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On Fund Mapping Regressions Applied to Segregated Funds Hedging Under Regime-Switching Dynamics

Denis-Alexandre Trottier [1], Frédéric Godin [2,3,*] and Emmanuel Hamel [3]

1 Faculté des Sciences de l'Administration, Université Laval, Québec, QC G1V 0A6, Canada; denis-alexandre.trottier.1@ulaval.ca
2 Department of Mathematics and Statistics, Concordia University, Montréal, QC H3G 1M8, Canada
3 École d'Actuariat, Université Laval, Québec, QC G1V 0A6, Canada; emmanuel.hamel.1@ulaval.ca
* Correspondence: frederic.godin@concordia.ca; Tel.: +1-514-848-2424 (ext. 3494)

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Abstract: Insurers issuing segregated fund policies apply dynamic hedging to mitigate risks related to guarantees embedded in such policies. A typical industry practice consists of using fund mapping regressions to represent basis risk stemming from the imperfect correlation between the underlying fund and its corresponding hedging instruments. The current work discusses the implications of using fund mapping regressions when the joint dynamics of the underlying and hedging assets is a regime-switching process. The potential underestimation of capital requirements stemming from the use of a fund mapping regression under such dynamics is discussed. The magnitude of the latter phenomenon is quantified through simulations calibrated on market data.

Keywords: basis risk; hedging; segregated funds; variable annuities; risk measures; risk management; regime-switching

1. Introduction

Variable annuities are hybrid investment and insurance contracts issued by insurers to individual investors. Such products involve guarantees whose payoff is contingent on equity price movements, interest rates and mortality. As indicated by Zhang (2010), within hedged portfolios of variable annuities held by insurers, basis risk was a major source of losses during the recent financial crisis. Canadian insurers issuing segregated fund policies face similar challenges related to basis risk when applying dynamic hedging. Segregated funds are the Canadian version of variable annuities.

The main objective of this short note is to assess the adequacy of the use by insurers of fund mapping regressions within hedging schemes applying to segregated fund policies. This is achieved through three main contributions.

- The presence of basis risk between underlying assets of segregated funds and their corresponding hedging instruments is assessed empirically. This fills an important gap in the literature as the presence of basis risk is overlooked in the majority of papers treating of variable annuities, with a few exceptions such as Ankirchner et al. (2014).
- Novel parallels between fund mapping-based hedges and minimal variance hedges are drawn. Such conceptual parallels indicate that fund mapping regressions are likely to produce downward biased estimates of capital requirements in the context of regime-switching models.
- The presence of such downward biases in this context are confirmed through simulation.

The paper is divided as follows. In Section 2, the empirical assessment of the magnitude of basis risk between funds underlying segregated funds guarantees in Canada and their corresponding hedging instrument is performed through the calibration of econometric models to joint segregated funds and index futures return time series. The current study considers Canadian and US equity segregated funds.

Then, in Section 3, the adequacy of fund mapping regressions used by insurers to represent basis risk is discussed in the context of regime-switching models. We show that assuming a constant relationship between the underlying and hedging assets throughout the various market regimes leads to constraints which can bias the estimated impact of basis risk. Investigating regime-switching models is relevant due to their high popularity for modeling segregated funds policies, see for instance Hardy (2003). Other papers from the literature applying various versions of the regime-switching models in the context of equity-linked products, variable annuities or segregated funds include Hardy (1999, 2002), Lin et al. (2009), Jin et al. (2011), Ng and Li (2011), Ngai and Sherris (2011), Augustyniak and Boudreault (2012, 2015), Gaillardetz et al. (2012) and Azimzadeh et al. (2014).

Finally, in Section 4, this note illustrates that fund mapping procedures applied within are a particular case of the family of hedging strategies developed by Trottier et al. (2018). The impact induced by fund mapping regressions on hedging errors is discussed. Under regime-switching models, fund mapping regressions are shown to lead to an under-estimation of capital requirements, the latter being particularly severe if basis risk is omitted. This finding could have managerial implications since it points towards the addition of a layer of conservatism over capital requirement estimates associated with segregated funds hedging schemes if fund mapping regressions are used to represent basis risk. Although the current results are presented in the context of segregated funds, the under-estimation of risk provided by fund mapping regressions in regime-switching contexts might also apply to hedging procedures for other long-term financial derivatives involving basis risk.

2. Assessment of the Basis Risk Magnitude for Segregated Funds

This section empirically assesses the importance of basis risk in the context of segregated funds hedging. We consider Canadian and US equity funds. A sample of segregated funds issued by insurers is selected. The TSX 60 and S&P 500 index futures are designated as the hedging instruments respectively for Canadian and US equity funds.

The joint dynamics of each segregated fund along the hedging instrument is represented by a bivariate regime-switching model that is estimated on historical data. This model is used among others by Trottier et al. (2018). Regime switching processes are very popular in the segregated funds literature to model equity return dynamics, see for instance Hardy (2001, 2003). Such models are particularly well-suited for long-term liabilities; over the long-run, economic conditions can go through multiple states throughout the life of the segregated fund policies.

2.1. Regime-Switching Model

Define discrete time steps $\mathcal{T} \equiv \{0, 1, \ldots, T\}$. Consider $\{h_t\}_{t\in\mathcal{T}}$ the homogeneous Markov chain representing the market regime; h_t takes values in $\{1, 2\}$. Transition probabilities from state i to state j are denoted by $P_{i,j}$. Define F_t and S_t as the time-t value of the mutual fund underlying the segregated fund and the hedging asset, respectively. For $i = 1, 2$, define $\mu_i^{(F)}, \mu_i^{(S)}, \sigma_i^{(F)}, \sigma_i^{(S)}$ and ρ_i being respectively the conditional log-return expectations, standard deviations, and the correlation for both assets given the currently prevailing regime is $h_t = i$. The regime-switching market assumes that

$$R_{t+1}^{(F)} \equiv \log\left(\frac{F_{t+1}}{F_t}\right) = \mu_{h_t}^{(F)} + \sigma_{h_t}^{(F)} z_{t+1}^{(F)}, \qquad R_{t+1}^{(S)} \equiv \log\left(\frac{S_{t+1}}{S_t}\right) = \mu_{h_t}^{(S)} + \sigma_{h_t}^{(S)} z_{t+1}^{(S)},$$
$$z_{t+1} \equiv \begin{bmatrix} z_{t+1}^{(F)} \\ z_{t+1}^{(S)} \end{bmatrix} \sim N_2\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{h_t} \\ \rho_{h_t} & 1 \end{bmatrix}\right), \tag{1}$$

where $N_2(\mu, \Sigma)$ is the bivariate normal distribution with mean vector μ and covariance matrix Σ, and $z = \{z_t\}_{t\in\mathcal{T}}$ is a strong standardized Gaussian bivariate white noise. See Trottier et al. (2018) for additional properties of this model. An implicit feature of this model is that conditional on being in a given regime, the dependence between the two assets is driven by the Gaussian copula.

