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# Immunization Strategies for Funding Multiple Inflation-Linked Retirement Income Benefits

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**Abstract:** Protecting against unexpected yield curve, inflation, and longevity shifts are some of the most critical issues institutional and private investors must solve when managing post-retirement income benefits. This paper empirically investigates the performance of alternative immunization strategies for funding targeted multiple liabilities that are fixed in timing but random in size (inflation-linked), i.e., that change stochastically according to consumer price or wage level indexes. The immunization procedure is based on a targeted minimax strategy considering the M-Absolute as the interest rate risk measure. We investigate to what extent the inflation-hedging properties of ILBs in asset liability management strategies targeted to immunize multiple liabilities of random size are superior to that of nominal bonds. We use two alternative datasets comprising daily closing prices for U.S. Treasuries and U.S. inflation-linked bonds from 2000 to 2018. The immunization performance is tested over 3-year and 5-year investment horizons, uses real and not simulated bond data and takes into consideration the impact of transaction costs in the performance of immunization strategies and in the selection of optimal investment strategies. The results show that the multiple liability immunization strategy using inflation-linked bonds outperforms the equivalent strategy using nominal bonds and is robust even in a nearly zero interest rate scenario. These results have important implications in the design and structuring of ALM liability-driven investment strategies, particularly for retirement income providers such as pension schemes or life insurance companies.

**Keywords:** pensions; interest rate risk; immunization; duration; M-Absolute; inflation risk; life insurance



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## 1. Introduction

Continuous longevity improvements, population aging, volatile labour markets, a prolonged ultra-low interest rate scenario, falling stock returns, deflation caps, the long-run wedge between inflation measures, together with the increasing requirements on solvency (e.g., Solvency II), and accounting (IAS 19, IFRS 9) have put significant strain on public and private pension funds, insurance companies and annuity providers, whose liabilities consist of current and future guaranteed or envisaged (targeted, promised) benefit payments to plan participants or policyholders Ayuso et al. (2021a), Bravo and Herce (2020). Protecting against unexpected yield curve, inflation and longevity shifts are some of the most critical issues institutional and private investors must solve when managing post-retirement income benefits or fixed income portfolios, given the size of their exposure to money market, fixed income (including inflation-linked) instruments and the uncertainty surrounding both future inflation and interest rates and the size and timing of the liability cash flow stream, also inflation-dependent. In final salary schemes, particularly defined benefit (DB) and hybrid schemes which still represent almost two thirds of total assets, a significant proportion of the scheme's liabilities (typically 50–80%) is linked to (consumer price, retail price, limited price) inflation. This means that changes to a member's remuneration package over their period of employment and the plan's benefit indexation rules will alter their pension entitlements (OECD 2019). In most countries, bills and bond holdings are one of the two main asset classes (together with equities) in which retirement savings are invested (directly or indirectly through collective investment schemes), accounting often for more than half of investments (OECD 2020).

For retirement income providers, where the duration of liabilities typically exceeds that of assets, a period of prolonged low interest rates and systematic longevity risk poses challenges for Asset Liability Management (ALM) liability-driven investing (LDI) in that actual portfolio returns and longevity developments fail to meet their expected path at contract inception and embedded (financial and biometric) options become more valuable Bravo and Freitas (2018), Bravo (2019, 2020). The risk is that yield curve, longevity, inflation, and credit spreads shifts have an asymmetric impact on the asset and the liability side of the balance sheet, resulting in a deterioration of the solvency position of providers.

Retirement income providers can address these risks in multiple ways, for instance, by holding longer maturity (and duration) investment-grade fixed income securities, possibly complemented with interest rate derivatives, by reducing (or eliminating where possible) interest rate, inflation and longevity guarantees, particularly in new contracts, by switching to participating (or with-profit) contracts, by renegotiating some of the pension plan parameters (e.g., retirement age, contribution rate, pension indexation, sustainability factors) or by switching from defined benefit (DB) to defined contribution (DC) or Collective Defined Contribution (CDC) plans (Bravo and Ayuso 2020; Bravo et al. 2021; Ayuso et al. 2021b).

Interest rate risk immunization is an ALM LDI strategy with a long tradition in both the financial and actuarial literature in the optimal selection of bond portfolios to match single or multiple future pension and insurance liabilities that have bond-like characteristics, regardless of the course of interest rates. Alternative strategies include cash flow matching (dedication). The recent wave of regulatory and accounting changes that requires insurance companies and pension funds to value their liabilities on a market basis and future cash commitments be discounted using a risk-free rate adjusted for a liquidity premium has created an additional incentive to the development of LDI strategies (CFGS 2011). Compared to, e.g., investment funds, the ALM activities of retirement income providers are less focused on replicating or beating a certain market-based benchmark. Instead, they are much dependent on the risk structure of the respective promised liabilities, which are uncertain both in the timing and the nature of payments (e.g., inflation-linked, stochastic).

The traditional approach to interest rate risk immunization of a company's net worth is Redington's theory of immunization which is based on a small deterministic shock to a flat yield curve Redington (1952). Fisher and Weil (1971) extended the analysis to a non-flat yield curve impacted by small parallel shifts Extensions considering alternative assumptions for the dynamics of interest rates and different risk measures can be found in Bierwag (1977), Khang (1979), Khang (1983) and Babbel (1983) or, in equilibrium interest rate models, in Cox et al. (1979), Ingersoll et al. (1978), Brennan and Schwartz (1983), Nelson and Schaefer (1983) and Wu (2000). In standard immunization problems considering a single fixed liability with known due date, necessary and sufficient conditions for immunization are achieved by setting up a duration-matching bond portfolio such that its Fisher-Weil duration matches the length of the investment horizon. Although the single liability problem is interesting, it is of limited practical value for real world LDI strategies. For instance, pension funds and insurance companies typically manage long-term multiple liability payments occurring at different and often uncertain times in the future and of, frequently, random size (e.g., inflation-linked) at investment inception. Inflation-hedging is crucial for pension funds in those cases in which pension benefits are indexed with respect to consumer price or wage level indexes (e.g., UK pension schemes). Moreover, investors must deal with multiple interest rate shocks, possibly of non-parallel nature.

An important drawback of the duration-matching approach is that it exposes the balance sheet to immunization risk, the risk measures used are not consistent with an arbitrage-free term structure model and the strategy fails to perform in the presence of non-parallel (slope, curvature) yield curve shifts, a critical limitation since most empirical studies reject the hypothesis that interest rates are perfectly correlated along the yield curve (see, e.g., Soto 2004; Bravo and Silva 2006). Alternative single-factor (see, e.g., the M-Squared (Fong and Vasicek 1983b, 1984; Fong and Fabozzi 1985; Nawalkha and Chambers 1997; Nawalkha et al. 2003) and the M-Absolute (Nawalkha and Chambers 1996; Balbás and Ibanez 1998; Balbás et al. 2002) linear cash flow dispersion measures) and multiple-factor interest rate risk models have been developed and empirically tested to minimize immunization risk (see, e.g., Nawalkha and Chambers 1997; Balbás and Ibanez 1998; Balbás et al. 2002; Nawalkha et al. 2003) and to consistently measure and manage interest rate risk in the presence of non-parallel yield curve shifts (see, e.g., the duration vector model of Garbade 1985; Chambers et al. 1988; Prisman and Shores 1988; de la Grandville 2003; De La Peña Esteban et al. 2021), the parametric duration and convexity models of Willner (1996), Diebold et al. (2006) and Bravo (2007), the key rate, directional duration and partial duration models of Ho (1992), Reitano (1990, 1991a, 1991b, 1992), Reitano (1996) and Poitras (2013), the principal component analysis model of Güultekin and Rogalski (1984), Elton et al. (1990), Garbade (1986), Litterman and Scheinkman (1991), Knez et al. (1994), D'Ecclesia and Zenios (1994), Barber and Copper (1996) and Nawalkha and Soto (2009) and the multi-factor arbitrage models proposed by Au and Thurston (1995), Agca (2005) and Oliveira et al. (2014) among others.

Theoretical extensions of interest rate immunization problem to multiple liabilities and non-parallel shifts have been proposed in the literature (see, e.g., Bierwag et al. 1983; Fong and Vasicek 1983a; Shiu 1987, 1988, 1990; Uberti 1997; Hürlimann 2002; Kaluszka and Kondratiuk-Janyska 2004; Kondratiuk-Janyska and Kaluszka 2006a, 2006b, 2009; Gajek and Krajewska 2013; Cesari and Mosco 2018) but usually assume the particular case of non-random liabilities with fixed timing and depreciate the importance of transaction costs in the selection of the optimal strategy and how they can significantly impact the performance of immunization strategies. Recently, immunization has been applied to mortality and longevity risk management in insurance see, e.g., Wang et al. (2010), Luciano et al. (2012), Lin and Tsai (2013), Tsai and Chung (2013), Liu and Sherris (2017), Fung et al. (2019), Bravo and Nunes (2021).

Despite its importance, up to our knowledge no empirical research has been conducted on the hedging performance of multiple liability immunization strategies using inflation-linked bonds (ILBs) considering for inflation-indexed (e.g., retirement income) liabilities. In this case, the liability risk depends both on interest rate and inflation risks. Interest rate risk immediately impacts liabilities through the discount factor, particularly those due later in time. Inflation risk, differently, influences the present value of the liability outgo through a deferred impact on the cash flows. If in the short-term interest rate risk dominates inflation risk, in the medium- and long-term inflation risk dominates interest rate risk, particularly for investment strategies that minimize the difference between the investment horizon and the liability due date.

This paper addresses these gaps and empirically investigates the performance of alternative immunization strategies for funding targeted multiple liabilities that are fixed in timing but random in size (inflation-linked), i.e., that change stochastically in time according to observed inflation rates. The immunization procedure is based on a targeted minimax strategy whereby we minimize the difference between the M-Absolute risk measure of the asset portfolio and the residual maturity of the multiple liability cash flow stream. We use two alternative datasets comprising daily closing prices for on-the-run U.S. Treasuries and U.S. Treasury Inflation-Protected Securities (TIPS) from January 2000 to December 2018. This is a rich and long sample in which the yield curve displayed moments of both high and low volatility and diverse shapes providing an interesting setting in which to analyse multi-period, multi-liability immunization. The immunization performance is investigated in two different periods, including a nearly zero interest rate sub-period from December 2008 to December 2018. The immunization performance is tested over 3-year and 5-year investment horizons, uses real and not simulated bond data and takes into consideration the importance of transaction costs.

