Mathematical and Computational Applications, Vol. 10, No. 1, pp. 35-44, 2005.

AN INVESTIGATION FOR SEMI-RIGID FRAMES BY DIFFERENT CONNECTION MODELS

Ali Uğur Öztürk and Mutlu Seçer
Department of Civil Engineering, Dokuz Eylul University, Izmir, Turkey

Abstract- A review is made of the development of models used to examine the dynamic response of semi-rigid frames. The connection flexibility is modeled by linear elastic rotational springs. The reducing coefficients and the lateral rigidity values are determined by using a computer program. Response characteristics of frames are compared with reference to their modal attributes.
Keywords- Semi-rigid, Reducing Coefficient, Lateral Rigidity

1. INTRODUCTION

The purpose of engineering design is to produce a structure capable of withstanding the environmental loading to which it may subject to. In conventional frame design, engineers assume that the beam-to-column connections are ideally fixed or frictionless pinned [1,2]. In practice, beam-to-column connections behave between these extremes.

To define the real behavior, two connection models were developed. In the first model, linear elastic springs which represent flexible connection behavior are located at the intersection of beam and column. In the second model, linear elastic springs are located at the ends of the beam. The main difference between these models is the location of the linear elastic springs. This difference affects behavior, dynamic properties and model attributes of frames.

2. SEMI-RIGID FRAME MODELS AND REDUCING COEFFICIENTS

The first semi-rigid frame model is shown in Figure 1. The model includes a beam with moment of inertia I_b and length L, and two columns with moment of inertia I_c, and, length H. The modulus of elasticity E is the same in all frame elements.

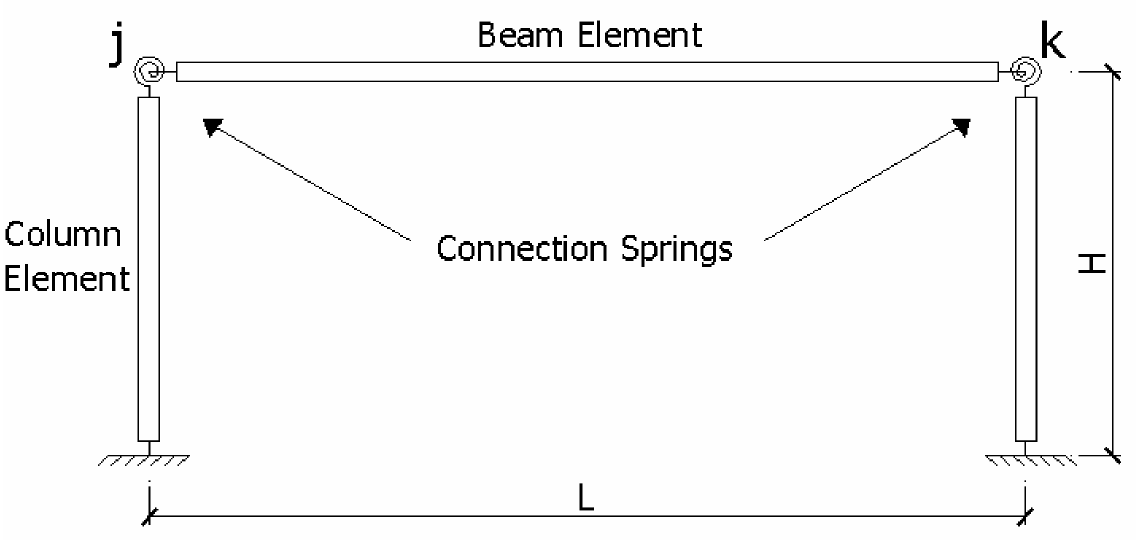

Figure 1. First Semi-Rigid Model

The connections are modeled as rotational springs at beam-to-column joints. Rotations and the lateral displacement are incorporated in the study. One can determine rigidity at the ends of frame elements by the term of rigidity index. For the connection with the hinge, rigidity index is zero, and flexural moments do not occur at the ends of frame elements. For a rigid connection, this value is infinite, and flexural moments occur at the ends of frame elements [3]. Flexural moments at the two ends for a frame element, with spring coefficients represented by $C_{\theta,j}$ and $C_{\theta,k}$, can be given by

