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# The Accuracy of Risk Measurement Models on Bitcoin Market during COVID-19 Pandemic

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**Abstract:** Since late 2019, during one of the largest pandemics in history, COVID-19, global economic recession has continued. Therefore, investors seek an alternative investment that generates profits during this financially risky situation. Cryptocurrency, such as Bitcoin, has become a new currency tool for speculators and investors, and it is expected to be used in future exchanges. Therefore, this paper uses a Value at Risk (VaR) model to measure the risk of investment in Bitcoin. In this paper, we showed the results of the predicted daily loss of investment by using the historical simulation VaR model, the delta-normal VaR model, and the Monte Carlo simulation VaR model with the confidence levels of 99%, 95%, and 90%. This paper displayed backtesting methods to investigate the accuracy of VaR models, which consisted of the Kupiec's POF and the Kupiec's TUFF statistical testing results. Finally, Christoffersen's independence test and Christoffersen's interval forecasts evaluation showed effectiveness in the predictions for the robustness of VaR models for each confidence level.

**Keywords:** risk measures; value at risk; COVID-19; cryptocurrency; bitcoin; backtesting; Monte-Carlo simulation

## 1. Introduction

Currently, the trend of alternative investment in digital currency or cryptocurrency is undertaken by investors and company owners. They believe that these cryptocurrencies will become a future global currency. The well-known cryptocurrency, Bitcoin (BTC), is the first digital currency and was introduced in 2008 (Wright 2008). BTC created a widespread interest during the great economic recession caused by the COVID-19 pandemic. The COVID-19 pandemic is an unprecedented event that has caused a slowdown in every sector of the world economy (Sohrabi et al. 2020). Investors in financial markets are limiting their investments and want to escape from traditional investments, which are depreciating. Therefore, investors seek new approaches to provide successful returns to compensate for such an adverse risk. They believe that BTC is a new financial instrument and an alternative investment (Sukamulja and Sikora 2018). The price of BTC, however, cannot be described by classical financial theories (Kristoufek 2015). The study by Vukovic et al. (2021) found that the COVID-19 pandemic had a positive impact on the efficiency of the cryptocurrency market. Investors reacted positively to the cryptocurrency market, in which BTC's price exceeded USD 60,000 in April 2020 during the outbreak of the pandemic. Cryptocurrencies have presented many periods of pronounced price volatility during their rapid growth and development. There is evidence to suggest that cryptocurrencies played a new role as potential safe-haven currencies during periods of substantial financial market stress (Corbet et al. 2021).

In this paper, we attempt to conduct a scientific impact assessment of the pandemic on the BTC market in terms of risk. In financial risk management, we can measure a

market risk by standard deviation, beta, and value-at-risk (VaR), etc. These measures can be used to distinguish a different dimension of risk. The standard deviation method shows the dispersion of returns of an asset; Baur and Dimpfl (2017) revealed that the standard deviation for measuring the volatility of BTC's daily returns could be used. The beta shows a systematic risk of the asset; Mehta and Afzelius (2017) demonstrated that the beta of Bitcoin cannot be a significant risk measurement. VaR is an important player in determining capital requirements, which were proposed by the Basel Committee (Cuoco and Liu 2006). Das and Rout (2020) analyzed the impact of COVID-19 on the stock market indices by using VaR. They found that COVID-19 significantly affected the volatility of stock markets. Furthermore, the market correlation was the highest in the COVID-19 period. Corbet et al. (2019) stated before the COVID-19 pandemic that BTC and the prices and dynamics of other cryptocurrencies had a high volatility. During the pre-COVID-19 era, Likitratcharoen et al. (2018) found that VaR could be used as a risk measurement tool for cryptocurrencies, which shows the probable losses for a given horizontal time and probability. Although VaR is useful, the situation was changed by the COVID-19 pandemic, since the pandemic caused market stress. After the COVID-19 outbreak, Corbet et al. (2021) identified significant and substantial interactions between cryptocurrency prices and liquidity effects. Before the COVID-19 outbreak, the shocks determined through liquidity shifts had significant effects on the volatility of price changes. Therefore, VaR is likely violated because of the market condition. This paper focuses on the fluctuation of BTC during the COVID-19 pandemic to investigate whether VaR can still be used to quantify the number of adverse losses. We collaborate with a historical simulation VaR model, a delta-normal VaR model, and a Monte Carlo simulation VaR model by using the logarithmic daily return.

## 2. Literature Review

This literature review is divided into three parts: cryptocurrency market efficiency, market risk and the VaR model, and COVID-19 and the BTC market.

### 2.1. Cryptocurrency Markets Efficiency: Anomalies, Predictability, Weak-Form Efficient Testing

As the efficient market hypothesis is a popular study in the financial literature, there are also many studies covering topics that criticize this hypothesis, such as the bandwagon effect, information asymmetry, market overreaction and underreaction, etc. Some studies also related that returns are predictable (Rudolf et al. 2021); Malkiel (2003) summarized the predictable returns of the stock market. The paper mentioned that there is short-term momentum in the market, which is caused by the psychological process of investment. Giudici and Pagnottoni (2019) presented the dynamic nature of returns over time. Resta et al. (2020) recommended the use of intraday and daily trading rules to estimate the Bitcoin price series. Chen et al. (2020) examined the temporal and spatial effect on Bitcoin trading volumes.

According to the hypothesis of an efficient market of Bitcoin (BTC), there were a number of recent studies, which discovered that the Bitcoin market was not an efficient market because it was more predictable than the stock markets (Latif et al. 2017). Kurihara and Fukushima (2017) showed that the Bitcoin (BTC) market was not a weak-form market since the price did not behave randomly. Empirical evidence also showed that there might be speculative bubbles in the cryptocurrencies market, which can be categorized as either rational or irrational (Dale et al. 2005; Agosto and Cafferata 2020). Cheah and Fry (2015) explained that rational bubbles are from the self-fulfilling, mispricing of fundamentals, and the endowment of irrelevant exogenous variables with asset pricing value. Irrational bubbles are formed by psychological factors unrelated to the asset's fundamental value. In contrast, Nadarajah and Chu (2017) studied the weak-form efficiency of Bitcoin. The results revealed that there is no serial correlation of Bitcoin's return—in other words, the market is weak-form efficient. In addition, Bartos (2015) found that the Bitcoin market is efficient because the price can reflect all known information immediately, and no one could outperform by using the same information.

### 2.2. Market Risk and VaR Model

A recent study by Bouri et al. (2020) examined the safe-haven property and the hedging of the downside risk of commodities considering Bitcoin, gold, and a commodity index by using a wavelet coherency approach for specific stock market indices. The results showed overall weak dependence among all, with Bitcoin being the least dependent. The study examined the diversification benefits through wavelet value-at-risk (VaR) and revealed the superior position of Bitcoin over both gold and commodities. Conlon et al. (2020) studied the safe-haven characteristics of Bitcoin (BTC), Ethereum (ETH), and Tether (USDT) cryptocurrencies. They found the inclusion of BTC, and ETH increases portfolio downside risk or VaR, proving that these assets were not a safe-haven for the majority of international equity markets during the COVID-19 pandemic. Conlon and McGee (2020) investigated the safe-haven properties of BTC for a US investor investing domestically during the COVID-19 pandemic and found that a portfolio allocation to BTC increased rather than decreased downside risk exposure.