2.2. Estimation Results

The selected funds are listed in Table 1. Since the majority of funds have a very short history which does not not include data from the crisis, only funds with at least ten years of data were selected. For each fund, the joint estimation with the hedging instrument is performed by maximum likelihood on the entire data history that is available. From a consistency standpoint, it could have been desirable to perform a joint estimation of parameters for all mutual funds with a common Markov chain driving the whole market. However, such an estimation procedure would be highly non-trivial due to its high dimensionality. We therefore do not attempt such a joint estimation and use a separate estimation for each fund. Estimation results are given in Table 2. We observe that state 1 corresponds to a bull market regime whereas state 2 represents a bear market regime. The correlation coefficients ρ_1, ρ_2 for both regimes range from 65.9% to 98.2% across the various funds.

Table 1. List of funds considered for basis risk assessment. The three letters between brackets for each fund is an identifier for further referral.

[GCE] GWL Canadian Equity (GWLIM) BEL	
Issuer:	Great-West Life
Strategy type:	Canadian equity growth (active)
[GCV] GWL Canadian Value (FGP) NL	
Issuer:	Great-West Life
Strategy type:	Canadian equity growth (active)
[GCI] GWL Equity Index (GWLIM) BEL	
Issuer:	Great-West Life
Strategy type:	Canadian equity index (passive)
[MPC] Manulife Canadian Small Cap Segregated Funds—Cap Category B	
Issuer:	Manulife
Strategy type:	Canadian equity small cap (active)
[MRA] Manulife Canadian Growth Segregated Funds—Series R Category A	
Issuer:	Manulife
Strategy type:	Canadian equity growth (active)
[MNA] Canadian Equity Segregated Funds—NAL/VISTA	
Issuer:	Manulife
Strategy type:	Canadian equity large cap (active)
[RCE] RBC Canadian Equity GIF Series 1	
Issuer:	Royal Bank of Canada
Strategy type:	Canadian equity growth (active)
[AVL] Assumption/Louisbourg Canadian Equity Fund Series A	
Issuer:	Assomption Vie
Strategy type:	Canadian equity growth (active)
[LCG] LL Canadian Equity Growth (CC&L) BEL	
Issuer:	London Life
Strategy type:	Canadian equity growth (active)
[LCE] LL SRI Canadian Equity (GWLIM) BEL	
Issuer:	London Life
Strategy type:	Canadian equity growth (active)
[RUS] RBC U.S. Equity GIF Series 1	
Issuer:	Royal Bank of Canada
Strategy type:	US equity (active)
[AUS] Assumption/Louisbourg U.S. Equity Fund Series A	
Issuer:	Assomption Vie
Strategy type:	US equity (active)

Table 2. Maximum likelihood estimation results for the bivariate lognormal two-state regime-switching model (1) for each respective mutual fund and the corresponding hedging instrument. Fund descriptions are provided in Table 1.

	GCE	GCV	GCI	MPC	MRA	MNA	RCE	AVL	LCG	LCE	RUS	AUS
Mutual fund												
$\mu_1^{(F)}$	0.0083	0.0104	0.0091	0.0168	0.0139	0.0078	0.0100	0.0060	0.0113	0.0088	0.0050	0.0162
	(0.0024)	(0.0022)	(0.0022)	(0.0034)	(0.0037)	(0.0021)	(0.0025)	(0.0030)	(0.0029)	(0.0026)	(0.0029)	(0.0055)
$\sigma_1^{(F)}$	0.0330	0.0236	0.0321	0.0356	0.0375	0.0330	0.0297	0.0368	0.0308	0.0315	0.0306	0.0326
	(0.0019)	(0.0019)	(0.0018)	(0.0026)	(0.0029)	(0.0018)	(0.0018)	(0.0021)	(0.0020)	(0.0018)	(0.0021)	(0.0045)
$\mu_2^{(F)}$	−0.0080	−0.0110	−0.0135	−0.0206	−0.0119	−0.0105	−0.0224	0.0023	−0.0089	−0.0094	−0.0096	−0.0031
	(0.0104)	(0.0061)	(0.0119)	(0.0128)	(0.0097)	(0.0104)	(0.0125)	(0.0105)	(0.0064)	(0.0099)	(0.0049)	(0.0032)
$\sigma_2^{(F)}$	0.0734	0.0493	0.0776	0.0971	0.0772	0.0760	0.0745	0.0783	0.0525	0.0664	0.0350	0.0375
	(0.0081)	(0.0041)	(0.0085)	(0.0094)	(0.0061)	(0.0081)	(0.0089)	(0.0081)	(0.0048)	(0.0072)	(0.0037)	(0.0022)
TSX 60 index futures (for CAD funds) / S&P 500 index futures (for US funds)												
$\mu_1^{(S)}$	0.0085	0.0117	0.0092	0.0116	0.0113	0.0082	0.0106	0.0076	0.0100	0.0081	0.0102	0.0193
	(0.0026)	(0.0025)	(0.0025)	(0.0024)	(0.0027)	(0.0024)	(0.0024)	(0.0025)	(0.0028)	(0.0027)	(0.0028)	(0.0036)
$\sigma_1^{(S)}$	0.0348	0.0286	0.0345	0.0252	0.0288	0.0353	0.0293	0.0315	0.0310	0.0328	0.0297	0.0208
	(0.0022)	(0.0023)	(0.0021)	(0.0018)	(0.0025)	(0.0020)	(0.0018)	(0.0018)	(0.0020)	(0.0019)	(0.0019)	(0.0026)
$\mu_2^{(S)}$	−0.0134	−0.0114	−0.0190	−0.0109	−0.0095	−0.0178	−0.0224	−0.0078	−0.0057	−0.0084	−0.0145	−0.0028
	(0.0127)	(0.0093)	(0.0146)	(0.0105)	(0.0074)	(0.0141)	(0.0130)	(0.0115)	(0.0074)	(0.0105)	(0.0062)	(0.0034)
$\sigma_2^{(S)}$	0.0858	0.0764	0.0924	0.0849	0.0616	0.0938	0.0791	0.0856	0.0661	0.0761	0.0470	0.0397
	(0.0097)	(0.0071)	(0.0104)	(0.0077)	(0.0055)	(0.0114)	(0.0093)	(0.0087)	(0.0055)	(0.0076)	(0.0042)	(0.0023)
Correlations												
ρ_1	0.9439	0.8727	0.9402	0.7349	0.8123	0.9001	0.9815	0.9016	0.9410	0.9366	0.9734	0.8208
	(0.0090)	(0.0230)	(0.0091)	(0.0448)	(0.0415)	(0.0146)	(0.0034)	(0.0151)	(0.0109)	(0.0101)	(0.0054)	(0.0565)
ρ_2	0.9068	0.7120	0.9069	0.9232	0.6585	0.7486	0.9824	0.9080	0.8684	0.9306	0.9292	0.9455
	(0.0269)	(0.0584)	(0.0279)	(0.0210)	(0.0654)	(0.0716)	(0.0068)	(0.0252)	(0.0318)	(0.0226)	(0.0229)	(0.0090)
Transition matrix												
$P_{1,1}$	0.9767	0.9375	0.9788	0.9364	0.9272	0.9728	0.9749	0.9885	0.9754	0.9877	0.9848	0.9604
	(0.0137)	(0.0284)	(0.0114)	(0.0254)	(0.0313)	(0.0130)	(0.0143)	(0.0084)	(0.0148)	(0.0091)	(0.0117)	(0.0360)
$P_{2,1}$	0.0850	0.1129	0.0906	0.1202	0.1377	0.1386	0.1009	0.0287	0.0373	0.0297	0.0254	0.0074
	(0.0527)	(0.0610)	(0.0507)	(0.0492)	(0.0545)	(0.0619)	(0.0578)	(0.0237)	(0.0237)	(0.0250)	(0.0205)	(0.0081)

