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Constant or Variable? A Performance Analysis among Portfolio Insurance Strategies

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Abstract: In this paper, we propose a comparison among three portfolio insurance strategies, namely the constant proportion portfolio insurance, the time-invariant portfolio protection, and the exponential proportion portfolio insurance, via an in-depth performance analysis. We aim to ascertain whether strategies characterized by variable parameters can outperform those with constant parameters by measuring potential returns, investment riskiness, downside protection capability, and ability to capture market upside. The results, achieved in a model-free framework by exploiting bootstrapping techniques, advise that no winning strategy exists overall, even when considering different volatility regimes, rebalancing frequencies, and protection levels.

Keywords: portfolio insurance; performance analysis; bootstrap

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1. Introduction

By definition, risk-averse investors prefer to protect their capital. For this reason, the asset management industry has looked for suitable investment opportunities for decades. The most prominent examples are given by the *portfolio insurance* (PI) strategies introduced by Leland and Rubinstein (1988) within the pension funds framework. The authors motivate their intuition by observing that "*after the decline of 1973–1974, many pension funds had withdrawn from the market only to miss the rally in 1975*" and by arguing that "*if only insurance were available, those funds could be attracted back to the market*". Portfolio insurance strategies are conceived to deliver a minimum level of wealth at maturity and guarantee participation in potential gains relative to a reference portfolio.

At an institutional investment level, portfolio insurance strategies might be used as an umbrella against potential large losses. At a private investment level, portfolio insurance strategies could help investors who would like to safeguard their initial investment. The literature offers several methodologies to safeguard the risky investment against large losses, such as the *synthetic put strategy*, the *stop-loss strategy*, or the *constant proportion portfolio insurance* (CPPI) strategy to name but a few; see, e.g., Leland and Rubinstein (1988). Benninga and Blume (1985) show theoretically that the optimality of portfolio insurance strategies depends on the investor's utility function. Moreover, assuming that the returns of the risky asset are independently and normally distributed, Brooks and Levy (1993) also show that risk-averse investors may prefer portfolio insurance strategies to buy-and-hold strategies. However, as well documented in Cont (2001), financial returns are not normally distributed. More specifically, their variance clusters in time, and both conditional and unconditional distributions display tails fatter than the normal distribution. Unfortunately, when the Gaussian assumption of returns distribution is violated, it is no longer possible to find analytical results witnessing the goodness of portfolio insurance strategies in a standard expected utility maximization setup.

A more recent strand of the literature finds empirical evidence suggesting the effectiveness of portfolio insurance strategies. In particular, Cesari and Cremonini (2003) show that the benefits of portfolio insurance strategies depend on the market features: using several performance measures the CPPI strategy dominates all other strategies in bear and sideways markets.

More recently, Annaert et al. (2009) found that portfolio insurance strategies outperform a buy-and-hold strategy regarding downside protection but provide lower excess returns. Moreover, a comparison based on stochastic dominance criteria reveals that no dominance relations between portfolio insurance strategies and buy-and-hold can be identified. Since the potential lower return is sufficiently compensated by lower risk, Annaert et al. (2009) conclude that portfolio insurance strategies might be valuable alternatives for risk-averse investors. Dichtl and Drobetz (2011) analyze portfolio insurance strategies within the framework of cumulative prospect theory. Their results indicate that loss aversion contributes to making portfolio insurance strategies a preferred investment strategy for a prospect theory investor. The attractiveness of portfolio insurance strategies in terms of cumulative prospect theory is also confirmed in Dierkes et al. (2010).

When deciding to implement portfolio insurance strategies, a crucial point is the identification of the most suitable procedure to use. Our study aims to enrich the above-mentioned contributions through a comparison among three different portfolio insurance strategies, namely the constant proportion portfolio insurance (CPPI) strategy, the *time-invariant portfolio protection* (TIPP) strategy, and the *exponential proportion portfolio insurance* (EPPI) strategy. The TIPP and EPPI strategies can be considered generalizations of the standard CPPI. The TIPP strategy is the first variant of CPPI, proposed in Estep and Kritzman (1988) and incorporates a stochastic time-varying floor, which increases as the market goes up, allowing to capture any equity market participation. Therefore, the investor guarantees the present value of the future guarantee (the so-called *floor*) and can increase such a value by incorporating intra-period gains. According to Estep and Kritzman (1988), introducing an *ad hoc* process may furnish better downside protection against adverse market movements.

The EPPI strategy was proposed by Lee et al. (2008) and belongs to the broader class of *variable proportions portfolio insurance* (VPPI) strategies. Both for CPPI and VPPI strategies, the exposure to the risky asset is proportional to the difference between the portfolio value and the floor. While a CPPI strategy relies on a proportionality factor assumed to be constant throughout the investment time horizon, a VPPI strategy allows for changes in such a proportionality factor. Hence, the literature has shown an increasing interest in implementing VPPI strategies where the multiplier either reduces or increases according to volatile or stable scenarios, respectively; see, e.g., Hamidi et al. (2014) and Zieling et al. (2014). However, as argued in Dichtl et al. (2017), the primacy of these strategies in providing better risk-adjusted returns compared to the standard CPPI strategy strongly depends on the forecasting techniques exploited by the investors. Conversely, the EPPI strategy keeps the implementation as simple as possible since it is rule-based. In other words, the dynamics of the multiplier depend on market fluctuations through a *dynamic multiplier adjustment factor* (DMAF), fixed a priori.

In the present paper, we provide a detailed comparison between the CPPI strategy and its generalization via traditional mean-variance performance measures and other indicators, such as Value-at-Risk, the Expected Shortfall, and Omega measures. Along the lines of Annaert et al. (2009) and Dichtl et al. (2017), we design a simulation experiment without imposing a (log)normal return distribution. Our approach differs from the proposal by Lee et al. (2008), where an extensive comparison between CPPI and EPPI is carried out by assuming a Black–Scholes framework. We simulate from the empirical distributions of four different market indices returns, to better capture some stylized facts. To this aim, we employ a bootstrapping simulation approach. In particular, we repeatedly draw with replacement a block of 252 returns (one year) from our dataset. One of the main advantages of applying such a procedure relies on carrying out a performance analysis of portfolio insurance strategies for different market scenarios. We follow a three-step procedure in our performance evaluation. First, we examine whether TIPP and EPPI outperform the CPPI strategy both in terms of protecting against losses and guaranteeing extra returns. Then, we show how choosing different protection levels affects their performances. Finally, we measure the impact of different (discrete) rebalancing assumptions.