According to the market risk measures, market risk and VaR models are used to measure a distinct characteristic of risk. Stock markets seem to negatively react to the news of the COVID-19 outbreak where investors have been facing with adverse losses. Olsen (1997) emphasized that a primary concern of investors is the potential for extreme losses from an investment or downside risk. Downside risk has been shown to be priced in the marketplace, with investors requiring higher returns on stocks exhibiting greater downside risk (Ang et al. 2006). VaR is used to measure the downside risk which was proposed by JP Morgan in 1994 and became a popular method for risk measurements. The model determines the potential loss of an investment that might happen with a given probability and time-period. The interpretation of the model is how much the expected maximum loss on investment should be subject to a confidence level.

Bouri et al. (2017) explained that Bitcoin has limited hedging properties and has safe-haven characteristics for Asian stocks only. Baur et al. (2018) demonstrated that Bitcoin is mainly employed as a speculative investment. BTC prices have a high volatility among other assets such as commodities or stocks (Rudolf et al. 2021). Its standard deviation is 100 times higher than fiat currencies in FX markets. For these reasons, many participants in the market treat Bitcoin likely as a speculative asset than a currency (Baur and Dimpfl 2017). An extreme volatility may generate a satisfying return for investors or speculators, but it may also result in extreme losses. Hence, this paper will use VaR model to be a downside risk measure of BTC. The previous study the pre-COVID-19 pandemic showed that VaR can be an appropriate risk measurement for cryptocurrency markets (Likitratcharoen et al. 2018). The overall results of the study discovered that the historical simulation VaR and the delta-normal VaR models are appropriate risk measurement methods, while the historical simulation VaR model provides more accuracy than the delta-normal VaR model. This paper aims to add the Monte Carlo simulation VaR model that is far the most flexible and powerful, rather than assuming the linearity of the delta-normal VaR model. This model can apply over longer holding periods, underly risk factors, and incorporate all desirable distributional properties.

### 2.3. COVID-19 and BTC Market

The COVID-19 pandemic is directly the main cause of global economic recession. It has led to panics and the temporary closure of businesses in most economies as the number of positive coronavirus cases has increased (Okorie and Lin 2021). Governments have implemented lockdowns to stop the spreading of the virus in which has economically different effects on each productive type (Sadowski et al. 2021).

Dow Jones and S&P500 had undergone as much as a 30% decrease in values during March 2020 (Iqbal et al. 2021). However, BTC has reacted opposite to the news of the COVID-19 outbreak and seems to outperform other assets (Iqbal et al. 2021). Goodell and Goutte (2021) indicated that such pandemic positively affects BTC prices. Mariana et al. (2021) found that BTC returns are negatively correlated with S&P500 returns. Béjaoui et al.

(2021) studied the short- and long-term evidence of the nexus between BTC price, social media metrics and the intensity of the COVID-19 pandemic. They found that the COVID-19 health crisis has significantly influenced social media networks and BTC prices.

Lahmiri and Bekiros (2020) found that the level of stability in cryptocurrency markets has significantly diminished while the irregularity level significantly augmented. Cryptocurrencies thus became more volatile, more instability, and more irregularity during the COVID-19 pandemic compared to stock markets. They stated that cryptocurrencies could be considered riskier as opposed to equities. Moreover, Vojtko and Cisár (2020) concluded that BTC prices are not stable because of its high volatility. Kayal and Rohilla (2021) found that BTC prices have more volatile than other currencies, particularly for US dollars, Euros, and Yen. To examine its volatility, Rudolf et al. (2021) studied volatility stability based on the return levels of each Bitcoin on major indexes traded with BTC and gold by using GARCH model. Castillo et al. (2021) studied the impact of the COVID-19 pandemic on the conditional variance of stock returns by using the EGARCH-D-ST model. They showed the VaR-backtesting performance comparison between the EGARCH-D-ST model. They concluded that the sudden changes in volatility are accounted for the persistence in volatility diminishes considerably. The EGARCH-D-ST model delivered realized number of exceptions closer to expected number of exceptions.

## 3. Data and Methodology

### 3.1. Data

First, the data of BTCs daily closed price between 28 April 2020 and 30 April 2021 were collected from Yahoo Finance as shown in Figure 1. It indicated that the price has risen against the stock indices during the COVID-19 pandemic which WHO announced it on 11 March 2020.



**Figure 1.** Bitcoin's daily closing price 28 April 2020–30 April 2021.

Second, the daily logarithmic return of BTC is calculated. It can be expressed by

$$R_t = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where $p_t$ and $p_{t-1}$ = prices on day $t$ and $t$–1, respectively.

These daily logarithmic returns are used for testing the VaR models. Figure 2 shows the noise of the one-year daily return between 28 April 2020 and 30 April 2021. It ranges between −0.10 and 0.10.
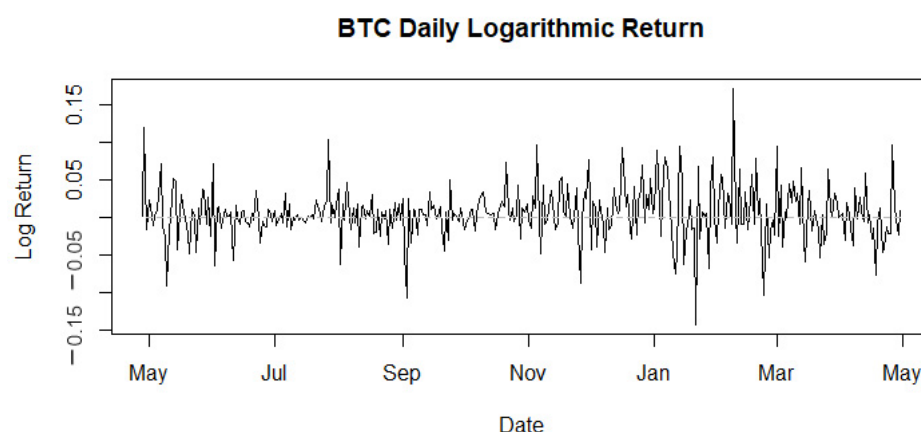
**BTC Daily Logarithmic Return**



**Figure 2.** Bitcoin's daily return between 28 April 2020–30 April 2021.

### 3.1.1. Historical Simulation Value at Risk

The historical simulation VaR model simply arranges the return data together and calculates the percentile value for each given confidence level (alpha). This model hypothesizes that the behavior of future returns on investment should be replicated with historical data. The VaR value is defined by

$$\text{VaR}_{t+1}^{1-\alpha} = Q^{\alpha}\left(\{R_{it}\}_{t=1}^{n}\right) \tag{2}$$

where $\text{VaR}_{t+1}^{1-\alpha}$ = a value at risk at $1 - \alpha$ for time $t + 1$;

    $Q^{\alpha}\left(\{R_{it}\}_{t=1}^{n}\right)$ = a quantile at $\alpha$ of $\{R_{it}\}_{t=1}^{n}$;

    $\{R_{it}\}_{t=1}^{n}$ = a return of asset $i$;

    when the time equals to $t$ between $t = 1$ to $n$.

