

Computational Experience with Piecewise-Linear Relaxations for Petroleum Refinery Planning

Zaid Ashraf Rana¹, Cheng Seong Khor^{1,2*}, Haslinda Zabiri^{1,3}

¹ Chemical Engineering Department, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak Darul Ridzuan, Malaysia; zaid_20000015@utp.edu.my, zaidashrf@gmail.com

² Centre for Systems Engineering, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak Darul Ridzuan, Malaysia; chengseong.khor@utp.edu.my; khorchengseong@gmail.com

³ Centre for CO₂ Research, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak Darul Ridzuan, Malaysia; haslindazabiri@utp.edu.my

* Correspondence: chengseong.khor@utp.edu.my (C. S. Khor), khorchengseong@gmail.com (C. S. Khor), haslindazabiri@utp.edu.my (H. Zabiri)

Supplementary Material

Appendix A: Mathematical Formulation

A.1. Crude Distillation Unit

Total crude oil feed to CDU is given by:

$$\sum_p Q_{u,p} \leq \sum_{cr} F_{cr}, u = \text{CDU} \quad (\text{A1})$$

where F_{cr} = flow rate of crude oil type cr and $Q_{u,p}$ = flow rate of CDU fraction (i.e., cut) p .

CDU capacity is described by:

$$L_u \leq cp_u^{\max}, \quad \forall u \quad (\text{A2})$$

where L_u = load of unit u and cp_u^{\max} = maximum capacity of u .

CDU outlet flow rate of cut p is given by:

$$Q_{u,p} = L_u W_p, \quad u = \text{CDU}, \quad \forall p \quad (\text{A3})$$

where W_p = weight transfer ratio of p that is determined based on true boiling point data of cr .

Weight transfer ratio of p sums to unity:

$$\sum_p W_p = 1 \quad (\text{A4})$$

Middle-of-point (or midpoint) weight transfer ratio MW_p of fraction p is given by:

$$MW_p = 100 \left(\sum_{p'} W_{p'} + \frac{1}{2} W_p \right), \quad p' < p, \forall p: (p, p') \neq \text{BR}. \quad (\text{A5})$$

A.2. Fluid Catalytic Cracking Unit

CDU cut of bottom residue is fed to FCC to be converted into more valuable products. FCC outlet flow rate of product fraction f is given by:

$$Q_{\text{FCC},f} = L_u Y_f, \quad u = \text{FCC}, \forall f \quad (\text{A6})$$

where $Q_{\text{FCC},f}$ = flow rate of f from FCC and Y_f = weight transfer ratio of f from FCC.

All weight transfer ratios of f sum to unity:

$$\sum_f Y_f = 1. \quad (\text{A7})$$

Y_f is determined using the following regression-based relation:

$$Y_f = a0_f + a1_f(\text{conv} - z_f) + a2_f(\text{conv} - z_f)^2, \quad \forall f \quad (\text{A8})$$

where regression coefficients given by $a0_f$, $a1_f$, and $a2_f$ are known constants.

To achieve a desired FCC conversion level, part of its outlet flow of total gas oil (TGO) is recycled (as Q_p^R) and mixed with total inlet feed (Q_u^T):

$$Q_{\text{FCC}}^T = Q_{\text{CDU},\text{BR}} + Q_{\text{TGO}}^R \quad (\text{A9})$$

where Q_{FCC}^T = total inlet flow rate to FCC, $Q_{\text{CDU, BR}}$ = flow rate of bottom residue (BR) outlet stream from CDU, and Q_{TGO}^R = flow rate of TGO recycle stream.

FCC load is equal to its inlet flow rate:

$$L_{\text{FCC}} = Q_{\text{FCC}}^T \quad (\text{A10})$$

TGO recycle stream flow rate are bounded (from above) by the following constraints:

$$Q_{\text{TGO}}^R \leq \frac{1}{2} Q_{\text{CDU, BR}} \quad (\text{A11})$$

$$Q_{\text{TGO}}^R \leq Q_{\text{FCC, FHO}}^P \quad (\text{A12})$$

Remaining TGO stream (after split for recycle) is sold as heavy oil (FHO):

$$Q_{\text{FCC, FHO}} = Q_{\text{FCC, TGO}} - Q_{\text{TGO}}^R \quad (\text{A13})$$

where $Q_{\text{FCC, FHO}}$ = flow rate of FHO product from FCC.

A.3. Gasoline Blending Unit

Lighter CDU fractions of GO and HN are processed further to improve their for gasoline blending to meet required research octane number (RON) specifications:

$$Q_{\text{CDU}, p} = \sum_{g \in G} F_{p,g}^P, \quad \forall p \in P_g \quad (\text{A14})$$

where $F_{p,g}^P$ = flow rate of gasoline product grade g , G = set of gasoline product grades with RON of 90 (g_{90}) and 93 (g_{93}), and P_g = set of CDU fractions for gasoline blending.

To improve product quality, additives (e.g., MTBE) are mixed with blended CDU fractions according to the following relation:

$$Q_p^P = \sum_{g \in G} F_{p,g}^P + \sum_r Q_r^A, \quad \forall p \quad (\text{A15})$$

where Q_r^A = flow rate of additive r and Q_p^P = flow rate of final product p .

FCC gasoline fraction called F_{gas} is blended to improve its quality. The flow rate of FCC blended fraction equals the sum of flow rates of its respective blended products, as follows:

$$Q_{t,\text{Fgas}}^{fprod} = \sum_g F_{\text{Fgas},g}^{iprod} \quad (\text{A16})$$

where $Q_{t,f}^{fprod}$ = flow rate of final product fraction of F_{gas} , $F_{f,g}^{iprod}$ is the flow rate of intermediate blended product g , which is produced by blending flow stream F_{gas} from FCC.

The gasoline final products g90 and g93 are sold to customers. Their flow rates are calculated using equation (A17).

$$Q_g^{fprod} = F_{f,g}^{iprod} + \sum_r F_{r,g}^{iprod} + \sum_p F_{p,g}^{iprod}, f = \text{Fgas}, p \in P_g, \forall g \quad (\text{A17})$$

where Q_g^{fprod} represents flow of final product g .

A.4. Diesel Blending Unit

Heavier CDU fractions LD and HD are blended in the DB to improve their properties such as, pour point. Flow rate of each CDU fraction p to DB equals the sum of flow rates of its respective diesel blended products (d0 and d10). These blended products are called as $iprod$, and modelled using equation (A18).

$$Q_{u,p} = \sum_d F_{p,d}^{iprod}, u = \text{CDU}, \forall p \in P_d \quad (\text{A18})$$

where $F_{p,d}^{iprod}$ is the flow rate of intermediate product d from the DB, which is produced by blending feed p (LD, HD) from the CDU.

The final products d10 and d0, from the DB are sold to customers. Their flow rates are calculated using equation (A19).

$$Q_d^{fprod} = \sum_p F_{p,d}^{iprod}, p \in P_d, \forall d \quad (\text{A19})$$

where Q_d^{fprod} represents flow rate of final product d from DB.

A.5. Quality Specifications

Octane numbers of light CDU fractions GO and HN and pour points of CDU heavy fractions LD and HD are calculated using property correlations from the literature:

$$Pr_{j,p} = a0_p + a1_p(MW_p - z_p) + a2_p(MW_p - z_p)^2, \forall j, \forall p \in (P_g \cup P_d) \quad (\text{A20})$$

Minimum octane number specifications for gasoline blended product g90 and g93 are given by:

$$\text{RON}_g Q_g^{fprod} \leq Pr_{j,p} F_{p,g}^{iprod} + Pr_{j,r} F_{r,g}^{iprod} + Pr_{j,p'} F_{p',g}^{iprod} + Pr_{j,f} F_{f,g}^{iprod}, \quad (\text{A21})$$

$$j = \text{ON}, p = \text{GO}, p' = \text{HN}, f = \text{Fgas}, \forall g, r$$

where RON_g is the research octane number of gasoline blended products g90 and g93.

A.6. Demand Requirement

Market demand of final products s (g90, g93, d0, d10, FHO, C24) is written as:

$$Q_s^{fprod} \leq D_s^{\max}, \forall s \quad (\text{A22})$$

where D_s^{\max} is the maximum demand of final product s .

A.7. Variable Bounds

FCC conversion level is bounded by equation (A23).

$$conv^{\text{LO}} \leq conv \leq conv^{\text{UP}} \quad (\text{A23})$$

A.8. Objective Function

The objective function for refinery profit is defined as:

*Profit = Price of valuable products – crude oil cost – additive raw materials cost
– operational cost of units*

$$Profit = \sum_s Q_s^{fprod} C_s^{fprod} - \sum_{cr} L_{u'} C_{cr} - \sum_r F_{r,g}^{iprod} C_r - \sum_u L_u C_u, u' = CDU, \forall g \quad (A24)$$

where C_s^{fprod} is the price of sellable products, C_{cr} is the cost of crude oil, C_r is the cost of additive raw materials, C_u is the operating cost of process unit u .

Reformulation of Nonlinear Equations

Total 21 bilinear terms are identified in the NLP model equations. All the equations containing bilinear terms are first reformulated to apply LP/MILP relaxations on each bilinear term.

Equation (A3) is reformulated as equation (A25).

$$Q_{u,p} = RL1_p, u = CDU, \forall p \quad (A25)$$

where $RL1_p$ replaces every occurrence of bilinear term $L_u W_p$ (when $u=CDU$).

Equation (A6) is reformulated as equation (A26).

$$Q_{FCC,f} = RL2_f, \forall f \quad (A26)$$

where $RL2_f$ replaces every occurrence of bilinear term $L_u Y_f$ (when $u=FCC$)

Equation (A8) is reformulated as equation (A27).

$$Y_f = a0_f + a1_f(conv - z_f) + a2_f(RL3 - 2conv \cdot z_f + z_f^2), \forall f \quad (A27)$$

where $RL3$ replaces every occurrence of bilinear term $conv^2$.

Equation (A20) is reformulated as equation (A28).

$$Pr_{j,p} = a0_p + a1_p(MW_p - z_p) + a2_p(RL4_p - 2z_p MW_p + z_p^2), j = ON, \forall p \in P_g \quad (A28)$$

where $RL4_p$ replaces every occurrence of bilinear term MW_p^2 (when $p \in P_g$).

Also,

$$Pr_{j,p} = a0_p + a1_p(MW_p - z_p) + a2_p(RL5_p - 2z_pMW_p + z_p^2), j = \text{PP}, \forall p \in P_d \quad (\text{A29})$$

where $RL5_p$ replaces every occurrence of bilinear term MW_p^2 (when $p \in P_d$).