Our paper is of course not the first to empirically analyse the long-term ALM problem in insurance and pension funds in the presence of interest rate and/or inflation risk and alternative investments rather than ILBs may offer attractive inflation-hedging benefits (e.g., commodities, real estate). Chambers et al. (1988) perform immunization tests for the U.S. market over single and multiperiod horizons and conclude that the improvement in the immunization performance is considerable with the addition of at least four interest rate risk measures. Similar results were obtained by Nawalkha and Chambers (1997) and Soto (2001) using, respectively, US and Spanish data. Bravo and da Bravo and Silva (2006) empirically investigate the performance of the M-Vector model using Portuguese government bond data and conclude that three duration constraints are enough to guarantee nearly perfect immunization and that additional constraints beyond the third can even cause the hedging performance to deteriorate because of more frequent reallocations and transaction costs. Soto (2004) suggests that the number of risk factors considered in the immunization strategy is more important than the stochastic model used. Theobald and Yallup (2010) investigated the problem of constructing an index tracking portfolio in the UK Gilt market using a vector of immunization moments as interest rate risk measures. The authors consider the case where multiple liabilities are fixed in size and timing, use fixed coupon bonds only, depreciate the importance of transaction costs and analyse a relatively short period of time, from 15 December 1997 to 15 December 2003. Agca (2005) compares the performance of both traditional and single-factor HJM stochastic interest rate risk measures and concludes that simpler models provide, in most scenarios, at least the same immunization performance when compared to more complex ones. Different results were obtained by Oliveira et al. (2014) using German government bond data. Platanakis and Sutcliffe (2017) discuss a parallel literature on alternative techniques for deriving optimal ALM strategies for pension funds (e.g., stochastic programming, dynamic programming, portfolio theory) and use robust optimization techniques to derive optimal investment policies for the Universities Superannuation Scheme and conclude that the model is superior to traditional benchmarks. Klingler and Sundaresan (2019) suggest that underfunded pension funds prefer to hedge their duration risk through interest rate swaps rather than setting up immunization strategies using nominal or ILB bonds. Brennan and Xia (2002) and Campbell et al. (2009) conclude that the introduction of ILBs in ALM strategies allows investors with liabilities indexed to price changes to reduce their portfolio the long-term risk more substantially when compared to other asset classes.

We complete and expand previous literature in several directions. First, we investigate the performance of immunization strategies in a multi-period and random multiple liability setting in which the liability cash flow stream is dependent on the uncertain path of future inflation rates. The lack of capacity of inflation-linked bond markets and increasing concerns over counterparty risk of derivatives-based hedging solutions (e.g., inflation swaps) expose retirement income providers to non-hedgeable inflation risk. Contrary to previous approaches, this immunization problem is of real practical value for liability-driven ALM, since the liability profile considered resembles that of pension funds in which benefits are indexed to cost-of-living-adjustment factors to keep up with inflation, or that of life insurance companies offering inflation-protected annuities, designed to help mitigate both inflation and longevity risks to a retiree's income. This contrasts with previous studies in which liabilities are fixed in size and timing (see, e.g., Theobald and Yallup 2010).

Second, we investigate to what extent the inflation-hedging properties of ILBs in ALM strategies targeted to immunize multiple liabilities of random size are superior to that of nominal bonds. To this end we test the performance of immunization strategies using two alternative datasets of nominal and inflation-linked bonds (TIPS), respectively, and try to answer to the conjecture that the main risk sources of pension plans—changes in the interest rates used to discount future liabilities and fluctuation in the dedicated asset portfolio—can be tackled by reallocating the pension assets to ILBs and nominal bonds (Siegel and Waring 2004). Inflation-linked bonds can perfectly hedge inflation-linked cash flows at any given maturity. Although the inflation-hedging properties of ILBs are clear, it is well known that the sensitivity of ILB prices to changes in inflation and interest rates is not equal. Compared to regular nominal bonds, ILB prices are relatively insensitive to changes in nominal interest rates, completely insensitive to changes in inflation but highly sensitive to changes in real interest rates. ILBs offer inflation protection, tend to exhibit low or negative return correlation with other asset classes and have long duration with respect to real interest rates, making them attractive for matching future liabilities, diversifying a portfolio or guaranteeing a retirement income stream. In addition, ILBs (real assets in general) are interesting in the performance-seeking component of investors' portfolios due to their positive risk premia and their relatively low correlations with stocks and bonds (Martellini et al. 2015).

Third, we evaluate the stability and liquidity of immunizing portfolios by analyzing whether the inclusion of transaction costs significantly affects the performance of immunization strategies and the selection of the optimal investment strategy. In real world ALM strategies, transaction costs increase asset purchase prices and reduce sale prices in portfolio restructuring. Moreover, principal and coupon repayments also generate transaction costs. It is thus of utmost importance to replicate the real and usual market constraints investors' face and analyse to what extent transaction costs can influence the design and performance of immunization strategies, particularly in a multiple liability setting.

Fourth, following previous studies that analyzed the importance of portfolio design in the performance of single- and multiple-factor immunization strategies considering a single liability, namely those that favored the inclusion of the so-called "maturity bond" in the optimal portfolio Fooladi and Roberts (1992), Bierwag et al. (1983), Soto (2001, 2004), Nawalkha et al. (2003) and Bravo and Silva (2006), in this paper we extend the analysis and test the performance of the immunization strategy against that of a naive ladder portfolio comprising only bonds with maturity close to each one of the multiple liabilities due date.

Fifth, we investigate the immunization performance in two alternative sample periods, including a nearly zero interest rate sub-period in which negative real interest rates were observed. Our empirical results show that using ILBs to immunize multiple liabilities against unexpected changes in interest rates and price levels offers superior protection when compared to alternative strategies using nominal bonds only or more naive strategies. This is both because their cash flow profile of ILBs resembles that of future planned or targeted liabilities in LDI investing, but also because the inflation accrual leverages the portfolio returns significantly, particularly in a scenario in which high inflation rates are compounding into future cash flows and real and nominal interest rates are low. These results have important implications in the design and structuring ALM liability-driven investment strategies, particularly for financial intermediaries such as pension schemes or life insurance companies.

The remaining part of the paper is organized as follows. Section 2 gives a brief description of the term structure specification, of traditional interest rate risk measures, outlines the security design and presents the M-absolute risk measures developed for nominal and inflation-linked bonds. Section 3 describes the alternative investment strategies tested, the testing methodology and the performance measures considered to be well as the datasets used. Section 4 compares the immunization performance of the alternative investment strategies in the baseline scenario. In Section 5 we test for the robustness of the immunization strategy in different interest rate scenarios, particularly in the nearly zero interest rate scenario that characterizes the post-December 2008 period. Section 6 concludes.

## 2. Theoretical Framework

In this section, we describe the theoretical framework used in this paper, comprising the term structure specification, the interest rate risk and immunizations models considered and the security design approach used.

### 2.1. Term Structure Specification

We use the Nelson and Siegel (1987) parametric approach to estimate the term structure of nominal and real interest rates using U.S. government bond data from 2000 to 2018 as an input to compute interest rate risk measures and set up immunizing portfolios. The approach is selected due to its mathematical tractability and economic intuition and because it can be used consistently across all the datasets analyzed, thus mitigating estimation errors. The approach and its extension by Svensson (1995) are widely used by central banks and other market participants for benchmark yield curve estimation, allowing us to directly extract the discount function, to identify the nature of a given shift in the term structure of interest rates and serves the main purpose of testing for immunization strategies, while still promptly capturing distressed events (e.g., the U.S. subprime crisis) that significantly impact interest rates and challenge the effectiveness of immunization strategies.

Let $y_N(0,\tau)$ and $y_R(0,\tau)$ denote, respectively, the nominal and real continuously compounded spot interest rate over the interval $[0,\tau]$. The corresponding nominal and real discount factors are represented by $\delta_N(0,\tau)$ and $\delta_R(0,\tau)$ respectively. The well known Nelson and Siegel (1987) functional form for the spot rate curve is defined as

$$y(0,\tau) = \gamma_0 + \gamma_1 \left( \frac{1 - exp\left(\frac{-t}{\gamma_3}\right)}{\frac{t}{\gamma_3}} \right) + \gamma_2 \left( \frac{1 - exp\left(\frac{-t}{\gamma_3}\right)}{\frac{t}{\gamma_3}} - exp\left(\frac{-t}{\gamma_3}\right) \right), \tag{1}$$

where $y(0,\tau)$ is a function of both the time to maturity $\tau$ and a vector of parameters $\gamma_{0,\ ...,}\gamma_3$ to be estimated by solving a non-linear optimization procedure to data observed on a trading day. The parameters in Equation (1) have economic meaning and good asymptotical characteristics Martellini et al. (2003) and Nawalkha et al. (2005).

To estimate the Nelson and Siegel (1987) parameters, we minimize the weighted sum of the squared deviations of the fitted prices from the quoted mid prices for a set of bonds used in the estimation procedure:

$$\min_{\gamma_0,\gamma_1,\gamma_2,\gamma_3} \frac{\sum_{j=1}^{m} \left[P^j_{mid}(t) - B^j(t)\right]^2}{m}, \tag{2}$$

where $P^j_{mid} = \frac{\left(P^j_{ask}+P^j_{bid}\right)}{2}$ is the corresponding dirty mid-price for each bond at time $t$ and

$$B(t) = \sum_{i=1}^{n} cf_i \times \delta(t,\tau_i,), \tag{3}$$

and where $B(t)$ the estimated fair value at time $t$ for each coupon bond maturing at $\tau_n$, with $cf$ cash flow payments (coupons or/and principal redemption) at date $\tau_i$, with $t \leq \tau_1 \leq \ldots < \tau_n$, and $\delta(t,\tau_i)$ is the discount function derived from the estimated term structure.

### 2.2. Interest Rate Risk Measures

Despite their limitations, the traditional duration measures proposed by Fisher and Weil (1971) and Bierwag (1977) continue to serve as a benchmark in single- and multiple liability immunization problems and will be the starting point of our analysis (Nawalkha et al. 2005;

Bravo and Silva 2006). The Fisher and Weil (1971) duration of a coupon bond at time $t$ is given by

$$D_{FW}(t) = \sum_{i=1}^{n} (\tau_i - t) \times w_i, \quad w_i = \frac{[cf_i \times \delta(t, \tau_i)]}{B(t)}, \tag{4}$$

Theoretically, for single liability immunization problems matching the Fisher-Weil duration of the immunizing bond portfolio with the investment horizon H provides accurate interest rate risk protection under parallel shifts in the yield curve. For non-infinitesimal parallel changes in the yield curve, second order (convexity) adjustments must be taken into account. However, convexity per se does not address non-parallel shifts in the term structure. To immunize against more complex yield curve shifts, alternative interest rate risk measures have been derived. Bond portfolios selected with minimum M-Square cluster cash flows around the investment horizon and minimize the immunization risk resulting from non-parallel yield curve shifts but, like convexity, require two interest rate risk measures for hedging (duration and M-square).