The historical simulation VaR model uses unconditional data where focuses on the left tail and therefore the sharp of distributed return is not concerned.

### 3.1.2. Delta-Normal Value at Risk

The delta-normal VaR model or variance-covariance VaR model uses historical data to calculate the main parameters: the mean, the standard deviation, and the correlation. This model is assumed that the distribution of collected data is symmetrical normal distribution where the skewness is zero and the kurtosis is three. However, the model might lack prediction if the log-returns of data used to predict the investment are not normally distributed. Although the delta-normal VaR model has a flaw which is incorporated with normal distribution, its simplicity is an advantage of this model. We can mathematically express the form as

$$\text{VaR}_{1-\alpha} = m + Z_{\alpha}S \tag{3}$$

where $\text{VaR}_{1-\alpha}$ = value at risk at $1 - \alpha$;

    $m$ = an average historical log-returns on investment;

    $Z_{\alpha}$ = a standardized score of normal distribution at $\alpha$;

    $S$ = a standard deviation of log-returns on investment.

### 3.1.3. Monte Carlo Simulation Value at Risk

The Monte Carlo simulation expects that the movements of risk factors are generated by drawings from some prespecified distribution. The method consists of sampling pseudorandom numbers which are sorted to produce the desired VaR. This method is the most flexible but requires an enormous computational burden and assumptions of the stochastic process (Jorion 2011). In this paper, the Monte Carlo simulation for $N = 1,000,000$ times will be performed to approach the central limit theorem.

Assume the behavior of price, $S_t$, at time $t$ is following a geometric Brownian motion where $\mu$ and $\sigma$ are constants and its process is determined as

$$dS_t = \mu S_t dt + \sigma S_t dW_t \qquad (4)$$

where $\mu$ is the drift, $\sigma$ is the volatility, and $W_t \sim N(0, t)$ is a Wiener process.

By Itô's Lemma, a price return process is following

$$d \ln S_t = \left( \mu - \frac{1}{2}\sigma^2 \right) dt + \sigma dW_t \qquad (5)$$

Equation (5) is used to do the Monte Carlo simulation. Since $W_t$ is assumed by normal distribution, we can standardize to be $W_t = \sqrt{t} Z_t$ where $Z_t \sim N(0, 1)$. We use the mathematical programming language, MATLAB, for this simulation.

### 3.2. The Backtesting

#### 3.2.1. Kupiec's POF Test

The Kupiec's POF test is also known as the proportion of failure. It is applied to VaR models to prove that the proportion of failure is statistically equal to the suggested proportion of failure of the confidence interval. In other words, if the observed rate of failure from a model differs significantly from the suggested rate of failure, test results would reject the null hypothesis which means that the model is inaccurate. However, in case that the test results are accepted, it indicates that a model is accurate (Halilbegovic and Vehabovic 2016). The null hypothesis of the test can be written as

$$H_0 \,:\, p = \hat{p}$$
$$H_1 \,:\, p = \hat{p}$$

where $p$ = proportion of failure suggested by a certain confidence interval;
$\hat{p}$ = observed failure rate.

The formula of this test can be expressed by

$$LR_{POF} = -2ln\left( \frac{(1-p)^{n-X} p^x}{\left(1-\left(\frac{x}{n}\right)\right)^{n-x} \left(\frac{x}{n}\right)^x} \right) \qquad (6)$$

where $LR_{POF}$ = a likelihood ratio of proportion of failure test statistic;
$p$ = probability of expected exceptions;
$n$ = number of observations;
$x$ = number of realized exceptions.

#### 3.2.2. Kupiec's TUFF Test

The Kupiec's TUFF test is similar to the Kupiec's POF test. It concerns whether the probability of expected exceptions is equal to the inverse of the time until first exception, which should also be equal the observed rate of failure (Halilbegovic et al. 2020). If the first error has happened too early, the test result would reject the model as the model underestimated the risks. Additionally, if the first exception occurs with delay, it would mean that the model overestimated the risks. The hypothesis of this test can be written as

$$H_0 \,:\, p = \frac{1}{v}$$
$$H_1 \,:\, p \neq \frac{1}{v}$$

where $p$ = probability of expected exceptions;
$v$ = time until first exception.

The formula of this test can be expressed by

$$LR_{TUFF} = -2\ln\left(\frac{p(1-p)^{v-1}}{\left(\frac{1}{v}\right)\left(1-\frac{1}{v}\right)^{v-1}}\right) \tag{7}$$

where $LR_{TUFF}$ = a likelihood ratio of time until first failure.

### 3.2.3. Christoffersen's Independence Test

The Christoffersen's independence test sets up an indicated function. If there is an exception occurs in that day, then the value of the indicator is 1. In contrast, if there is no exception in that day, then the value of the indicator is 0 (Christoffersen 1998).

$$LR_{ind} = \begin{cases} 0 \; if \; \text{VaR}_\alpha \; is \; not \; breached, \\ 1 \; otherwise \end{cases} \tag{8}$$

where $LR_{ind}$ = a test indicator;
$\quad R_{it}$ = return of asset $i$ on time $t$;
$\quad \text{VaR}_\alpha$ = value at risk at a given $\alpha$.

Hence, we construct the $LR_{ind}$ table with the 2 consecutive days. Each day has two possible states: 0 and 1 as shown in Table 1. We obtain 4 conditions which are 00, 01, 10, and 11.

**Table 1.** Contingency table of independence test indicator (Halilbegovic et al. 2020).

| Indicator Test | $LR_{ind,t-1} = 0$ | $LR_{ind,t-1} = 1$ | |
| --- | --- | --- | --- |
| $LR_{ind,t} = 0$ | $n_{00}$ | $n_{10}$ | $n_{00} + n_{10}$ |
| $LR_{ind,t} = 1$ | $n_{01}$ | $n_{11}$ | $n_{01} + n_{11}$ |
| | $n_{00} + n_{01}$ | $n_{10} + n_{11}$ | $N$ |

where $LR_{ind,t}$ = a test indicator on day $t$.
$\quad n_{LR_{ind,t-1},LR_{ind,t}}$ = numbers of days when two conditions are met for day $t$ and day $t-1$.
The formula of the test statistic is

$$LR_M = -2ln\left(\frac{(1-\pi)^{n_{00}+n_{01}} + \pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right) \tag{9}$$

where $LR_M$ = an independence test statistic;
$\quad n_{LR_{ind,t-1},LR_{ind,t}}$ = numbers of days when two conditions are met for day $t$ and day $t-1$.
$\quad \pi_0$ = probability of 01 occurring when $LR_{ind,t-1}$ is 0, which follows