Our results exhibit a couple of interesting features, namely that (i) TIPP outperforms CPPI in terms of downside protection but is not able to attain the same equity market participation within upward market scenarios, and (ii) EPPI provides equity market participation comparable to the CPPI one but worsens the downside protection.

For the sake of completeness, we show a comparison of our work with the previous literature, as shown in Table 1. From such a comparison, we argue that we provide a performance analysis of all dynamic rule-based portfolio insurance strategies in a model-free setting.

The remaining part of the paper is as follows. Section 2 describes the main properties of the rule-based portfolio insurance strategies. In Section 3, we describe the simulation method, while Section 4 shows the numerical experiments. Section 5 concludes.

Table 1. The relevant literature: a comparison.

Reference	Dynamic PI Strategies			Static PI	Methodology
	CPPI	TIPP	EPPI	Strategies	(Model-Free)
Cesari and Cremonini (2003)	✓			✓	
Lee et al. (2008)	✓		✓		
Annaert et al. (2009)	✓			✓	✓
Dichtl and Drobetz (2011)	✓	✓		✓	
Hamidi et al. (2014)		✓			
Zieling et al. (2014)	✓				
Ardia et al. (2016)	✓	✓			✓
Dichtl et al. (2017)	✓	✓		✓	
Chen et al. (2022)	✓	✓			
Our work	✓	✓	✓		✓

2. Portfolio Insurance Strategies: Definitions and Features

The main idea behind portfolio insurance strategies is to offer participation from positive stock market movements while limiting potential losses to a given level. Accordingly, the resulting return distribution becomes asymmetric and right-skewed. Various investment strategies that provide loss protection are suggested in the literature; see, e.g., Ho et al. (2010) for a detailed review. In the present paper, we focus on the three prominent examples, namely *(i)* the constant proportion portfolio insurance (CPPI) strategy, *(ii)* the time-invariant portfolio protection (TIPP) strategy, and *(iii)* the exponential proportion portfolio insurance (EPPI) strategy. In this section, we provide a detailed description of these strategies.

2.1. Constant Proportion Portfolio Insurance Strategy

The constant proportion portfolio insurance (CPPI), introduced for the first time in the literature by Black and Jones (1987, 1988), is based on the allocation among two financial assets: a riskless asset, denoted by B, which provides a cash reserve, and a risky asset, denoted by S, which is usually a stock index. We assume that the rebalancing dates are discrete along the management period $[0, T]$, since the discrete-time dynamics are more realistic than continuous-time trading for portfolio management. Let τ denote a sequence of equidistant refinements of the management period $[0, T]$, i.e., $\tau = \{t_0 = 0 < t_1 < \cdots < t_{n-1} < t_n = T\}$, where $t_{k-1} - t_k = \frac{T}{n}$ for $k = 0, \ldots, n-1$. In particular, we restrict that trading is only possible immediately after $t_k \in \tau$, for $k = 0, \ldots, n-1$. This implies that the number of shares held in the risky asset is constant on the intervals $(t_k, t_{k+1}]$ for $k = 0 \ldots n-1$. Denote by $V_{t_k}^{CPPI}$ the portfolio value at time t_k. The CPPI strategy must satisfy the following conditions as a portfolio insurance method. First, the portfolio value must be higher than the guaranteed amount. Second, the investor must benefit from market rises. In order to meet these purposes, the standard CPPI is based on the following:

- The choice of a floor, F_{t_k} which represents the minimum value of the portfolio which is acceptable for an investor at any instant of time during the management period $[0, T]$. Its initial value, F_{t_0}, capitalized at the non-risky rate, must be equal to a predetermined percentage of the initial capital deposit;
- The choice of a dynamic investment rule on the risky asset defined as follows: the total amount E_{t_k} (the *exposure*) invested into the underlying asset S_{t_k} is equal to a fixed proportion m (the *multiplier*) of the difference between the portfolio value $V_{t_k}^{CPPI}$ and the floor F_{t_k}. Such a difference is called the *cushion* and is denoted by C_{t_k}. Since the strategy results to be self-financing, the remaining amount, $V_{t_k}^{CPPI} - F_{t_k}$, is invested into the riskless asset B_{t_k}, such as a money market account or a government bond, with log-return r_{t_k} for each period $[t_{k-1}, t_k]$.

A fundamental point in implementing the CPPI strategy is the choice of the multiplier, which is a non-negative constant. This key parameter has to be higher than 1 in order to provide a convex payoff, which allows making profits from substantial market rises. The value of the floor F_{t_k} gives the dynamically insured amount. The floor is assumed to evolve at the same rate as the riskless asset B, namely

$$F_{t_k} - F_{t_{k-1}} = F_{t_{k-1}} \exp\{r_{t_k}\}, \quad k = 1, \ldots, n, \tag{1}$$

whose initial value is $F_{t_0} = G \cdot \exp\{-r_{t_0}(T - t_0)\}$. The parameter G is the *guarantee*, defined as the product of the capital initially invested $V_{t_0}^{CPPI}$ and the percentage of $V_{t_0}^{CPPI}$ that the investor aims to recover at maturity T (the so-called *protection level* $PL \in (0, 1]$). Both the protection level PL and the multiplier m are functions of the investor risk tolerance. Indeed, the higher the multiplier m (resp., the lower the protection level PL), the higher the amount E_{t_k} invested into the risky asset. Therefore, an aggressive investor would choose high values for m and low values for PL. Nevertheless, the corresponding portfolio is riskier, and the guarantee may no longer hold.