The M-Absolute was developed to address this shortcoming, by condensing the ability to immunize against non-parallel term structure shifts in a single measure, while partially immunizing against level shifts. The M-Absolute measure is computed as a weighted average of the absolute distances of the bond's cash flows payment dates from a planning horizon, i.e.,

$$M^A(t) = \sum_{i=1}^{n} abs(\tau_i - H) \times w_i, \quad w_i = \frac{[cf_i \times \delta(t, \tau_i)]}{B(t)}, \tag{5}$$

Similar to the M-Squared model, bond portfolios selected with minimum M-Absolute cluster cash flows around the planned liability horizon. This means the relative performance of duration-matching and M-Absolute models will depend on the relative importance of level shifts against slope, curvature, and other higher order term structure shifts Nawalkha et al. (2005).

### *2.3. Security Design*

We use on-the-run U.S. Treasuries and U.S. Treasury Inflation-Protected Securities (TIPS) data for the period between 2000 and 2018 to conduct our empirical investigation. U.S. Treasuries are bonds that pay a fixed coupon and principal amount and whose implied rate of return is nominal (i.e., its rate accounts for both real investment return and inflation accrual). The estimated fair value at time t of a U.S. Treasury ($B_{UST}$) maturing at time T can be expressed as

$$B_{UST}(t) = \sum_{i=1}^{n} c_i^N \times \delta^N(t, \tau_i) + F \times \delta^N(t, \tau_n), \tag{6}$$

where $c^N$ is the nominal coupon, $F$ is the principal redemption value, $\delta^N(t, \tau_i)$ is the nominal discount function for maturity $\tau_i$ and the remaining variables keep their previous meanings.

Inflation-linked bonds (ILB) are debt instruments which allow for inflation risk protection, since they provide a fixed real interest rate return plus a floating return indexed to a broad inflation measure, in both coupon and principal payments. An investor buying ILBs will earn a return that is not only interest rate driven but also protects him from observed inflation, thus maintaining the purchasing power of money throughout the investment horizon. ILBs can be seen as a combination of two different instruments: one with a real deterministic component and another that accounts for inflation. The estimated real fair value of a U.S. TIPS ($B_{TIPS_R}$) at time t can be expressed as

$$B_{TIPS_R}(t) = \sum_{i=1}^{n} c_i^R \times \delta^R(t, \tau_i) + F \times \delta^R(t, \tau_n), \tag{7}$$

where $c^R$ is the real coupon, $\delta^R(t, \tau_i)$ is the real discount function for maturity $\tau_i$ and the remaining variables keep their previous meanings.

Market quotes for U.S. TIPS do not include the inflation accrual, thus allowing for a straightforward computation of the real term structure of interest rates using the Nelson and Siegel (1987) parametric approach. The estimate in Equation (7) has to be adjusted to include the inflation accrual (please refer to Appendix A for a description of the computation of the Index Ratio), to compute the nominal fair value of a U.S. TIPS, i.e., we need to take into account both the real and inflation components as follows

$$B_{TIPS}(t) = \sum_{i=1}^{n} c_i^R \times IR_i \times \delta^N(t, \tau_i) + F \times max\{1, IR_N\}\mathbf{1}_{\{i=n\}} \times \delta^N(t, \tau_n), \tag{8}$$

where $IR_i$ is the inflation index factor at time i expressing the change in the U.S. consumer price index (CPI) and $max\{1; IR_N\}$ is an embedded option offering face value deflation protection at maturity. The study of the deflation option is beyond the scope of this article. For a meaningful research contribution regarding the embedded deflation option, see e.g., Grishchenko et al. (2016).

Equation (4) can be used to compute the nominal duration of U.S. Treasuries and the real duration of U.S. TIPS. However, to be able to compare both datasets results, we need to compute the nominal duration of U.S. TIPS. Siegel and Waring (2004) discuss the dual duration characteristic of TIPS. ILBs and nominal bonds have two duration measures, the inflation duration, and the real interest rate duration. The two duration measures can be separately defined and seen as the decomposition of a bond's nominal duration. In the case of nominal bonds, the difference is not relevant because both measures are similar to each other and equal to the regular, or nominal, duration since any change in nominal interest rates influences the nominal bond price similarly, whether it arises from changes in inflation or changes in real interest rate. This way, by investing in nominal bonds, an investor is unable to hedge independently against changes in real interest rates and in inflation.

For ILBs, the distinction between nominal and real duration is more relevant since the two risk measures are not identical, and not equal to other assets or liabilities. Furthermore, since ILBs' inflation duration is close to zero, the nominal duration is mostly explained by the real interest rate duration. Therefore, if the target liabilities are indexed to inflation, we do not know the final value of the liability ex-ante and it seems clear that immunization strategies should use real interest rate duration as the appropriate risk measure. This way, the investor would be able to hedge directly against real interest rate risk while accounting for (and naturally hedging) inflation risk in both variable components of inflation-linked bonds and inflation-linked liabilities. However, one must account for the indexation lag that exists in all inflation-linked bonds in order to make the formulas consistent with reality. Siegel and Waring (2004) note that the duration measures of nominal bonds and ILBs are not directly comparable. This means that in order to compare the immunization performance of duration-matching strategies using these two types of bonds the interest rate risk measures have to be adjusted.

Other authors have also focused on the comparison of the real and nominal durations of TIPS. Roll (2004) documents the correlation of TIPS returns with nominal bonds and equity returns and the relationship between TIPS real and effective durations through the estimation of the yield beta (β), by regressing TIPS returns on current changes in nominal yields using data on U.S. TIPS from 1997 to 2003. By studying the evolution of the yield $\beta$ parameter throughout the sample, Cocci (2013) shows that the correlation between nominal and real interest rate yields dropped during the financial crisis, implying a weaker link between these markets in a distressed environment. This result has serious implications on the usefulness of the yield $\beta$ measure to estimate TIPS' effective duration, thus confirming Roll (2004) findings.

Laatsch and Klein (2005) take a different approach and study the applications of the effective duration of TIPS while setting up mixed bond portfolios. The authors show that the relationship between effective durations and real durations of TIPS is not constant and has to consider investors' expectations regarding the future evolution of both nominal and real interest rates and inflation rates, as portrayed by the Fisher (1930) equation.

To be able to compare the results of multiple liability immunization strategies using both datasets, we follow Siegel and Waring (2004) and Laatsch and Klein (2005) and compute the effective duration of ILBs by adjusting the bonds' real cash flows considering the expected inflation rate over the life of the bond and then discounting cash flows using nominal rates. This way, for a non-flat interest rate term structure the ILBs real duration at time $t$, $D_j^R(t)$, will be computed as follows

$$DFW^R(t) = \sum_{i=1}^{n}(\tau_i - t) \times w_i, \quad w_i = \frac{\left[cf_i^R \times \delta^R(t,\tau_i)\right]}{B_{TIPS_R}(t)}, \tag{9}$$

To compute the effective duration of ILBs, we need to take into account the inflation accrual. For this purpose, we assume that the best estimator of future inflation is the year-on-year actual inflation rate (depicted as $\pi$) and recall the classical "Fisherian" decomposition of nominal interest rates:

$$(1 + y_N(0,\tau)) = (1 + y_R(0,\tau)) \times (1 + \pi). \tag{10}$$

We assume that the inflation risk premium required for investing in inflation-linked bonds is negligible. This assumption is consistent with empirical studies showing that the magnitude of the one-year inflation risk premium in the U.S. TIPS market is estimated to be between one and two basis points (see e.g., Deacon et al. 2004, p. 80; Ang et al. 2008; Chen et al. 2010). To amend the duration in Equation (4), we need to adjust both the numerator and the denominator. The denominator adjustment is straightforward since we have estimated a continuous function for the nominal term structure of interest rates. The numerator adjustment will be done by multiplying it by the inflation accrual, which is translated through the ILB's Index Ratio ($IR_i$) computed assuming that future year-on-year inflation $\pi$ will be equal to the latest observed year-on-year inflation rate (please refer to Equation (A2) in Appendix A). This hypothesis is consistent with the absence of arbitrage opportunities between both securities at time $t$. Formally, the U.S. TIPS effective duration ($ED$) is computed as follows:

$$ED(t) = \sum_{i=1}^{n}(\tau_i - t) \times w_i, \quad w_i = \frac{c_i^R \times IR_i \times \delta^N(t,\tau_i) + F \times max\{1, IR_N\}\mathbf{1}_{\{i=n\}} \times \delta^N(t,\tau_n)}{B_{TIPS}(t)}. \tag{11}$$

Although it can be argued that the inflation indexation factor affects Equations (8) and (11) in the same manner and, hence, the inflation adjustment is redundant that is not entirely true due to the indexation lag, i.e., future inflation expectations are reflected instantaneously in the nominal interest rates but reflected with a three-month lag in the coupon and principal valuation. Anyway, although not irrelevant, we can conjecture that the difference between the effective duration and the nominal duration measures is expected to be small.

To compute the M-Absolute of bond portfolios including U.S. TIPS we use a similar procedure to account for both the inflation accrual and real interest rate accrual, i.e.,

$$M^A(t) = \sum_{i=1}^{n} abs(\tau_i - H) \times w_i, \quad w_i = \frac{\left[c_i^R \times IR_i + F \times max\{1, IR_n\}\mathbf{1}_{\{i=n\}}\right] \times \delta^N(t,\tau_i)}{B_{TIPS}(t)}, \tag{12}$$

where all variables keep their previous meanings.

Kondratiuk-Janyska and Kaluszka (2006a) note that this measure should only be used in single liability immunization problems. To adapt it to a multiple liability immunization context, the authors derive the following M-Absolute measure of a multiple asset and liability portfolio in a continuous-time setting:

$$M^A_{ptf} = \int_0^T abs(A(t) - A(T) + E[L(T) - L(t)])dt, \quad (13)$$

where for a given time interval $[t,\ T]$, $A(t) \geq 0$ denotes the asset inflows occurring at fixed time $t$ required to cover multiple expected liabilities $L(t)$ due at dates $(t < \tau_1 < \tau_2 < \ldots < T)$. Please note that the asset inflows can occur before the liabilities are due because we are using real market data. If this is the case and the inflow occurs at time $t_\omega < t_d$ this amount will be compounded in a time deposit, whose rate is given by $y_N(t_\omega, t_d)$.

## 3. Empirical Study

In this section, we describe the main features of the empirical study, namely the datasets used, the portfolio design, the liability cash flow stream to hedge, the testing methodology and the immunization performance metrics. A benchmarking strategy, based on a naive ladder porfolio, serves as benchmark to assess relative performance of the multiple liability M-Absolute strategy.