$$M_{jf} = C_{\theta,j} x \overline{\Phi}_{\mathrm{j}} \ ; \ M_{kf} = C_{\theta,k} x \overline{\Phi}_{\mathrm{k}} \tag{1}$$

where M_{jf} and M_{kf} are flexural moments, respectively, at j and k ends of a frame element , $\overline{\Phi}_{\mathrm{j}}$ and $\overline{\Phi}_{\mathrm{k}}$ are rotations occurred by rotational springs.

The relationship between spring coefficients and rigidity index can be written by

$$R_j = \frac{C_{\theta,j} L_i}{EI_x} \quad ; \quad R_k = \frac{C_{\theta,k} L_i}{EI_x} \tag{2}$$

where R_j and R_k are rigidity index at two ends of a frame element, respectively.

The stiffness matrix of a semi-rigid column element in Figure 1 can be written by

$$[K_{cf}] = \begin{bmatrix} \frac{12EI_c}{h^3}\gamma_1 & 0 & -\frac{6EI_c}{h^2}\gamma_2 & -\frac{12EI_c}{h^3}\gamma_1 & 0 & -\frac{6EI_c}{h^2}\gamma_3 \\ 0 & \frac{A_cE}{h} & 0 & 0 & -\frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2}\gamma_2 & 0 & \frac{4EI_c}{h}\beta_1 & \frac{6EI_c}{h^2}\gamma_2 & 0 & \frac{2EI_c}{h}\beta_2 \\ -\frac{12EI_c}{h^3}\gamma_1 & 0 & \frac{6EI_c}{h^2}\gamma_2 & \frac{12EI_c}{h^3}\gamma_1 & 0 & \frac{6EI_c}{h^2}\gamma_3 \\ 0 & -\frac{A_cE}{h} & 0 & 0 & \frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2}\gamma_3 & 0 & \frac{2EI_c}{h}\beta_2 & \frac{6EI_c}{h^2}\gamma_3 & 0 & \frac{4EI_c}{h}\beta_3 \end{bmatrix} \tag{3}$$

where ;

$$\gamma_1 = \frac{\beta_1 + \beta_2 + \beta_3}{3} \ ; \ \gamma_2 = \frac{2\beta_1 + \beta_2}{3} \ ; \ \gamma_3 = \frac{2\beta_3 + \beta_2}{3} \tag{4}$$

$$\beta_1 = \frac{3\lambda_1\lambda_2}{\left(4\lambda_1{}^2\lambda_2 - \lambda_1\right)} \quad ; \quad \beta_2 = \frac{3}{\left(4\lambda_1\lambda_2 - 1\right)} \quad ; \quad \beta_3 = \frac{3\lambda_1}{\left(4\lambda_1\lambda_2 - \lambda_1\right)} \tag{5}$$

$$\lambda_1 = \left(1 + \frac{3}{R_j}\right) \quad ; \quad \lambda_2 = \left(1 + \frac{3}{R_k}\right) \tag{6}$$

The stiffness matrix of a semi-rigid beam element in Figure 1 can be written by

$$[K_{bf}] = \begin{bmatrix} \frac{A_b E}{L} & 0 & 0 & -\frac{A_b E}{L} & 0 & 0 \\ 0 & -\frac{12EI_b}{L^3}\gamma_1 & \frac{6EI_b}{L^2}\gamma_3 & 0 & -\frac{12EI_b}{L^3}\gamma_1 & \frac{6EI_b}{L^2}\gamma_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_2 & \frac{4EI_b}{L}\beta_1 & 0 & -\frac{6EI_b}{L^2}\gamma_1 & \frac{2EI_b}{L}\beta_2 \\ -\frac{A_b E}{L} & 0 & 0 & \frac{A_b E}{L} & 0 & 0 \\ 0 & -\frac{12EI_b}{L^3}\gamma_1 & -\frac{6EI_b}{L^2}\gamma_1 & 0 & \frac{12EI_b}{L^3}\gamma_1 & -\frac{6EI_b}{L^2}\gamma_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_3 & \frac{2EI_b}{L}\beta_2 & 0 & -\frac{6EI_b}{L^2}\gamma_3 & \frac{4EI_b}{h}\beta_3 \end{bmatrix} \tag{7}$$