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}$$

$\pi_1$ = probability of $\alpha_{11} = 1$ occurring when $LR_{ind,t-1}$ is 1, which follows

$$\pi_1 = \frac{n_{11}}{n_{10} + n_{11}}$$

$\pi$ = probability of having an exception in the previous day

$$\pi = \frac{n_{11}}{n_{10} + n_{11}}.$$

### 3.2.4. Christoffersen's Interval Forecasts or Joint Test

The Christoffersen's interval forecasts or joint test is a combination of the independence statistic with Kupiec's POF test. This test measures not only the correct failure

rate but also the independence of violations. This method can solve the aforementioned accuracy problem. This test will show both the unconditional coverage and independence properties. Its capacity to detect a VaR measure that just violates one of the two characteristics is, however, limited. The joint test finds it more difficult to discover the inadequacy of the VaR measure if one of the two conditions is satisfied. This test combines $LR_{POF}$ and $LR_M$ to be $LR_{CC}$ with Chi-squared critical value with 2 degrees of freedom.

$$LR_{CC} = LR_{POF} + LR_M \tag{10}$$

where $LR_{CC}$ = Christoffersen's interval forecasts test statistics;
   $LR_{POF}$ = Kupiec's POF-test statistics;
   $LR_M$ = the independence test statistics.

## 4. Results

The results from three models are presented here.

### 4.1. Historical Simulation VaR Model

First, we show the noise of the daily log-return of BTC, which is compared with lines of daily historical simulation VaR value in Figure 3. The interpretation of this figure is that if the noise is over the reference VaR line, it means those days have losses over the VaR model, and it is counted to be the number of extreme losses or the realized number of exceptions that occur. These results will be used to be a parameter for Kupiec's POF test in Table 2.
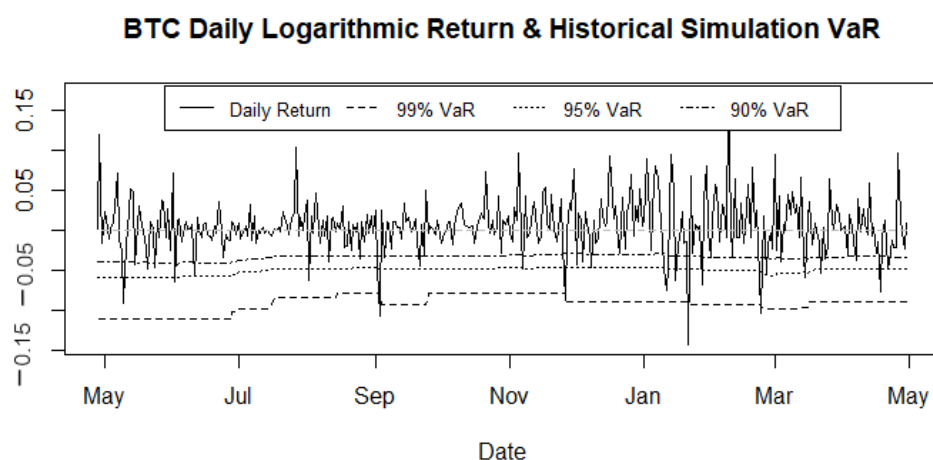


**Figure 3.** BTC daily return and historical simulation VaR.

**Table 2.** Historical simulation VaR Kupiec's POF-test.

| Kupiec's POF Test | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Expected Number of Exceptions | Realized Number of Exceptions | Test Statistic $LR_{POF}$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 4 | 0.0329 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 18 | 19 | 0.0320 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.90 | 365 | 37 | 37 | 0.0076 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |

In Table 2, we obtain test statistic by Chi-squared test to examine the occurrence of realized extreme losses. If the number of extreme losses approaches to the expected number of exceptions, then the historical simulation VaR model will be useful to capture the losses

during the COVID-19 pandemic. The results shows that daily loss of BTC can be predicted for all confidence levels of the historical simulation VaR model, which measures a risk on BTC during the COVID-19 pandemic. In addition, Table 3 shows the performance of the historical simulation VaR of BTC. The Kupiec's TUFF test is another method to test VaR models. The statistical results $LR_{TUFF}$ claimed with $\chi^2_{1-\alpha,\nu}$ that the historical simulation VaR model on BTC can be the risk measurement. Both results from Tables 1 and 2 emphasize that the historical simulation VaR model can be used to predict the extreme losses occurring during the pandemic.

**Table 3.** Historical simulation VaR Kupiec's TUFF-test.

| | | | | Kupiec's TUFF Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Time Until First Failure | Test Statistic $LR_{TUFF}$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 66 | 0.1528 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 12 | 0.2359 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.90 | 365 | 8 | 0.0519 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |

Table 4 shows the results of the independence test of historical simulation VaR model. The results proved that for all confidence levels historical simulation VaR models are statistically accepted. We can interpret that those extreme losses do not occur dependably. These results will incorporate Kupiec's POF results to test the model effectiveness by Christoffersen's interval forecasts.

**Table 4.** Historical simulation VaR independence test.

| | | | | Independence Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Realized Number of Exceptions | Test Statistic $LR_M$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 0.0886 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 19 | 3.1149 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Reject |
| 0.90 | 365 | 37 | 0.4767 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |

The results from Christoffersen's interval forecasts are shown in Table 5 where $LR_{CC} = LR_{POF} + LR_M$. Overall, the Chi-squared tests are all accepted, which means that historical simulation VaR model is effective to use for predicting losses on BTC's investment during the COVID-19 pandemic. On the other hand, the historical simulation VaR model on BTC's investment is effective and robust.

**Table 5.** Historical simulation Christoffersen's Interval Forecast Test.

| | | | | | | Independence Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Realized Number of Exceptions | Test Statistic $LR_{POF}$ | Test Statistic $LR_M$ | Test Statistic $LR_{CC}$ | Critical Value $\chi^2$ (Chi-Squared) (2;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (2;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (2;0.90) | |
| | | | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 0.0329 | 0.0886 | 0.1215 | 9.2103 | Accept | 5.9915 | Accept | 4.6052 | Accept |
| 0.95 | 365 | 19 | 0.0320 | 3.1149 | 3.1470 | 9.2103 | Accept | 5.9915 | Accept | 4.6052 | Accept |
| 0.9 | 365 | 37 | 0.0076 | 0.4767 | 0.4843 | 9.2103 | Accept | 5.9915 | Accept | 4.6052 | Accept |