Note: Standard errors are given in parentheses.

Trottier et al. (2018) develop a dynamic hedging scheme applicable to segregated funds policies where an insurer makes periodic injections of liquidities in its hedging portfolio to offset gains and losses on the fund guarantee value. In their model, the proportion of the hedging cash flow injection standard deviation that can be eliminated through an Ederington (1979)-type local minimal variance hedge is a regime-switching version of the thumb rule $1 - \sqrt{1 - \rho^2}$ where ρ is the correlation between the mutual fund and the hedging asset.[1] Such values are tabulated for different values of ρ in Table 3. By comparing the latter to the estimated correlation coefficients ρ_1, ρ_2 from Table 2, one can deduce that the magnitude of basis risk within hedging procedures is substantial.

Table 3. Proportion $1 - \sqrt{1 - \rho^2}$ of the standard deviation of cash flow injections that can be eliminated through a local minimal variance hedge for a correlation between the mutual fund and the hedging asset of ρ.

ρ	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
$1 - \sqrt{1 - \rho^2}$	0.240	0.286	0.339	0.400	0.473	0.564	0.688	1

[1] See Proposition 3.3 of Trottier et al. (2018).

3. Representation of Basis Risk Through Fund Mapping Regressions

The usual industry practice to represent basis risk between a segregated fund and its hedging instrument is to use a fund mapping regression, see for instance Roncalli and Teïletche (2007) for such mappings. The fund mapping regression assumes the following relationship between returns of F and S:

$$\log\left(\frac{F_{t+1}}{F_t}\right) = \beta_0 + \beta_1 \log\left(\frac{S_{t+1}}{S_t}\right) + \sigma_M z_{t+1}, \tag{2}$$

where z is a strong standardized Gaussian white noise and σ_M is the relationship noise volatility. This model implicitly assumes the linkage between returns of assets F and S remains constant across regimes. Coefficients β_0 and β_1 are computed through ordinary least squares regression on historical data. Results of such regressions are presented in Table 4 for the various funds considered in the previous section.

Table 4. Linear regression results for the fund mapping model (2) for each respective fund and the corresponding hedging instrument. Mutual fund descriptions are provided in Table 1. For Canadian equity funds, the hedging instrument is the TSX 60 index futures. For US equity funds, the hedging instrument is the S&P 500 index futures.

	GCE	GCV	GCI	MPC	MRA	MNA	RCE	AVL	LCG	LCE	RUS	AUS
β_0	0.0017	0.0010	0.0016	0.0009	0.0012	0.0022	−0.0004	0.0016	0.0011	0.0012	−0.0015	−0.0008
	(0.0012)	(0.0016)	(0.0012)	(0.0022)	(0.0028)	(0.0018)	(0.0007)	(0.0016)	(0.0013)	(0.0011)	(0.0010)	(0.0011)
β_1	0.8159 *	0.5321 *	0.8052 *	1.0822 *	0.9188 *	0.6921 *	0.9485 *	0.8837 *	0.7938 *	0.8605 *	0.8342 *	0.9197 *
	(0.0247)	(0.0314)	(0.0232)	(0.0438)	(0.0621)	(0.0350)	(0.0149)	(0.0307)	(0.0304)	(0.0240)	(0.0283)	(0.0307)
σ_M	0.0175	0.0233	0.0172	0.0288	0.0403	0.0258	0.0079	0.0222	0.0180	0.0142	0.0107	0.0144
	(0.0009)	(0.0011)	(0.0008)	(0.0015)	(0.0020)	(0.0013)	(0.0005)	(0.0011)	(0.0009)	(0.0007)	(0.0014)	(0.0017)

Note: Standard errors are given in parentheses. For β_0 and β_1, "*" indicates 5% confidence level significance.