Assuming that the risky asset evolves according to

$$S_{t_k} = S_{t_0} \prod_{l=1}^{k} \left(1 + \frac{\Delta S_{t_l}}{S_{t_{l-1}}}\right), \tag{2}$$

where ΔS_{t_l} denotes the difference $S_{t_l} - S_{t_{l-1}}$, we deduce that the portfolio value is the solution of

$$V_{t_k}^{CPPI} = V_{t_{k-1}}^{CPPI} + E_{t_{k-1}}^{CPPI} \frac{\Delta S_{t_k}}{S_{t_{k-1}}} + \left(V_{t_{k-1}}^{CPPI} - E_{t_{k-1}}^{CPPI}\right) \exp\{r_{t_k}\}, \quad k = 0, \ldots, n, \tag{3}$$

with

$$E_{t_{k-1}}^{CPPI} = \max\left\{m \cdot \left(V_{t_{k-1}}^{CPPI} - F_{t_{k-1}}\right), 0\right\}. \tag{4}$$

2.2. Time Invariant Portfolio Protection Strategy

As argued in Estep and Kritzman (1988), the performance of the CPPI strategy is price dependent: any gains at a particular point in time may still be lost if the underlying risky asset subsequently falls. The authors introduced the time-invariant portfolio protection (TIPP) strategy with a ratchet mechanism to lock in all interim capital gains. In particular, the TIPP strategy can be considered a generalization of the standard CPPI strategy described in Section 2.1. The main difference between the CPPI and TIPP is in the stochastic time-varying definition of the floor. The TIPP floor is the maximum between the usual CPPI floor and the percentage PL of the maximum past portfolio values. Now, the floor $\tilde{F}_{t_k}$ satisfies

$$\tilde{F}_{t_k} = \max\left\{ F_{t_k}, PL \cdot \sup_{s \le k} V_{t_s}^{TIPP} \right\}, \quad k = 1, \ldots, n. \tag{5}$$

Thanks to the mechanism depicted in Equation (5), the floor jumps up with the portfolio value to reduce risky asset allocation when the market peaks. As for the CPPI allocation mechanism, the TIPP is a self-financing strategy. Hence, also in this case, the portfolio value is the solution of

$$V_{t_k}^{TIPP} = V_{t_{k-1}}^{TIPP} + E_{t_{k-1}}^{TIPP} \frac{\Delta S_{t_k}}{S_{t_{k-1}}} + \left(V_{t_{k-1}}^{TIPP} - E_{t_{k-1}}^{TIPP} \right) \exp\{ r_{t_k} \}, \quad k = 0, \ldots, n, \tag{6}$$

with

$$E_{t_{k-1}}^{TIPP} = \max\left\{ m \cdot \left(V_{t_{k-1}}^{TIPP} - \tilde{F}_{t_{k-1}} \right), 0 \right\}. \tag{7}$$

2.3. Exponential Proportion Portfolio Insurance Strategy

A second generalization of the CPPI strategy is proposed in Lee et al. (2008). Such a novel strategy allows the multiplier m to vary at any time, according to market fluctuations. In particular, the authors argue that a dynamic multiplier m_{t_k}, linked to the fluctuations of the underlying risky asset S, can improve the convex nature of the CPPI strategy. In particular, when the stock price increases, the multiplier increases, determining an improvement in the upside capture. By contrast, when the stock price reduces its value, a lower multiplier induces higher downside protection. Moreover, to keep the implementation as simple as possible, a particular version of the VPPI strategy is introduced, the so-called exponential proportion portfolio insurance (EPPI) strategy. As for the CPPI strategy, the EPPI portfolio must be self-financing, and the insured amount evolves according to Equation (1). However, the multiplier presents the following dynamics:

$$m_{t_k} = m_{t_{k-1}} + a \cdot \exp\left\{ a \ln\left(\frac{S_{t_k}}{S_{t_{k-1}}} \right) \right\} \frac{\Delta S_{t_k}}{S_{t_{k-1}}}, \quad t = 0, \ldots, n-1, \tag{8}$$

where $\eta > 1$ is an arbitrary constant, and $\exp\left\{ a \ln\left(\frac{S_{t_k}}{S_{t_{k-1}}} \right) \right\}$ is the so called dynamic multiplier adjustment factor (DMAF). The parameter a in Equation (8) amplifies the multiplier. The parameter is set greater than 1 to account for both the *enlargement effect* or the *shrinkage effect* in case of bullish or bearish markets, respectively. As for CPPI and the TIPP strategy, the evolution of the EPPI portfolio is given by

$$V_{t_k}^{EPPI} = V_{t_{k-1}}^{EPPI} + E_{t_{k-1}}^{EPPI} \frac{\Delta S_{t_k}}{S_{t_{k-1}}} + \left(V_{t_{k-1}}^{EPPI} - E_{t_{k-1}}^{EPPI} \right) \exp\{ r_{t_k} \}, \quad k = 0, \ldots, n, \tag{9}$$

with

$$E_{t_{k-1}}^{EPPI} = \max\left\{ m_{t_{k-1}} \cdot \left(V_{t_{k-1}}^{EPPI} - F_{t_{k-1}} \right), 0 \right\}, \tag{10}$$

$$m_{t_{k-1}} = \eta + a \sum_{l=1}^{k-1} \exp\left\{ a \ln\left(\frac{S_{t_l}}{S_{t_{l-1}}} \right) \right\} \frac{\Delta S_{t_l}}{S_{t_{l-1}}}. \tag{11}$$

2.4. Practical Issues for the Implementation of Portfolio Insurance Strategies

To implement a portfolio insurance strategy, we need to consider a couple of fundamental aspects. First of all, Equations (4), (7), and (10) imply that the investment in the risky asset might be potentially unbounded. For this reason, the common market practice suggests that the portfolio insurance strategies should usually be implemented such that the exposure to the risky asset varies between 0% and a *maximum leverage factor* $L_{\max}$ fixed between 100% and 200% of the current portfolio value. This implies that short sales and leverage are limited. Hence, Equations (4), (7), and (10) are modified as follows:

$$E_{t_k}^j = \max\Big\{\min\Big\{m \cdot C_{t_k}^j, L_{\max} \cdot V_{t_k}^j\Big\}, 0\Big\}, \quad k = 0, \dots, n\,, \tag{12}$$