### 4.2. Delta-Normal VaR Model

Similarly, Figure 4 shows the noise of the daily log-return of BTC which is compared with lines of daily delta-normal VaR value.
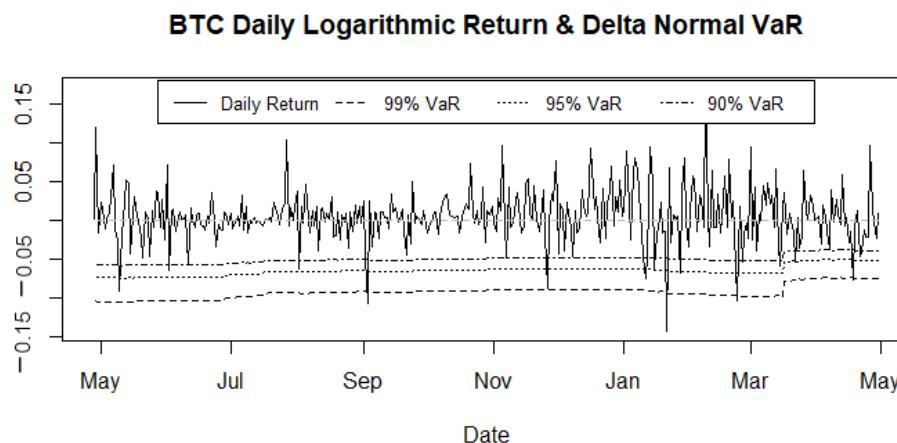
**BTC Daily Logarithmic Return & Delta Normal VaR**



**Figure 4.** BTC daily return and Delta-Normal VaR.

The results from Table 6 imply that the 90% delta-normal VaR is not the most accurate risk measurement method for BTC investments during the COVID-19 pandemic as the test result for all confidence levels rejected it since the model overestimated the negative risks of BTC which causes the realized number of failure to be significantly lower than the expected number of failure. However, we found out that the 99% delta-normal VaR is an accurate risk measurement tool for BTC since the Kupiec's POF had accepted at it all statistical confidence. This means that the model is accurate because it did not overestimate or underestimate the risks.

**Table 6.** Delta-Normal VaR Kupiec's POF-test.

| | | | | | Kupiec's POF Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Obser-vations | Expected Number of Exceptions | Realized Number of Exceptions | Test Statistic $LR_{POF}$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 4 | 0.0329 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 18 | 9 | 6.0197 | 6.6349 | Accept | 3.8415 | Reject | 2.7055 | Reject |
| 0.90 | 365 | 37 | 19 | 11.1070 | 6.6349 | Reject | 3.8415 | Reject | 2.7055 | Reject |

Nevertheless, as shown in Table 7 the delta-normal VaR models accept the null hypothesis of Kupiec's TUFF test for all confidence levels in which the results proved that the delta-normal model is accurate for all confidence levels. Therefore, it is doubtful whether the measurement at 95% and 90% has accuracy due to the contrary results between Kupiec's POF and Kupiec's TUFF tests. However, the 99% delta-normal VaR was proven that it is an accurate risk measurement method due to the consistent results of Kupiec's POF and Kupiec's TUFF tests.

Table 8 shows independent properties results of exception in the delta-normal VaR models. All Chi-squared tests are accepted, that is all exceptions of each model have independent properties. In other words, there is no replicated pattern occurring in the exceptional cases. Therefore, the delta-normal VaR model is efficient to predict losses of investment on BTC except for the 90% delta-normal VaR at 90% statistical confidence level in which the results are rejected. It would be implied that independence properties of

exceptions from the 90% delta-normal VaR are violated at 90% confidence level which indicating that it is not the best confidence level for the model.

**Table 7.** Delta-Normal VaR Kupiec's TUFF-test.

| Confidence Level | Number of Observa-tions | Time Until First Failure | Test Statistic $LR_{TUFF}$ | Independence Test | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 12 | 2.5474 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 12 | 0.2359 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.90 | 365 | 8 | 0.0519 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |

**Table 8.** Independence test on Delta-Normal VaR.

| Confidence Level | Number of Observa-tions | Realized Number of Exceptions | Test Statistic $LR_M$ | Independence Test | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 0.0886 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 9 | 0.4551 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.90 | 365 | 19 | 3.1149 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Reject |

Table 9 shows the results of Christoffersen's interval forecasts of the delta-normal VaR at each confidence level. The Chi-squared test of the 90% delta-normal VaR were rejected for all statistical confidence levels. We can conclude that 90% delta normal VaR is not the best for measuring risks of Bitcoin. Likewise, the results of the 95% delta-normal VaR were also rejected at 95% and 90% statistical confidence levels. This shows that it may not be the most robust method for risk measurement on BTC during the COVID-19 pandemic. The model overestimated the risks, and the independence properties were violated at 95% and 90% confidence levels, but the model is robust at the 99% statistical confidence level. However, the test results at all statistical confidence levels for the 99% delta-normal VaR revealed that it is effective as the results were all accepted.

**Table 9.** Christoffersen's Interval Forecast test on Delta-Normal VaR.

| Confidence Level | Number of Observa-tions | Realized Number of Exceptions | Test Statistic $LR_{POF}$ | Test Statistic $LR_M$ | Test Statistic $LR_{CC}$ | Joint Test—Christoffersen's Interval Forecast Test | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | | | Critical Value $\chi^2$ (Chi-Squared) (2;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (2;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (2;0.90) | |
| | | | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 0.0329 | 0.0886 | 0.1215 | 9.2103 | Accept | 5.9915 | Accept | 4.6052 | Accept |
| 0.95 | 365 | 9 | 6.0197 | 1.0064 | 6.4748 | 9.2103 | Accept | 5.9915 | Reject | 4.6052 | Reject |
| 0.90 | 365 | 19 | 11.1070 | 2.4795 | 14.2219 | 9.2103 | Reject | 5.9915 | Reject | 4.6052 | Reject |

### 4.3. Monte Carlo Simulation VaR Model

In this final section, we simulate the daily returns 1,000,000 times to calculate the Monte Carlo simulation VaR. The simulation uses pseudo-random numbers through the normal distribution where the mean is average historical daily log-return, the standard deviation is historical daily volatility, and the price process follows a geometric Brownian motion. Figure 5 shows the noise of the daily log-return of BTC, which is compared with

lines of daily Monte Carlo simulation VaR value. The graph showed 17 exceptions of 90% VaR, 8 exceptions of 95% VaR, and 4 exceptions of 99% VaR.
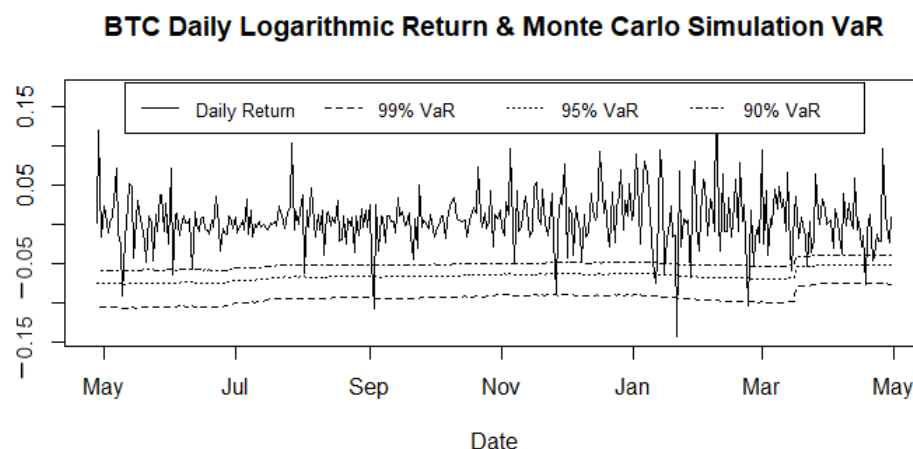