It can be shown that the above model can be expressed in the form of the bivariate regime-switching model (1) under the following constraints for $i \in \{1,2\}$:

$$\mu_i^{(F)} = \beta_0 + \beta_1 \mu_i^{(S)}, \quad \sigma_i^{(F)} = \left[\left(\beta_1 \sigma_i^{(S)}\right)^2 + \sigma_M^2\right]^{1/2}, \quad \rho_i = \beta_1 \sigma_i^{(S)} \left[\left(\beta_1 \sigma_i^{(S)}\right)^2 + \sigma_M^2\right]^{-1/2}. \tag{3}$$

Applying such constraints to the parameters of the mutual fund dynamics yields the new parameters presented in Table 5, where it is presumed for simplicity that the parameters of the hedging instrument and the Markov chain remain unchanged from their previously estimated values presented in Table 2. One can notice a considerable worsening of the underlying fund dynamics in the first regime (the bull market regime) for the purpose of hedging; its expected return is generally lower, its volatility is generally higher, and its correlation with the hedging instrument is lower. For instance, when an insurer issues a Guaranteed Minimal Maturity Benefit (GMMB) segregated funds policy, it has a short position on an implicit put option over the underlying fund. Since the delta of a put option is negative, the short put option translates into an implicit long position on the underlying fund. A lower expected return on the fund F therefore is a negative outcome for the insurer. A higher volatility is also undesirable as it will require holding additional short positions on the hedging futures S which entail shorting the equity risk premium and earning a smaller expected return. Finally, a lower correlation is clearly negative for hedging as it increases the impact of basis risk. Conversely, an improvement is noted for the underlying fund dynamics in the second regime; its expected return is generally higher, its volatility is generally lower, and its correlation with the hedging instrument is higher.

Table 5. Calculation of the fund mapping parameters obtained under constraints (3) for the various funds described in Table 1. The difference compared to the unconstrained model (see Table 2) is given in brackets. Teal brackets indicate an improvement from the perspective of the insurer performing the hedge (i.e., higher mean, lower volatility, or higher correlation), and red brackets indicate a worsening (i.e., lower mean, higher volatility, or lower correlation).

	GCE	GCV	GCI	MPC	MRA	MNA	RCE	AVL	LCG	LCE	RUS	AUS
						First regime						
$\mu_1^{(F)}$	0.0086 [+0.0003]	0.0072 [−0.0032]	0.0090 [−0.0001]	0.0135 [−0.0033]	0.0116 [−0.0023]	0.0079 [+0.0001]	0.0097 [−0.0003]	0.0084 [+0.0024]	0.0090 [−0.0023]	0.0082 [−0.0006]	0.0070 [+0.0020]	0.0170 [+0.0008]
$\sigma_1^{(F)}$	0.0333 [+0.0003]	0.0278 [+0.0042]	0.0327 [+0.0006]	0.0397 [+0.0041]	0.0482 [+0.0107]	0.0355 [+0.0025]	0.0289 [−0.0008]	0.0356 [−0.0012]	0.0305 [−0.0003]	0.0316 [+0.0001]	0.0270 [−0.0036]	0.0240 [−0.0086]
ρ_1	0.8515 [−0.0924]	0.5469 [−0.3260]	0.8501 [−0.0901]	0.6872 [−0.0477]	0.5492 [−0.2631]	0.6872 [−0.2129]	0.9624 [−0.0192]	0.7813 [−0.1203]	0.8076 [−0.1334]	0.8937 [−0.0429]	0.9186 [−0.0548]	0.7985 [−0.0223]
						Second regime						
$\mu_2^{(F)}$	−0.0093 [−0.0013]	−0.0050 [+0.0060]	−0.0137 [−0.0002]	−0.0109 [+0.0097]	−0.0076 [+0.0043]	−0.0101 [+0.0004]	−0.0216 [+0.0008]	−0.0053 [−0.0076]	−0.0034 [+0.0055]	−0.0060 [+0.0034]	−0.0136 [−0.0040]	−0.0033 [−0.0002]
$\sigma_2^{(F)}$	0.0722 [−0.0012]	0.0469 [−0.0024]	0.0764 [−0.0012]	0.0963 [−0.0008]	0.0695 [−0.0077]	0.0699 [−0.0061]	0.0754 [+0.0009]	0.0788 [+0.0005]	0.0555 [+0.0030]	0.0670 [+0.0006]	0.0406 [+0.0056]	0.0393 [+0.0018]
ρ_2	0.9702 [+0.0634]	0.8676 [+0.1556]	0.9743 [+0.0674]	0.9541 [+0.0309]	0.8148 [+0.1563]	0.9292 [+0.1806]	0.9946 [+0.0122]	0.9594 [+0.0514]	0.9461 [+0.0777]	0.9774 [+0.0468]	0.9650 [+0.0358]	0.9301 [−0.0154]

The higher (lower) correlation observed in the bear (bull) market for fund mapping parameters in comparison to the unconstrained case is due to the last constraint in (3); the function ρ_i defined therein is increasing in $\sigma_i^{(S)}$ for $\beta_1 > 0$. This constraint could lead to an upward biased estimate for the correlation in the bear market (high volatility) regime. Therefore, the hedging efficiency might be overestimated during crises, a time at which hedges of segregated funds are the most crucial. One might suspect that using the fund mapping parameters will yield downward biased estimators of quantile-based risk metrics applied to hedging errors since the tail thickness of the loss side of the distribution is mainly driven by the bear market regime which is too optimistic. The impact of such bias on capital requirement estimates is investigated in the next section.

4. Hedging of Variable Annuities

We consider the variable annuities hedging framework developed in Trottier et al. (2018), which is recalled in Appendix A. In the latter setup, an insurance company hedges the risk pertaining to a GMMB contract on the underlying mutual fund F. The latter scheme considers risk on a standalone basis for a single policy at a time. The incorporation of portfolio effects where risk associated with a given policy can be partially offset by other policies is not considered in the current study and is left out for further research.

The insurer performs a cross-hedge by taking positions on the futures S, which creates basis risk since the assets F and S are not perfectly correlated. The insurer sets up a hedging portfolio taking positions in two assets: the risk-free asset $B = \{B_t\}_{t\in\mathcal{T}}$ and the risky equity futures contract $S = \{S_t\}_{t\in\mathcal{T}}$. The risk-free asset is assumed to grow at a constant risk-free rate r, i.e., $B_t = e^{rt}$. The number of long positions[2] within the hedging portfolio during the time interval $(t, t+1]$ are respectively denoted by $\theta_{t+1}^{(B)}$ and $\theta_{t+1}^{(S)}$, with the convention $\theta_0^{(B)} = \theta_0^{(S)} = 0$. The insurer performs periodic injections or withdrawals of liquidities from the hedging portfolio at each time step. The injection at time t is denoted by I_t (negative amounts correspond to withdrawals). Such injections are characterized by Proposition 2.1 of Trottier et al. (2018). As shown in the latter work, they can be approximated by

$$I_{t+1} \approx \Theta_t + \Delta_t \delta F_t - \theta_{t+1}^{(S)} \delta S_t, \tag{4}$$

where $\delta F_t \equiv F_{t+1} - F_t$, $\delta S_t \equiv S_{t+1} - S_t$, and Θ_t and Δ_t are the time-t Greeks of the GMMB guarantee as defined in Equation (A4). The Insurer must hold reserves and capital to meet future variable annuity guarantee liabilities. The Total Gross Capital Required (TGCR) at time $t = 0$ is given by

$$\text{TGCR} = \text{CVaR}_{0.95}^{\mathbb{P}} \left[\sum_{t=1}^{T} e^{-rt} I_t \right] \tag{5}$$

where $\text{CVaR}_{0.95}^{\mathbb{P}}$ denotes the Conditional Value-at-Risk measure under the physical measure $\mathbb{P}$ at the 95% confidence level, see Rockafellar and Uryasev (2002).