Equation (A21) is reformulated as follows:

$$\text{RON}_g Q_g^{fprod} \leq RL6 + Pr_{j,r} F_{r,g}^{iprod} + RL7 + Pr_{j,f} F_{f,g}^{iprod}, \quad (\text{A30})$$

$$g = \text{g90}, f = \text{Fgas}, j = \text{ON}, r = \text{MTBE}$$

where

$$RL6 = Pr_{j,p} F_{p,g}^{iprod}, j = \text{ON}, p = \text{GO}, g = \text{g90}$$

$$RL7 = Pr_{j,p'} F_{p',g}^{iprod}, j = \text{ON}, p' = \text{HN}, g = \text{g90}$$

Also,

$$\text{RON}_g Q_g^{fprod} \leq RL8 + Pr_{j,r} F_{r,g}^{iprod} + RL9 + Pr_{j,f} F_{f,g}^{iprod} \quad (\text{A31})$$

$$g = \text{g93}, f = \text{Fgas}, j = \text{ON}, r = \text{MTBE}$$

where

$$RL8 = Pr_{j,p} F_{p,g}^{iprod}, j = \text{ON}, p = \text{GO}, g = \text{g93}$$

$$RL9 = Pr_{j,p'} F_{p',g}^{iprod}, j = \text{ON}, p' = \text{HN}, g = \text{g93}$$

A.9. mc Relaxation

The **mc** relaxations for all the reformulated equations are as follows. Here LO and UP refer to lower and upper bounds of the variables. The resulting relaxed model gives rise to a LP.

$$RL1_p \geq L_u^{\text{LO}} W_p + L_u W_p^{\text{LO}} - L_u^{\text{LO}} W_p^{\text{LO}} \quad (\text{A32})$$

$$RL1_p \geq L_u^{\text{UP}} W_p + L_u W_p^{\text{UP}} - L_u^{\text{UP}} W_p^{\text{UP}} \quad (\text{A33})$$

$$RL1_p \leq L_u^{\text{UP}} W_p + L_u W_p^{\text{LO}} - L_u^{\text{UP}} W_p^{\text{LO}} \quad (\text{A34})$$

$$RL1_p \leq L_u^{\text{LO}} W_p + L_u W_p^{\text{UP}} - L_u^{\text{LO}} W_p^{\text{UP}} \quad (\text{A35})$$

$$u = \text{CDU}, \forall p$$

$$RL2_f \geq L_u^{\text{LO}} Y_f + L_u Y_f^{\text{LO}} - L_u^{\text{LO}} Y_f^{\text{LO}} \quad (\text{A36})$$

$$RL2_f \geq L_u^{\text{UP}} Y_f + L_u Y_f^{\text{UP}} - L_u^{\text{UP}} Y_f^{\text{UP}} \quad (\text{A37})$$

$$RL2_f \leq L_u^{\text{UP}} Y_f + L_u Y_f^{\text{LO}} - L_u^{\text{UP}} Y_f^{\text{LO}} \quad (\text{A38})$$

$$RL2_f \leq L_u^{\text{LO}} Y_f + L_u Y_f^{\text{UP}} - L_u^{\text{LO}} Y_f^{\text{UP}} \quad (\text{A39})$$

$$u = \text{FCC}, \forall f$$

$$RL3 \geq conv^{\text{LO}} \cdot conv + conv \cdot conv^{\text{LO}} - conv^{\text{LO}} \cdot conv^{\text{LO}} \quad (\text{A40})$$

$$RL3 \geq conv^{\text{UP}} \cdot conv + conv \cdot conv^{\text{UP}} - conv^{\text{UP}} \cdot conv^{\text{UP}} \quad (\text{A41})$$

$$RL3 \leq conv^{\text{UP}} \cdot conv + conv \cdot conv^{\text{LO}} - conv^{\text{UP}} \cdot conv^{\text{LO}} \quad (\text{A42})$$

$$RL3 \leq conv^{\text{LO}} \cdot conv + conv \cdot conv^{\text{UP}} - conv^{\text{LO}} \cdot conv^{\text{UP}} \quad (\text{A43})$$

$$RL4_p \geq MW_p^{\text{LO}} \cdot MW_p + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{LO}} \cdot MW_p^{\text{LO}} \quad (\text{A44})$$

$$RL4_p \geq MW_p^{\text{UP}} \cdot MW_p + MW_p \cdot MW_p^{\text{UP}} - MW_p^{\text{UP}} \cdot MW_p^{\text{UP}} \quad (\text{A45})$$

$$RL4_p \leq MW_p^{\text{UP}} \cdot MW_p + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{UP}} \cdot MW_p^{\text{LO}} \quad (\text{A46})$$

$$RL4_p \leq MW_p^{\text{LO}} \cdot MW_p + MW_p \cdot MW_p^{\text{UP}} - MW_p^{\text{LO}} \cdot MW_p^{\text{UP}} \quad (\text{A47})$$

$$\forall p \in P_g$$

$$RL5_p \geq MW_p^{\text{LO}} \cdot MW_p + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{LO}} \cdot MW_p^{\text{LO}} \quad (\text{A48})$$

$$RL5_p \geq MW_p^{\text{UP}} \cdot MW_p + MW_p \cdot MW_p^{\text{UP}} - MW_p^{\text{UP}} \cdot MW_p^{\text{UP}} \quad (\text{A49})$$

$$RL5_p \leq MW_p^{\text{UP}} \cdot MW_p + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{UP}} \cdot MW_p^{\text{LO}} \quad (\text{A50})$$

$$RL5_p \leq MW_p^{\text{LO}} \cdot MW_p + MW_p \cdot MW_p^{\text{UP}} - MW_p^{\text{LO}} \cdot MW_p^{\text{UP}} \quad (\text{A51})$$

$$\forall p \in P_d$$

$$RL6 \geq F_{p,g}^{iprod,\text{LO}} Pr_{j,p} + Pr_{j,p}^{\text{LO}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{LO}} \quad (\text{A52})$$

$$RL6 \geq F_{p,g}^{iprod,\text{UP}} Pr_{j,p} + Pr_{j,p}^{\text{UP}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} Pr_{j,p}^{\text{UP}} \quad (\text{A53})$$

$$RL6 \leq F_{p,g}^{iprod,\text{UP}} Pr_{j,p} + Pr_{j,p}^{\text{LO}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} Pr_{j,p}^{\text{LO}} \quad (\text{A54})$$

$$RL6 \leq F_{p,g}^{iprod,\text{LO}} Pr_{j,p} + Pr_{j,p}^{\text{UP}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{UP}} \quad (\text{A55})$$

$$p = \text{GO}, j = \text{ON}, g = \text{g90}$$

$$RL7 \geq F_{p',g}^{iprod,\text{LO}} Pr_{j,p'} + Pr_{j,p'}^{\text{LO}} F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{LO}} Pr_{j,p'}^{\text{LO}} \quad (\text{A56})$$

$$RL7 \geq F_{p',g}^{iprod,\text{UP}} Pr_{j,p'} + Pr_{j,p'}^{\text{UP}} F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{UP}} Pr_{j,p'}^{\text{UP}} \quad (\text{A57})$$

$$RL7 \leq F_{p',g}^{iprod,\text{UP}} Pr_{j,p'} + Pr_{j,p'}^{\text{LO}} F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{UP}} Pr_{j,p'}^{\text{LO}} \quad (\text{A58})$$

$$RL7 \leq F_{p',g}^{iprod,\text{LO}} Pr_{j,p'} + Pr_{j,p'}^{\text{UP}} F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{LO}} Pr_{j,p'}^{\text{UP}} \quad (\text{A59})$$

$$p' = \text{HN}, j = \text{ON}, g = \text{g90}$$

$$RL8 \geq F_{p,g}^{iprod,\text{LO}} Pr_{j,p} + Pr_{j,p}^{\text{LO}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{LO}} \quad (\text{A60})$$

$$RL8 \geq F_{p,g}^{iprod,\text{UP}} Pr_{j,p} + Pr_{j,p}^{\text{UP}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} Pr_{j,p}^{\text{UP}} \quad (\text{A61})$$

$$RL8 \leq F_{p,g}^{iprod,\text{UP}} Pr_{j,p} + Pr_{j,p}^{\text{LO}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} Pr_{j,p}^{\text{LO}} \quad (\text{A62})$$

$$RL8 \leq F_{p,g}^{iprod,\text{LO}} Pr_{j,p} + Pr_{j,p}^{\text{UP}} F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{UP}} \quad (\text{A63})$$

$$p = \text{GO}, j = \text{ON}, g = \text{g93}$$

$$RL9 \geq F_{p',g}^{iprod,LO} Pr_{j,p'} + Pr_{j,p'}^{LO} F_{p',g}^{iprod} - F_{p',g}^{iprod,LO} Pr_{j,p'}^{LO} \quad (A64)$$

$$RL9 \geq F_{p',g}^{iprod,UP} Pr_{j,p'} + Pr_{j,p'}^{UP} F_{p',g}^{iprod} - F_{p',g}^{iprod,UP} Pr_{j,p'}^{UP} \quad (A65)$$

$$RL9 \leq F_{p',g}^{iprod,UP} Pr_{j,p'} + Pr_{j,p'}^{LO} F_{p',g}^{iprod} - F_{p',g}^{iprod,UP} Pr_{j,p'}^{LO} \quad (A66)$$

$$RL9 \leq F_{p',g}^{iprod,LO} Pr_{j,p'} + Pr_{j,p'}^{UP} F_{p',g}^{iprod} - F_{p',g}^{iprod,LO} Pr_{j,p'}^{UP} \quad (A67)$$

$$p' = GO, j = ON, g = g93$$

A.10. **bm** Relaxation

The **bm** relaxations for all the reformulated equations are as follows. Here M1, M2,...M9 are the big-M parameters, $\lambda 1, \lambda 2, \dots, \lambda 9$ are the binary variables that are equal to one when the relaxed variable is activated in the subdomain n , and k1, k2,...k9 are the grid points of segments n for the relaxed variable. The resulting relaxed model gives rise to an MILP.