The second semi-rigid frame model is shown in Figure 2. The model includes a beam with moment of inertia I_b and length L, and two columns with moment of inertia I_c, and length H. The modulus of elasticity E is the same in all frame elements.

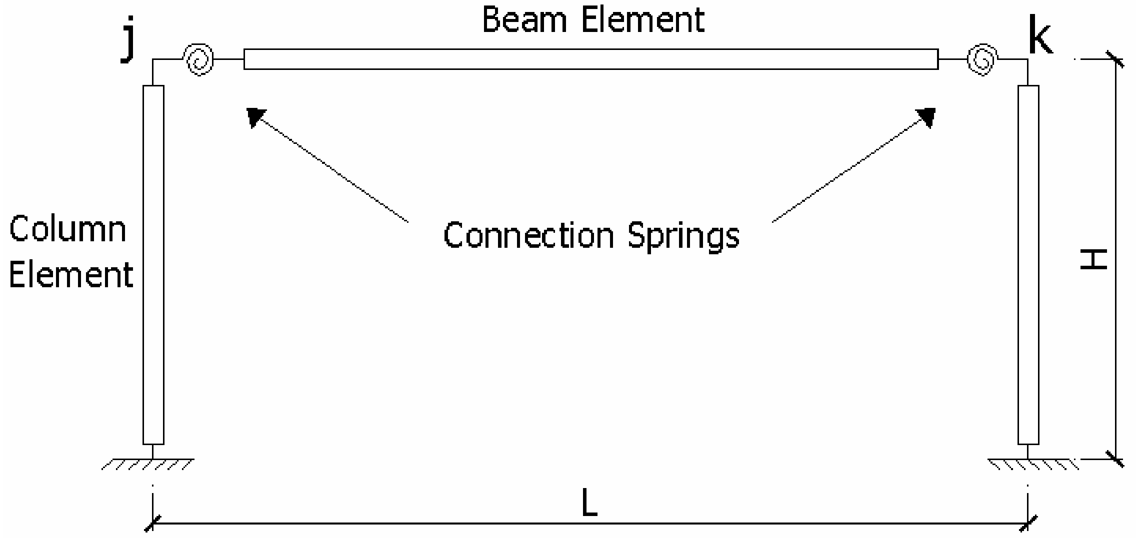

Figure 2. Second Semi-Rigid Model

The connections are modeled by rotational springs near beam-to-column joints and their presence will introduce relative rotations of $\overline{\Phi}_j$ and $\overline{\Phi}_k$ at the ends of the beam [4]. Denoting the stiffness of connections at the ends of beam $C_{\theta,j}$ and $C_{\theta,k}$, respectively, the relative rotation between the joint and the beam end (rotational deformation of the connection) can be given by Equation (1).

If we denote the joint rotation at the ends of the beam by ϕ_j and ϕ_k respectively, the slope-deflection equations for the beam modified for presence of connections can be given by [5,6]

$$M_A = \frac{EI}{L}\left[s_{ii}\left(\theta_j - \frac{M_A}{C_{\theta,j}} \right) + s_{ij}\left(\theta_k - \frac{M_B}{C_{\theta,k}} \right) \right] \tag{8}$$

$$M_B = \frac{EI}{L}\left[s_{ij}\left(\theta_j - \frac{M_A}{C_{\theta,j}} \right) + s_{jj}\left(\theta_k - \frac{M_B}{C_{\theta,k}} \right) \right] \tag{9}$$