where $C_{t_k}^j$ is the cushion at time t_k, for all $j = \{CPPI, TIPP, EPPI\}$. Moreover, we need to take into account transaction costs. It is worth noting that, because of the guarantee constraint, it is not possible to change the risk-free position of the portfolio insurance method (which is needed without transaction costs). Rather, the transaction costs are to be financed by reducing the asset exposure arising in the case without transaction costs. Along the lines of Balder et al. (2009), we assume the transaction cost to be proportional to the value of shares traded. The investor will rebalance the portfolio at dates t_k, an instant before rebalancing, which we denote by t_k-; the prices are all assumed to be known, and the modification of the portfolio will take place instantaneously and without price impact so that just after rebalancing, at time t_k, the weights are adjusted, and the value of the portfolio has evolved according to

$$V_{t_k}^j = V_{t_k-}^j - \text{transaction costs}, \quad k = 0, \dots, n\,,$$

for all $j = \{CPPI, TIPP, EPPI\}$. Denoting by $\varphi_{t_{k-1}}^{S,j} := \frac{E_{t_{k-1}}^j}{S_{t_{k-1}}}$ (resp., $\varphi_{t_{k-1}}^{B,j} := \frac{V_{t_{k-1}}^j - E_{t_{k-1}}^j}{B_{t_{k-1}}}$) the number of units of risky (resp., riskless) asset at time t_{k-1}, we can write the value of the portfolio before rebalancing as follows:

$$V_{t_k-}^j = \varphi_{t_{k-1}}^{S,j} S_{t_k} + \varphi_{t_{k-1}}^{B,j} B_{t_k}, \quad k = 1, \dots, n,$$

for all $j = \{CPPI, TIPP, EPPI\}$. The proportional transaction cost for risky asset S is given by $\left|\varphi_{t_k}^{S,j} - \varphi_{t_{k-1}}^{S,j}\right| S_{t_k}\theta$, where θ denotes the proportionality factor. Hence,

$$V_{t_k}^j = V_{t_k-}^j - \left|\varphi_{t_k}^{S,j} - \varphi_{t_{k-1}}^{S,j}\right| S_{t_k}\theta, \,, k = 1, \dots, n\,, \tag{13}$$

for all $j = \{CPPI, TIPP, EPPI\}$.

3. Simulation Setup

3.1. Data and Design of Empirical Analysis

To examine the performance of the strategies, we conduct a bootstrap simulation on a dataset containing equity return from the following market indexes: S&P500, Hang Seng, Nikkei 225, and FTSE 100. In particular, daily returns are retrieved from Bloomberg for 1 December 1989–24 December 2019, thereby deleting all non-trading days. The U.S. 3-month Treasury Bill yields are also downloaded for the same period. We do not consider stock data without the corresponding risk-free rate. Summary statistics are reported in Table 2.

Table 2. Summary statistics for S&P500, Hang Seng, Nikkei 225, and FTSE 100 index returns. Each time series contains 7511 returns. The daily average returns are reported on an annual basis using $\bar{r}_{annual} = (1 + \bar{r}_{daily})^{252} - 1$. Daily standard deviations are transformed into annual standard deviations using $\sigma_{annual} = \sigma_{daily} \cdot \sqrt{252}$, where we assume 252 trading days per year.

Series	**Average Return (%)**	**Standard Deviation (σ) (%)**	**Skewness**	**Kurtosis**	***p*-Value Autocorrelation (Ljung-Box Test)**	***p*-Value Heteroscedasticity (Engle's ARCH Test)**
UKX	6.044	17.288	0.120	10.374	0.000	0.000
HSI	11.322	25.268	−0.120	18.748	0.000	0.000
NKY	1.362	23.415	−0.006	9.169	0.001	0.000
SPX	9.449	17.518	−0.142	11.814	0.000	0.000

Our preliminary statistical analysis shows that, apart from the FTSE 100 index, the time series exhibit fat tails and negative skewness. The Ljung–Box test detects significant serial correlation in all the time series under investigation. Moreover, we find significant heteroscedasticity in each time series.

We resort to bootstrapping to generate simulated return series without making any assumption regarding the return distribution but including the aforementioned statistical features, as in Annaert et al. (2009) and Dichtl et al. (2017). To encompass autocorrelation and heteroscedasticity, we use block bootstrapping, as suggested in Sanfilippo (2003). Therefore, our simulation procedure can be divided into the following steps:

1. We randomly draw a market index (S&P500, Hang Seng, Nikkei 225 or FTSE 100) with replacement;
2. We draw with replacement a starting date;
3. Starting from the initial date obtained in Step 2, we analyze the one-year performance of CPPI, TIPP, and EPPI strategies for the drawn market, i.e., the 252 days following the starting date are used to evaluate the different portfolio insurance strategies;
4. The procedure (Step 1–Step 3) is repeated 20,000 times.

The previous procedure provides the return distribution for each portfolio insurance strategy we investigate. Such distributions are the objective of our performance analysis.

3.2. Performance Measures and Statistical Tests

We compare specific moments or other statistics that characterize the returns distributions of the different portfolio insurance strategies described in Section 2. Moreover, for each performance indicator, we test the existence of significant differences between a given portfolio insurance strategy and another one chosen as the benchmark. In particular, we focus on the *average returns* in excess w.r.t. the risk-free rate $(\bar{r} - r_f)$, the return volatility (σ), and the Sharpe ratio SR (see, Sharpe 1966) which is defined as follows:

$$SR = \frac{\bar{r} - r_f}{\sigma}, \tag{14}$$

where r^f is the annual risk-free rate. From the simulation procedure described in Section 3, we obtain N paths of the daily risky asset returns and daily risk-less asset log-returns. For all $n = 1, \ldots, N$, the corresponding annual rate of return of PI strategies and the risk-free annual rate are given by $r_n = \frac{V_{t_n}}{V_{t_0}} - 1$ and $r_n^f = \exp\left\{\sum_{k=1}^{252} r_{t_{k,n}}\right\} - 1$, respectively. Hence, $\bar{r} = \frac{1}{N}\sum_{n=1}^{N} r_n$ and $r^f = \frac{1}{N}\sum_{n=1}^{N} r_n^f$.

We apply (i) the t-test to check for the difference in average excess return among the implemented strategies, (ii) the Levene (1960) test to test for equality in variance, (iii) the Jobson and Korkie (1981) test to test for difference in Sharpe ratio.