4.1. Minimal Variance Hedging

The minimal variance strategy is formulated as the solution to the problem

$$\theta_{t+1}^{(S)*} = \underset{\theta_{t+1}^{(S)}}{\arg\min}\, \text{Var}^{\mathbb{P}} \left[I_{t+1} | \mathcal{F}_t \right], \qquad t \in \{0, \ldots, T-1\} \tag{6}$$

where $\mathcal{F}_t \equiv \sigma(S_u, F_u : u = 0, \ldots, t)$ is the information available at time t. As shown in Trottier et al. (2018), the solution under the approximation (4) is

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2 A negative number of long positions represents a short position.

$$\theta_{t+1}^{(S)*} = \Delta_t \frac{\mathrm{Cov}^{\mathbb{P}}[F_{t+1}, S_{t+1}|\mathcal{F}_t]}{\mathrm{Var}^{\mathbb{P}}[S_{t+1}|\mathcal{F}_t]}. \tag{7}$$

Interestingly, in the absence of basis risk, i.e., if S_t and F_t are perfectly correlated, the minimal variance strategy coincides with the usual form of delta hedging.

4.2. Fund Mapping Delta Hedging

A common industry practice is to set up the hedging portfolio which neutralizes the sensitivity of the guarantee with respect to risk factors (the Greeks). The application of the usual delta hedging procedure is not straightforward in the presence of basis risk; theoretical positions in the mutual fund F suggested by delta hedging must be translated into positions on the futures S. The common industry practice is to use the fund mapping regression relationship (2) to obtain the number $\theta_{t+1}^{(S)}$ of futures contract positions to be included in the hedging portfolio between time t and $t+1$, see for instance Fredricks et al. (2010). Equation (2) implies that

$$\begin{aligned} F_t &= F_0 \exp\left(\beta_0 t + \sum_{k=1}^{t} \sigma_M z_k\right) \left(\frac{S_t}{S_0}\right)^{\beta_1}. \\ \Rightarrow \frac{\partial F_t}{\partial S_t} &= F_0 \exp\left(\beta_0 t + \sum_{k=1}^{t} \sigma_M z_k\right) \frac{\beta_1}{S_0} \left(\frac{S_t}{S_0}\right)^{\beta_1 - 1} = \frac{\beta_1 F_t}{S_t}. \end{aligned}$$

Define the delta of a segregated fund guarantee by $\Delta_t^{(\Pi)} \equiv \frac{\partial \Pi_t}{\partial F_t}$ where Π_t is the time-t value of the guarantee. This leads to the use of the following delta-hedge strategy:[3]

$$\theta_{t+1}^{(S)} = \frac{\partial \Pi_t}{\partial S_t} = \frac{\partial \Pi_t}{\partial F_t}\frac{\partial F_t}{\partial S_t} = \Delta_t^{(\Pi)} \frac{\beta_1 F_t}{S_t}. \tag{8}$$

4.3. Links between Fund Mapping and Minimal Variance Hedging

Under the fund mapping regression model, it can be shown that the delta hedging strategy (8) is actually a minimal variance hedge based on a coarse approximation.

First, define the function $f(s,u) \equiv (1+u)s^{\beta_1}$, $s > 0$, $u \in \mathbb{R}$, whose first-order Taylor expansion centered on (s_0, u_0) is given by

$$f(s,u) \approx (1+u_0)s_0^{\beta_1} + (s - s_0)\beta_1(1+u_0)s_0^{\beta_1 - 1} + (u - u_0)s_0^{\beta_1}. \tag{9}$$

Defining

$$U_{t+1} \equiv e^{\beta_0 + \sigma_M z_{t+1}} - 1, \tag{10}$$

the fund mapping relationship (2) can be expressed as

$$F_{t+1} = F_t e^{\beta_0 + \sigma_M z_{t+1}} \left(\frac{S_{t+1}}{S_t}\right)^{\beta_1} = \frac{F_t}{S_t^{\beta_1}} f(S_{t+1}, U_{t+1}). \tag{11}$$

Using the Taylor approximation (9) with $(s_0, u_0) = (S_t, 0)$ in (11) yields

$$F_{t+1} \approx F_t + \delta S_t \frac{\beta_1 F_t}{S_t} + F_t U_{t+1} \quad \iff \quad \delta F_t \approx \delta S_t \frac{\beta_1 F_t}{S_t} + F_t U_{t+1}. \tag{12}$$

Hence, the injection approximation formula (4) can be further approximated:

$$I_{t+1} \approx \Theta_t + \Delta_t \delta F_t - \theta_{t+1}^{(S)} \delta S_t \approx \Theta_t + \delta S_t \left[\Delta_t \frac{\beta_1 F_t}{S_t} - \theta_{t+1}^{(S)}\right] + \Delta_t F_t U_{t+1}.$$

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[3] There exists a very slight difference between the definition of $\Delta_t^{(\Pi)}$ and Δ_t which are two versions of the guarantee's delta, see Appendix A. However this difference is negligible in practice, i.e., $\Delta_t^{(\Pi)} \approx \Delta_t$. It can thus be overlooked.

Using the fact that δS_t and U_{t+1} are independent, it is straightforward to show that the minimal variance strategy is given by

$$\theta_{t+1}^{(S)*} = \Delta_t \frac{\beta_1 F_t}{S_t}. \tag{13}$$

This matches the fund mapping delta hedging strategy (8) up to a slightly different definition of the GMMB contract's delta, i.e., $\Delta_t^{(\Pi)}$ versus Δ_t. To sum up, the fund mapping delta hedge is the minimal variance hedging strategy under the approximation (12) for δF_t, which is usually quite accurate for monthly returns. Hence, both methods lead to the same hedging strategy if the fund mapping constraints (3) are satisfied.

