

# Special Issue “Actuarial Mathematics and Risk Management”

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Among the most important implementations of the principles of enterprise risk management (ERM), the risk management process (RMP) involves various quantitative phases, usually encompassed under the label of quantitative risk management (QRM).

The RMP starts with defining the objectives (of an organization or a line of business) and then proceeding through the phases of risk identification, risk assessment, impact assessment, analysis of actions, choice of actions, and monitoring.

The whole RMP can benefit from the adoption of appropriate quantitative tools. In particular, the risk and impact assessments necessarily involve either stochastic evaluations (frequently implemented via Monte Carlo simulation procedures) or deterministic evaluations, such as sensitivity analysis and stress testing. The costs and efficiency of the alternative actions can be better understood in a quantitative framework. Statistical procedures are required for the monitoring phase, when observations must be elaborated and merged with initial assumptions, yielding updated input for a new cycle of the RMP.

Actuarial mathematics principles and tools can provide substantial support when implementing QRM phases, in particular when facing new risks or risks with changing features. Examples are provided by the development of products suitable for protecting individuals or organizations from emerging risks, the assessment of insurance product and portfolio risk profiles, the modeling of new risks or the revised modeling of traditional risks, and the study of effective risk measures. This background suggests that many areas of modeling and managing risks can benefit from novel research, aiming at both methodological and application innovation, in the insurance (life and non-life) context as well as in other economic sectors.

This Special Issue contributes in this regard with ten high-quality research papers addressing the following specific topics:

1. The design of post-retirement benefits (Chen et al. 2022; Pitacco and Tabakova 2022);
2. Designs of life and health insurance policies against new risks (Jędrzykowska 2022; Marciniuk and Zmysłona 2022);
3. Advancements in mortality modeling (Awad et al. 2022; Spreeuw 2022);
4. Advancements in risk measures (Faroni et al. 2022) and risk models (Pesenti 2022);
5. Reserving disclosure tools (Breuer and Staudt 2022);
6. Innovative approximation formulae for the mean duration (Orfanos 2022).

In detail, Awad et al. (2022) discuss an extension of the Lee–Carter model. In particular, they propose a generalization of the Poisson log-bilinear Lee–Carter-type model by introducing a new class of families of counting distributions, namely, the ABM class, which belongs to a wider class of natural exponential families. This class is characterized by its variance functions and contains the Poisson and negative binomial distributions as special cases, offering an infinite class of additional counting distributions to be considered within the Lee–Carter framework. The results of a numerical study demonstrate that when fitting mortality data using this new class of distribution, superior results with respect to more traditional assumptions can be obtained in a number of situations.

Breuer and Staudt (2022) focus on equalization reserves, an insurance liability with features of own capital, with particular regard to the Swiss regulation. Although, according



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to the local GAAP, Swiss reinsurers and non-life insurers must report equalization reserves in their statutory accounts, the solvency regulation does not admit them. As a result, the information about the equalization reserve is not fully disclosed. The purpose of the study is to recover that information and investigate the relationship between the equalization reserves and the publicly available technical account items. A generalized additive model (GAM) and a generalized linear model (GLM) were applied; based on publicly available data, the former proved to work better for reinsurers, whereas the latter worked better for nonlife insurers. The authors obtained encouraging results but also identified areas to be further investigated, such as the opportunity to link the equalization reserves to the insurance/reserving risk assessed from capital modeling.

[Chen et al. \(2022\)](#) address tontines as an alternative retirement product to conventional annuities. In particular, the authors introduce unit-linked tontines, which provide payments linked to an underlying financial asset. Two alternative designs are considered, differing with respect to the guarantee provided. First, the price is obtained using the risk-neutral approach; second, the attractiveness of the products is studied for a utility-maximizing individual. The findings of the numerical assessments stress the main difficulties of implementing retirement products.