The beam stiffness matrix can be written as follows:

$$[K_{bf}] = \begin{bmatrix} \frac{A_b E}{L} & 0 & 0 & -\frac{A_b E}{L} & 0 & 0 \\ 0 & -\frac{EI_b}{L^3}\Psi_1 & \frac{EI_b}{L^2}\Psi_2 & 0 & -\frac{12EI_b}{L^3}\Psi_1 & \frac{EI_b}{L^2}\Psi_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_2 & \frac{EI_b}{L}\Psi_4 & 0 & -\frac{EI_b}{L^2}\Psi_2 & \frac{2EI_b}{L}\Psi_5 \\ -\frac{A_b E}{L} & 0 & 0 & \frac{A_b E}{L} & 0 & 0 \\ 0 & -\frac{EI_b}{L^3}\Psi_1 & -\frac{6EI_b}{L^2}\Psi_2 & 0 & \frac{12EI_b}{L^3}\Psi_1 & -\frac{EI_b}{L^2}\Psi_3 \\ 0 & \frac{EI_b}{L^2}\Psi_3 & \frac{2EI_b}{L}\Psi_5 & 0 & -\frac{6EI_b}{L^2}\gamma_3 & \frac{EI_b}{h}\Psi_5 \end{bmatrix} \tag{10}$$

$$\Psi_1 = s_{ii} + 2\times s_{ij} + s_{jj}\,;\quad \Psi_2 = s_{ii} + s_{ij}\,;\quad \Psi_3 = s_{ij} + s_{jj}\,;\quad \Psi_4 = s_{ii}\,;\quad \Psi_5 = s_{jj} \tag{11}$$

$$s_{ii} = \frac{4 + \frac{12\times EI}{L\times C_{\theta,k}}}{C^*}\,;\quad s_{jj} = \frac{4 + \frac{12\times EI}{L\times C_{\theta,j}}}{C^*}\,;\quad s_{ij} = s_{ji} = \frac{2}{C^*} \tag{12}$$

$$C^* = \left(1 + \frac{4\times EI}{L\times C_{\theta,j}} \right)\times\left(1 + \frac{4\times EI}{L\times C_{\theta,k}} \right) - \left(\frac{EI}{L} \right)^2 \times \frac{4}{C_{\theta,j}\times C_{\theta,k}} \tag{13}$$

The column stiffness matrix can be written as follows:

$$[K_{cf}]=\begin{bmatrix} \frac{12EI_c}{h^3} & 0 & -\frac{6EI_c}{h^2} & -\frac{12EI_c}{h^3} & 0 & -\frac{6EI_c}{h^2} \\ 0 & \frac{A_cE}{h} & 0 & 0 & -\frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2} & 0 & \frac{4EI_c}{h} & \frac{6EI_c}{h^2} & 0 & \frac{2EI_c}{h} \\ -\frac{12EI_c}{h^3} & 0 & \frac{6EI_c}{h^2} & \frac{12EI_c}{h^3} & 0 & \frac{6EI_c}{h^2} \\ 0 & -\frac{A_cE}{h} & 0 & 0 & \frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2} & 0 & \frac{2EI_c}{h} & \frac{6EI_c}{h^2} & 0 & \frac{4EI_c}{h} \end{bmatrix} \qquad (14)$$

The structure stiffness matrix is obtained by assembling the column and beam stiffness matrices described above according to conventional stiffness matrix analysis procedure [7]. One obtains a 6x6 stiffness matrix for the frame of Figure 3.