$$\sum_n \lambda 1_{n,p} = 1, \forall p \quad (A68)$$

$$W_p \geq W_p^{LO} + (k1_{n-1,p} - W_p^{LO})\lambda 1_{n,p}, \forall n, p \quad (A69)$$

$$W_p \leq W_p^{UP} - (W_p^{UP} - k1_{n,p})\lambda 1_{n,p}, \forall n, p \quad (A70)$$

$$RL1_p \geq W_p L_u^{LO} + k1_{n-1,p} (L_u - L_u^{LO}) - M1_p (1 - \lambda 1_{n,p}) \quad (A71)$$

$$RL1_p \geq W_p L_u^{UP} + k1_{n,p} (L_u - L_u^{UP}) - M1_p (1 - \lambda 1_{n,p}) \quad (A72)$$

$$RL1_p \leq W_p L_u^{UP} + k1_{n-1,p} (L_u - L_u^{UP}) + M1_p (1 - \lambda 1_{n,p}) \quad (A73)$$

$$RL1_p \leq W_p L_u^{LO} + k1_{n,p} (L_u - L_u^{LO}) + M1_p (1 - \lambda 1_{n,p}) \quad (A74)$$

$$u = CDU, \forall n, p$$

$$\sum_n \lambda 2_{n,f} = 1, \forall f \quad (A75)$$

$$Y_f \geq Y_f^{\text{LO}} + (k2_{n-1,f} - Y_f^{\text{LO}})\lambda2_{n,f}, \forall n, f \quad (\text{A76})$$

$$Y_f \leq Y_f^{\text{UP}} - (Y_f^{\text{UP}} - k2_{n,f})\lambda2_{n,f}, \forall n, f \quad (\text{A77})$$

$$RL2_f \geq Y_f L_u^{\text{LO}} + k2_{n-1,f}(L_u - L_u^{\text{LO}}) - M2_f(1 - \lambda2_{n,f}) \quad (\text{A78})$$

$$RL2_f \geq Y_f L_u^{\text{UP}} + k2_{n,f}(L_u - L_u^{\text{UP}}) - M2_f(1 - \lambda2_{n,f}) \quad (\text{A79})$$

$$RL2_f \leq Y_f L_u^{\text{UP}} + k2_{n-1,f}(L_u - L_u^{\text{UP}}) + M2_f(1 - \lambda2_{n,f}) \quad (\text{A80})$$

$$RL2_f \leq Y_f L_u^{\text{LO}} + k2_{n,f}(L_u - L_u^{\text{LO}}) + M2_f(1 - \lambda2_{n,f}) \quad (\text{A81})$$

$$u = \text{FCC}, \forall n, f$$

$$\sum_n \lambda3_n = 1 \quad (\text{A82})$$

$$conv \geq conv^{\text{LO}} + (k3_{n-1} - conv^{\text{LO}})\lambda3_n, \forall n \quad (\text{A83})$$

$$conv \leq conv^{\text{UP}} - (conv^{\text{UP}} - k3_n)\lambda3_n, \forall n \quad (\text{A84})$$

$$RL3 \geq conv \cdot conv^{\text{LO}} + k3_{n-1}(conv - conv^{\text{LO}}) - M3(1 - \lambda3_n), \forall n \quad (\text{A85})$$

$$RL3 \geq conv \cdot conv^{\text{UP}} + k3_n(conv - conv^{\text{UP}}) - M3(1 - \lambda3_n), \forall n \quad (\text{A86})$$

$$RL3 \leq conv \cdot conv^{\text{UP}} + k3_{n-1}(conv - conv^{\text{UP}}) + M3(1 - \lambda3_n), \forall n \quad (\text{A87})$$

$$RL3 \leq conv \cdot conv^{\text{LO}} + k3_n(conv - conv^{\text{LO}}) + M3(1 - \lambda3_n), \forall n \quad (\text{A88})$$

$$\sum_n \lambda4_{n,p} = 1, \forall p \in P_g \quad (\text{A89})$$

$$MW_p \geq MW_p^{\text{LO}} + (k4_{n-1,p} - MW_p^{\text{LO}})\lambda4_{n,p}, \forall n, p \in P_g \quad (\text{A90})$$

$$MW_p \leq MW_p^{\text{UP}} - (MW_p^{\text{UP}} - k4_{n,p})\lambda4_{n,p}, \forall n, p \in P_g \quad (\text{A91})$$

$$RL4_p \geq MW_p \cdot MW_p^{\text{LO}} + k4_{n-1,p}(MW_p - MW_p^{\text{LO}}) - M4_p(1 - \lambda4_{n,p}) \quad (\text{A92})$$

$$RL4_p \geq MW_p \cdot MW_p^{\text{UP}} + k4_{n,p}(MW_p - MW_p^{\text{UP}}) - M4_p(1 - \lambda4_{n,p}) \quad (\text{A93})$$

$$RL4_p \leq MW_p \cdot MW_p^{\text{UP}} + k4_{n-1,p}(MW_p - MW_p^{\text{UP}}) + M4_p(1 - \lambda4_{n,p}) \quad (\text{A94})$$

$$RL4_p \leq MW_p \cdot MW_p^{\text{LO}} + k4_{n,p}(MW_p - MW_p^{\text{LO}}) + M4_p(1 - \lambda4_{n,p}) \quad (\text{A95})$$

$$\forall n, p \in P_g$$

$$\sum_n \lambda 5_{n,p} = 1, \forall p \in P_d \quad (\text{A96})$$

$$MW_p \geq MW_p^{\text{LO}} + (k5_{n-1,p} - MW_p^{\text{LO}}) \lambda 5_{n,p}, \forall n \quad (\text{A97})$$

$$MW_p \leq MW_p^{\text{UP}} - (MW_p^{\text{UP}} - k5_{n,p}) \lambda 5_{n,p}, \forall n \quad (\text{A98})$$

$$RL5_p \geq MW_p \cdot MW_p^{\text{LO}} + k5_{n-1,p} (MW_p - MW_p^{\text{LO}}) - M5_p (1 - \lambda 5_{n,p}) \quad (\text{A99})$$

$$RL5_p \geq MW_p \cdot MW_p^{\text{UP}} + k4_{n,p} (MW_p - MW_p^{\text{UP}}) - M5_p (1 - \lambda 5_{n,p}) \quad (\text{A100})$$

$$RL5_p \leq MW_p \cdot MW_p^{\text{UP}} + k4_{n-1,p} (MW_p - MW_p^{\text{UP}}) + M5_p (1 - \lambda 5_{n,p}) \quad (\text{A101})$$

$$RL5_p \leq MW_p \cdot MW_p^{\text{LO}} + k5_{n,p} (MW_p - MW_p^{\text{LO}}) + M5_p (1 - \lambda 5_{n,p}) \quad (\text{A102})$$

$$\forall n, p \in P_d$$

$$\sum_n \lambda 6_{n,p} = 1, p = \text{GO} \quad (\text{A103})$$

$$Pr_{j,p} \geq Pr_{j,p}^{\text{LO}} + (k6_{n-1,p} - Pr_{j,p}^{\text{LO}}) \lambda 6_{n,p}, p = \text{GO}, j = \text{ON}, \forall n \quad (\text{A104})$$

$$Pr_{j,p} \leq Pr_{j,p}^{\text{UP}} - (Pr_{j,p}^{\text{UP}} - k6_{n,p}) \lambda 6_{n,p}, p = \text{GO}, j = \text{ON}, \forall n \quad (\text{A105})$$

$$RL6 \geq Pr_{j,p} F_{p,g}^{iprod,\text{LO}} + k6_{n-1,p} (Pr_{j,p} - Pr_{j,p}^{\text{LO}}) - M6_p (1 - \lambda 6_{n,p}) \quad (\text{A106})$$

$$RL6 \geq Pr_{j,p} F_{p,g}^{iprod,\text{UP}} + k6_{n,p} (Pr_{j,p} - Pr_{j,p}^{\text{UP}}) - M6_p (1 - \lambda 6_{n,p}) \quad (\text{A107})$$

$$RL6 \leq Pr_{j,p} F_{p,g}^{iprod,\text{UP}} + k6_{n-1,p} (Pr_{j,p} - Pr_{j,p}^{\text{UP}}) + M6_p (1 - \lambda 6_{n,p}) \quad (\text{A108})$$

$$RL6 \leq Pr_{j,p} F_{p,g}^{iprod,\text{LO}} + k6_{n,p} (Pr_{j,p} - Pr_{j,p}^{\text{LO}}) + M6_p (1 - \lambda 6_{n,p}) \quad (\text{A109})$$

$$p = \text{GO}, j = \text{ON}, g = \text{g90}, \forall n$$

$$\sum_n \lambda 7_{n,p'} = 1, p' = \text{HN} \quad (\text{A110})$$

$$Pr_{j,p'} \geq Pr_{j,p'}^{\text{LO}} + (k7_{n-1,p'} - Pr_{j,p'}^{\text{LO}}) \lambda 7_{n,p'}, p' = \text{HN}, j = \text{ON}, \forall n \quad (\text{A111})$$

$$Pr_{j,p'} \leq Pr_{j,p'}^{\text{UP}} - (Pr_{j,p'}^{\text{UP}} - k7_{n,p'})\lambda7_{n,p'}, p' = \text{HN}, j = \text{ON}, \forall n \quad (\text{A112})$$

$$RL7 \geq Pr_{j,p'} F_{p',g}^{i\text{prod},\text{LO}} + k7_{n-1,p'}(Pr_{j,p'} - Pr_{j,p'}^{\text{LO}}) - M7_{p'}(1 - \lambda7_{n,p'}) \quad (\text{A113})$$

$$RL7 \geq Pr_{j,p'} F_{p',g}^{i\text{prod},\text{UP}} + k7_{n,p'}(Pr_{j,p'} - Pr_{j,p'}^{\text{UP}}) - M7_{p'}(1 - \lambda7_{n,p'}) \quad (\text{A114})$$

$$RL7 \leq Pr_{j,p'} F_{p',g}^{i\text{prod},\text{UP}} + k7_{n-1,p'}(Pr_{j,p'} - Pr_{j,p'}^{\text{UP}}) + M7_{p'}(1 - \lambda7_{n,p'}) \quad (\text{A115})$$

$$RL7 \leq Pr_{j,p'} F_{p',g}^{i\text{prod},\text{LO}} + k7_{n,p'}(Pr_{j,p'} - Pr_{j,p'}^{\text{LO}}) + M7_{p'}(1 - \lambda7_{n,p'}) \quad (\text{A116})$$

$$p' = \text{HN}, j = \text{ON}, g = \text{g90}, \forall n$$

$$\sum_n \lambda8_{n,p} = 1, p = \text{GO} \quad (\text{A117})$$

$$Pr_{j,p} \geq Pr_{j,p}^{\text{LO}} + (k8_{n-1,p} - Pr_{j,p}^{\text{LO}})\lambda8_{n,p}, p = \text{GO}, j = \text{ON}, \forall n \quad (\text{A118})$$

$$Pr_{j,p} \leq Pr_{j,p}^{\text{UP}} - (Pr_{j,p}^{\text{UP}} - k8_{n,p})\lambda8_{n,p}, p = \text{GO}, j = \text{ON}, \forall n \quad (\text{A119})$$

$$RL8 \geq Pr_{j,p} F_{p,g}^{i\text{prod},\text{LO}} + k8_{n-1,p}(Pr_{j,p} - Pr_{j,p}^{\text{LO}}) - M8_p(1 - \lambda8_{n,p}) \quad (\text{A120})$$

$$RL8 \geq Pr_{j,p} F_{p,g}^{i\text{prod},\text{UP}} + k8_{n,p}(Pr_{j,p} - Pr_{j,p}^{\text{UP}}) - M8_p(1 - \lambda8_{n,p}) \quad (\text{A121})$$

$$RL8 \leq Pr_{j,p} F_{p,g}^{i\text{prod},\text{UP}} + k8_{n-1,p}(Pr_{j,p} - Pr_{j,p}^{\text{UP}}) + M8_p(1 - \lambda8_{n,p}) \quad (\text{A122})$$

$$RL8 \leq Pr_{j,p} F_{p,g}^{i\text{prod},\text{LO}} + k8_{n,p}(Pr_{j,p} - Pr_{j,p}^{\text{LO}}) + M8_p(1 - \lambda8_{n,p}) \quad (\text{A123})$$

$$p = \text{GO}, j = \text{ON}, g = \text{g93}, \forall n$$

$$\sum_n \lambda9_{n,p'} = 1, p' = \text{HN} \quad (\text{A124})$$

$$Pr_{j,p'} \geq Pr_{j,p'}^{\text{LO}} + (k9_{n-1,p'} - Pr_{j,p'}^{\text{LO}})\lambda9_{n,p'}, p' = \text{HN}, j = \text{ON}, \forall n \quad (\text{A125})$$