4.4. Impact of the Fund Mapping Constraints

In this section, we perform capital requirements estimation under the fund mapping model and compare the results with the unconstrained regime-switching model to determine which is most conservative. The simulation setup and parameters are as described in Trottier et al. (2018). Parameters are recalled in Appendix B and the simulation approach is described below.

The performance of the minimal variance strategy under both dynamics (the unconstrained model and the fund mapping regression) is studied. In the first numerical experiment, the data generating mechanism for F is given by the values in Table 2 (the unconstrained model) and the hedging strategy is performed consistently with this assumption. This simulation/hedging step is repeated 50,000 times to yield CVaR estimates for the discounted sum of injections. The CVaR risk measure is considered since this risk metric is used to determine reserves and capital requirements both in Canada and the US, see OSFI (2002) and AAA (2011). The injections are simulated according to their dynamics described in Appendix A. The same experiment is carried out for the data generating process of Table 5 (under fund mapping constraints), with the hedging strategy performed consistently with this new assumption. As shown in the preceding section, the usual fund mapping delta hedge then coincides with the minimal variance strategy.

The simulation results are presented in Table 7. For all twelve funds considered the risk statistics are higher under the unconstrained model, compared to the fund mapping constrained model (with the exception of the standard deviation for AUS). Hence, the fund mapping representation of basis risk leads to less conservative estimators of capital requirements. In other words, the constraints (3) result in downward biased risk measures.

4.5. Impact of Neglecting the Error Term in the Fund Mapping Regression

In this last section, the aim is to study the impact of neglecting the error term in the fund mapping regression model. In Canada, the Canadian Institute of Actuaries recognizes that an error term should be added to reflect basis risk,[4] but insurers are not required to do so in practice. Under such an assumption, the parameters are calculated using the constraints (3) with the fund mapping regression coefficients of Table 4 except for the standard error parameter which is set to $\sigma_M = 0$. This yields the parameters in Table 6. Compared to the unconstrained model, one can see that the correlation between F and S is now 100% in both regimes (indeed we assume no basis risk). The other parameters characterizing the underlying fund F dynamics (drifts and volatilities) are also generally biased towards values that are favorable for the insurer (i.e., higher expected returns and smaller volatilities). Indeed, carrying out the minimal variance hedging strategy under such parameters, i.e., by using them both to calculate the hedging strategy and as the data-generating model for simulated paths, yields much lower capital requirements as it can be seen in Table 7 where the statistics are estimated from 50,000 simulation runs. The omission of basis risk in a segregated funds hedging framework therefore leads to a severe underestimation of risk.

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4 See Section 5.2 of the report "Reflection of Hedging in Segregated Fund Valuation. Document 212027" (http://www.cia-ica.ca/docs/default-source/2012/212027e.pdf) by the Canadian Institute of Actuaries.

Table 6. Calculation of the fund mapping parameters obtained under constraints (3) assuming no basis risk (i.e., $\sigma_M = 0$) for the various funds described in Table 1. The difference compared to the unconstrained model (see Table 2) is given in brackets. Teal brackets indicate an improvement from the perspective of the insurer performing the hedge (i.e., higher mean, lower volatility, or higher correlation), and red brackets indicate a worsening (i.e., lower mean, higher volatility, or lower correlation).

	GCE	GCV	GCI	MPC	MRA	MNA	RCE	AVL	LCG	LCE	RUS	AUS
						First regime						
$\mu_1^{(F)}$	0.0086 [+0.0003]	0.0072 [−0.0032]	0.0090 [−0.0001]	0.0135 [−0.0033]	0.0116 [−0.0023]	0.0079 [+0.0001]	0.0097 [−0.0003]	0.0084 [+0.0024]	0.0090 [−0.0023]	0.0082 [−0.0006]	0.0070 [+0.0020]	0.0170 [+0.0008]
$\sigma_1^{(F)}$	0.0284 [−0.0046]	0.0152 [−0.0084]	0.0278 [−0.0043]	0.0273 [−0.0083]	0.0265 [−0.0110]	0.0244 [−0.0086]	0.0278 [−0.0019]	0.0278 [−0.0090]	0.0246 [−0.0062]	0.0282 [−0.0033]	0.0248 [−0.0058]	0.0191 [−0.0135]
ρ_1	1.0000 [+0.0561]	1.0000 [+0.1273]	1.0000 [+0.0598]	1.0000 [+0.2651]	1.0000 [+0.1877]	1.0000 [+0.0999]	1.0000 [+0.0185]	1.0000 [+0.0984]	1.0000 [+0.0590]	1.0000 [+0.0634]	1.0000 [+0.0266]	1.0000 [+0.1792]
						Second regime						
$\mu_2^{(F)}$	−0.0093 [−0.0013]	−0.0050 [+0.0060]	−0.0137 [−0.0002]	−0.0109 [+0.0097]	−0.0076 [+0.0043]	−0.0101 [+0.0004]	−0.0216 [+0.0008]	−0.0053 [−0.0076]	−0.0034 [+0.0055]	−0.0060 [+0.0034]	−0.0136 [−0.0040]	−0.0033 [−0.0002]
$\sigma_2^{(F)}$	0.0700 [−0.0034]	0.0407 [−0.0086]	0.0744 [−0.0032]	0.0919 [−0.0052]	0.0566 [−0.0206]	0.0649 [−0.0111]	0.0750 [+0.0005]	0.0756 [−0.0027]	0.0525 [−0.0001]	0.0655 [−0.0009]	0.0392 [+0.0042]	0.0365 [−0.0010]
ρ_2	1.0000 [+0.0932]	1.0000 [+0.2880]	1.0000 [+0.0931]	1.0000 [+0.0768]	1.0000 [+0.3415]	1.0000 [+0.2514]	1.0000 [+0.0176]	1.0000 [+0.0920]	1.0000 [+0.1316]	1.0000 [+0.0694]	1.0000 [+0.0708]	1.0000 [+0.0545]

Table 7. Results for the minimal variance hedge applied under the unconstrained model (top panel), under the fund mapping constraints (middle panel), and under the fund mapping constraints with $\sigma_M = 0$ (bottom panel). Funds are described in Table 1. The statistics are for the discounted sum of injections: $\sum_{t=1}^{T} e^{-rt} I_t$.