Furthermore, it is well-known that the Sharpe ratio is useful when the standard deviation is the reference risk measure. As argued in Annaert et al. (2009), this is not the case, since a high standard deviation can be to due positive outliers in excess return distributions, which would attract rather than repel investors. For this reason, we consider further *naive* performance measures, namely, the number of negative excess returns and the average negative excess return, which are respectively given by

$$\tilde{N} = \frac{1}{N}\sum_{n=1}^{N} \mathbb{1}_{\left\{r_n - r_n^f < 0\right\}}, \tag{15}$$

$$\tilde{r} = \frac{1}{\tilde{N}}\sum_{n=1}^{N} \left(r_n - r_n^f\right) \mathbb{1}_{\left\{r_n - r_n^f < 0\right\}}. \tag{16}$$

Again, to check for statistical differences between the frequency of negative excess returns and the negative average excess return among the various portfolio insurance strategies, we employ a t-test.

Furthermore, within the portfolio insurance strategies framework, the investors are also interested in monitoring the potential losses, the latter being included in the left tail of the return distribution. Hence, we consider the Value-at-Risk (VaR); see, e.g., Ibragimov and Walden (2007). Formally, the VaR at a confidence level α can be defined as follows:

$$VaR_\alpha(R) = \inf\{x \in \mathbb{R} | F_R(x) \geq \alpha\} \, , \tag{17}$$

where $F_R(x)$ is the cumulative return distribution. VaR denotes the maximum loss at a certain confidence level. We perform an unconditional coverage test (see, e.g., Christoffersen (2003)) for testing statistical differences in VaR. Under the null, the VaR of strategy A equals the VaR of strategy B. We fix a 95% confidence level, so that we expect 5% of the observations of strategy B, say r_n^B, to exceed the VaR of strategy A, say $VaR_\alpha(R^A)$, under the null. This is formally tested by calculating the fraction of strategy A VaR violations (i.e., the *hit ratio*) and performing a t-test as follows:

$$\frac{\frac{1}{N}\sum_{n=1}^{N} Hit_n - \alpha}{\sqrt{\frac{\alpha(1-\alpha)}{N}}} \rightarrow \mathcal{N}(0,1) \text{ as } N \rightarrow \infty \, , \tag{18}$$

where N is the sample size, $\alpha = 5\%$, and

$$Hit_n = \begin{cases} 1, & \text{if } r_n^B < VaR_\alpha(R^A) \\ 0, & otherwise \end{cases} .$$

To overcome the drawbacks related to the VaR as a non-coherent risk measure, we consider the expected shortfall (ES), to measure the average loss below the VaR; see, e.g., Acerbi and Tasche (2002) for further details. Formally, the ES at a confidence level of α can be defined as

$$ES_\alpha(R) = \mathbb{E}[R | R \leq VaR(\alpha)] \, , \tag{19}$$

where $\mathbb{E}[R|R \leq VaR(\alpha)]$ is the conditional expected value of the returns given that the returns are less than or equal to the VaR at a confidence level equal to α. For testing differences in ES, we resort to the method proposed by Annaert et al. (2009), exploiting the bootstrap technique. Given the sample return distributions of size 20,000, both for the reference strategy and the one to be compared, we randomly select 10,000 returns pairwise from these distributions. Thus, two return distributions of size 10,000 are obtained, one for each strategy. Then, we calculate ES for each distribution. We repeat such a procedure 10,000 times, ensuring the attainment of two distributions for ES, one for each strategy. Finally, to test for statistical difference, we use a paired sample t-test.

As discussed in Annaert et al. (2009), higher confidence levels will entail more negative VaR and ES values. Hence, choosing a higher confidence level indicates a higher risk aversion. However, linking these two concepts is possible when returns are normally distributed. For this reason, we include in our analysis also the skewness. A more significant skewness makes protection strategies more appealing; see, e.g., Harvey and Siddique (2000); Post et al. (2008).

We use a procedure similar to the one exploited for ES for testing differences in skewness, along the lines of Annaert et al. (2009). First, for each strategy, from the i-th return r_i, $i = 1, \ldots,$ 20,000 of the bootstrapped distributions, we create a new *synthetic* return $\tilde{r}_i$, $i = 1, \ldots,$ 20,000, such that $\tilde{r}_i = -(r_i + \bar{r}) + \bar{r}$. Then, we add the new i-th return to the initial distribution. Thereby, we obtain symmetric distributions. Next, we randomly draw 10,000 returns pairwise from the modified samples of the two portfolio insurance strategies and calculate the empirical skewness difference. Repeating this 10,000 times, we obtain an approximation of the distribution, which is then used to compute the p-value of obtaining an even more extreme skewness difference than the empirical one.

Finally, we consider the Omega measure introduced by Keating and Shadwick (2002), an indicator that takes into account the whole distribution and is thus a valuable tool to characterize non-Gaussian distributions. It is worth mentioning that if X is a continuous r.v. with support $(a, b) \in \mathbb{R}$ and CDF $F_X(x)$, then the Omega measure is

$$\Omega(r^*) = \frac{\int_{r^*}^{b}(1 - F_R(x))\mathrm{d}x}{\int_{a}^{r^*} F_R(x)\mathrm{d}x}, \tag{20}$$

with r^* being a reference threshold fixed a priori. The numerator of Equation (20) indicates the probability-weighted gains, while the denominator represents the probability-weighted losses. Thus, a high value of the Omega measure indicates a more profitable investment and, consequently, a more attractive strategy to investors.

4. Performance Measurement Results

To begin with, we compare the three portfolio insurance strategies based on the risk metrics and performance indicators introduced in Section 3.2. We recall that we are discussing dynamic strategies characterized by some parameters. Such parameters are the rebalancing frequency, the protection level (PL), the multiplier, and the maximum leverage (L_{max}).