**Figure 5.** BTC daily return and Monte Carlo simulation VaR.

Table 10 shows the results of Kupiec's POF test. The results proved that the 99% Monte Carlo simulation VAR provides the best accuracy since the results were all accepted for each confidence level. In contrast, the 95% and 90% Monte Carlo simulation VaR are all rejected. The reason they were rejected was because the risks were overestimated at these confidence levels, resulting in an incorrect failure rate. In other words, the estimation is too conservative for the risk of BTC's prices during the pandemic.

**Table 10.** Monte Carlo simulation VaR Kupiec's POF-test.

| Kupiec's POF-TEST | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Expected Number of Exceptions | Realized Number of Exceptions | Test Statistic $LR_{POF}$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 4 | 0.0329 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 18 | 8 | 7.6045 | 6.6349 | Reject | 3.8415 | Reject | 2.7055 | Reject |
| 0.90 | 365 | 37 | 17 | 14.1559 | 6.6349 | Reject | 3.8415 | Reject | 2.7055 | Reject |

Next, we performed the Kupiec's TUFF test which emphasizes on the first failure. The results are presented in Table 11. The results revealed that the VaR calculated from the Monte Carlo simulation method is accurate as the test results are all accepted. As mentioned earlier, the test only concerns about the time until first failure, it ignores a lot of information.

**Table 11.** Monte Carlo simulation VaR Kupiec's TUFF-test.

| Kupiec's TUFF-TEST | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Time Until First Failure | Test Statistic $LR_{TUFF}$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 13 | 2.4006 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 13 | 0.1716 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.90 | 365 | 9 | 0.0120 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |

The independent properties of exceptions of the Monte Carlo simulation VaR models are again tested by the independence test shown in Table 12. The results showed that each exception is independent for all confidence levels of the Monte Carlo simulation VaR and Chi-squared test. These mean that the exceptions from the model on any given day is not dependent on the outcome of the previous day. Therefore, it means that the model is reliable due to the independence test results.

**Table 12.** Monte Carlo simulation VaR Independence Test.

| | | | | Independence Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Realized Number of Exceptions | Test Statistic $LR_M$ | Critical Value $\chi^2$ (Chi-Squared) (1;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (1;0.90) | |
| | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 0.0889 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.95 | 365 | 8 | 0.3596 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |
| 0.90 | 365 | 17 | 1.4719 | 6.6349 | Accept | 3.8415 | Accept | 2.7055 | Accept |

Finally, Christoffersen's interval forecasts adjusted the error from Kupiec's POF test which could be failed to reject the null hypothesis because of unconditional data, as shown in Table 13. The test proved that the 99% Monte Carlo simulation VaR is an appropriate risk measurement considering that the model has the correct failure rate, and the independence properties are not violated. Nonetheless, the results of the 95% Monte Carlo simulation VaR were rejected at $\chi^2_{0.95,1}$ and $\chi^2_{0.90,1}$ but it was accepted at $\chi^2_{0.99,1}$. This means that the 95% Monte Carlo simulation VaR is not reliable at 95% and 90% statistical confidence levels, but it is reliable at 99% confidence level. Moreover, the 90% Monte Carlo simulation were rejected by the Chi-squared test statistics for all confidence levels which means that the model is not robust for BTC's daily return during the pandemic due to the failure rate and the independence properties of the failures.

**Table 13.** Monte Carlo simulation VaR Christoffersen's Interval Forecast test.

| | | | | | | Joint Test—Christoffersen's Interval Forecast Test | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Confidence Level | Number of Observations | Realized Number of Exceptions | Test Statistic $LR_{POF}$ | Test Statistic $LR_M$ | Test Statistic $LR_{CC}$ | Critical Value $\chi^2$ (Chi-Squared) (2;0.99) | | Critical Value $\chi^2$ (Chi-Squared) (2;0.95) | | Critical Value $\chi^2$ (Chi-Squared) (2;0.90) | |
| | | | | | | $\chi^2_{0.99,1}$ | Test Result | $\chi^2_{0.95,1}$ | Test Result | $\chi^2_{0.90,1}$ | Test Result |
| 0.99 | 365 | 4 | 0.0329 | 0.0889 | 0.1218 | 9.2103 | Accept | 5.9915 | Accept | 4.6052 | Accept |
| 0.95 | 365 | 8 | 7.6045 | 0.3596 | 7.9641 | 9.2103 | Accept | 5.9915 | Reject | 4.6052 | Reject |
| 0.90 | 365 | 17 | 14.1559 | 1.4719 | 15.6278 | 9.2103 | Reject | 5.9915 | Reject | 4.6052 | Reject |

## 5. Discussion

The COVID-19 pandemic has caused the world's economy to slow down significantly due to lockdown policy (Sohrabi et al. 2020; Okorie and Lin 2021). Global financial markets reacted unprecedentedly to the pandemic news. In contrast, BTC reacted positively to the pandemic (Iqbal et al. 2021; Goodell and Goutte 2021) and it was proven to have positive correlations with the S&P500 during the pandemic (Mariana et al. 2021). As a result, BTC can generate excessive returns and outperform other classes of asset. Nevertheless, greater returns come along with greater risks. Therefore, we attempt to investigate that the VaR can be a risk quantifier for BTC during the COVID-19 pandemic.

This paper studies the accuracy and effectiveness of VaR models on BTC market during the COVID-19 pandemic. We use three VaR models: the historical simulation VaR model, the delta-normal VaR model, and the Monte Carlo simulation VaR model. The accuracy is tested by Kupiec's POF and Kupiec's TUFF tests where they provide the number

of extreme losses that occur during the study period and the first day where the extreme losses occur, respectively. The study period is one year between 28 April 2020 and 30 April 2021 where each VaR period contains 356 days. Hence, the realized number of the extreme losses for 99%, 95%, and 90% VaR should not significantly distinguish from 4, 18, and 37, respectively. We then investigate the independence property of those extreme losses. If the losses are not independent, the VaR model will fail to be the effectiveness model. Finally, we combine both Kupiec's POF results and independence results to be Christoffersen's interval forecasts. This test can interpret whether VaR model is effective or not to predict the extreme losses during the COVID-19 pandemic.