$$Pr_{j,p'} \leq Pr_{j,p'}^{\text{UP}} - (Pr_{j,p'}^{\text{UP}} - k9_{n,p'})\lambda9_{n,p'}, p' = \text{HN}, j = \text{ON}, \forall n \quad (\text{A126})$$

$$RL9 \geq Pr_{j,p'} F_{p',g}^{i\text{prod},\text{LO}} + k9_{n-1,p'}(Pr_{j,p'} - Pr_{j,p'}^{\text{LO}}) - M9_{p'}(1 - \lambda9_{n,p'}) \quad (\text{A127})$$

$$RL9 \geq Pr_{j,p'} F_{p',g}^{i\text{prod},\text{UP}} + k9_{n,p'}(Pr_{j,p'} - Pr_{j,p'}^{\text{UP}}) - M9_{p'}(1 - \lambda9_{n,p'}) \quad (\text{A128})$$

$$RL9 \leq Pr_{j,p'} F_{p',g}^{iprod,UP} + k9_{n-1,p'} (Pr_{j,p'} - Pr_{j,p'}^{UP}) + M9_{p'} (1 - \lambda 9_{n,p'}) \quad (\text{A129})$$

$$RL9 \leq Pr_{j,p'} F_{p',g}^{iprod,LO} + k9_{n,p'} (Pr_{j,p'} - Pr_{j,p'}^{LO}) + M9_{p'} (1 - \lambda 9_{n,p'}) \quad (\text{A130})$$

$$p' = \text{HN}, j = \text{ON}, g = g93, \forall n$$

A.11. **nf5** Relaxation

The **nf5** relaxations for all the reformulated equations are as follows. Here $\theta 1, \theta 2, \dots, \theta 9$ are the binary variables that are equal to one when the relaxed variable is activated in the n th subdomain, $q1, q2, \dots, q9$ are the partition lengths for the relaxed variable, defined as $q_{n,p} = [(n/N)^\gamma - ((n-1)/N)^\gamma](x^{UP} - x^{LO})$, and the continuous variables comprising $dU_n \in [0,1]$, $dQ_n \in [0, y^{UP} - y^{LO}]$, $n = 1, 2, \dots, N$, and $dV_n \in [0, y^{UP} - y^{LO}]$, $n = 1, 2, \dots, N-1$. The resulting relaxed model gives rise to an MILP.

$$W_p = W_p^{LO} + \sum_n q1_{n,p} dU1_{n,p}, \quad \forall p \quad (\text{A131})$$

$$dU1_{n,p} \geq \theta 1_{n,p}, \quad \forall n < N, p \quad (\text{A132})$$

$$dU1_{n,p} \leq \theta 1_{n-1,p}, \quad \forall n > 1, p \quad (\text{A133})$$

$$RL1_p = W_p L_u^{LO} + W_p^{LO} L_u - L_u^{LO} W_p^{LO} + \sum_n q1_{n,p} dQ1_{n,p} \quad (\text{A134})$$

$$u = \text{CDU}, \forall p$$

$$dQ1_{n,p} \geq (L_u^{UP} - L_u^{LO}) dU1_{n,p} + L_u - L_u^{UP}, \quad n = 1, u = \text{CDU}, \forall p \quad (\text{A135})$$

$$dQ1_{n,p} \geq dV1_{n,p}, \quad \forall n < N, p \quad (\text{A136})$$

$$dQ1_{n,p} \geq (L_u^{UP} - L_u^{LO})(dU1_{n,p} - \theta 1_{n-1}) + dV1_{n-1,p}, \quad u = \text{CDU}, \forall n > 1, p \quad (\text{A137})$$

$$dQ1_{n,p} \leq (L_u - L_u^{LO}), \quad n = 1, u = \text{CDU}, \forall p \quad (\text{A138})$$

$$dQ1_{n,p} \leq (L_u^{UP} - L_u^{LO})(dU1_{n,p} - \theta 1_{n,p}) + dV1_{n,p}, \quad u = \text{CDU}, \forall n < N, p \quad (\text{A139})$$

$$dQ1_{n,p} \leq dV1_{n-1,p}, \quad \forall n > 1, p \quad (\text{A140})$$

$$dQ1_{n,p} \leq (L_u^{\text{UP}} - L_u^{\text{LO}})dU1_{n,p}, \quad n = \text{N}, u = \text{CDU}, \forall p \quad (\text{A141})$$

$$Y_f = Y_f^{\text{LO}} + \sum_n q2_{n,f} dU2_{n,f}, \quad \forall f \quad (\text{A142})$$

$$dU2_{n,f} \geq \theta2_{n,f}, \quad \forall n < N, f \quad (\text{A143})$$

$$dU2_{n,f} \leq \theta2_{n-1,f}, \quad \forall n > 1, f \quad (\text{A144})$$

$$RL2_f = Y_f L_u^{\text{LO}} + L_u Y_f^{\text{LO}} - L_u^{\text{LO}} Y_f^{\text{LO}} + \sum_n q2_{n,f} dQ2_{n,f}, \quad u = \text{CDU}, \forall f \quad (\text{A145})$$

$$dQ2_{n,f} \geq (L_u^{\text{UP}} - L_u^{\text{LO}})dU2_{n,f} + L_u - L_u^{\text{UP}}, \quad n = 1, u = \text{CDU}, \forall f \quad (\text{A146})$$

$$dQ2_{n,f} \geq dV2_{n,f}, \quad \forall n < N, f \quad (\text{A147})$$

$$dQ2_{n,f} \geq (L_u^{\text{UP}} - L_u^{\text{LO}})(dU2_{n,f} - \theta2_{n-1,f}) + dV2_{n-1,f}, \quad u = \text{CDU}, \forall n > 1, f \quad (\text{A148})$$

$$dQ2_{n,f} \leq (L_u - L_u^{\text{LO}}), \quad n = 1, u = \text{CDU}, \forall f \quad (\text{A149})$$

$$dQ2_{n,f} \leq (L_u^{\text{UP}} - L_u^{\text{LO}})(dU2_{n,f} - \theta2_{n,f}) + dV2_{n,f}, \quad u = \text{CDU}, \forall n < N, f \quad (\text{A150})$$

$$dQ2_{n,f} \leq dV2_{n-1,f}, \quad \forall n > 1, f \quad (\text{A151})$$

$$dQ2_{n,f} \leq (L_u^{\text{UP}} - L_u^{\text{LO}})dU2_{n,f}, \quad n = \text{N}, u = \text{CDU}, \forall f \quad (\text{A152})$$

$$conv = conv^{\text{LO}} + \sum_n q3_n dU3_n \quad (\text{A153})$$

$$dU3_n \geq \theta3_n, \quad \forall n < N \quad (\text{A154})$$

$$dU3_n \leq \theta3_{n-1}, \quad \forall n > 1 \quad (\text{A155})$$

$$RL3 = conv \cdot conv^{\text{LO}} + conv \cdot conv^{\text{LO}} - conv^{\text{LO}} \cdot conv^{\text{LO}} + \sum_n q3_n dQ3_n \quad (\text{A156})$$

$$dQ3_n \geq (conv^{\text{UP}} - conv^{\text{LO}})dU3_n + conv - conv^{\text{UP}}, \quad n = 1 \quad (\text{A157})$$

$$dQ3_n \geq dV3_n, \quad \forall n < N \quad (\text{A158})$$

$$dQ3_n \geq (conv^{\text{UP}} - conv^{\text{LO}})(dU3_n - \theta3_{n-1}) + dV3_{n-1}, \quad \forall n > 1 \quad (\text{A159})$$

$$dQ3_n \leq (conv - conv^{\text{LO}}), \quad n = 1 \quad (\text{A160})$$

$$dQ3_n \leq (conv^{\text{UP}} - conv^{\text{LO}})(dU3_n - \theta3_n) + dV3_n, \quad \forall n < N \quad (\text{A161})$$

$$dQ3_n \leq dV3_{n-1}, \forall n > 1 \quad (\text{A162})$$

$$dQ3_n \leq (conv^{\text{UP}} - conv^{\text{LO}})dU3_n, n = N \quad (\text{A163})$$

$$MW_p = MW_p^{\text{LO}} + \sum_n q4_{n,p} dU4_{n,p}, \forall p \in P_g \quad (\text{A164})$$

$$dU4_{n,p} \geq \theta4_{n,p}, \forall n < N, p \in P_g \quad (\text{A165})$$

$$dU4_{n,p} \leq \theta4_{n-1,p}, \forall n > 1, p \in P_g \quad (\text{A166})$$

$$RL4_p = MW_p \cdot MW_p^{\text{LO}} + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{LO}} \cdot MW_p^{\text{LO}} + \sum_n q4_{n,p} dQ4_{n,p} \quad (\text{A167})$$

$$\forall p \in P_g$$

$$dQ4_{n,p} \geq (MW_p^{\text{UP}} - MW_p^{\text{LO}})dU4_{n,p} + MW_p - MW_p^{\text{UP}}, n = 1, \forall p \in P_g \quad (\text{A168})$$

$$dQ4_{n,p} \geq dV4_{n,p}, \forall n < N, p \in P_g \quad (\text{A169})$$

$$dQ4_{n,p} \geq (MW_p^{\text{UP}} - MW_p^{\text{LO}})(dU4_{n,p} - \theta4_{n-1,p}) + dV4_{n-1,p}, \forall n > 1, p \in P_g \quad (\text{A170})$$

$$dQ4_{n,p} \leq (MW_p - MW_p^{\text{LO}}), n = 1, \forall p \in P_g \quad (\text{A171})$$

$$dQ4_{n,p} \leq (MW_p^{\text{UP}} - MW_p^{\text{LO}})(dU4_{n,p} - \theta4_{n,p}) + dV4_{n,p}, \forall n < N, p \in P_g \quad (\text{A172})$$

$$dQ4_{n,p} \leq dV4_{n-1,p}, \forall n > 1, p \in P_g \quad (\text{A173})$$

$$dQ4_{n,p} \leq (MW_p^{\text{UP}} - MW_p^{\text{LO}})dU4_{n,p}, n = N, \forall p \in P_g \quad (\text{A174})$$

$$MW_p = MW_p^{\text{LO}} + \sum_n q5_{n,p} dU5_{n,p}, \forall p \in P_d \quad (\text{A175})$$

$$dU5_{n,p} \geq \theta5_{n,p}, \forall n < N, p \in P_d \quad (\text{A176})$$

$$dU5_{n,p} \leq \theta5_{n-1,p}, \forall n > 1, p \in P_d \quad (\text{A177})$$

$$RL5_p = MW_p \cdot MW_p^{\text{LO}} + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{LO}} \cdot MW_p^{\text{LO}} + \sum_n q5_{n,p} dQ5_{n,p} \quad (\text{A178})$$

$$\forall p \in P_d$$

$$dQ5_{n,p} \geq (MW_p^{\text{UP}} - MW_p^{\text{LO}})dU5_{n,p} + MW_p - MW_p^{\text{UP}}, n = 1, \forall p \in P_d \quad (\text{A179})$$