	GCE	GCV	GCI	MPC	MRA	MNA	RCE	AVL	LCG	LCE	RUS	AUS
Unconstrained model												
Mean	3.13	6.78	3.39	6.10	3.07	2.91	8.00	1.42	8.24	6.23	14.31	10.48
Std.Dev.	4.97	8.24	5.19	9.62	10.17	6.91	4.11	8.05	8.21	6.78	6.06	4.71
$\text{CVaR}^{\mathbb{P}}_{0.70}$	9.16	17.27	9.79	17.76	15.65	11.36	13.06	10.29	18.66	14.80	21.69	16.07
$\text{CVaR}^{\mathbb{P}}_{0.80}$	10.73	19.58	11.46	20.22	18.75	13.60	14.05	12.19	20.82	16.65	23.28	17.41
$\text{CVaR}^{\mathbb{P}}_{0.90}$	13.04	22.73	13.90	23.76	23.12	16.93	15.48	15.11	23.77	19.32	25.48	19.53
$\text{CVaR}^{\mathbb{P}}_{0.95}$	15.04	25.26	16.01	26.78	26.54	19.78	16.73	17.61	26.23	21.61	27.21	21.51
$\text{CVaR}^{\mathbb{P}}_{0.99}$	18.96	29.90	20.16	32.60	32.66	25.23	19.22	22.19	31.02	26.10	30.17	25.75
Fund mapping model												
Mean	2.46	4.32	2.71	2.83	1.67	0.84	7.47	2.34	3.73	3.62	10.77	8.70
Std.Dev.	4.86	6.93	4.85	7.54	10.15	6.53	3.78	6.21	5.16	4.50	4.36	4.85
$\text{CVaR}^{\mathbb{P}}_{0.70}$	8.36	12.95	8.60	11.80	14.04	8.81	12.08	9.88	10.05	9.13	15.86	14.52
$\text{CVaR}^{\mathbb{P}}_{0.80}$	9.79	14.92	10.02	13.82	17.03	10.84	12.99	11.60	11.54	10.36	16.88	15.67
$\text{CVaR}^{\mathbb{P}}_{0.90}$	11.92	17.72	12.13	16.78	21.35	13.90	14.28	14.14	13.74	12.14	18.41	17.33
$\text{CVaR}^{\mathbb{P}}_{0.95}$	13.75	20.02	13.95	19.36	24.90	16.49	15.38	16.29	15.61	13.64	19.70	18.75
$\text{CVaR}^{\mathbb{P}}_{0.99}$	17.27	24.29	17.36	24.38	31.40	21.36	17.49	20.42	19.14	16.53	22.16	21.49
Fund mapping model without basis risk												
Mean	2.91	5.15	3.16	3.89	3.61	1.75	7.60	3.19	4.21	4.04	10.97	9.03
Std.Dev.	2.01	2.15	2.13	2.26	1.65	1.68	2.71	2.89	2.10	2.45	3.02	2.97
$\text{CVaR}^{\mathbb{P}}_{0.70}$	5.41	7.63	5.79	6.58	5.65	3.85	10.88	6.77	6.81	7.11	14.27	11.91
$\text{CVaR}^{\mathbb{P}}_{0.80}$	5.97	7.96	6.38	7.23	6.04	4.37	11.40	7.54	7.27	7.73	14.70	12.18
$\text{CVaR}^{\mathbb{P}}_{0.90}$	6.86	8.45	7.31	8.31	6.62	5.18	12.18	8.75	7.95	8.67	15.19	12.53
$\text{CVaR}^{\mathbb{P}}_{0.95}$	7.70	8.89	8.19	9.35	7.16	6.00	12.91	9.86	8.57	9.53	15.50	12.81
$\text{CVaR}^{\mathbb{P}}_{0.99}$	9.61	9.90	10.14	11.69	8.42	7.71	14.45	12.03	9.88	11.29	16.03	13.37

5. Conclusions

The current work illustrates that basis risk is highly material within hedging procedures associated with segregated funds guarantees for most of the considered funds among a sample of Canadian and US equity mutual funds that was selected. A bivariate regime-switching model is utilized to represent the joint dynamics of the fund and the hedging asset. Under this particular framework, the impact of using a simplifying fund mapping regression to quantify the relation between the mutual fund and the hedging asset under the bivariate regime switching model is assessed. The current work shows that, assuming the unconstrained regime switching model is the true model driving the assets dynamics, the use of fund mapping regressions can generate biases in parameter estimates which can translate into a downward bias for capital requirement estimates. Moreover, the omission of basis risk within fund mapping regression-based hedging is shown to lead to a severe under-estimation of capital requirements. Although fund mapping regressions might continue being utilized in practice due to their convenience, keeping in mind their limitations is important. This points out towards the need of developing a convenient estimation method applicable to regime-switching frameworks for the joint dynamics of the underlying fund and the hedging instrument which is not subject to biases induced by fund mapping regressions.

An interesting question for further research would be to determine if estimated capital requirements are also biased downward when using fund mapping regressions in conjunction with other models and assess if results presented in the current paper are idiosyncratic to regime-switching models. Intuitively, other models producing scenarios where simultaneously the mutual fund's drift is highly negative, its volatility is very large and its correlation is not larger than usual could produce similar effects than the regime-switching model outlined in this paper; fund mapping regressions which reflect the stationary (i.e., average) relationship between both the underlying and hedging assets might not be able to capture such highly unfavorable scenarios which are the ones driving capital requirements. A multivariate GARCH-in-mean with a highly negative volatility premium for the mutual fund would be an example of a model possessing such features.

It would also be relevant to investigate portfolio effects related to insurer capital requirements associated segregated funds; the basis risk associated with a given policy could be partially mitigated by offsetting risk associated with other policies.