We test the appropriate measure of downside risk of VaR during the COVID-19 pandemic. We found different results from the pre-COVID. The historical simulation VaR model is still a useful measure of the downside risk of BTC, but the delta-normal VaR and Monte Carlo simulation VaR models fail to capture the downside risk at 95% and 90% confidence levels (Tables 9 and 13). However, this failure is from Kupiec's POF test since the realized numbers of exceptions are less than the expected numbers. It means that there are a few downside risks of BTC during the COVID-19 pandemic. Therefore, this study agrees with Vukovic et al. (2021) that the COVID-19 pandemic has a positive impact on the BTC market. On the other hand, both VaR models are overestimated the extreme losses at 95% and 90% confidence levels. At 99% confidence level, investors should, however, tradeoff between accepting the losses at 99% where is enormous but accurate or maintaining cutoff losses at 95% and 90% confidence levels.

## 6. Conclusions

During the COVID-19 pandemic, many investors or market participants have been greedily exploring new investments especially on the cryptocurrency markets. Bitcoin (BTC) is the most famous cryptocurrency which accounts for around 70 percent of the cryptocurrency markets. To determine whether the VaR models can be a risk measurement tool for BTC, and whether they are reliable, we tested three types of VaR models: the historical simulation VaR model, the delta-normal VaR model, and the Monte Carlo simulation VaR model with 99%, 95%, and 90% confidence levels, respectively.

The unconditional tests, Kupiec's POF test showed that the historical simulation VaR method is the most effective among all three models. The results showed that it has the correct failure rate. In other words, it neither overestimates nor underestimates the price risk of BTC for VaR at all confidence levels. The test results for the delta-normal VaR and the Monte Carlo simulation VaR methods proved that they provide the best estimates at 99% confidence level. The results of Kupiec's TUFF test showed that all these models are reliable. However, for some problem of accuracy that cannot be concluded, we suggested that it could find solutions from other VaR tests.

Furthermore, we discussed the independent properties of these exceptions, which should follow the Markovian process. The independence test of VaR models showed that almost all results are independent except for the 95% historical simulation VaR which was rejected at $\chi^2_{0.90,1}$. It means that the independence properties of the model were doubted at 90% statistical confidence level. Note that the independence test results of Monte Carlo simulation VaR model must be all accepted because we generated the pseudo-random number with independent and identically distributed property which is one of assumptions of Markovian process.

Finally, we tried to manage errors from the unconditional data with Christoffersen's interval forecasts. The test results implied that historical simulation VaR model is the most effective risk measurement. For the delta-normal VaR and Monte Carlo simulation VaR models, the test proved that they are effective at 99% confidence level.

In conclusion, all three types of the VaR models are almost accurate and effective to be the risk measurement of extreme losses for BTC during the COVID-19 pandemic. The historical simulation VaR method provides the best performance at all confidence levels in which are the most accurate among three methods. The delta-normal VaR method provides

the most accurate result at 99% confidence level. The results for the Monte Carlo simulation VaR method are similar to the delta-normal VaR method, which were proven to be excellent at 99% confidence level. Users should realize that the models are studied especially special world event at the fixed time horizon. If the situation changes, then the data could be different, and the performance of these three models would probably change. However, we suggested that the unflawed models should be selected to predict those losses that could occur by market anomalies with user's decision and other tools. Furthermore, if we consider an Expected Shortfall (ES) model which calculates average losses that are above a confidence level, it will give number of rejections more than VaR since the realized number of exceptions are likely lower than the expected number of exceptions.

## References

Agosto, Arianna, and Alessia Cafferata. 2020. Financial Bubbles: A Study of Co-Explosivity in the Cryptocurrency Market. *Risks* 8: 34. [CrossRef]

Ang, Andrew, Joseph Chen, and Yuhang Xing. 2006. Downside Risk. *The Review of Financial Studies* 19: 1191–239. [CrossRef]

Bartos, Jakub. 2015. Does Bitcoin Follow The Hypothesis of Efficient Market. *Journal of Economics Sciences* 4: 10–23. [CrossRef]

Baur, Dirk G., and Thomas Dimpfl. 2017. Realized Bitcoin Volatility. working paper. *SSRN Electronic Journal* 1–40. Available online: https://ssrn.com/abstract=2949754 (accessed on 15 July 2021).

Baur, Dirk G., KiHoon Hong, and Adrian D. Lee. 2018. Bitcoin: Medium of exchange or speculative assets? *Journal of International Financial Markets, Institutions and Money* 54: 177–89. [CrossRef]

Béjaoui, Azza, Nidhal Mgadmi, Wajdi Moussa, and Tarek Sadraoui. 2021. A short-and long-term analysis of the nexus between Bitcoin, social media and Covid-19 outbreak. *Heliyon* 7: 1–7. [CrossRef] [PubMed]

Bouri, Elie, Rangan Gupta, Aviral Kumar Tiwari, and David Roubaud. 2017. Does Bitcoin hedge global uncertainty? Evidence from wavelet-based quantile-in-quantile regressions. *Finance Research Letters* 23: 87–95. [CrossRef]

Bouri, Elie, Syed Jawad Hussain Shahzad, David Roubaud, Ladislav Kristoufek, and Brian Lucey. 2020. Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis. *The Quarterly Review of Economics and Finance* 77: 156–64. [CrossRef]

Castillo, Brenda, Ángel León, and Trino-Manuel Ñíguez. 2021. Backtesting VaR under the COVID-19 sudden changes in volatility. *Finance Research Letters* 43: 1–9. [CrossRef]

Cheah, Eng-Tuck, and John Fry. 2015. Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economics Letters* 130: 32–36. [CrossRef]

Chen, Conghui, Lanlan Liu, and Ningru Zhao. 2020. Fear sentiment, uncertainty, and Bitcoin price dynamics: The case of COVID-19. *Emerging Markets Finance and Trade* 56: 2298–309. [CrossRef]

Christoffersen, Peter F. 1998. Evaluating interval forecasts. *International Economic Review* 39: 841–62. [CrossRef]

Conlon, Thomas, and Richard McGee. 2020. Safe haven or risky hazard? Bitcoin during the COVID-19 bear market. *Finance Research Letters* 35: 101607. [CrossRef] [PubMed]

Conlon, Thomas, Shaen Corbet, and Richard J. McGee. 2020. Are cryptocurrencies a safe haven for equity markets? An international perspective from the COVID-19 pandemic. *Research in International Business and Finance* 54: 101248. [CrossRef] [PubMed]

Corbet, Shaen, Brian Lucey, Andrew Urquhart, and Larisa Yarovaya. 2019. Cryptocurrencies as a financial asset: A systematic analysis. *International Review of Financial Analysis* 62: 182–99. [CrossRef]

Corbet, Shaen, Yang Hou, Yang Hu, Charles Larkin, Brian Lucey, and Les Oxley. 2021. Cryptocurrency liquidity and volatility interrelationships during the COVID-19 pandemic. *Finance Research Letters*, 102137. [CrossRef]

Cuoco, Domenico, and Hong Liu. 2006. An Analysis of VaR-based Capital Requirements. *Journal of Financial Intermediation* 15: 362–394. [CrossRef]