$$dQ5_{n,p} \geq dV5_{n,p}, \forall n < N, p \in P_d \quad (\text{A180})$$

$$dQ5_{n,p} \geq (MW_p^{\text{UP}} - MW_p^{\text{LO}})(dU5_{n,p} - \theta5_{n-1,p}) + dV5_{n-1,p}, \quad \forall n > 1, p \in P_d \quad (\text{A181})$$

$$dQ5_{n,p} \leq (MW_p - MW_p^{\text{LO}}), \quad n = 1, \forall p \in P_d \quad (\text{A182})$$

$$dQ5_{n,p} \leq (MW_p^{\text{UP}} - MW_p^{\text{LO}})(dU5_{n,p} - \theta5_{n,p}) + dV5_{n,p}, \quad \forall n < N, p \in P_d \quad (\text{A183})$$

$$dQ5_{n,p} \leq dV5_{n-1,p}, \quad \forall n > 1, p \in P_d \quad (\text{A184})$$

$$dQ5_{n,p} \leq (MW_p^{\text{UP}} - MW_p^{\text{LO}})dU5_{n,p}, \quad n = N, \forall p \in P_d \quad (\text{A185})$$

$$Pr_{j,p} = Pr_{j,p}^{\text{LO}} + \sum_n q6_{n,p} dU6_{n,p}, \quad j = \text{ON}, p = \text{GO} \quad (\text{A186})$$

$$dU6_{n,p} \geq \theta6_{n,p}, \quad p = \text{GO}, \forall n < N \quad (\text{A187})$$

$$dU6_{n,p} \leq \theta6_{n-1,p}, \quad p = \text{GO}, \forall n > 1 \quad (\text{A188})$$

$$RL6 = Pr_{j,p} F_{p,g}^{iprod,\text{LO}} + F_{p,g}^{iprod} Pr_{j,p}^{\text{LO}} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{LO}} + \sum_n q6_{n,p} dQ6_{n,p} \quad (\text{A189})$$

$$p = \text{GO}, g = g90, j = \text{ON}$$

$$dQ6_{n,p} \geq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})dU6_{n,p} + F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} \quad (\text{A190})$$

$$n = 1, p = \text{GO}, g = g90$$

$$dQ6_{n,p} \geq dV6_{n,p}, \quad p = \text{GO}, \forall n < N \quad (\text{A191})$$

$$dQ6_{n,p} \geq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})(dU6_{n,p} - \theta6_{n-1,p}) + dV6_{n-1,p} \quad (\text{A192})$$

$$p = \text{GO}, g = g90, \forall n > 1$$

$$dQ6_{n,p} \leq (F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}}), \quad n = 1, p = \text{GO}, g = g90 \quad (\text{A193})$$

$$dQ6_{n,p} \leq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})(dU6_{n,p} - \theta6_{n,p}) + dV6_{n,p} \quad (\text{A194})$$

$$p = \text{GO}, g = g90, \forall n < N$$

$$dQ6_{n,p} \leq dV6_{n-1,p}, \quad p = \text{GO}, \forall n > 1 \quad (\text{A195})$$

$$dQ6_{n,p} \leq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})dU6_{n,p}, \quad n = N, p = \text{GO}, g = g90 \quad (\text{A196})$$

$$Pr_{j,p'} = Pr_{j,p'}^{\text{LO}} + \sum_n q7_{n,p'} dU7_{n,p'}, \quad j = \text{ON}, p' = \text{HN} \quad (\text{A197})$$

$$dU7_{n,p'} \geq \theta7_{n,p'}, \quad p' = \text{HN}, \forall n < N \quad (\text{A198})$$

$$dU7_{n,p'} \leq \theta7_{n-1,p'}, \quad p' = \text{HN}, \forall n > 1 \quad (\text{A199})$$

$$RL7 = Pr_{j,p'} F_{p',g}^{iprod,\text{LO}} + F_{p',g}^{iprod} Pr_{j,p'}^{\text{LO}} - F_{p',g}^{iprod,\text{LO}} Pr_{j,p'}^{\text{LO}} + \sum_n q7_{n,p'} dQ7_{n,p'} \quad (\text{A200})$$

$$p' = \text{HN}, g = \text{g90}, j = \text{ON}$$

$$dQ7_{n,p'} \geq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU7_{n,p'} + F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{UP}} \quad (\text{A201})$$

$$n = 1, p' = \text{HN}, g = \text{g90}$$

$$dQ7_{n,p'} \geq dV7_{n,p'}, \quad p' = \text{GO}, \forall n < N \quad (\text{A202})$$

$$dQ7_{n,p'} \geq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) (dU7_{n,p'} - \theta7_{n-1,p'}) + dV7_{n-1,p'} \quad (\text{A203})$$

$$p' = \text{HN}, g = \text{g90}, \forall n > 1$$

$$dQ7_{n,p'} \leq \left(F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{LO}} \right), \quad n = 1, p' = \text{HN}, g = \text{g90} \quad (\text{A204})$$

$$dQ7_{n,p'} \leq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) (dU7_{n,p'} - \theta7_{n,p'}) + dV7_{n,p'} \quad (\text{A205})$$

$$p' = \text{HN}, g = \text{g90}, \forall n < N$$

$$dQ7_{n,p'} \leq dV7_{n-1,p'}, \quad p' = \text{HN}, \forall n > 1 \quad (\text{A206})$$

$$dQ7_{n,p'} \leq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU7_{n,p'}, \quad n = \text{N}, p' = \text{HN}, g = \text{g90} \quad (\text{A207})$$

$$Pr_{j,p} = Pr_{j,p}^{\text{LO}} + \sum_n q8_{n,p} dU8_{n,p}, \quad j = \text{ON}, p = \text{GO} \quad (\text{A208})$$

$$dU8_{n,p} \geq \theta8_{n,p}, \quad p = \text{GO}, \forall n < N \quad (\text{A209})$$

$$dU8_{n,p} \leq \theta8_{n-1,p}, \quad p = \text{GO}, \forall n > 1 \quad (\text{A210})$$

$$RL8 = Pr_{j,p} F_{p,g}^{iprod,\text{LO}} + F_{p,g}^{iprod} Pr_{j,p}^{\text{LO}} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{LO}} + \sum_n q8_{n,p} dQ8_{n,p} \quad (\text{A211})$$

$$p = \text{GO}, g = \text{g93}, j = \text{ON}$$

$$dQ8_{n,p} \geq \left(F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}} \right) dU8_{n,p} + F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} \quad (\text{A212})$$

$$n = 1, p = \text{GO}, g = \text{g93}$$

$$dQ8_{n,p} \geq dV8_{n,p}, \quad p = \text{GO}, \forall n < N \quad (\text{A213})$$

$$dQ8_{n,p} \geq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})(dU8_{n,p} - \theta8_{n-1,p}) + dV8_{n-1,p} \quad (\text{A214})$$

$$p = \text{GO}, g = \text{g93}, \forall n > 1$$

$$dQ8_{n,p} \leq (F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}}), \quad n = 1, p = \text{GO}, g = \text{g93} \quad (\text{A215})$$

$$dQ8_{n,p} \leq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})(dU8_{n,p} - \theta8_{n,p}) + dV8_{n,p} \quad (\text{A216})$$

$$p = \text{GO}, g = \text{g93}, \forall n < N$$

$$dQ8_{n,p} \leq dV8_{n-1,p}, \quad p = \text{GO}, \forall n > 1 \quad (\text{A217})$$

$$dQ8_{n,p} \leq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}})dU8_{n,p}, \quad n = \text{N}, p = \text{GO}, g = \text{g93} \quad (\text{A218})$$

$$Pr_{j,p'} = Pr_{j,p'}^{\text{LO}} + \sum_n q9_{n,p'} dU9_{n,p'}, \quad j = \text{ON}, p' = \text{HN} \quad (\text{A219})$$

$$dU9_{n,p'} \geq \theta9_{n,p'}, \quad p' = \text{HN}, \forall n < N \quad (\text{A220})$$

$$dU9_{n,p'} \leq \theta9_{n-1,p'}, \quad p' = \text{HN}, \forall n > 1 \quad (\text{A221})$$

$$RL9 = Pr_{j,p'} F_{p',g}^{iprod,\text{LO}} + F_{p',g}^{iprod} Pr_{j,p'}^{\text{LO}} - F_{p',g}^{iprod,\text{LO}} Pr_{j,p'}^{\text{LO}} + \sum_n q9_{n,p'} dQ9_{n,p'} \quad (\text{A222})$$

$$p' = \text{HN}, g = \text{g93}, j = \text{ON}$$

$$dQ9_{n,p'} \geq (F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}}) dU9_{n,p'} + F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{UP}} \quad (\text{A223})$$

$$n = 1, p' = \text{HN}, g = \text{g93}$$

$$dQ9_{n,p'} \geq dV9_{n,p'}, \quad p' = \text{GO}, \forall n < N \quad (\text{A224})$$

$$dQ9_{n,p'} \geq (F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}})(dU9_{n,p'} - \theta9_{n-1,p'}) + dV9_{n-1,p'} \quad (\text{A225})$$

$$p' = \text{HN}, g = \text{g93}, \forall n > 1$$

$$dQ9_{n,p'} \leq (F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{LO}}), \quad n = 1, p' = \text{HN}, g = \text{g93} \quad (\text{A226})$$

$$dQ9_{n,p'} \leq (F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}})(dU9_{n,p'} - \theta9_{n,p'}) + dV9_{n,p'} \quad (\text{A227})$$

$$p' = \text{HN}, g = g93, \forall n < N$$

$$dQ9_{n,p'} \leq dV9_{n-1,p'}, p' = \text{HN}, \forall n > 1 \quad (\text{A228})$$

$$dQ9_{n,p'} \leq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU9_{n,p'}, n = \text{N}, p' = \text{HN}, g = g93 \quad (\text{A229})$$

A.12. **nf6t** Relaxation

The **nf6t** relaxations for all the reformulated equations are as follows. The variable definitions of θ , dU_n and dQ_n are the same as that of **nf5**. It is smaller in size than the **nf5** and does not include dV_n variables. The resulting relaxed model gives rise to an MILP.