### *3.1. Portfolio Design and Testing Methodology*

In this paper, we focus on the performance of targeted multiple liability immunization portfolios considering liabilities that are inflation-linked, i.e., that growth stochastically in time according to observed consumer prices. To achieve appropriate immunization results the hedging portfolio should be liability-driven (Fabozzi 2000; Siegel and Waring 2004). To this end, we can conjecture ex-ante that for unknown (index-linked) future liability amounts the inclusion of floating-rate bonds that match a series of current or future expected cash payments is expected to deliver good immunization results. We assume the liability portfolio can be seen as a portfolio of zero coupon bonds with differing annual maturities. Let $H$ be an investor planning horizon with $0 < H < T$. The liabilities $L$ are defined as multiple annual payments for each planning horizon $H$ until the final laibility is due in $T$. These liabilities have embedded and annual growth rate given by the year-on-year inflation portrayed by $\pi$. The present nominal value of each liability is given by

$$L_H(t, T) = l_H \times (1 + \pi_H)^H \times\ \delta^N(t, \tau_i), \quad (14)$$

where $l_H$ is the real amount of each liability, $(1 + \pi_H)^H$ is the annual year on year inflation for each planning horizon $H$ and the remaining variables keep their previous meanings. Please note that the real value of each liability will be given by $L^R_H(t, T) = l_H \times\ \delta^R(t, \tau_i)$.

The liability profile resembles a zero coupon bond, which means that their estimated durations and M-Absolute can be computed by adjusting Equations (9), (11) and (12) term $w_i$ with the liability profile described in Equation (14).

Similar to Barber and Copper (1998) and Kaluszka and Kondratiuk-Janyska (2004), to ensure that we are able to comply with every yearly payment, we decompose the overall liability into subsets of single liabilities and immunize $\frac{1}{H}$ of the overall portfolio against each upcoming payment. This means, for instance, that for the 3-year and 5-year investment horizons considered each sub-portfolio will comprise, respectively, $\frac{1}{3}$ or $\frac{1}{5}$ of the overall portfolio value. These sub-portfolios are always immunized in every rebalancing period in accordance with the strategy described above. The M-Absolute strategy is set up to minimize the difference between the M-Absolute of the asset portfolio and the residual maturities of the liability cash flow stream while maximizing the value of the portfolio in each rebalancing period. Formally, the following linear programming optimization problem is used Bierwag et al. (1983), Fong and Vasicek (1983a):

$$\min_{k} \sum_{j=1}^{m} k_j M_j^A \tag{15}$$

$$
\begin{gathered}
s.t. \sum_{j=1}^{m} k_j = 1 \\
k_j \geq 0,\ \forall j = 1\ldots, m \\
(i)\ A(t) = E[L(t)],\ \forall t = 0\ldots, T \\
(ii) \begin{bmatrix} M_{t_1}^A \\ \vdots \\ M_H^A \end{bmatrix} = \begin{bmatrix} t_1 \\ \vdots \\ H \end{bmatrix} \\
(iii)\ \sum_{t=1}^{H} M_t^A = \sum_{t=1}^{H} t,
\end{gathered}
$$

where $k_j$ denotes the portfolio weight of the $j$th bond and $m$ is the number of bonds in the portfolio, $M_j^A$ denotes the M-Absolute measure of the asset portfolio (which depends on the j bonds included), $A(t)$ is the present value of the asset portfolio and $E[L(t)]$ is the expected value of the liability portfolio at time t, so that in the first set-up date, as well as the following rebalancing dates we ensure that the asset sub-portfolios set-up will cover the expected liabilities.

The constraints considered in Equation (15) to compute the immunization portfolio ensure that:

(i) the present value of the asset portfolio matches that of the expected liability cash flow stream, i.e., the immunization portfolios are self-financing;
(ii) For each sub-portfolio, the vector of the M-Absolute measures matches the residual maturity of each liability;
(iii) The sum of the M-Absolute interest rate risk measures has to be equal to the sum of the residual maturities of each liability due.

In a multiple liability setting, the M-Absolute measures the dispersion of the overall asset portfolio around the liability portfolio, considering that each sub-portfolio cash flows are clustered around the corresponding liability payment dates. By definition, the M-Absolute is a dispersion measure that takes into account the portfolio's composition. This means, for instance, that duration-matching bullet and barbell portfolios will have the same duration but exhibit quite different M-Absolute values. This is because the cash flows of bullet portfolios are nested around the liability date (i.e., they have low M-Absolute), whereas for the barbell portfolios cash flows are spreaded out (i.e., they exhibit high M-Absolute).

Minimizing the M-Absolute of a portfolio is equivalent to minimizing the absolute duration gap between the asset and liability cash flows streams Kondratiuk-Janyska and Kaluszka (2006b). This reduces the importance of the asset–liability dispersion measures based on duration mentioned in Bierwag et al. (1983). In Equation (15) constraints (i) to (iii) ensure that the dispersion of the asset portfolio is no shorter than the dispersion of the liability portfolio as required by the third immunizing condition in Fong and Vasicek (1983a). Note, however, that we need to enforce these conditions ex-ante since we are immunizing $\frac{1}{H}$ of the portfolio against each liability due. In standard immunization problems considering a single liability the third immunizing condition in Fong and Vasicek (1983a) is trivial. In a multiple immunization setting, to guarantee that the bond portfolio complies with all the conditions stated in Fong and Vasicek (1983a) we need to enforce conditions (ii) and (iii) simultaneously. In brief, the immunizing constraints in Equation (15) are both necessary and sufficient conditions in a multiple liability immunization minimax problem with the M-Absolute strategy.

The immunization performance of the alternative investment strategies is analyzed over three- and five-year holding periods. The testing procedure is as follows. Using January 2000 as a starting point, for the three- and five-year holding periods bond portfolios were set up on the first business day of each quarter considering, respectively, eight or ten different bonds with residual maturities up to five years after the horizon date. To provide a sufficient number of simulations in which to empirically test the models, overlapping holding periods were considered. Thus, in the period January 2000–December 2018, 64 three-year and 56 five-year overlapping investment periods were constructed, each starting one quarter after the previous period. For the purposes of multi-period immunization, all portfolios are rebalanced quarterly. Coupon and principal payments are reinvested with the same strategy on the first reallocating date available after the payment. At the end of the planning horizon, the portfolios are liquidated at market prices. Short-selling was not allowed.

To replicate the usual market constraints investors' face, we considered transaction costs, namely bid-ask spreads. Moreover, the same set of bonds is used in each rebalancing period. This eliminates any potential bias in the performance of a given immunization strategy resulting from different portfolio compositions.

To assess the relative performance of the M-Absolute multiple liability strategy, a naive ladder portfolio strategy is set up, comprising only bonds with maturity close to (but below) the payment date of each of the multiple liabilities. Since we are using real bonds, in some cases the maturity of the bond used will be slightly lower than the maturity of the liability. When this happens, cash is reinvested in a time deposit with maturity equal to the liability payment date.

To make strategies comparable when using either the U.S. Treasuries subset or the U.S. TIPS subset, the final amount of the liability was adjusted in order to account for the year-on-year inflation, as measured by the Consumer Price Index for all urban consumers (CPI-U henceforth). By applying the same inflation rate to both the liabilities and the bonds considered in the immunization process we eliminate any inflation bias that could influence the strategies performance. Throughout the sample period, year-on-year inflation rates range from 2.15 to 2.35%. As for the cumulative inflation rates, for the 3-year immunization period prices grew by 6.41% on average while for the 5-year immunization prices grew by 11.17% on average. This means that each year successfully immunized portfolios have to generate, on average, at least the cumulative liability growth projected for that year.

### *3.2. Performance Metrics*

A portfolio is considered to be successfully immunized if the portfolio generates sufficient cash flows to cover the multiple liability payments. To assess the effectiveness of each particular immunization strategy, absolute and relative performance measures were computed. The absolute measures are straightforward and aim to assess liability coverage and return. We first use an average liability coverage measure ($\overline{LC}$) to assess whether, on average, the bond portfolio cash flows are sufficient to cover the liability stream at the end of the projected liability horizon $h$. This measure is defined as

$$\overline{LC}_h = \frac{\sum_{\lambda=1}^{\Lambda}\left(\frac{V_h^\lambda}{V_0^\lambda} - (1+\pi)^h\right)}{\Lambda}, \tag{16}$$

where $V_0^\lambda$ and $V_h^\lambda$ denote, respectively, the value of the immunizing portfolio for a given investment strategy at inception and a given liability horizon $h$ and $\pi$ is the estimated (target) inflation rate for the same period. The number of possible overlapping simulations is denoted by $\Lambda$. Positive values for $\overline{LC}_h$ mean the immunization strategy was successful in covering liability payments.

We computed a second absolute performance measure called average excess return. The excess return of the portfolio, ($\overline{ER}$), measures to what extent, on average, the portfolio's annual return (including reinvested coupon payments) is above (or below) the yield on a zero-coupon bond maturing at the end of the holding period. Stated differently, $\overline{ER}$ assesses whether the immunizing portfolio outperforms the naive strategy of making a continuously compounded time deposit of h years done at time 0. This measure is defined as

$$\overline{ER}_h = \frac{\sum_{\lambda=1}^{\Lambda}\left(\frac{ln\left(\frac{V_h^{\lambda}}{V_0^{\lambda}}\right)}{h}\right) - y(0,h)}{\Lambda}. \quad (17)$$

To evaluate the stability and liquidity of the immunizing portfolios, the average turnover ($\overline{T}$) and the average transaction costs ($\overline{TC}$) have been computed. For a given investment strategy, the average turnover is defined as

$$\overline{T}_h = \frac{\sum_{j=1}^{m} abs\left(Q^j_{\alpha_z} - Q^j_{\alpha_{z-1}}\right)}{m \times z}, \quad (18)$$

where $z$ denotes the total number of portfolio rebalancing quarters for each strategy, such that $\alpha_z$ stands for the current quarterly rebalancing period for the portfolio and $Q^j$ are the bond units of bond j in the portfolio. If the $j$-th bond was not held in the portfolio before a given quarterly rebalancing period, the respective transaction amount is set to zero. The amount to be reinvested includes any coupons received since the last rebalancing period and any amount bought or sold at the rebalancing date. The average turnover measure assesses the frequency and extent to which immunizing portfolios have to be rebalanced (buy/sell trades, coupon, and principal reinvestment trades) in order to match immunization constraints.

The average transaction costs measure is computed as the absolute value of the product between the bid-offer spread ($P^j_{ask} - P^j_{bid}$) and the transaction amount, in units, for each bond in the portfolio, i.e.,

$$\overline{TC}_h = \frac{\sum_{j=1}^{m} abs\left(Q^j_{\alpha_z} - Q^j_{\alpha_{z-1}}\right) \times \left(P^j_{ask} - P^j_{bid}\right)}{m \times z}. \quad (19)$$

These two measures are positively correlated since a higher (average) absolute turnover (expressed in units) will lead to higher (average) absolute transaction costs (expressed as a percentage of each bond's value).