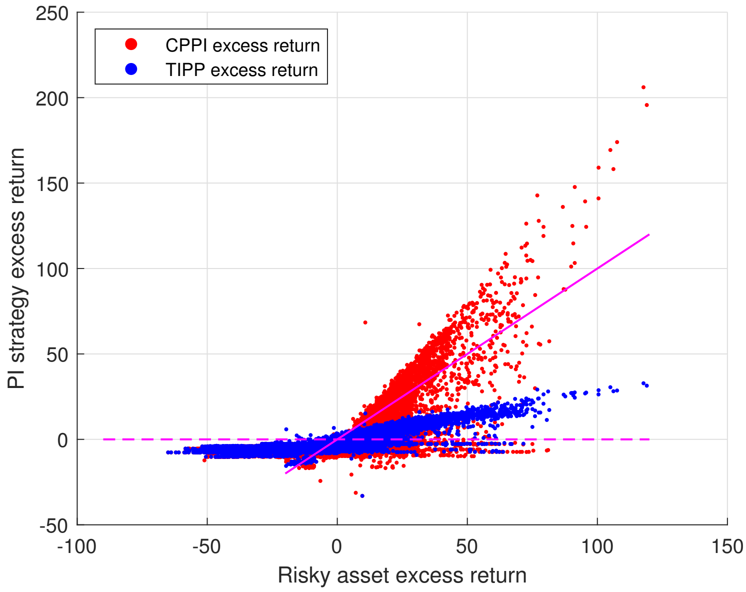
To obtain all the performance results, we apply a block-bootstrapping procedure over a time horizon equal to one year. Moreover, we consider transaction costs in our analysis, according to Equation (13), setting $\theta = 0.1\%$ Finally, we set $r^* = 0$ for the Omega measure.

4.1. Constant Proportion Portfolio Insurance vs Its Generalizations

We start by comparing TIPP and EPPI strategies with the CPPI one, referred to as the benchmark. As a first attempt, we consider the corresponding payoff functions. The top chart in Figure 1 compares CPPI and TIPP in terms of the excess returns. We observe that TIPP outperforms CPPI in terms of downside protection, but the former cannot fully capture potential market raising, as shown by the most convex shape of the latter. Such an empirical behavior is also confirmed by the outcomes of the performance analysis, shown in Table 3.

Table 3. Comparison among CPPI, TIPP, and EPPI strategies. We denote by *, ** and *** the significant difference between the reference strategy (CPPI) and the TIPP (resp. EPPI) strategy, at a 10%, 5%, and 1% confidence level, respectively.

Portfolio Insurance Strategy	CPPI	TIPP	EPPI (a = 5)	EPPI (a = 10)	EPPI (a = 20)
Rebalancing discipline	Daily	Daily	Daily	Daily	Daily
Protection level (%)	97.5	97.5	97.5	97.5	97.5
Multiplier	14	14	-	-	-
η	-	-	14	14	14
Initial equity allocation (%)	35.27	35.27	35.27	35.27	35.27
Average excess return	1.482	0.213 ***	1.501	1.519	1.559
Standard deviation	15.616	5.666 ***	15.828	16.095 ***	16.660 ***
Sharpe ratio	0.095	0.038 ***	0.095	0.094	0.094
% < 0	75.115	53.660 ***	76.010 **	76.955 ***	77.910 ***
Average negative excess return	−4.753	−4.076	−4.786	−4.855 ***	−5.087 ***
VaR 5%	−8.488	−7.584 ***	−8.508 *	−8.644 ***	−9.456 ***
ES 5%	−10.324	−8.626 ***	−10.419 ***	−10.535 ***	−10.823 ***
Skewness	3.525	0.688	3.493	3.452	3.368
Omega measure	1.413	1.096	1.410	1.404	1.391



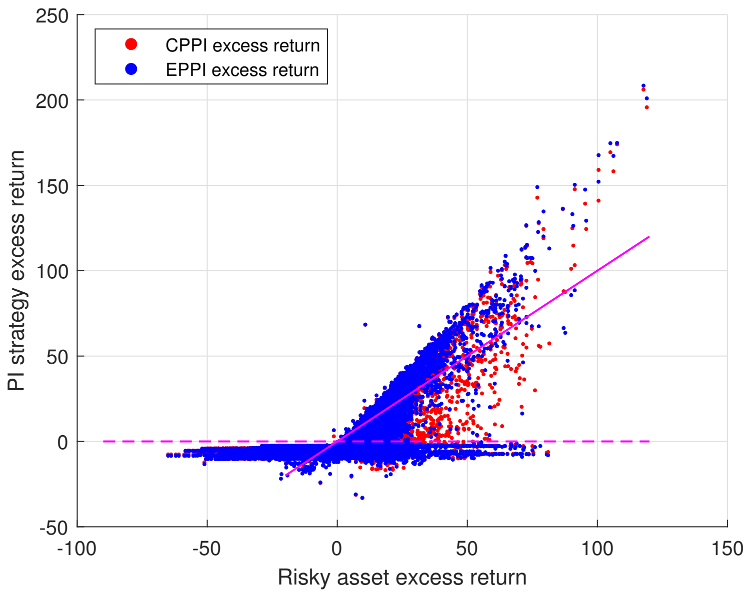


Figure 1. Comparison between the payoff functions of CPPI and its generalizations, with daily rebalancing and 0.1% transaction costs. The top chart compares CPPI and TIPP in terms of excess returns, and the bottom chart compares CPPI and EPPI in terms of excess returns. The dashed horizontal line indicates the floor values and the solid line is introduced to facilitate the interpretation.

More precisely, TIPP outperforms CPPI regarding risk reduction while suffering from lower excess returns. We observe that the standard deviation is three times lower than for CPPI, and VaR and ES within the TIPP strategy are reduced by 10% and 19%, respectively. Moreover, our analysis does not lead to rejecting the hypothesis that the CPPI skewness exceeds or equals the TIPP one. This should not be surprising as TIPP has a solid protective slant that protects against downside risk. On the other hand, the mechanism embedded in the TIPP strategy forces the risky asset exposure to be systematically lower than the CPPI one. This causes a worsening in terms of upside capture: in the presence of market upside, the TIPP cannot guarantee effective participation in the stock market. The Omega measure further supports such behavior.