$$W_p = W_p^{\text{LO}} + \sum_n q1_{n,p} dU1_{n,p}, \quad \forall p \quad (\text{A230})$$

$$dU1_{n,p} \geq \theta1_{n,p}, \quad \forall n < N, p \quad (\text{A231})$$

$$dU1_{n,p} \leq \theta1_{n-1,p}, \quad \forall n > 1, p \quad (\text{A232})$$

$$RL1_p = W_p L_u^{\text{LO}} + W_p^{\text{LO}} L_u - L_u^{\text{LO}} W_p^{\text{LO}} + \sum_n q1_{n,p} dQ1_{n,u} \quad (\text{A233})$$

$$u = \text{CDU}, \forall p$$

$$dQ1_{n,u} \geq (L_u^{\text{UP}} - L_u^{\text{LO}}) dU1_{n,p} + L_u - L_u^{\text{UP}}, u = \text{CDU}, \forall n, p \quad (\text{A234})$$

$$dQ1_{n,u} \leq L_u - L_u^{\text{LO}}, n = 1, u = \text{CDU} \quad (\text{A235})$$

$$dQ1_{n,u} \leq dQ1_{n-1,u}, u = \text{CDU}, \forall n > 1 \quad (\text{A236})$$

$$dQ1_{n,u} \leq (L_u^{\text{UP}} - L_u^{\text{LO}}) dU1_{n,p}, u = \text{CDU}, \forall n, p \quad (\text{A237})$$

$$Y_f = Y_f^{\text{LO}} + \sum_n q2_{n,f} dU2_{n,f}, u = \text{FCC}, \forall f \quad (\text{A238})$$

$$dU2_{n,f} \geq \theta2_{n,f}, \forall n < \text{N}, f \quad (\text{A239})$$

$$dU2_{n,f} \leq \theta2_{n-1,f}, \forall n > 1, f \quad (\text{A240})$$

$$RL2_f = Y_f L_u^{\text{LO}} + Y_f^{\text{LO}} L_u - L_u^{\text{LO}} Y_f^{\text{LO}} + \sum_n q2_{n,f} dQ2_{n,u}, u = \text{FCC}, \forall f \quad (\text{A241})$$

$$dQ2_{n,u} \geq (L_u^{\text{UP}} - L_u^{\text{LO}})dU2_{n,f} + L_u - L_u^{\text{UP}}, u = \text{FCC}, \forall n, f \quad (\text{A242})$$

$$dQ2_{n,u} \leq L_u - L_u^{\text{LO}}, n = 1, u = \text{FCC} \quad (\text{A243})$$

$$dQ2_{n,u} \leq dQ2_{n-1,u}, u = \text{FCC}, \forall n > 1 \quad (\text{A244})$$

$$dQ2_{n,u} \leq (L_u^{\text{UP}} - L_u^{\text{LO}})dU2_{n,f}, u = \text{FCC}, \forall n, f \quad (\text{A245})$$

$$conv = conv^{\text{LO}} + \sum_n q3_n dU3_n \quad (\text{A246})$$

$$dU3_n \geq \theta 3_n, \forall n < N \quad (\text{A247})$$

$$dU3_n \leq \theta 3_{n-1}, \forall n > 1 \quad (\text{A248})$$

$$RL3 = conv \cdot conv^{\text{LO}} + conv \cdot conv^{\text{LO}} - conv^{\text{LO}} \cdot conv^{\text{LO}} + \sum_n q3_n dQ3_n \quad (\text{A249})$$

$$dQ3_n \geq (conv^{\text{UP}} - conv^{\text{LO}})dU3_n + conv - conv^{\text{UP}}, \forall n \quad (\text{A250})$$

$$dQ3_n \leq conv - conv^{\text{LO}}, n = 1 \quad (\text{A251})$$

$$dQ3_n \leq dQ3_{n-1}, \forall n > 1 \quad (\text{A252})$$

$$dQ3_n \leq (conv^{\text{UP}} - conv^{\text{LO}})dU3_n, \forall n \quad (\text{A253})$$

$$MW_p = MW_p^{\text{LO}} + \sum_n q4_{n,p} dU4_{n,p}, \forall p \in P_g \quad (\text{A254})$$

$$dU4_{n,p} \geq \theta 4_{n,p}, \forall n < N, p \in P_g \quad (\text{A255})$$

$$dU4_{n,p} \leq \theta 4_{n-1,p}, \forall n > 1, p \in P_g \quad (\text{A256})$$

$$RL4_p = MW_p \cdot MW_p^{\text{LO}} + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{LO}} \cdot MW_p^{\text{LO}} + \sum_n q4_{n,p} dQ4_{n,p} \quad (\text{A257})$$

$$\forall p \in P_g$$

$$dQ4_{n,p} \geq (MW_p^{\text{UP}} - MW_p^{\text{LO}})dU4_{n,p} + MW_p - MW_p^{\text{UP}}, \forall n, p \in P_g \quad (\text{A258})$$

$$dQ4_{n,p} \leq MW_p - MW_p^{\text{LO}}, n = 1, \forall p \in P_g \quad (\text{A259})$$

$$dQ4_{n,p} \leq dQ4_{n-1,p}, \forall n > 1, p \in P_g \quad (\text{A260})$$

$$dQ4_{n,p} \leq (MW_p^{\text{UP}} - MW_p^{\text{LO}})dU4_{n,p}, \forall n, p \in P_g \quad (\text{A261})$$

$$MW_p = MW_p^{\text{LO}} + \sum_n q5_{n,p} dU5_{n,p}, \forall p \in P_d \quad (\text{A262})$$

$$dU5_{n,p} \geq \theta5_{n,p}, \forall n < N, p \in P_d \quad (\text{A263})$$

$$dU5_{n,p} \leq \theta5_{n-1,p}, \forall n > 1, p \in P_d \quad (\text{A264})$$

$$RL5_p = MW_p \cdot MW_p^{\text{LO}} + MW_p \cdot MW_p^{\text{LO}} - MW_p^{\text{LO}} \cdot MW_p^{\text{LO}} + \sum_n q5_{n,p} dQ5_{n,p} \quad (\text{A265})$$

$$\forall p \in P_d$$

$$dQ5_{n,p} \geq (MW_p^{\text{UP}} - MW_p^{\text{LO}}) dU5_{n,p} + MW_p - MW_p^{\text{UP}}, \forall n, p \in P_d \quad (\text{A266})$$

$$dQ5_{n,p} \leq MW_p - MW_p^{\text{LO}}, n = 1, \forall p \in P_d \quad (\text{A267})$$

$$dQ5_{n,p} \leq dQ5_{n-1,p}, \forall n > 1, p \in P_d \quad (\text{A268})$$

$$dQ5_{n,p} \leq (MW_p^{\text{UP}} - MW_p^{\text{LO}}) dU5_{n,p}, \forall n, p \in P_d \quad (\text{A269})$$

$$Pr_{j,p} = Pr_{j,p}^{\text{LO}} + \sum_n q6_{n,p} dU6_{n,p}, j = \text{ON}, p = \text{GO} \quad (\text{A270})$$

$$dU6_{n,p} \geq \theta6_{n,p}, p = \text{GO}, \forall n < N \quad (\text{A271})$$

$$dU6_{n,p} \leq \theta6_{n-1,p}, p = \text{GO}, \forall n > 1 \quad (\text{A272})$$

$$RL6 = Pr_{j,p} F_{p,g}^{iprod,\text{LO}} + F_{p,g}^{iprod} Pr_{j,p}^{\text{LO}} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{LO}} + \sum_n q6_{n,p} dQ6_{n,p} \quad (\text{A273})$$

$$p = \text{GO}, g = \text{g90}, j = \text{ON}$$

$$dQ6_{n,p} \geq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}}) dU6_{n,p} + F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} \quad (\text{A274})$$

$$p = \text{GO}, g = \text{g90}, \forall n$$

$$dQ6_{n,p} \leq F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}}, n = 1, p = \text{GO}, g = \text{g90} \quad (\text{A275})$$

$$dQ6_{n,p} \leq dQ6_{n-1,p}, p = \text{GO}, g = \text{g90}, \forall n > 1 \quad (\text{A276})$$

$$dQ6_{n,p} \leq (F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}}) dU6_{n,p}, p = \text{GO}, g = \text{g90}, \forall n \quad (\text{A277})$$

$$Pr_{j,p'} = Pr_{j,p'}^{\text{LO}} + \sum_n q7_{n,p'} dU7_{n,p'}, j = \text{ON}, p' = \text{HN} \quad (\text{A278})$$

$$dU7_{n,p'} \geq \theta7_{n,p'}, p' = \text{HN}, \forall n < N \quad (\text{A279})$$

$$dU7_{n,p'} \leq \theta7_{n-1,p'}, \quad p' = \text{HN}, \forall n > 1 \quad (\text{A280})$$

$$RL7 = Pr_{j,p'} F_{p',g}^{iprod,\text{LO}} + F_{p',g}^{iprod} Pr_{j,p'}^{\text{LO}} - F_{p',g}^{iprod,\text{LO}} Pr_{j,p'}^{\text{LO}} + \sum_n q7_{n,p'} dQ7_{n,p'} \quad (\text{A281})$$

$$p' = \text{HN}, g = \text{g90}, j = \text{ON}$$

$$dQ7_{n,p'} \geq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU7_{n,p'} + F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{UP}} \quad (\text{A282})$$

$$p' = \text{HN}, g = \text{g90}, \forall n$$

$$dQ7_{n,p'} \leq F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{LO}}, \quad n = 1, p' = \text{HN}, g = \text{g90} \quad (\text{A283})$$

$$dQ7_{n,p'} \leq dQ7_{n-1,p'}, \quad p' = \text{HN}, g = \text{g90}, \forall n > 1 \quad (\text{A284})$$

$$dQ7_{n,p'} \leq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU7_{n,p'}, \quad p' = \text{HN}, g = \text{g90}, \forall n \quad (\text{A285})$$

$$Pr_{j,p} = Pr_{j,p}^{\text{LO}} + \sum_n q8_{n,p} dU8_{n,p}, \quad j = \text{ON}, p = \text{GO} \quad (\text{A286})$$

$$dU8_{n,p} \geq \theta8_{n,p}, \quad p = \text{GO}, \forall n < N \quad (\text{A287})$$

$$dU8_{n,p} \leq \theta8_{n-1,p}, \quad p = \text{GO}, \forall n > 1 \quad (\text{A288})$$

$$RL8 = Pr_{j,p} F_{p,g}^{iprod,\text{LO}} + F_{p,g}^{iprod} Pr_{j,p}^{\text{LO}} - F_{p,g}^{iprod,\text{LO}} Pr_{j,p}^{\text{LO}} + \sum_n q8_{n,p} dQ8_{n,p} \quad (\text{A289})$$

$$p = \text{GO}, g = \text{g93}, j = \text{ON}$$

$$dQ8_{n,p} \geq \left(F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}} \right) dU8_{n,p} + F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{UP}} \quad (\text{A290})$$

$$p = \text{GO}, g = \text{g93}, \forall n$$

$$dQ8_{n,p} \leq F_{p,g}^{iprod} - F_{p,g}^{iprod,\text{LO}}, \quad n = 1, p = \text{GO}, g = \text{g93} \quad (\text{A291})$$

$$dQ8_{n,p} \leq dQ8_{n-1,p}, \quad p = \text{GO}, g = \text{g93}, \forall n > 1 \quad (\text{A292})$$

$$dQ8_{n,p} \leq \left(F_{p,g}^{iprod,\text{UP}} - F_{p,g}^{iprod,\text{LO}} \right) dU8_{n,p}, \quad p = \text{GO}, g = \text{g93}, \forall n \quad (\text{A293})$$

$$Pr_{j,p'} = Pr_{j,p'}^{\text{LO}} + \sum_n q9_{n,p'} dU9_{n,p'}, \quad j = \text{ON}, p' = \text{HN} \quad (\text{A294})$$

$$dU9_{n,p'} \geq \theta9_{n,p'}, \quad p' = \text{HN}, \forall n < N \quad (\text{A295})$$

$$dU9_{n,p'} \leq \theta 9_{n-1,p'}, \quad p' = \text{HN}, \forall n > 1 \quad (\text{A296})$$

$$RL9 = Pr_{j,p'} F_{p',g}^{iprod,\text{LO}} + F_{p',g}^{iprod} Pr_{j,p'}^{\text{LO}} - F_{p',g}^{iprod,\text{LO}} Pr_{j,p'}^{\text{LO}} + \sum_n q9_{n,p'} dQ9_{n,p'} \quad (\text{A297})$$

$$p' = \text{HN}, g = \text{g93}, j = \text{ON}$$

$$dQ9_{n,p'} \geq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU9_{n,p'} + F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{UP}} \quad (\text{A298})$$

$$p' = \text{HN}, g = \text{g93}, \forall n$$

$$dQ9_{n,p'} \leq F_{p',g}^{iprod} - F_{p',g}^{iprod,\text{LO}}, \quad n = 1, p' = \text{HN}, g = \text{g93} \quad (\text{A299})$$

$$dQ9_{n,p'} \leq dQ9_{n-1,p'}, \quad p' = \text{HN}, g = \text{g93}, \forall n > 1 \quad (\text{A300})$$

$$dQ9_{n,p'} \leq \left(F_{p',g}^{iprod,\text{UP}} - F_{p',g}^{iprod,\text{LO}} \right) dU9_{n,p'}, \quad p' = \text{HN}, g = \text{g93}, \forall n \quad (\text{A301})$$