Finally, to better assess the strategies' performance we computed for each liability horizon a Reward-to-Risk Ratio, ($\frac{R}{R_h}$), defined as the ratio between the portfolio's excess return $\overline{ER}(S)$ and the portfolio returns' volatility (standard deviation of the portfolio returns) $\sigma_S$,

$$\frac{R}{R_h} = \frac{\overline{ER}(S)}{\sigma_S}. \quad (20)$$

The objective is to rank strategies controlling for the volatility of their returns. This allows us to identify the most efficient immunization strategy, i.e., the strategy that maximizes return for every unit of risk incurred.

### *3.3. Data*

The datasets used in this study comprise, for the period between January 2000 and December 2018, U.S. government bond data, U.S. Treasuries and Treasury Inflation-Protected Securities (TIPS) gathered from Bloomberg, nominal and real interest rates obtained from the U.S. Department of the Treasury (https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/default.aspx, last accessed on 4 January 2019) and inflation data (non-seasonally adjusted Consumer Price Index for all urban consumers) retrieved from the Bureau of Labor Statistics (http://data.bls.gov/pdq/SurveyOutputServlet, last accessed on 4 January 2019).

The bond datasets comprise daily bid and offer prices for 74 U.S. Treasuries and 26 U.S. TIPS with maturities ranging between 2 to 30 years paying semi-annual coupons, selected considering the total amount issued and the bid-offer spread. Only on-the-run bonds with a high degree of liquidity were considered in the analysis. The most liquid U.S. Treasury issuances are included in the delivery basket of futures contracts issued on these bonds and, for this dataset, this criteria was crucial to select the most liquid (on-the-run) bonds. Zero coupon bonds, principal or interest rate strips and bonds with embedded options (i.e., callable and puttable bonds) were discarded. Nominal coupon sizes ranged from 0.25 to 11.625%, a challenging feature for any immunization strategy and a sign of the significant yield curve shifts observed during this period.

As for the U.S. TIPS, the selection process was simpler because its issuance is more recent and less widespread than U.S. Treasuries. These bonds address the needs of a specific segment of investors, such as insurance companies and pension funds, whose primary investment objective is to protect long-term investments from changes in the inflation rate. This also explains their low liquidity in the market since these investors tend to follow a buy-and-hold strategy. Consequently, all the 26 on-the-run bonds alive from January 2000 to December 2014 were selected (29 bonds were excluded due to severe liquidity issues). The annual coupon sizes range between 0.125% and 4.25%. Although the coupon size range is smaller than that of U.S. Treasuries, in the period under review we observe a downward trend in TIPS coupon rates that mimics the overall term structure of interest rates evolution.

Nominal zero coupon yields were reconstructed from prior estimates of the parameters of the Nelson-Siegel (NS) term structure specification, estimated using daily interest rates for residual maturities ranging between 6 months and 30 years obtained from the U.S. Department of the Treasury. From this, the continuous discount function curve is derived and discount factors for non-standard and non-published maturities are computed. To assess the quality of the fit, we computed both the daily correlation between fitted and observed interest rates and the t-test of equality of means for all maturities considered in the estimation procedure, for both nominal and real interest rates. The null hypothesis that the two mean values are equal was not rejected at 53 level of significance for all maturities. Moreover, the correlation coefficients between fitted and observed interest rates are high and statistically significant.

Regarding the estimation of real interest rates, the lower liquidity, the reduced number of available bonds and the lack of real interest rates for maturities of less than five years posed a problem and made the use of NS unfeasible. The lack of estimates for maturities below the 5-year benchmark is explained by the use of on-the-run bonds for the estimation (as is done for the U.S. Treasuries) by the U.S. Department of the Treasury. As a result, the term structure of real interest rates has been estimated using U.S. TIPS data instead of the rates provided by the U.S. Department of the Treasury. The procedure involves minimizing the weighted sum of the squared deviations of the fitted prices from the quoted prices as depicted in Equation (2).

The mean square price error (MSPE) obtained for the overall sampling period and dataset are 0.0071 basis points and the standard deviation of this measure is 0.0114 basis points. The MSPE between the observed and the estimated prices ranges from 0 to 0.0079 basis points and the standard deviation varies between 0 and 0.1664 basis points. The correlations between estimated and real prices for each U.S. TIPS are fairly high, ranging from 77.23 to 99.75%. To assess the quality of the fit, we additionally computed both the daily correlation between fitted and observed interest rates and the t-test of equality of means for all maturities considered in the estimation procedure. For all maturities and a 95% confidence level, the test results fail to reject the null hypothesis that observed and fitted interest rates are equal.

As for the evolution of the term structure of interest rates and inflation, we briefly analyze the evolution of each set in the sample period. We can observe a clear negative trend in the dynamics interest rates until 2012, with particular emphasis between January

2000 and January 2004 and between January 2007 and December 2008, where a significant downward movement occurred, namely in the 1- and 3-year maturities. From 2013 onwards a tendency for interest rate increases is visible, first in the longer maturities and, after 2014, also in the 1-year maturity. The huge adjustment in nominal interest rates occurred in the short end of the yield curve (i.e., maturities below 5 years), where the downward movements have been more pronounced, allied with a flattening movement in the term structure of interest rates.

Figure 1 contains the nominal yields for the 1, 3, 5 and 10-year maturities between January 2000 and December 2017. We observe a considerable decrease in interest rates between 2000 and 2002, followed by an increase for all maturities until year-end 2006 in a highly volatile environment, namely in the 1-year maturity. From 2006 to 2008 we see another significant decrease in interest rates due to the subprime crisis. Apart from the 1-year maturity that stays in low levels until year-end 2014, interest rates rebound for a while in 2009 but subsequently resume their decreasing tendency until 2012. From that year onwards, interest rate above the 3-year maturity tend to increase until the end of the sampling period, while the 1-year interest rate only starts increasing from 2014 onwards.

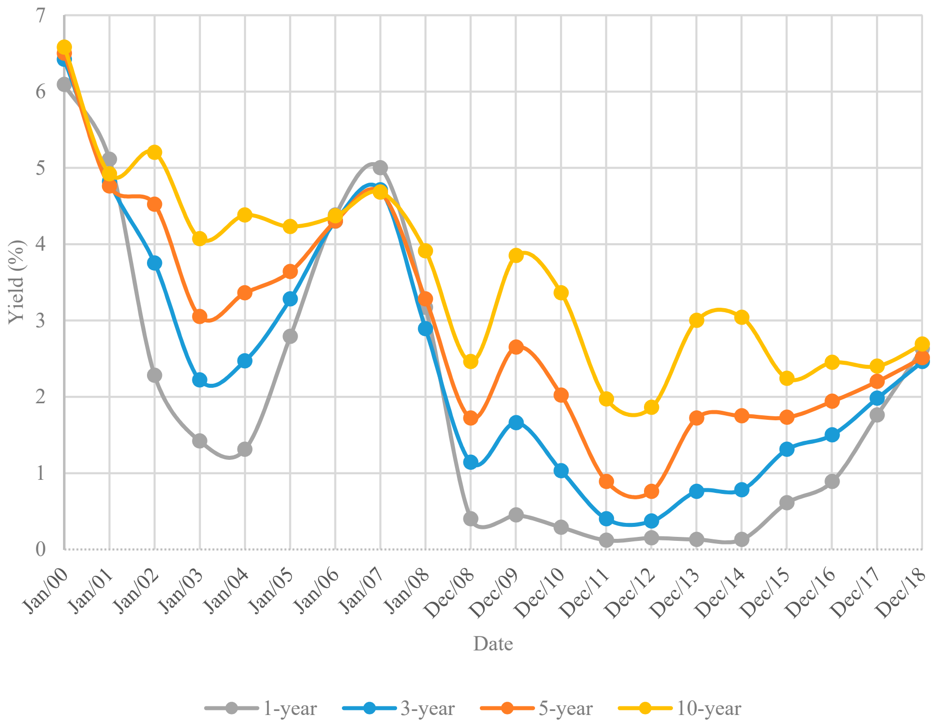


**Figure 1.** U.S. Treasuries Interest Rates.

Figure 2 contains the real yields for the 1, 3, 5 and 10-year maturities between January 2000 and December 2017. The interest rate evolution hints at a quasi-flat structure until year-end 2012. It is visible a downward tendency in interest rates between 2000 and 2005 with some stability from 2002 to 2004. From 2006 onwards an increasing tendency sets is and interest rates peak in 2008. Afterwards, the downward tendency resumes until December 2012 and into negative territory. Rates increase and decouple from 2012 onwards, even though they remain negative in the 1-year maturity until mid-2014. From 2015 onwards we only negative interest rates in the 1- and 3-year maturities. In the end of the sample period all interest rates maturities are positive.

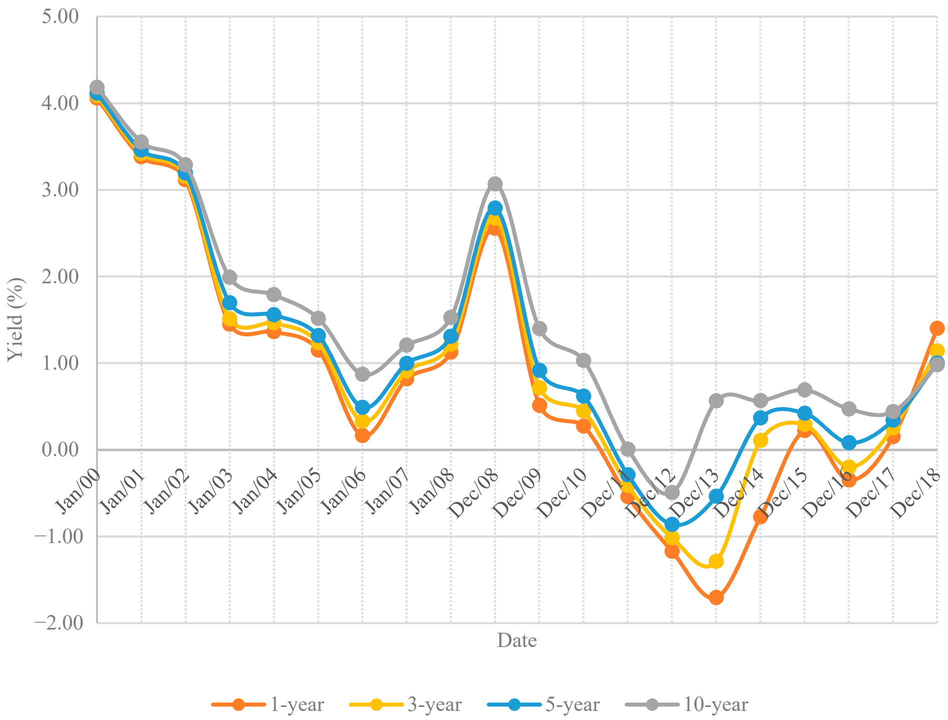


**Figure 2.** U.S. TIPS Interest Rates.

The underlying tendency toward low interest rates is also visible for real interest rates. However, the pattern is not similar. The periods where real interest rates decrease sharply are between 2000 and 2005 and again between 2009 and 2012. The difference between the two term structures is the evolution of inflation, which increased between 2004 and 2006. The term structures of interest rates also show this pattern. However, the decrease seems to be similar throughout the whole term structure of interest rates except for 2014 onwards, where an inversion toward interest rate increases with a flattening movement in the term structure of interest rates occurs.