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Appendix A. The Trottier et al. (2018) Hedging Framework

In the Trottier et al. (2018) hedging framework, a GMMB policy with guaranteed value K is considered. The post-fee policy account value $\{A_t\}_{t\in\mathcal{T}}$ evolves according to

$$A_{t+1} = A_t(1-\omega_{tot})\frac{F_{t+1}}{F_t}, \qquad t \in \{0,\ldots,T-1\},$$

where A_0 is the initial account value and ω_{tot} is the periodic total percentage of fees charged to the policyholder. Let ω_{opt} be the periodic fee rate used for hedging purposes (e.g., removing profits and overhead costs) and ℓ_t being the proportion of policies that are still active at time t i.e., whose policyholder did not lapse nor die before or at time t. The proportion of policies that are still active at time t is assumed to be given by

$$\ell_t = (1-b)^t {}_tp_x, \qquad t \in \mathcal{T}, \tag{A1}$$

where b is a constant monthly lapse rate and ${}_tp_x$ is defined as the probability that a policyholder aged x months at time 0 survives t months.

At each time t, the net cash outflow for the insurer is given by

$$CF_t = -\frac{\omega_{opt}}{1-\omega_{tot}}A_t\ell_{t-1} + \mathbb{1}_{\{t=T\}}\max(0, K-A_T)\ell_T, \qquad t \in \{1,\ldots,T\},$$

where the first term corresponds to fees charged to policyholders that are still active (a negative outflow means an inflow to the insurer), and the second term corresponds to the benefit at maturity. Using a risk-neutral evaluation, the time-t GMMB guarantee contract value Π_t is given by

$$\Pi_t = B_t\,\mathbb{E}^{\mathbb{Q}}\left[\sum_{j=t+1}^{T}\frac{CF_j}{B_j}\middle|\mathcal{F}_t\right] \qquad t \in \mathcal{T}.$$

where $\mathcal{F}_t$ is the information available at time t and $\mathbb{Q}$ is a suitable martingale measure. The chosen martingale measure is model invariant; the asset dynamics are identical between the physical measure $\mathbb{P}$ and the martingale measure $\mathbb{Q}$, except that drifts are translated to the risk-free rate (or zero for the futures contract) minus a convexity correction in both regimes. Π_t is the post-cash flow value as it does not take into account CF_t, the cash flow at time t. The fee rate ω_{opt} is assumed to be a fair fee rate which is set such that $\Pi_0 = 0$.

The insurer holds a hedging portfolio to mitigate the risk associated with the GMMB guarantee. The hedging portfolio is invested in the trading strategy $\theta = \{\theta_t^{(B)}, \theta_t^{(S)}\}_{t\in\mathcal{T}}$ where $\theta_{t+1}^{(B)}$ and $\theta_{t+1}^{(S)}$ denote respectively the number of long positions within the hedging portfolio of the risk-free asset and the risky hedging asset during the time interval $(t, t+1]$, with the convention $\theta_0^{(B)} = \theta_0^{(S)} = 0$. At each time t, the insurer performs an injection or a withdrawal of liquidities from the hedging portfolio. The time-t injection is denoted by I_t, where negative amounts correspond to withdrawals. Injections are made such that the hedging portfolio values exactly tracks the guarantee value i.e., that its value and the guarantee value are exactly equal after the injection. Defining V_{t-}^{θ} as the hedging portfolio value before the injection I_t and the cash flow CF_t, we obtain

$$I_t = \Pi_t - V_{t-}^{\theta} + CF_t, \qquad t \in \mathcal{T}. \tag{A2}$$

Since liquidities and futures margin amounts are assumed to be invested at the risk-free asset, we have $\theta_{t+1}^{(B)} = \frac{\Pi_t}{B_t}$ for $t = 0, \ldots, T$. Therefore, the choice of the trading strategy θ by the insurer is characterized by choosing the number of hedging asset positions $\theta_{t+1}^{(S)}$ taken in each period t. The portfolio value evolves according to

$$V_{(t+1)-}^{\theta} = \theta_{t+1}^{(B)} B_{t+1} + \theta_{t+1}^{(S)} (S_{t+1} - S_t), \tag{A3}$$

which is obtained by summing the value of the position in the risk-free asset and gains from the hedging futures position. Trottier et al. (2018) show that using a first order Taylor expansion approximations, one obtains

$$I_{t+1} \approx \Theta_t + \Delta_t \delta F_t - \theta_{t+1}^{(S)} \delta S_t$$

where the option Greeks (discrete-time theta and delta) are given by

$$\begin{aligned} \gamma_t &\equiv \frac{A_0}{F_0}(1-\omega_{tot})^t \ell_t, \\ \Theta_t &\equiv \Pi_t(1-e^r) + \gamma_T \left[g\left(t+1, F_t, \eta_{1,t}^{\mathbb{Q}}\right) - g\left(t, F_t, \eta_{1,t}^{\mathbb{Q}}\right) \right], \\ \Delta_t &\equiv -\omega_{opt} \sum_{j=t+1}^{T} \gamma_{j-1} + \gamma_T \frac{\partial g}{\partial F}\left(t+1, F_t, \eta_{1,t}^{\mathbb{Q}}\right), \end{aligned} \tag{A4}$$

with $\eta_{1,t}^{\mathbb{Q}} \equiv \mathbb{Q}[h_t = 1 | \mathcal{F}_t]$ and g being the GMMB benefit pricing functional such that

$$\begin{aligned} g\left(t, F_t, \eta_{1,t}^{\mathbb{Q}}\right) &= B_t \, \mathbb{E}^{\mathbb{Q}} \left[\left. \frac{\max(0, \tilde{K} - F_T)}{B_T} \right| \mathcal{F}_t \right], \\ \tilde{K} &\equiv \frac{K F_0}{A_0 (1-\omega_{tot})^T}. \end{aligned}$$

Appendix B. The Trottier et al. (2018) Simulation Setup

Parameters used for simulations of the GMMB hedging portfolio injections are provided in Table A1. Policyholders aged 55 years at time $t = 0$ purchasing an at-the-money GMMB segregated fund with a 10-year maturity ($T = 120$) are considered. Survival probabilities are obtained using the Canadian Institute of Actuaries' recommended methodology; base mortality rates are drawn from the CPM2014 table, see CIA (2014). Mortality improvements are projected with rates proposed in Appendix C of CIA (2010). Monthly mortality rates are obtained from annual rates by assuming a constant force of mortality throughout the year.

Table A1. Baseline parameters in monthly frequency used in simulations of Sections 4.4 and 4.5.

Maturity (in months)	T	120
Survival probability	${}_tp_{660}$	Projected CPM2014
Lapse rate	b	0.34%
Total fee rate	ω_{tot}	0.29%
Risk-free rate	r	0.25%
GMMB guarantee	K	100
Initial value of F and A	$F_0 = A_0$	100
Initial value of S	S_0	100

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