Dale, Richard S., Johnnie E. V. Johnson, and Leilei Tang. 2005. Financial markets can go mad: Evidence of irrational behaviour during the South Sea bubble. *Economic History Review* 58: 233–271. [CrossRef]

Das, Nupur Moni, and Bhabani Sankar Rout. 2020. Impact of COVID-19 on market risk: Appraisal with value-at-risk models. *The Indian Economic Journal* 68: 396–416. [CrossRef]

Giudici, Paolo, and Paolo Pagnottoni. 2019. High Frequency Price Change Spillovers in Bitcoin Markets. *Risks* 7: 111. [CrossRef]

Goodell, John W., and Stephane Goutte. 2021. Co-movement of COVID-19 and Bitcoin: Evidence from wavelet coherence analysis. *Finance Research Letters* 38: 101625. [CrossRef]

Halilbegovic, Sanel, and Mia Vehabovic. 2016. Backtesting Value at Risk Forecast: The Case of Kupiec Pof-Test. *European Journal of Economic Studies* 17: 393–404. [CrossRef]

Halilbegovic, Sanel, Nadim Čelebić, and Adisa Omerbegovic Arapovic. 2020. Backtesting Value at Risk Models. Available online: https://www.researchgate.net/publication/340444014_Backtesting_Value_at_Risk_Models (accessed on 15 July 2021).

Iqbal, Najaf, Zeeshan Fareed, Guangcai Wan, and Farrukh Shahzad. 2021. Asymmetric nexus between COVID-19 outbreak in the world and cryptocurrency market. *International Review of Financial Analysis* 73: 101613. [CrossRef]

Jorion, Philippe. 2011. *Financial Risk Manager Handbook Plus Test Bank: FRM Part I/Part II*, 6th ed. Hoboken: John Wiley & Sons, Inc.

Kayal, Parthajit, and Purnima Rohilla. 2021. Bitcoin in the literature of economics and finance: A survey. *SN Business & Economics* 88. [CrossRef]

Kristoufek, Ladislav. 2015. What Are the Main Drivers of the Bitcoin Price? Evidence from Wavelet Coherence Analysis. *PLoS ONE* 10: e0123923. [CrossRef] [PubMed]

Kurihara, Yutaka, and Akio Fukushima. 2017. The Market Efficiency of Bitcoin: A Weekly Anomaly Perspective. *Journal of Applied Finance & Banking* 7: 57–64. Available online: http://www.scienpress.com/Upload/JAFB%2fVol%207_3_4.pdf (accessed on 15 April 2021).

Lahmiri, Salim, and Stelios Bekiros. 2020. The impact of COVID-19 pandemic upon stability and sequential irregularity of equity and cryptocurrency markets. *Chaos, Solitons & Fractals* 138: 109936. [CrossRef]

Latif, Saiful Reeza, Muhammad Azri Mohd, Mohd Nazrul Mohd Amin, and Arwin Idham Mohamad. 2017. Testing the Weak Form of Efficient Market in Cryptocurrency. *Journal of Engineering and Applied Science* 12: 22285–88. Available online: https://www.researchgate.net/profile/Muhammad-Azri-Mohd/publication/341536510_Testing_the_weak_form_of_efficient_market_in_cryptocurrency/links/5ec5db7292851c11a87ae7ea/Testing-the-weak-form-of-efficient-market-in-cryptocurrency.pdf (accessed on 1 May 2021).

Likitratcharoen, Danai, Teerasak Na Ranong, Ratikorn Chuensuksomboon, Norrasate Sritanee, and Ariyapong Pansriwong. 2018. Value at Risk Performance In Cryptocurrencies. *The Journal of Risk Management and Insurance* 22: 11–28. Available online: https://jrmi.au.edu/index.php/jrmi/article/view/171 (accessed on 1 November 2019).

Malkiel, Burton G. 2003. The Efficient Market Hypothesis and Its Critics. *Journal of Economic Perspectives* 17: 59–82. [CrossRef]

Mariana, Christy Dwita, Irwan Adi Ekaputra, and Zaäfri Ananto Husodo. 2021. Are Bitcoin and Ethereum safe-havens for stocks during the COVID-19 pandemic? *Finance Research Letters* 38: 101798. [CrossRef]

Mehta, Suhant, and Douglas Afzelius. 2017. Gotta CAPM' All: An Empirical Study on the Validity of the CAPM Against Four Unique Assets. *SSRN Electronic Journal*. [CrossRef]

Nadarajah, Saralees, and Jeffrey Chu. 2017. On the efficiency of Bitcoin. *Economics Letters* 150: 6–9. [CrossRef]

Okorie, David Iheke, and Boojang Lin. 2021. Stock markets and the COVID-19 fractal contagion effects. *Finance Research Letters* 38: 101640. [CrossRef] [PubMed]

Olsen, Robert A. 1997. Investment Risk: The Experts' Perspective. *Financial Analysts Journal* 53: 62–66. [CrossRef]

Resta, Marina, Paolo Pagnottoni, and Maria Elena De Giuli. 2020. Technical Analysis on the Bitcoin Market: Trading Opportunities or Investors' Pitfall? *Risks* 8: 44. [CrossRef]

Rudolf, Karl Oton, Samer Ajour El Zein, and Nicola Jackman Lansdowne. 2021. Bitcoin as an Investment and Hedge Alternative. A DCC MGARCH Model Analysis. *Risks* 9: 154. [CrossRef]

Sadowski, Adam, Zbigniew Galar, Robert Walasek, Grzegorz Zimon, and Per Engelseth. 2021. Big data insight on global mobility during the Covid-19 pandemic lockdown. *Journal of Big Data* 8: 78. [CrossRef]

Sohrabi, Catrin, Zaid Alsafi, Niamh O'Neill, Mehdi Khan, Ahmed Kerwan, Ahmed Al-Jabir, Christos Iosifidis, and Riaz Agha. 2020. World Health Organization declares global emergency: A review of the 2019 novel coronavirus (COVID-19). *International Journal of Surgery* 76: 71–76. [CrossRef]

Sukamulja, Sukmawati, and Comelia Olivia Sikora. 2018. The new era of financial innovation: The determinants of Bitcoin's price. *Journal of Indonesian Economy and Business* 33: 46–64. Available online: https://core.ac.uk/download/pdf/207923915.pdf (accessed on 1 May 2021). [CrossRef]

Vojtko, Radovan, and Dominik Cisár. 2020. Bitcoin in a Time of Financial Crisis. Working paper. *SSRN Electronic Journal*. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3557575 (accessed on 1 May 2021).

Vukovic, Darko, Moinak Maiti, Zoran Grubisic, Elena M. Grigorieva, and Michael Frömmel. 2021. COVID-19 Pandemic: Is the Crypto Market a Safe Haven? The Impact of the First Wave. *Sustainability* 13: 8578. [CrossRef]

Wright, Craig S. 2008. Bitcoin: A Peer-to-Peer Electronic Cash System. working paper. *SSRN Electronic Journal*. [CrossRef]