Finally, in order to allow for U.S. TIPS to account for the inflation evolution, we used the CPI-U as an inflation proxy for these bonds. These values were used to compute the year-on-year inflation rate and the Index Ratios applied to U.S. TIPS. There is an upward tendency between 2002 and 2006 and 2007–2008 that is followed by a severe deflation process that arises as a consequence of the Quantitative Easing process carried out by the U.S. Federal Reserve in the wake of the subprime crisis. Actually, from 2009 to 2012 inflation picks up again, even though it never reaches the around 4% levels from 2008; from 2012 onwards, the underlying tendency is for a steady and continuous decline in year-on-year inflation. We observe an increase in inflation from 2014 onwards.

Figure 3 contains the year-on-year inflation rate, computed between January 2000 and December 2017, taking into account the monthly U.S. CPI-U unrevised index statistics, as published by the U.S. Bureau of Labor Statistics. The year-on-year inflation is quite volatile during the sample period, without showing a clear tendency. The only significant movement occurs between 2007 and 2009 where after an increasing tendency and consequent upper bound above 53, a sharp decrease occurs into negative inflation of around 23 during 2009, mainly due to the U.S. Federal Reserve Quantitative Easing measures. In late 2009 inflation starts increasing again into positive territory. From 2011 to 2013 the inflation rate varied between 13 and 23. After declining to zero in 2014, inflation picked up and displayed a positive trend from 2016 until mid-2018, decreasing from that point onwards.

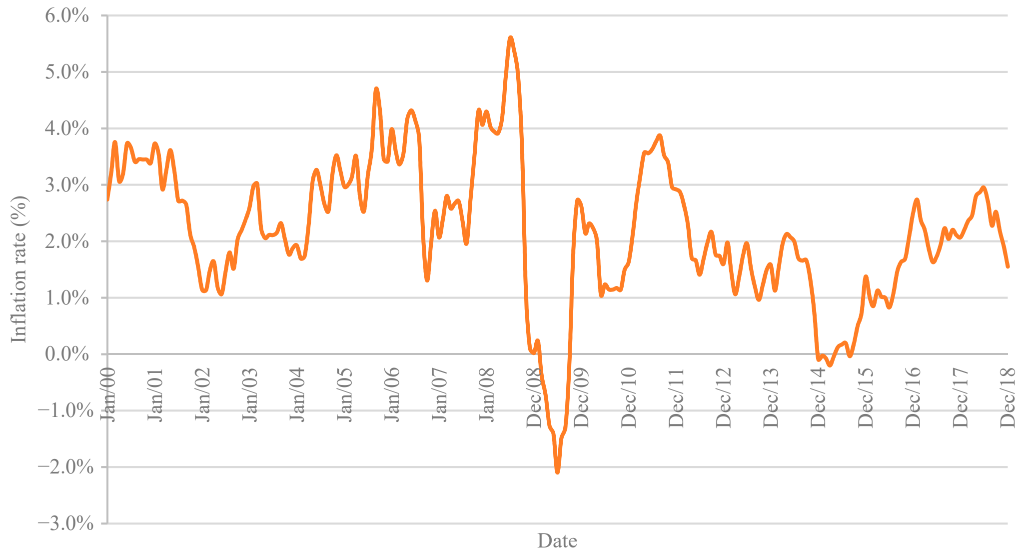


**Figure 3.** U.S. Year-on-year Consumer Price Index Rates.

## 4. Results

In Table 1 we report the immunization results of bond portfolios comprising U.S. Treasuries only for both the three- and five-year holding periods and corresponding sub-portfolio horizons. For the M-Absolute portfolio comprising U.S. Treasuries only, the average initial duration ranged from 1.23 (1.18) to 2.97 (4.49) years for the three-(five) year investment horizon, respectively. The initial weight of the maturity bond in the immunizing portfolio ranged from 75% (76%) to 80% (84%) for the three-(five) year investment horizon, respectively, converging as expected towards 100% as we approach the horizon end. The average quarterly turnover of the three-year investment horizon immunizing portfolio was 0.6%, whereas for the five-year horizon the turnover ranged from 0.64% to 0.77%. The results show that in all scenarios tested the immunization strategy succeeded in generating sufficient cash flows to cover the multiple liability payments. These results are very significant given the 2.14% average year-on-year inflation growth observed during this period. On average, the asset portfolios' final value exceeded the liability value by an amount between 0.48% and 6.70% and, consequently, average excess returns were always positive.

The results also show that for the 1-, 2- and 4-year maturities the M-Absolute strategy performs better when compared with the Naive ladder portfolios, while for the 3- and 5-year maturities the Naive portfolios' performance is higher, as seen by the Reward-to-Risk Ratios of 52.97% and 37.63%, respectively (against the 29.29% and 27.06% of the M-Absolute portfolios). The negative result in the 4-year Naive strategy is explained by the relative importance of bonds maturing before the liability due date and the need to reinvest in time deposits, whose return declined during this period due to the decreasing interest rate environment. This is essentially explained by the lower turnover of Naive portfolios in these maturities when compared to that of M-Absolute strategies that often require substantial portfolio restructuring at each quarterly rebalancing period. This result shows that transaction costs have the potential to erode strategies' overall performance.

The results in Table 1 also highlight that for both strategies there is an inverse relationship between the average return and transaction costs, since portfolios with higher average transaction costs tend to have lower average excess returns. The average turnover is generally lower for long holding periods, suggesting that in shorter investment horizons immunizing portfolios tend to be more volatile and require more frequent rebalancing to achieve perfect immunization. These two factors are related and can be explained by the fact that most U.S. Treasuries and U.S. TIPS bonds have maturities above five years.

**Table 1.** U.S. Treasuries Immunization Results.

| Panel A—3-Year Immunization Horizon | | | | | |
|---|---|---|---|---|---|
| **M-Absolute—Sub-Portfolio Horizon** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $R/R_h$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 0.48% | 0.64% | 45.01% | 1.66% | 28.93 |
| 2-year | 1.53% | 1.13% | 61.49% | 1.26% | 22.34 |
| 3-year | 1.94% | 0.59% | 29.29% | 0.86% | 20.01 |
| **Naive—Sub-portfolio horizon** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $R/R_h$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 0.09% | 0.30% | 24.10% | 0.86% | 7.84 |
| 2-year | 0.83% | 0.46% | 29.41% | 0.77% | 27.92 |
| 3-year | 2.50% | 1.06% | 52.97% | 0.45% | 15.73 |
| **Panel B—5-year immunization horizon** | | | | | |
| **M-Absolute—Sub-portfolio horizon** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $R/R_h$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 0.56% | 0.67% | 45.39% | 1.78% | 29.33 |
| 2-year | 1.90% | 1.27% | 69.10% | 1.90% | 20.25 |
| 3-year | 2.52% | 0.69% | 32.72% | 0.74% | 19.38 |
| 4-year | 5.27% | 1.36% | 54.23% | 0.70% | 14.72 |
| 5-year | 6.70% | 0.73% | 27.06% | 0.55% | 15.20 |
| **Naive—Sub-portfolio horizon** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $R/R_h$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 0.16% | 0.32% | 24.69% | 0.88% | 7.89 |
| 2-year | 1.09% | 0.53% | 32.06% | 0.64% | 29.97 |
| 3-year | 3.10% | 1.24% | 58.93% | 0.46% | 15.55 |
| 4-year | 3.85% | −0.01% | −0.26% | 0.85% | 29.39 |
| 5-year | 7.08% | 1.10% | 37.63% | 0.72% | 26.45 |

This table is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels report the same metrics for the M-Absolute and Naive Strategies. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio ($R/R_h$). As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

The results for the U.S. TIPS immunization strategy, taking into account the inflation accrual, are enlightening (Table 2).

The baseline hypothesis is that the immunization results using U.S. TIPS should be in line with those obtained using U.S. Treasuries. Our empirical results show, however that the average liability coverage of immunizing portfolios using U.S. TIPS is much higher. For the 3-year immunization portfolios, the average liability coverage for U.S. TIPS varies between 5.98% and 9.67%, while for the 5-year immunization portfolios the average liability values range from 7.04% to 20.36% in the 5-year sub-portfolio maturity. The superior performance of the M-Absolute strategy using U.S. TIPS is not accompanied by the Naive portfolio, whose average liability coverage is consistently negative for most maturities (i.e., liability payments would not be sufficiently covered by U.S. TIPS asset portfolio), despite the strategy's positive excess returns (ranging between 2.74% to 7.84% for the 3-year immunization portfolios and 0.77% to 2.15% for the 5-year immunization portfolios).

Considering that the U.S. TIPS and U.S. Treasuries datasets used in this study are similar and that the time to maturity of the bonds selected for the clustering process is also in line, the main explanation for the superior performance of the M-Absolute strategy using U.S. TIPS lies in the dynamics of inflation rates observed during this period. Recall that for either coupon or principal amounts the inflation accrual is computed through the Index Ratio, as stated in (A.2) that takes into account the evolution of the CPI-U index since the issuance of the bond. The year-on-year inflation estimates increased consistently during the sampling period, with an exception for 2008, resulting in upward trend for the CPI-U index.