[Faroni et al. \(2022\)](#) address the equivalence between VaR and TCE. The authors introduce a new risk indicator that extends TCE to consider higher-order risks. The quantiles of this indicator are compared with the quantiles of VaR in a simple Pareto framework and then in a generalized Pareto framework. The equivalence results between the quantiles of high-order TCEs are also examined.

The purpose of the study by [Jędrzykowska \(2022\)](#) is to describe the initial concept of household bridging insurance. After discussing the research gap regarding the insufficient protection of households against destabilization resulting from the lost personal contribution, the authors discuss the possibility of creating a new product, describing the desired features of its benefit structure.

[Marciniuk and Zmyślona \(2022\)](#) discuss products combining an equity release with a critical illness insurance; this is meant as a solution for protecting the living standards of individuals exposed to longevity risk. Two variants of the policy design are introduced: one addressed to couples and one to single individuals. The possible stream of benefits is analyzed for the two variants. The results suggest that the amount of cash flow related to reverse equity and critical illness insurance benefits depends on several factors, such as the spouse's economic status, age, and health condition.

[Orfanos \(2022\)](#) discusses issues related to the net present value of the cash flows exposed to interest rate risk. In particular, a new approximation formula for the Macaulay duration and convexity is described, which involves hyperbolic functions. The specific purpose of the study is to assess the reliability of each approximation formula under different scenarios. The results may be helpful in a number of actuarial implementations.

[Pesenti \(2022\)](#) proposes a reverse sensitivity analysis framework, which is model-free and allows for stresses on the output such as (a) the mean and variance, (b) any distortion risk measure including the value-at-risk and expected shortfall, and (c) expected utility type constraints. This framework is suitable for risk models. In particular, the author discusses a problem where a modeller needs to understand how a model consisting of random input factors, a corresponding random output of interest, and a baseline probability measure changes under a stress on the output's distribution. The findings not only provide a theoretical description of the stressed distribution but also show how to numerically efficiently calculate it.

[Pitacco and Tabakova \(2022\)](#) analyse special-rate life annuities, i.e., life annuity products rated considering the health status of the applicant. Better annuity rates are applied in the presence of poor health conditions, i.e., when an assumption of a shorter lifetime is acceptable. As the portfolio size should increase and as more potential annuitants can be attracted by more favorable annuity rates, a higher degree of heterogeneity of the portfolio follows as a result of including several risk classes. The pooling effect benefits from the

larger size but not from the increased heterogeneity. The purpose of the study is to analyze the impact on the variability of the total portfolio payout of extending the life annuity portfolio by selling special-rate life annuities. Numerical evaluations are performed by adopting deterministic and stochastic approaches, according to diverse assumptions concerning both lifetime distributions and the portfolio structure and size. The authors suggest that extending the annuity business by issuing special-rate annuities without significantly worsening the portfolio risk profile is possible.

Spreeuw (2022) introduces a new Archimedean copula family that is based on a link between Archimedean generators and utility functions. The family can well fit the mortality data of coupled lives. The parameter estimates suggest the possible existence of short-term dependence, i.e., the mortality of bereaved lives increases on bereavement but diminishes later.

All the papers part of this Special Issue underwent a refereeing process subject to the usual high standards of *Risks*. I would like to thank all the authors for their excellent contributions and all the referees for their thorough and timely reviews. I would also like to thank the MDPI Editorial Team for their active and timely support.

I have edited this Special Issue with Prof. Ermanno Pitacco. Our professional collaboration and our friendship have been lengthy and deep. The promotion of this Special Issue together is only one of the many projects that we had shared. Sadly and unexpectedly, Prof. Pitacco left us last September, when this Special Issue was still open. This was not the only project in which he was involved at that time. Although he was already retired, he was still very active in the field. During his whole professional life, he has deeply contributed to the development and dissemination of the actuarial culture, not only in Italy, but all over the world. He is greatly missed not only by his friends and family but also by the international actuarial community. He would have been proud to see how inspiring this Special Issue is.

**Conflicts of Interest:** The author declares no conflict of interest.

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