risks, we estimate the model for the following macroeconomic variables: domestic and foreign (euro) interest rates, real GDP growth, inflation, foreign exchange rate, and primary deficit. For the *ex-ante* scenario analyses, we use the model estimates to build stochastic simulations for the underlying macroeconomic variables, which we then use to build the corresponding debt path based on the framework of Equation (2).

The procedure to calculate the stochastic debt path start with a simulation, where the starting date is December 2006, and the parameters are based on the model estimations up to that point. Taking the covariance matrix, we run simulations about the possible path of the macroeconomic variables that follow their interrelationships driven by the model, while converging to a given *long-term* outcome. For the long-term outcome, we choose two scenarios, one of which we call *optimistic* (or something reflecting the expectations at the starting date of 2006), while the other scenario is called *pessimistic* (or something that proved to be realistic in retrospect for 2008).

The simulated path of the macroeconomic variables is driven by the model and reflects the economic forces in the macroeconomic framework. Although the target values are somewhat arbitrary, the speed and the shape of their convergence to it are driven by the model, hopefully reflecting the economic realities. The simulation helps us to map the stochastic properties of the debt dynamics, in the sense that we can draw confidence intervals within which we can expect the outcome to have a 90 percent probability. This means that the probability of the debt path that would fall below the lower bound or would break the upper bound is only 5 percent on each side. It is important to note that the debt is not modeled directly, and the variable for debt is not included in the variables of the model. The model variables, however, describe the economic movements. Based on these movements in each variable, we can calculate the effect on the debt as attributed in Equation (3). This procedure gives the model projection for the gross debt in Figure 2 for the optimistic assumptions, and in Figure 3 for the pessimistic assumptions.

The assumptions about the target (restriction) the economy is expected to reach in the future starting from 2006 in the optimistic scenario are as follows.

Long-run assumptions in the optimistic scenario:

- The *share of foreign debt* in gross debt: 30 percent.
- Long-run *interest rate* on both the domestic and foreign debt: 5.0 percent.
- Real GDP *growth rate* (annual): 4.0 percent.
- Primary fiscal *deficit*: 0.0 percent of GDP.
- The change in the Hungarian forint exchange rate to the euro (long-run): 0 percent annual change (the forint is stable).
- *Inflation rate*: 2.0 percent.

The speed of adjustment to the long-run expected values is not uniform for the variables but in about 8–10 quarters, all of them are reasonably close. Substituting the calculated path of the variables into Equation (2), where d_0 is the debt-to-GDP ratio at the end of 2006, we get the projection for debt in Figure 2. This line depicts the gross debt-to-GDP ratio, which increases at a slowing pace until about the time when the variables reach the values picked in the optimistic scenario. Going forward, the ratio starts its slow but steady decline. Based on the simulation, we added the intervals (dashed lines in Figure 2) within which the debt-to-GDP ratio would have a 90 percent probability. The comparison of the actual debt path with these intervals indicates that the actual debt dynamics before 2008 were even better than what would have been expected at a 90 percent likelihood in the optimistic scenario. However, even in this optimistic scenario, an increasing (destabilizing) debt dynamic could not have been ruled out, as the upper side of the interval shows. The crisis in 2008 brought about a jump in the debt-to-GDP ratio, as the debt increased and GDP fell, but even this jump bought the debt line above the upper side of the interval only for a short period. It is interesting to note that what happened in the debt-to-GDP ratio was well within the realm of the Hungarian macroeconomic conditions, which could not have been ruled out at 90 percent probability even in 2006. Such a gloomy possibility was not foreign, even in a rather optimistic scenario for the coming two to three years. We assumed a balanced primary fiscal position in this calculation. The fact that this assumption was breached in reality should have raised doubt about the expected stabilization.

The expectation expressed in this scenario might have been appealing in 2006 when it could have been seen even as a realistic one, but it turned out to be an overly optimistic picture of the future. Let us look at what the model would suggest if we had picked a scenario closer to reality.

Long-run assumptions in the pessimistic scenario:

- The *share of foreign debt* in gross debt: 30 percent.
- Long-run *interest rate* on both the domestic and foreign debt: 7.5 percent.

- Real GDP *growth rate* (annual): 2.5 percent.
- Primary fiscal *surplus*: 0.5 percent of GDP.
- The change in the Hungarian forint exchange rate to the euro (long-run): 2 percent annual depreciation of the forint.
- *Inflation rate*: 3.0 percent.

The result of the simulation exercise for this scenario is summarized in Figure 3. The projection and the actual debt path, as well as the estimated gross debt, which is the sum of the attributions based on the formula in Equation (3), were all promisingly close in the period after 2009. This might be good news for the model, but it is rather worrisome for the observer, because this pessimistic scenario, which proved to be realistic, depicts explosive debt dynamics. The interval that covers 90 percent of all possible outcomes in the simulation would not rule out such a positive surprise that would bring the debt-to-GDP ratio on a stabilizing path, but the likelihood of such outcomes is small. It is somewhat promising that the projection in Figure 3, although still on an increasing trend, looks like it is approaching a plateau and could take a stabilizing turn afterward.

8. Conclusions

The main factors driving the public debt-to-GDP ratio, including the interest rate, exchange rate, rate of growth, inflation, and primary fiscal deficit, are policy-dependent factors. We did not address the question of how different policies may alter the path of these factors. The analysis focused on the issues of how these variables drove the debt-to-GDP ratio between 1996 and 2010. We used the main macroeconomic variables, which had a direct impact on the debt dynamics, and pooled the information in a VECM. Simulations based on the model were used to analyze the stochastic properties of the debt dynamics.

Starting in 2006 and using the VECM estimates up to that point to calculate the stochastic properties of the economic development based on simulations, the result showed that even under an optimistic scenario, the unwelcome surprise in debt dynamics in the 2008 crisis was within the likely outcomes with 90 percent probability.

Doing the same exercise, but using a more realistic scenario, which we may have called pessimistic ex-ante, but which proved to be rather realistic ex-post, the crisis outcome of the debt-to-GDP ratio looks like the most probable outcome. It is very close to the central model projection.

Risks in debt dynamics are inherent, and assessing them is important. The stochastic characteristics of the main macroeconomic variables offer an approach to assessing macroeconomic risks.

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Notes

1. We focused on the Hungarian fiscal crisis in 2008 and its aftermath but did not go into the period when, as a result of unconventional monetary policy and the quantitative easing of fiscal financing, conditions changed fundamentally after 2010.
2. This is a shortcoming of the method used by Ábel and Kóbor (2011), which we hope to circumvent by following a different approach in this paper.

[3] This is in fact a shortcoming of the method used by Ábel and Kóbor (2011), which we hope to circumvent by following a different approach in this paper.

[4] All data were obtained from data sources used by the Hungarian central bank (Magyar Nemzeti Bank, MNB) for regular analysis. The advantage of using this source is that the consistency of this dataset is maintained on a regular basis. Updates are closely followed, and corrections are made accordingly. Magyar Nemzeti Bank (MNB) (2008, 2014) gives a detailed description of financial accounts covered by MNB statistics.

[5] For Croatia, a similar identification method was used by Ravnik and Žilić (2011) in an SVAR framework and by Rukelj (2009) in a VECM framework.

[6] The authors express gratitude to Tamás Kiss for his kind assistance in these calculations.

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