**Table 2.** U.S. TIPS with Inflation Accrual Immunization Results.

| **Panel A—3-Year Immunization Horizon** | | | | | |
|---|---|---|---|---|---|
| **M-Absolute—Sub-Portfolio Horizon** | $\overline{\mathrm{LC}}$ | $\overline{\mathrm{ER}}$ | $R/R_h$ | $\overline{\mathrm{TC}}$ | $\overline{\mathrm{T}}$ |
| 1-year | 5.98% | 9.03% | 209.29% | 0.61% | 20.49 |
| 2-year | 9.67% | 15.71% | 270.38% | 0.40% | 13.40 |
| 3-year | 8.98% | 18.05% | 298.01% | 0.31% | 10.68 |
| **Naive—Sub-portfolio horizon** | $\overline{\mathrm{LC}}$ | $\overline{\mathrm{ER}}$ | $R/R_h$ | $\overline{\mathrm{TC}}$ | $\overline{\mathrm{T}}$ |
| 1-year | −0.29% | 2.74% | 76.73% | 1.18% | 21.54 |
| 2-year | −0.22% | 5.76% | 99.06% | 0.96% | 19.78 |
| 3-year | −1.08% | 7.84% | 129.41% | 0.40% | 12.64 |
| **Panel B—5-year immunization horizon** | | | | | |
| **M-Absolute—Sub-portfolio horizon** | $\overline{\mathrm{LC}}$ | $\overline{\mathrm{ER}}$ | $R/R_h$ | $\overline{\mathrm{TC}}$ | $\overline{\mathrm{T}}$ |
| 1-year | 7.04% | 7.81% | 180.20% | 0.64% | 21.26 |
| 2-year | 11.38% | 12.49% | 210.23% | 0.43% | 14.69 |
| 3-year | 10.99% | 12.42% | 199.34% | 0.33% | 11.42 |
| 4-year | 10.96% | 12.47% | 190.87% | 0.26% | 8.38 |
| 5-year | 20.36% | 21.32% | 260.24% | 0.50% | 13.93 |
| **Naive—Sub-portfolio horizon** | $\overline{\mathrm{LC}}$ | $\overline{\mathrm{ER}}$ | $R/R_h$ | $\overline{\mathrm{TC}}$ | $\overline{\mathrm{T}}$ |
| 1-year | −0.10% | 0.77% | 20.68% | 1.32% | 23.93 |
| 2-year | 0.26% | 1.77% | 29.77% | 1.08% | 22.19 |
| 3-year | −0.53% | 1.52% | 24.34% | 0.42% | 13.69 |
| 4-year | −0.39% | 1.92% | 29.46% | 0.44% | 20.68 |
| 5-year | −0.72% | 2.15% | 61.18% | 0.47% | 15.24 |

This table is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels report the same metrics for the M-Absolute and Naive Strategies. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio ($R/R_h$). As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

Moreover, between 2004 and 2006, inflation rates maintained the upward slopingtrend, but real interest rates exhibited a negative trend. In this sense, the inclusion of the inflation accrual allied with the divergent behavior of inflation and real interest rates contributed to the outperformance of the M-Absolute strategy using U.S. TIPS.

Therefore, with Index Ratios for all the bonds increasing with time, higher inflation accruals are being incorporated as time passes. Their subsequent reinvestment in the immunizing portfolio leverages its coverage in a way that is similar to that arising from higher coupons being reinvested in the portfolio. In the counterfactual scenario in which the Index Ratio applied to U.S. TIPS coupon payments considered the inflation rate observed since the last coupon date and not since the bond's issuance, the inflation accrual would be lower and, in theory, this would allow for near-perfect hedging of the inflation accrual, since the only bias would come from the indexation lag, as depicted in Appendix A. In reality, this is not the way U.S. TIPS coupon payments are determined and, as such, considering this scenario is a good theoretical exercise but it cannot be replicated in real world portfolios.

Notwithstanding this, we constructed a theoretical exercise in which only the real component of U.S. TIPS is considered. This allows us to quantify the magnitude of inflation accrual effect in the immunizing portfolios. The M-Absolute immunization results using the U.S. TIPS dataset and considering the real interest rate component only are reported in Table 3. For the M-Absolute portfolio comprising U.S. TIPS, the average initial duration ranged from 1.48 (1.52) to 2.90 (4.43) years for the three-(five) year investment horizon, respectively. The initial weight of the maturity bond in the immunizing portfolio ranged in this case from 72% (74%) to 79% (82%) for the three-(five) year investment horizon, respectively, converging once again towards 100% as we approach the horizon end. The average quarterly turnover of the three-year investment horizon immunizing portfolio was 0.33%, whereas for the five-year horizon the turnover ranged from 0.35% to 0.42%. With no surprise, our results show that in this scenario the multiple liability immunization strategy fails to cover the liability cash flow stream by a value ranging between −0.27% to −3.36%. This is mostly explained by the decreasing interest rate environment, allied with high coupons paid and reinvested into the portfolio. Note, however, that for some holding periods negative excess returns are observed, an anticipated result since we are (artificially) forcing the real component of U.S. TIPS to surpass a liability that has a floating component.

**Table 3.** U.S. TIPS Immunization Results.

| **Panel A—3-Year Immunization Horizon** | | | | | |
|---|---|---|---|---|---|
| **Sub-Portfolio Horizon** | $\overline{\textbf{LC}}$ | $\overline{\textbf{ER}}$ | $R/R_h$ | $\overline{\textbf{TC}}$ | $\overline{\textbf{T}}$ |
| 1-year | −0.36% | 0.56% | 23.95% | 0.89% | 25.47 |
| 2-year | −1.11% | 0.62% | 23.52% | 0.54% | 17.12 |
| 3-year | −2.03% | 0.44% | 15.39% | 0.40% | 13.37 |
| **Panel B—5-year immunization horizon** | | | | | |
| **Sub-portfolio horizon** | $\overline{\textbf{LC}}$ | $\overline{\textbf{ER}}$ | $R/R_h$ | $\overline{\textbf{TC}}$ | $\overline{\textbf{T}}$ |
| 1-year | −0.27% | 0.61% | 26.32% | 0.89% | 26.57 |
| 2-year | −1.04% | 0.51% | 19.09% | 0.51% | 16.29 |
| 3-year | −1.81% | 0.28% | 9.58% | 0.38% | 12.85 |
| 4-year | −2.63% | −0.15% | −4.72% | 0.38% | 12.18 |
| 5-year | −3.36% | −0.38% | −11.16% | 0.43% | 12.92 |

This table is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels report the same metrics. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio ($R/R_h$)As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated. Since this is a theoretical exercise, only the M-Absolute strategy results are depicted.

With explained the good performance of the M-Absolute strategy, we still need to understand why this result does not translate into the Naive portfolios. First, we recall that there are fewer bonds in the U.S. TIPS dataset and that these bonds are less liquid than U.S. Treasuries, particularly during the financial crisis period in which TIPS' yields do not clearly followed real rates. Furthermore, the issuance of these bonds in the first decade of the 21st century was relatively scarce. In this sense, when building up the ladder portfolio, particularly for shorter investment horizons, we were often forced to adopt a time deposit strategy (using the nominal rates) since there were no bonds with residual maturity close to or below the sub-portfolio target maturities available in the market. Hence, for these cases, the added inflation accrual effect was not considered.

Furthermore, when analyzing the turnover and transaction costs of the Naive U.S. TIPS portfolios, depicted in Table 2, we observe that they exceed those of the M-Absolute strategy. This means that constraints on portfolio choice, namely the obligation to invest in a few bonds (often, a single one) also erodes the portfolio's performance due to the low liquidity of these bonds. In this sense, our results suggest that the M-Absolute strategy favors the construction of more efficient portfolios combining several bonds to cater for the investment in shorter maturities, thus taking advantage of the inflation accrual for these maturities as well. This effect is also exacerbated by the somewhat low maturity profile of these portfolios, i.e., we assume that for longer holding periods the performance of the Naive strategy is likely to be more in line with that of the M-Absolute strategy, but this hypothesis must be empirically validated.

To summarize, our results corroborate Fogler (1984) and Siegel and Waring (2004) and suggest that the performance of targeted multiple liability immunization (M-Absolute model) considering liabilities that are fixed in timing but random in size (inflation-linked) is superior when the immunizing portfolio includes inflation-linked bonds (U.S. TIPS), when compared to that obtained using nominal bonds (U.S. Treasuries). This conclusion does not hold when considering Naive portfolios since they do not allow investors to properly benefit from the features of U.S. TIPS (namely the inflation accrual).

## 5. Sensitivity Analysis: Zero Interest Rate Scenario

The sample period analyzed in this study is marked by the U.S. subprime crisis, the collapse of Lehman Brothers and the entrance of Fannie Mae & Freddie Mac into Conservatorship in September 2008, significant monetary policy stimulus, e.g., the implementation of an extensive quantitative easing toolkit which included outright purchase programs of several assets, extensive credit lines negotiated with other central banks (e.g., Bank of Japan, European Central Bank) and the maintenance of the fed funds rate between 0% and 0.25% for an extensive period of time (December 2008 to December 2015). This last policy measure is often denominated as the zero interest rate policy (ZIRP) and it will be the main focus of this sensitivity analysis.

The aim of this sensitivity analysis is to evaluate whether the implementation of the ZIRP, which stabilized nominal interest rates at historically low levels and stimulated inflation growth (thus rendering real interest rates into negative territory), has an impact on the overall performance of the multiple liability M-Absolute immunization strategy. This is done by splitting the immunization portfolios into two distinct periods: (i) the pre-crisis period, comprising portfolios set up before December 2008 and (ii) the ZIRP period, comprising all portfolios set up after that date. The average liability coverage during the pre-crisis period was 4.30% and 9.32% for 3- and 5-year horizons, respectively. The average liability coverage decreased significantly in the ZIRP period, to 0.43% and 1.20% for 3- and 5-year horizons, respectively.

The results for the 3- and 5-year immunization horizons for the Pre-crisis and ZIRP periods for portfolios including U.S. Treasuries only are depicted in Table 4. In the pre-crisis period the immunization strategy generates sufficient cash flows to cover the multiple liability cash flow stream but during the ZIRP period the strategy fails to guarantee the targeted benefit payments by an amount ranging between −1.15% and −1.98% in the 3-year holding period and between −1.21% and −2.34% in the 5-year holding period.

When analyzing risk and return figures, the average excess return and Reward-to-Risk ratios of immunizing portfolios in the pre-crisis period is generally better when compared to that achieved in the overall sample period. This is of course explained by the extremely low nominal and negative real interest rates observed during the ZIRP period. However, achieving positive excess returns is not the primary aim of an immunization strategy. We also observe an increase in transaction costs during the ZIRP period, which could be attributed to lower market liquidity recorded in this period. The immunization performance of the 3- and 5-year portfolios is fairly stable in the pre-crisis period for the early years of the holding period (up to the third year).

Table 5 reports the results for the U.S. TIPS immunization strategy considering the inflation accrual dataset. Contrary to portfolios formed using U.S. Treasuries, in this case the immunization strategy proved to be more robust and generated sufficient cash flows to cover the multiple liability cash flow stream in both periods.

These results are in line with those reported in the previous section for the U.S. TIPS portfolios, suggesting that the inclusion of the inflation accrual somehow compensates for the negative real interest rates observed in the ZIRP period. The transaction costs and turnover measures obtained in these scenarios mimic those reported for the overall sample period, further suggesting that the inclusion of transaction costs does not significantly affect the superiority of U.S. TIPS portfolios in immunizing inflation-linked liability cash flow streams.