We then proceeded to compare CPPI and EPPI. As before, we consider the corresponding payoff functions, depicted in the bottom chart of Figure 1. EPPI shows a more extreme attitude than CPPI: for positive excess returns of the portfolio insurance strategies (y-axis), the expected gains from implementing EPPI (blue dots) are slightly higher than those provided by CPPI (red dots), indicating a slightly stronger tendency to upside capture. Surprisingly, for the negative excess returns of the same strategies, the loss experienced by implementing an EPPI strategy (blue dots) is always greater than or equal to that provided by CPPI (red dots), indicating a weaker ability to hedge against downside risk. We investigate the performance indicators described in Table 3 to strengthen such a qualitative finding. For the EPPI strategy, we choose three different parameter values a. Considering the case where $a = 5$, the performance analysis shows slightly better average excess returns than CPPI, although this result is not statistically significant. EPPI also performs less efficiently than CPPI in downside risk mitigation: all the risk measures examined for EPPI are worse than those associated with CPPI. Such results are even more pronounced the larger the value of parameter a. Moreover, the skewness is positive but lower than the CPPI one, although not statistically significant, indicating that the strategy may be less attractive to investors. The Omega measure provides comparable values for the two strategies examined. In conclusion, we can argue that a powerful alternative strategy to the CPPI is the TIPP for investors more interested in protecting themselves against downside risk.

We then compare the performance of the three portfolio insurance strategies in three different market configurations, i.e., in low-, medium-, and high-volatility regimes, calculated according to the methodology proposed in Annaert et al. (2009); Ardia et al. (2016). More precisely, we split the simulated return paths into tercile groups, by using the realized volatility. The results are shown in Table 4. From the inter-regime comparison, we note that the highest returns are obtained when the market is calm (low volatility). At the same time, the worst situation occurs when the market is turbulent: in this case, excess returns become negative for all the strategies examined. In low volatility, TIPP shows the best performance in terms of risk hedging (lowest standard deviation, VaR, and ES) and exhibits the highest Omega measure. The findings also show that TIPP may induce losses in the portfolio since it displays negative skewness. However, we cannot reject the hypothesis that CPPI's skewness outperforms this. Finally, the CPPI strategy beats its variant with a stochastic multiplier. In the moderate volatility scenario, the picture remains unchanged in terms of risk measures. The CPPI (resp., TIPP) delivers the best (respective, worst) skewness. In the high-volatility regime, the skewness of the time-invariant strategy is positive but still lower than the constant parameter one.

Table 4. Comparison among CPPI, TIPP, and EPPI strategies for different volatility market scenarios with daily rebalancing and assuming a fixed protection level. The volatility is 9.66%, 17.90%, and 27.28% for the low-, medium-, and high-volatility scenarios, with 6667, 6666, and 6667 observations, respectively. We denote by *, ** and *** the significant difference between the reference strategy (CPPI) and the TIPP (resp. EPPI) strategy, at a 10%, 5%, and 1% confidence level, respectively.

Volatility Subgroup Rebalancing Discipline	**Low Volatility Regime Daily**			**Medium Volatility Regime Daily**			**High Volatility Regime Daily**		
Portfolio insurance strategy	CPPI	TIPP	EPPI	CPPI	TIPP	EPPI	CPPI	TIPP	EPPI
Protection level (%)	97.5	97.5	97.5	97.5	97.5	97.5	97.5	97.5	97.5
Multiplier	14	14	-	14	14	-	14	14	-
η	-	-	14	-	-	14	-	-	14
a	-	-	20	-	-	20	-	-	20
Initial equity allocation (%)	64.25	64.25	64.25	35.27	35.27	35.27	95.28	95.28	95.28
Average excess return	3.683	2.396 ***	3.657	2.375	0.687 ***	2.463	−1.613	−2.443 ***	−1.442
Standard deviation	11.286	3.426 ***	12.319 ***	17.326	5.494 ***	18.315 ***	16.996	6.508 ***	18.202 ***
Sharpe ratio	0.326	0.699 ***	0.297 ***	0.137	0.125	0.134	−0.095	−0.375 ***	−0.079 ***
%< 0	53.817	24.269 ***	60.132 ***	79.283	55.476 ***	81.263 ***	92.245	81.238 ***	92.335
Average negative excess return	−3.238	−2.036 ***	−3.835 ***	−4.792	−3.427 ***	−5.182 ***	−5.604	−5.128 ***	−5.819 ***
VaR 5%	−6.736	−2.783 ***	−7.335 ***	−8.877	−7.216 ***	−9.779 ***	−8.776	−8.350 ***	−9.989 ***
ES 5%	−8.933	−5.007 ***	−9.486 ***	−9.861	−7.683 ***	−10.318 ***	−11.781	−10.037 ***	−12.244 ***
Skewness	1.924	−0.078	1.722	2.511	0.635	2.390	5.197	1.812	4.970
Omega measure	3.108	5.836	2.581	1.622	1.360	1.583	0.684	0.412	0.732

4.2. Changing the Protection Level

In this section, we focus on the role of the protection level in determining the performances of each strategy along the lines described in Annaert et al. (2009). As described in Section 2, the protection level is linked to the value of the floor process. It is worth noting that the protection level affects the investment: it triggers the protection mechanism if the risky security suffers sharp declines, thus shifting the allocation to the non-risky security only. In particular, the lower the protection level, the lower the capital to be protected and the greater the investment in the security.

The results are shown in Table 5. In our analysis, we consider three different protection levels, i.e., we assume that, for all strategies, the capital to be protected at maturity is 90%, 95%, and 97.5% of the initial capital. Furthermore, for our statistical significance tests, we consider the case where $PL = 97.5\%$ is a benchmark for each strategy.

The left panel of Table 5 shows the results of the CPPI strategy. Based on such results, we deduce that a lower protection level guarantees higher returns, but higher risk, both in terms of standard deviation and VaR and ES. We cannot reject the hypothesis that the skewness of the return distribution of the strategy with the highest PL exceeds or equals the skewness of the other cases. The Omega measure is better when the floor is lower. The right panel of Table 5 shows the performance analysis of the EPPI strategy. The results obtained are similar to the previous case. Again, we cannot conclude that guaranteeing a higher or lower amount at maturity is a more compelling investment strategy, as the Omega measure results also confirm. For the TIPP strategy, the picture is slightly different: although even in this case, a lower protection level guarantees higher returns but higher risk when $PL = 95\%$, the return distribution exhibits the greatest skewness. As expected, the strategy with the lowest PL exhibits the higher values of the Omega measure.