A.13. **de** Relaxation

The **de** relaxations for all the reformulated equations are as follows. This formulation is based on eigen vector decomposition. It constructs the bounding over and underestimators on each portioned segment in the ξ and η directions and utilizes SOS Type 2 variables λ^ξ and λ^η , respectively to activate feasible solution domains of the relaxed variable.

$$RL1_p = \omega 1_p^\xi - \omega 1_p^\eta, \quad \forall p \quad (\text{A302})$$

$$\xi 1_p = (L_u + W_p)/2, \quad u = \text{CDU}, \forall p \quad (\text{A303})$$

$$\eta 1_p = (W_p - L_u)/2, \quad u = \text{CDU}, \forall p \quad (\text{A304})$$

$$\omega 1_p^\xi \geq 2\overline{\xi 1}_{j,p} \xi 1_p - \overline{\xi 1}_{j,p}^2, \quad j = 1, \dots, M^\xi, \forall p \quad (\text{A305})$$

$$\omega 1_p^\xi \leq \sum_n \xi 1_{n,p}^2 \lambda 1_{n,p}^\xi, \quad \forall p \quad (\text{A306})$$

$$\xi 1_p = \sum_n \xi 1_{n,p} \lambda 1_{n,p}^\xi, \quad \forall p \quad (\text{A307})$$

$$\sum_n \lambda 1_{n,p}^\xi = 1, \quad \forall p \quad (\text{A308})$$

$$\omega 1_p^\eta \geq 2\overline{\eta 1}_{j,p} \eta 1_p - \overline{\eta 1}_{j,p}^2, \quad j = 1, \dots, M^\eta, \forall p \quad (\text{A309})$$

$$\omega 1_p^\eta \leq \sum_n \eta 1_{n,p}^2 \lambda 1_{n,p}^\eta, \quad \forall p \quad (\text{A310})$$

$$\eta 1_p = \sum_n \eta 1_{n,p} \lambda 1_{n,p}^\eta, \quad \forall p \quad (\text{A311})$$

$$\sum_n \lambda 1_{n,p}^\eta = 1, \quad \forall p \quad (\text{A312})$$

$$RL2_f = \omega 2_f^\xi - \omega 2_f^\eta, \quad \forall f \quad (\text{A313})$$

$$\xi 2_f = (L_u + Y_f)/2, \quad u = \text{FCC}, \forall f \quad (\text{A314})$$

$$\eta 2_f = (Y_f - L_u)/2, \quad u = \text{FCC}, \forall f \quad (\text{A315})$$

$$\omega 2_f^\xi \geq 2\overline{\xi 2_{j,f}} \xi 2_f - \overline{\xi 2_{j,f}^2}, \quad j = 1, \dots, M^\xi, \forall f \quad (\text{A316})$$

$$\omega 2_f^\xi \leq \sum_n \xi 2_{n,f}^2 \lambda 2_{n,f}^\xi, \quad \forall f \quad (\text{A317})$$

$$\xi 2_f = \sum_n \xi 2_{n,f} \lambda 2_{n,f}^\xi, \quad \forall f \quad (\text{A318})$$

$$\sum_n \lambda 2_{n,f}^\xi = 1, \quad \forall f \quad (\text{A319})$$

$$\omega 2_f^\eta \geq 2\overline{\eta 2_{j,f}} \eta 2_f - \overline{\eta 2_{j,f}^2}, \quad j = 1, \dots, M^\eta, \forall f \quad (\text{A320})$$

$$\omega 2_f^\eta \leq \sum_n \eta 2_{n,f}^2 \lambda 2_{n,f}^\eta, \quad \forall f \quad (\text{A321})$$

$$\eta 2_f = \sum_n \eta 2_{n,f} \lambda 2_{n,f}^\eta, \quad \forall f \quad (\text{A322})$$

$$\sum_n \lambda 2_{n,f}^\eta = 1, \quad \forall f \quad (\text{A323})$$

$$RL3 = \omega 3^\xi - \omega 3^\eta \quad (\text{A324})$$

$$\xi 3 = (conv + conv)/2 \quad (\text{A325})$$

$$\eta 3 = (conv - conv)/2 \quad (\text{A326})$$

$$\omega 3^\xi \geq 2\overline{\xi 3_j} \xi 3 - \overline{\xi 3_j^2}, \quad j = 1, \dots, M^\xi \quad (\text{A327})$$

$$\omega 3^\xi \leq \sum_n \xi 3_n^2 \lambda 3_n^\xi, \quad \forall f \quad (\text{A328})$$

$$\xi 3 = \sum_n \xi 3_n \lambda 3_n^\xi \quad (\text{A329})$$

$$\sum_n \lambda 3_n^\xi = 1 \quad (\text{A330})$$

$$\omega 3^\eta \geq 2\overline{\eta 3}_j \eta 3 - \overline{\eta 3_j^2}, \quad j = 1, \dots, M^\eta \quad (\text{A331})$$

$$\omega 3^\eta \leq \sum_n \eta 3_n^2 \lambda 3_n^\eta \quad (\text{A332})$$

$$\eta 3 = \sum_n \eta 3_n \lambda 3_n^\eta \quad (\text{A333})$$

$$\sum_n \lambda 3_n^\eta = 1 \quad (\text{A334})$$

$$RL4_p = \omega 4_p^\xi - \omega 4_p^\eta, \quad \forall p \in P_g \quad (\text{A335})$$

$$\xi 4_p = (MW_p + MW_p)/2, \quad \forall p \in P_g \quad (\text{A336})$$

$$\eta 4_p = (MW_p - MW_p)/2, \quad \forall p \in P_g \quad (\text{A337})$$

$$\omega 4_p^\xi \geq 2\overline{\xi 4}_{j,p} \xi 4_p - \overline{\xi 4_{j,p}^2}, \quad j = 1, \dots, M^\xi, \forall p \in P_g \quad (\text{A338})$$

$$\omega 4_p^\xi \leq \sum_n \xi 4_{n,p}^2 \lambda 4_{n,p}^\xi, \quad \forall p \in P_g \quad (\text{A339})$$

$$\xi 4_p = \sum_n \xi 4_{n,p} \lambda 4_{n,p}^\xi, \quad \forall p \in P_g \quad (\text{A340})$$

$$\sum_n \lambda 4_{n,p}^\xi = 1, \quad \forall p \in P_g \quad (\text{A341})$$

$$\omega 4_p^\eta \geq 2\overline{\eta 4}_{j,p} \eta 4_p - \overline{\eta 4_{j,p}^2}, \quad j = 1, \dots, M^\eta, \forall p \in P_g \quad (\text{A342})$$

$$\omega 4_p^\eta \leq \sum_n \eta 4_{n,p}^2 \lambda 4_{n,p}^\eta, \quad \forall p \in P_g \quad (\text{A343})$$

$$\eta 4_p = \sum_n \eta 4_{n,p} \lambda 4_{n,p}^\eta, \quad \forall p \in P_g \quad (\text{A344})$$

$$\sum_n \lambda 4_{n,p}^\eta = 1, \quad \forall p \in P_g \quad (\text{A345})$$

$$RL5_p = \omega 5_p^\xi - \omega 5_p^\eta, \quad \forall p \in P_d \quad (\text{A346})$$

$$\xi 5_p = (MW_p + MW_p)/2, \quad \forall p \in P_d \quad (\text{A347})$$

$$\eta 5_p = (MW_p - MW_p)/2, \quad \forall p \in P_d \quad (\text{A348})$$

$$\omega 5_p^\xi \geq 2\overline{\xi 5}_{j,p} \xi 5_p - \overline{\xi 5_{j,p}^2}, \quad j = 1, \dots, M^\xi, \forall p \in P_d \quad (\text{A349})$$

$$\omega 5_p^\xi \leq \sum_n \xi 5_{n,p}^2 \lambda 5_{n,p}^\xi, \quad \forall p \in P_d \quad (\text{A350})$$

$$\xi 5_p = \sum_n \xi 5_{n,p} \lambda 5_{n,p}^\xi, \quad \forall p \in P_d \quad (\text{A351})$$

$$\sum_n \lambda 5_{n,p}^\xi = 1, \quad \forall p \in P_d \quad (\text{A352})$$

$$\omega 5_p^\eta \geq 2\overline{\eta 5_{J,p}} \eta 5_p - \overline{\eta 5_{J,p}^2}, \quad j = 1, \dots, M^\eta, \forall p \in P_d \quad (\text{A353})$$

$$\omega 5_p^\eta \leq \sum_n \eta 5_{n,p}^2 \lambda 5_{n,p}^\eta, \quad \forall p \in P_d \quad (\text{A354})$$

$$\eta 5_p = \sum_n \eta 5_{n,p} \lambda 5_{n,p}^\eta, \quad \forall p \in P_d \quad (\text{A355})$$

$$\sum_n \lambda 5_{n,p}^\eta = 1, \quad \forall p \in P_d \quad (\text{A356})$$

$$RL6_p = \omega 6_p^\xi - \omega 6_p^\eta, \quad p = \text{GO} \quad (\text{A357})$$

$$\xi 6_p = (Pr_{j,p} + F_{p,g}^{iprod})/2, \quad p = \text{GO}, j = \text{ON}, g = \text{g90} \quad (\text{A358})$$

$$\eta 6_p = (Pr_{j,p} - F_{p,g}^{iprod})/2, \quad p = \text{GO}, j = \text{ON}, g = \text{g90} \quad (\text{A359})$$

$$\omega 6_p^\xi \geq 2\overline{\xi 6_{J,p}} \xi 6_p - \overline{\xi 6_{J,p}^2}, \quad j = 1, \dots, M^\xi, p = \text{GO} \quad (\text{A360})$$