**Table 4.** U.S. Treasuries Sensitivity Analysis Immunization Results.

| Panel A—3-Year Immunization Horizon | | | | | |
|---|---|---|---|---|---|
| **Pre-Crisis** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 1.74% | 0.88% | 54.32% | 1.57% | 29.65 |
| 2-year | 4.26% | 1.78% | 82.93% | 0.92% | 19.05 |
| 3-year | 4.95% | 0.73% | 30.61% | 0.75% | 17.54 |
| **ZIRP** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | −1.15% | 0.33% | 15.26% | 1.77% | 28.07 |
| 2-year | −1.98% | 0.29% | 56.04% | 1.57% | 25.37 |
| 3-year | −1.92% | 0.41% | 42.49% | 0.98% | 21.78 |
| **Panel B—5-year immunization horizon** | | | | | |
| **Pre-crisis** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 1.74% | 0.88% | 54.32% | 1.57% | 29.67 |
| 2-year | 4.26% | 1.78% | 82.64% | 0.92% | 19.05 |
| 3-year | 4.95% | 0.73% | 30.61% | 0.75% | 18.44 |
| 4-year | 9.19% | 2.19% | 77.52% | 0.46% | 14.99 |
| 5-year | 11.09% | 1.55% | 51.08% | 0.41% | 15.15 |
| **ZIRP** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | −1.55% | 0.29% | 22.50% | 2.12% | 28.69 |
| 2-year | −2.34% | 0.34% | 67.21% | 3.31% | 22.00 |
| 3-year | −1.86% | 0.62% | 58.55% | 0.71% | 20.96 |
| 4-year | −1.79% | −0.14% | −10.14% | 1.26% | 16.68 |
| 5-year | −1.21% | −0.74% | −45.56% | 1.00% | 18.23 |

This table presents the results for the sensitivity analysis and is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels present the results for the Pre-crisis and ZIRP periods and report the same metrics. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio ($R/R_h$). As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

**Table 5.** U.S. TIPS with Inflation Accrual Sensitivity Analysis Immunization Results.

| Panel A—3-Year Immunization Horizon | | | | | |
|---|---|---|---|---|---|
| **Pre-Crisis** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 7.28% | 10.00% | 234.60% | 0.59% | 21.74 |
| 2-year | 14.52% | 19.76% | 343.34% | 0.44% | 16.80 |
| 3-year | 15.62% | 23.79% | 395.66% | 0.33% | 13.48 |
| **ZIRP** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 4.30% | 7.79% | 168.46% | 0.64% | 19.21 |
| 2-year | 3.42% | 10.50% | 167.90% | 0.39% | 10.36 |
| 3-year | 0.43% | 10.66% | 164.61% | 0.32% | 7.29 |
| **Panel B—5-year immunization horizon** | | | | | |
| **Pre-crisis** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 7.28% | 7.40% | 173.59% | 0.58% | 21.74 |
| 2-year | 14.52% | 14.24% | 247.48% | 0.44% | 16.80 |
| 3-year | 15.62% | 15.19% | 252.59% | 0.33% | 13.48 |
| 4-year | 17.72% | 16.89% | 259.45% | 0.29% | 10.85 |
| 5-year | 26.49% | 24.50% | 288.68% | 0.39% | 13.24 |
| **ZIRP** | $\overline{\mathbf{LC}}$ | $\overline{\mathbf{ER}}$ | $\boldsymbol{R/R_h}$ | $\overline{\mathbf{TC}}$ | $\overline{\mathbf{T}}$ |
| 1-year | 6.61% | 8.55% | 190.82% | 0.69% | 20.72 |
| 2-year | 5.73% | 9.34% | 148.98% | 0.43% | 11.57 |
| 3-year | 2.66% | 7.45% | 112.98% | 0.33% | 7.76 |
| 4-year | 1.20% | 4.51% | 69.42% | 0.20% | 3.68 |
| 5-year | 9.32% | 15.59% | 205.91% | 0.75% | 15.58 |

This table presents the results for the sensitivity analysis and is divided in two panels. Panel A contains the immunization results for the 3-year horizon and Panel B contains the results for the 5-year horizon. Both Panels present the results for the Pre-crisis and ZIRP periods and report the same metrics. Immunization Coverage and Performance metrics include the Average Liability Coverage ($\overline{LC}$), Average Excess Return ($\overline{ER}$) and the Risk-to-Reward ratio ($R/R_h$)As for the Immunization Costs metrics, Average Transaction Costs ($\overline{TC}$) and Average Turnover ($\overline{T}$) are calculated.

## 6. Conclusions

In this paper, we empirically investigated the performance of alternative immunization strategies for funding targeted multiple liabilities that are fixed in timing but random in size (inflation-linked) using two alternative datasets, comprising bond prices for on-the-run U.S. Treasuries and U.S. TIPS for the period from January 2000 to December 2018. The immunization procedure involves minimizing the difference between the M-Absolute risk measure of the asset portfolio and the residual maturity of the liability cash flow stream.

The results suggest that the M-Absolute strategy using U.S. TIPS outperforms the equivalent strategy using U.S. Treasuries in both the 3-year and 5-year investment horizons. The average liability coverage of immunizing portfolios using U.S. TIPS and considering the inflation accrual is much higher when compared to the alternative of using nominal bonds only. The inclusion of the inflation accrual allied with the divergent behavior of inflation and real interest rates observed in the analyzed sample period, mostly explain the outperformance of the M-Absolute strategy using U.S. TIPS.

The superior performance of the M-Absolute strategy using U.S. TIPS is not accompanied by the Naive portfolio, whose average liability coverage is consistently negative for most maturities. Contrary to previous studies that analyzed the importance of portfolio design in the performance of single- and multiple-factor immunization strategies considering a single liability, our results suggest that in multiple liability immunization problems with random (inflation-linked) liability payments the role of the maturity bond is less evident. Although the reduced depth and liquidity of the U.S. TIPS market, when compared to the

U.S. Treasuries market may partially justify the reduced importance of the maturity bond in immunizing portfolios found in this study. The security design of U.S. TIPS and the compounding inflation accrual effect lost by forcing immunizing portfolios to include bonds with maturity close to each of the multiple liability due dates is identified as the main explanation for this result.

The relative performance of the M-Absolute strategy against that of the alternative Naive ladder portfolios in the U.S. Treasuries dataset is mixed. Our results show that for the 1-, 2- and 4-year maturities the M-Absolute strategy performs better, while for the 3- and 5-year maturities the Naive portfolios' performance is higher. This is mostly explained by the higher turnover of M-Absolute strategies in some holding periods that often require substantial portfolio restructuring at each quarterly rebalancing period. These results also highlight the potential impact of transaction costs in the performance of multiple liability immunization strategies.

As expected, our results show an inverse relationship between the average return and transaction costs, with portfolios with higher average transaction costs exhibiting lower average excess returns. The average turnover is generally lower for long holding periods, suggesting that in shorter investment horizons the multiple liability immunization problem requires more frequent rebalancing to achieve a perfect immunization. This result can be partially explained by the fact that most U.S. Treasuries and U.S. TIPS bonds considered in this study have medium and long-term maturities (above 5 years).

The sensitivity analysis results show that the multiple liability M-Absolute immunization strategy using U.S. TIPS bonds is robust in extreme yield curve scenarios, generating sufficient cash flows in both the pre-crisis and the nearly zero interest rate periods. This is in contrast with M-Absolute immunization portfolios comprising only U.S. Treasuries that generated sufficient cash flows to cover the multiple liability payments in the relatively high interest rate scenarios in the pre-crisis period but significantly failed to guarantee the targeted benefit payments in the nearly zero interest rate scenarios during the after-crisis period.

Even though the tested holding periods can be seen as short from the perspective of retirement income benefits management, the M-Absolute results are shown to be optimal investment strategies. Building low turnover portfolios, with assets whose cash flow profile mimics the future liabilities in a persistent low-to-zero interest rate will likely pay off in the long run.

Although interest rates and inflation are currently expected to remain low over the near to medium term, the rapid and potentially unsustainable increase in the public debt burden of most countries because of COVID-19 pandemic outbreak could cause inflation to head structurally higher, a solution less socially unpopular and potentially counterproductive when compared with austerity measures or a default. In this scenario, the inflation carry is a return component that makes ILBs a great match for pension funds and life insurance providers offering inflation-linked retirement benefits.

The deterministic setup presented is also quite simple to replicate and could prove beneficial in portfolio management for pension funds and insurers where inflation-linked bonds are available, thus reaping the benefits of these instruments while applying different risk immunization measures that can attain better immunization results when compared with the traditional Fisher-Weil duration. In some countries with less developed financial markets and highly regulated retirement savings markets, setting up the immunization strategy may require additional optimization constraints. For instance, in some Central and Eastern Europe private pension fund markets, the lack of investment opportunities domestically, capital controls, the existence of an annual non-negative nominal guarantee to plan members and investment regulations requiring pension providers to invest a certain proportion of their assets in certain instruments would have to be accommodated.

Further empirical research should assess (i) the robustness of these results in different Treasury bond markets using alternative interest rate risk models, different inflation-linked bond profiles (e.g., year-on-year inflation accruals) and longer holding periods; (ii) the application of stochastic models to estimate the term structure of interest rates and to improve the portfolio immunization techniques to infer if the results obtained are sensitive to the models used and (iii) to adapt these strategies to the application of inflation derivatives to hedge inflation in highly regulated markets where inflation-linked-bonds might not be available (For a meaningful contribution regarding this issue, please refer to Anton et al. 2016).

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## Appendix A. U.S. TIPS Inflation Adjustment

Since time is needed to compile and publish the data for the index, the inflation adjustment is made with a lag, even though this lag is minimized by making it as short as possible. The indexation lag applied to U.S. TIPS is 3 months and the reference index is computed by linear interpolation, between index publications, as Equation (A1) shows.

$$I = CPI_{v-3} + \frac{(d-1)}{D_v} \times (CPI_{v-2} - CPI_{v-3}), \quad \text{(A1)}$$

where $I$ is the reference index for day $t$, $CPI_{v-3}$ is the value of the price index at time $v - 3$ months, $CPI_{v-2}$ is the value of the price index at time $v - 2$ months, $D_v$ is the number of days in month $v$, $d$ is the day of the month $v$ when settlement occurs and $v$ is the month on which settlement takes place.

Applying this formula to the day in which the inflation accrual for the bond begins, by substituting the $D_v$ day for the first day when the bond's inflation component starts to accrue (the base day), allows for the calculation of the base index ($I_{base}$). This way, it is possible to compute a daily Index Ratio to adjust for daily inflation changes in the bond and whenever it is traded, making the inflation accrual steadily over each month instead of adjusting only once a month, when the new figure of the price index is published. The daily adjusted Index Ratio is given by the expression

$$IR_t = \frac{I_t}{I_{base}}, \quad \text{(A2)}$$

where $IR_t$ stands for Index Ratio in day $t$. Both indices used to compute $IR_t$ are truncated to six decimal places and then rounded to five decimal places. To compute cash settlement amounts, real accrued interest is computed as done for nominal fixed rate bonds. Then, clean price and real accrued interest are each multiplied by the Index Ratio. As for coupon and principal amounts, the process is the same: each is multiplied by the Index Ratio computed with reference to the day when they are calculated. However, if at maturity the Index Ratio is less than one (this will happen if deflation occurs), the inflation floor will be triggered, and the principal amount will be redeemed at par value.

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