Table 5. Comparison among CPPI, TIPP, and EPPI strategies for different protection levels with daily rebalancing. Within each strategy, the benchmark is the one with the highest protection level (PL = 97.5%). We denote by *, ** and *** the significant difference between the strategies with lower protection levels and the reference one at a 10%, 5%, and 1% confidence level, respectively.

Portfolio Insurance Strategy		**CPPI**			**TIPP**			**EPPI**	
Rebalancing discipline		Daily			Daily			Daily	
Protection level (%)	90	95	97.5	90	95	97.5	90	95	97.5
Multiplier	14	14	-	14	14	-	14	14	-
η	-	-	14	-	-	14	-	-	14
a	-	-	20	-	-	20	-	-	20
Initial equity allocation (%)	140.25	70.27	35.27	140.25	70.27	35.27	150.00	75.28	37.79
Average excess return	3.644 ***	2.369 ***	1.482	2.015 ***	0.752 ***	0.213	3.739 ***	2.480 ***	1.550
Standard deviation	23.647 ***	19.174 ***	15.616	20.435 ***	10.474 ***	5.666	23.941 ***	19.740 ***	16.352
Sharpe ratio	0.154 ***	0.124 ***	0.095	0.099 ***	0.072 ***	0.038	0.156 ***	0.126 ***	0.095
% < 0	60.365 ***	70.455 ***	75.115	60.090 ***	56.995 ***	53.660	59.635 ***	70.600 ***	77.290
Average negative excess return	−11.455 ***	−6.924 ***	−4.753	−11.327 ***	−6.707 ***	−4.076	−11.864 ***	−7.259 ***	−4.932
VaR 5%	−15.713 ***	−10.881 ***	−8.488	−15.674 ***	−10.533 ***	−7.584	−15.863 ***	−11.006 ***	−8.929
ES 5%	−17.232 ***	−12.506 ***	−10.324	−17.229 ***	−12.046 ***	−8.626	−17.473 ***	−12.798 ***	−10.656
Skewness	1.955	2.793	3.525	1.828 ***	1.210 ***	0.688	1.901	2.712	3.442
Omega measure	1.526	1.484	1.413	1.295	1.196	1.096	1.527	1.482	1.404

4.3. Changing the Rebalancing Frequency

In this Section, we study the impact of portfolio rebalancing frequency. In particular, we explore the case of daily, weekly, and monthly frequencies. The results are presented in Table 6, where we compare daily, weekly, and monthly rebalancing mechanisms for each strategy. Intuitively, a higher rebalancing frequency leads to higher transaction costs, regardless of how the investment strategy works. Therefore, we expect a high-frequency strategy to show a lower excess return. Our findings support this intuition. On the other hand, low returns are compensated by significant downside protection levels, as shown by the standard deviation, VaR, and ES values. As the rebalancing frequency reduces, the time interval when the strategy is buy&hold-like becomes longer so that the chance of taking advantage of any bullish market phases diminishes. Consequently, the skewness of the return distribution shrinks when switching from daily to weekly and monthly investments. Therefore, investors who prefer a positive skewness find it more profitable to pay higher transaction costs to satisfy their need to hedge against possible losses. However, we cannot conclude that a strategy with daily rebalancing is preferable to one with more frequent rebalancing, as highlighted by the Omega measure.

Table 6. Comparison among CPPI, TIPP, and EPPI strategies for different rebalancing frequencies. Within each strategy, the benchmark is the one with the highest rebalancing frequency (daily). We denote by *, ** and *** the significant difference between the strategies with lower frequencies and the reference one at a 10%, 5%, and 1% confidence level, respectively.

Portfolio Insurance Strategy	**CPPI**			**TIPP**			**EPPI**		
Rebalancing discipline	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly
Protection level (%)	97.5	97.5	97.5	97.5	97.5	97.5	97.5	97.5	97.5
Multiplier	14	14	-	14	14	-	14	14	-
η	-	-	14	-	-	14	-	-	14
a	-	-	20	-	-	20	-	-	20
Initial equity allocation (%)	35.27	35.27	35.27	35.27	35.27	35.27	37.79	37.79	37.79
Average excess return	1.482	2.051 ***	2.383 ***	0.213	0.495 ***	0.805 ***	1.559	2.190 ***	2.480 ***
Standard deviation	15.616	16.866 ***	18.251 ***	5.666	6.273 ***	7.408 ***	16.660	17.462 ***	18.747 ***
Sharpe ratio	0.095	0.122 ***	0.131 ***	0.038	0.079 ***	0.109 ***	0.094	0.125 ***	0.132 ***
% <0	75.115	71.665 ***	66.850 ***	53.660 ***	51.280 ***	47.900 ***	77.910 ***	72.735 ***	67.525 ***
Average negative excess return	−4.753	−5.327 ***	−6.619 ***	−4.076 ***	−4.443 ***	−5.256 ***	−5.087	−5.527 ***	−6.845
VaR 5%	−8.488	−9.975 ***	−12.866 ***	−7.584	−8.009 ***	−9.319 ***	−9.458	−10.157 ***	−13.383 ***
ES 5%	−10.324	−12.523 ***	−18.520 ***	−8.626	−9.813 ***	−13.091 ***	−10.823	−13.178 ***	−19.160 ***
Skewness	3.525	3.153	2.626	0.688	0.600	0.332	3.368	3.042	2.529
Omega measure	1.413	1.535	1.536	1.096	1.216	1.318	1.391	1.542	1.535

5. Concluding Remarks

We propose a comparison among the constant proportion portfolio insurance, the time-invariant portfolio protection, and the exponential proportion portfolio insurance, measuring both the risk profile and the risk-return trade-off. We perform our study without specifying any model for the risky asset, exploiting bootstrapping methodology and considering several scenarios according to the volatility, the rebalancing frequency, and the protection level. Our results show that TIPP outreaches CPPI regarding downside protection, even though it cannot face potential upside capture. By contrast, we show that EPPI and CPPI are comparable in terms of equity market participation, but EPPI is less effective in hedging against downside risks. The performance of the variable strategies might be improved by further reducing the risky exposure of the TIPP with a stochastic version of the multiplier, as in EPPI. Moreover, a comparison among portfolio insurance strategies via the cumulative prospect theory may be extended to the non-Gaussian diffusion for the underlying. We leave such open problems for further research.

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