$$\omega 6_p^\xi \leq \sum_n \xi 6_{n,p}^2 \lambda 6_{n,p}^\xi, \quad p = \text{GO} \quad (\text{A361})$$

$$\xi 6_p = \sum_n \xi 6_{n,p} \lambda 6_{n,p}^\xi, \quad p = \text{GO} \quad (\text{A362})$$

$$\sum_n \lambda 6_{n,p}^\xi = 1, \quad p = \text{GO} \quad (\text{A363})$$

$$\omega 6_p^\eta \geq 2\overline{\eta 6_{J,p}} \eta 6_p - \overline{\eta 6_{J,p}^2}, \quad j = 1, \dots, M^\eta, p = \text{GO}, j = \text{ON}, g = \text{g90} \quad (\text{A364})$$

$$\omega 6_p^\eta \leq \sum_n \eta 6_{n,p}^2 \lambda 6_{n,p}^\eta, \quad p = \text{GO} \quad (\text{A365})$$

$$\eta 6_p = \sum_n \eta 6_{n,p} \lambda 6_{n,p}^\eta, \quad p = \text{GO} \quad (\text{A366})$$

$$\sum_n \lambda 6_{n,p}^\eta = 1, \quad p = \text{GO} \quad (\text{A367})$$

$$RL7_{p'} = \omega 7_{p'}^\xi - \omega 7_{p'}^\eta, \quad p' = \text{HN} \quad (\text{A368})$$

$$\xi7_{p'} = \left(Pr_{j,p'} + F_{p',g}^{iprod} \right) / 2, \quad p' = \text{HN}, j = \text{ON}, g = \text{g90} \quad (\text{A369})$$

$$\eta7_{p'} = \left(Pr_{j,p'} - F_{p',g}^{iprod} \right) / 2, \quad p' = \text{HN}, j = \text{ON}, g = \text{g90} \quad (\text{A370})$$

$$\omega7_{p'}^{\xi} \geq 2\overline{\xi7}_{J,p'}\xi7_{p'} - \overline{\xi7}_{J,p'}^2, \quad j = 1, \dots, M^{\xi}, p' = \text{HN} \quad (\text{A371})$$

$$\omega7_{p'}^{\xi} \leq \sum_n \xi7_{n,p'}^2 \lambda7_{n,p'}^{\xi}, \quad p' = \text{HN} \quad (\text{A372})$$

$$\xi7_{p'} = \sum_n \xi7_{n,p'} \lambda7_{n,p'}^{\xi}, \quad p' = \text{HN} \quad (\text{A373})$$

$$\sum_n \lambda7_{n,p'}^{\xi} = 1, \quad p' = \text{HN} \quad (\text{A374})$$

$$\omega7_{p'}^{\eta} \geq 2\overline{\eta7}_{J,p'}\eta7_{p'} - \overline{\eta7}_{J,p'}^2, \quad j = 1, \dots, M^{\eta}, p' = \text{HN}, j = \text{ON}, g = \text{g90} \quad (\text{A375})$$

$$\omega7_{p'}^{\eta} \leq \sum_n \eta7_{n,p'}^2 \lambda7_{n,p'}^{\eta}, \quad p' = \text{HN} \quad (\text{A376})$$

$$\eta7_{p'} = \sum_n \eta7_{n,p'} \lambda7_{n,p'}^{\eta}, \quad p' = \text{HN} \quad (\text{A377})$$

$$\sum_n \lambda p'^{\eta}_{n,p'} = 1, \quad p' = \text{HN} \quad (\text{A378})$$

$$RL8_p = \omega8_p^{\xi} - \omega8_p^{\eta}, \quad p = \text{GO} \quad (\text{A379})$$

$$\xi8_p = \left(Pr_{j,p} + F_{p,g}^{iprod} \right) / 2, \quad p = \text{GO}, j = \text{ON}, g = \text{g93} \quad (\text{A380})$$

$$\eta6_p = \left(Pr_{j,p} - F_{p,g}^{iprod} \right) / 2, \quad p = \text{GO}, j = \text{ON}, g = \text{g93} \quad (\text{A381})$$

$$\omega8_p^{\xi} \geq 2\overline{\xi8}_{J,p}\xi8_p - \overline{\xi8}_{J,p}^2, \quad j = 1, \dots, M^{\xi}, p = \text{GO} \quad (\text{A382})$$

$$\omega8_p^{\xi} \leq \sum_n \xi8_{n,p}^2 \lambda8_{n,p}^{\xi}, \quad p = \text{GO} \quad (\text{A383})$$

$$\xi8_p = \sum_n \xi8_{n,p} \lambda8_{n,p}^{\xi}, \quad p = \text{GO} \quad (\text{A384})$$

$$\sum_n \lambda8_{n,p}^{\xi} = 1, \quad p = \text{GO} \quad (\text{A385})$$

$$\omega8_p^{\eta} \geq 2\overline{\eta8}_{J,p}\eta8_p - \overline{\eta8}_{J,p}^2, \quad j = 1, \dots, M^{\eta}, p = \text{GO}, j = \text{ON}, g = \text{g93} \quad (\text{A386})$$

$$\omega8_p^{\eta} \leq \sum_n \eta8_{n,p}^2 \lambda8_{n,p}^{\eta}, \quad p = \text{GO} \quad (\text{A387})$$

$$\eta8_p = \sum_n \eta8_{n,p} \lambda8_{n,p}^{\eta}, \quad p = \text{GO} \quad (\text{A388})$$

$$\sum_n \lambda 8_{n,p}^\eta = 1, \quad p = \text{GO} \quad (\text{A389})$$

$$RL9_{p'} = \omega 9_{p'}^\xi - \omega 9_{p'}^\eta, \quad p' = \text{HN} \quad (\text{A390})$$

$$\xi 9_{p'} = \left(Pr_{j,p'} + F_{p',g}^{iprod} \right) / 2, \quad p' = \text{HN}, j = \text{ON}, g = \text{g93} \quad (\text{A391})$$

$$\eta 9_{p'} = \left(Pr_{j,p'} - F_{p',g}^{iprod} \right) / 2, \quad p' = \text{HN}, j = \text{ON}, g = \text{g93} \quad (\text{A392})$$

$$\omega 9_{p'}^\xi \geq 2\overline{\xi 9_{j,p'}} \xi 9_{p'} - \overline{\xi 9_{j,p'}^2}, \quad j = 1, \dots, M^\xi, p' = \text{HN} \quad (\text{A393})$$

$$\omega 9_{p'}^\xi \leq \sum_n \xi 9_{n,p'}^2 \lambda 9_{n,p'}^\xi, \quad p' = \text{HN} \quad (\text{A394})$$

$$\xi 9_{p'} = \sum_n \xi 9_{n,p'} \lambda 9_{n,p'}^\xi, \quad p' = \text{HN} \quad (\text{A395})$$

$$\sum_n \lambda 9_{n,p'}^\xi = 1, \quad p' = \text{HN} \quad (\text{A396})$$

$$\omega 9_{p'}^\eta \geq 2\overline{\eta 9_{j,p'}} \eta 9_{p'} - \overline{\eta 9_{j,p'}^2}, \quad j = 1, \dots, M^\eta, p' = \text{HN}, j = \text{ON}, g = \text{g93} \quad (\text{A397})$$

$$\omega 9_{p'}^\eta \leq \sum_n \eta 9_{n,p'}^2 \lambda 9_{n,p'}^\eta, \quad p' = \text{HN} \quad (\text{A398})$$

$$\eta 9_{p'} = \sum_n \eta 9_{n,p'} \lambda 9_{n,p'}^\eta, \quad p' = \text{HN} \quad (\text{A399})$$

$$\sum_n \lambda p'^\eta_{n,p'} = 1, \quad p' = \text{HN} \quad (\text{A400})$$

Nomenclature

Sets

CR	Crude oils
U	Process units {CDU, FCC}
P	CDU fractions {GO, HN, LD, HD, BR}
P_g	Feed to GB {GO, HN}
P_d	Feed to DB {LD, HD}
F	FCC fractions {Fgas, C24, FHO, Coke}

G	Products of GB {g90, g93}
D	Products of DB {d0, d10}
R	Additive raw materials {MTBE}
S	Sellable products {g90, g93, d0, d10, C24, FHO}
J	Quality properties, octane number and pour point {ON, PP}
N	Set containing grid-points/number of partitions{1,2,3,...,N}
RON_g	Research octane number of GB products {90,93}

Indices

u, u'	Refinery process units
p, p'	Material stream from CDU unit
f	Material stream from FCC unit
g	Material stream from gasoline blending unit
d	Material stream from diesel oil blending unit
r	Additive raw material stream for improving product quality
s	Sellable products
j	Property

Parameters

cp_u^{max}	Maximum capacity of refinery process unit u
F_{cr}	Flow rate of crude oil
$a0_p, a1_p, a2_p, z_p$	Correlation coefficients for CDU fractions p
$a0_f, a1_f, a2_f, z_f$	Correlation coefficients for FCC fractions f
C_s^{fprod}	Price of sellable product s
C_r	Cost of additive raw materials r
C_u	Operating cost of refinery units
C_{cr}	Cost of crude oil
D_s^{max}	Maximum demand of sellable product s

$Pr_{j,r}, Pr_{j,f}$ Property specification j of material streams r and f

Continuous Variables

L_u	Load of process unit u
$Q_{u,p}$	Flow rate of CDU fraction p from process unit u
$Q_{\text{FCC},f}$	Flow rate of fraction f from the FCC unit
W_p	Weight transfer ratio of CDU fraction p
Q_f^{fprod}	Flow rate FCC final product f
Y_f	Weight transfer ratio of FCC product streams f
conv	FCC conversion level
Q_{FCC}^T	Sum of flow rates of FCC recycle stream and FCC feed
Q_{TGO}^R	Flow rate of recycle TGO from the FCC unit
$F_{t,f}^{iprod}$	Flow rate of FCC intermediate product stream f , which is produced by total mixed feed stream t to FCC unit
$F_{p,g}^{iprod}$	Flow rate of intermediate product stream g (g90, g93) from GB, which is produced by blending CDU fraction p
Q_r^A	Flow rate of additive raw material to GB unit to improve blended products quality
$F_{r,g}^{iprod}$	Flow rate of stream g (g90, g93) from GB, which is produced by blending additive raw material stream r
Q_g^{fprod}	Final products flow rate (g90, g93) from GB
$F_{p,d}^{iprod}$	Flow rate of intermediate product stream d from DB, which is produced by blending CDU fraction p
Q_d^{fprod}	Final products flow rate (d0, d10) from DB
Q_s^{fprod}	Flow of final sellable products s
Q_p^P	Flow rate of final product p
MW_p	Mid-point weight transfer ratio of CDU fraction p
$Pr_{j,p}$	Property specification j of CDU fraction p

Profit Total profit of the refinery

Other – Subscripts and Superscripts

iprod Intermediate product

fprod Final product

Abbreviations

CDU Crude Distillation Unit

FCC Fluid Catalytic Cracking Unit

GB Gasoline Blending Unit

DB Diesel Oil Blending Unit

GO Gross Overhead

HN Heavy Naphtha

LD Light Distillate

HD Heavy Distillate

BR Bottom Residue