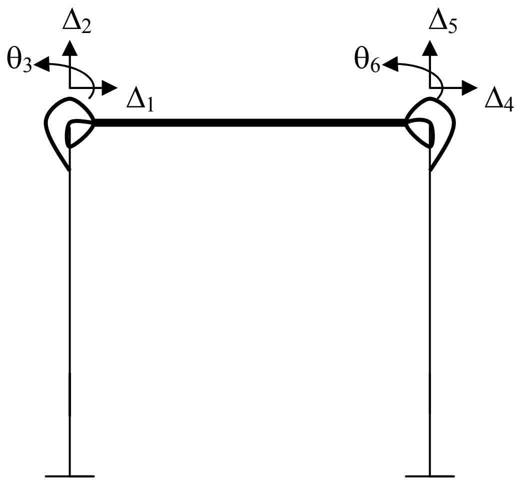

Figure 3. Degrees-of-freedom

By assuming that Δ_1 and Δ_4 are equal and the axial deformations Δ_2 and Δ_5 are relatively small, one can eliminate Δ_4, Δ_2 and Δ_5 from the frame of Figure 4. The reduced displacements are given by Figure 4. The remaining stiffness matrix is a 3x3 matrix [8].

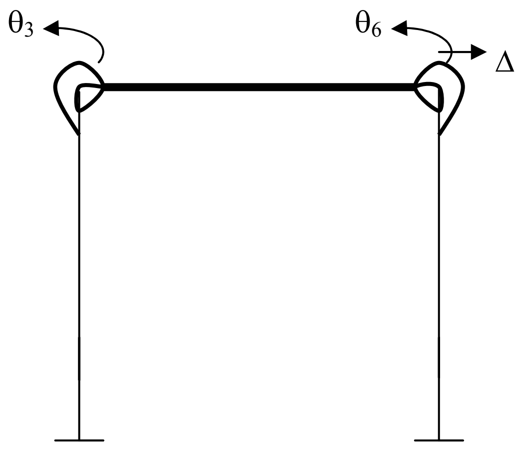

Figure 4. The reduced displacements

$$\{F\} = [Ksf] \times \{ \delta \} \tag{15}$$

The relationship between deformations and forces are given by equation (10). Solving the above matrix equation for displacements except Δ and back substituting the result into the first row, the one-degree-of–freedom system stiffness relationship can be written as

$$F=\frac{24EI_x}{h^3}\alpha_r \times \Delta \tag{16}$$

where Δ is the lateral displacement ,and F and α_r are the lateral force and reducing coefficient respectively.

3. DYNAMIC ANALYSIS AND NUMERICAL STUDIES

The primary objective of the present study is to investigate the dynamic characteristics of semi-rigid frames and how connection flexibility influences them. For a given frame in Figure 4, the equation of motion for a semi-rigid frame in free vibration is given by

$$[\,M\,]\,\{\ddot{v}\}+[k]\{\,v\,\}=\{0\} \tag{17}$$

where $\ddot{v}$ and v are, respectively, acceleration and displacement of a structure [9,10].

The dynamic characteristics of semi-rigid frames are determined by modal analysis. The frequency and period of a vibration will be investigated. The influence of connection flexibility will be studied.

In the present study, 3-story semi-rigid frames having four different spring coefficients were studied. The semi-rigid models for the present analysis are given in Figure 5. All frames have the same geometry, cross-section and material property to compare the influence of connection flexibility on dynamic characteristics. First, the reducing coefficients were determined by using a computer program. Then, lateral rigidity values were calculated for each frame. The reducing coefficients and periods are given in Table 1 below. The values are given in terms of ton, meter, radian, second which are represented by (t), (m), (rd),(sec) respectively

All frame elements are designed by using steel. The modulus of Elasticity E is 2.1×10^7 t/m^2 for all elements. The length of beam elements are chosen 6 meters and the length of column elements are chosen 3.5 meters. The cross-section (0.25 x 0.50) area of each beam is 0.125 m^2 and the cross-section (0.30 x 0.50) area of each column is 0.150 m^2. Therefore, the moment of inertia of each beam is 2.604×10^{-3} m^4 and the moment of inertia of each column is 3.125×10^{-3} m^4. According to the modal analysis procedures; w_1, w_2, w_3 show the lumped-masses of all stories, respectively.

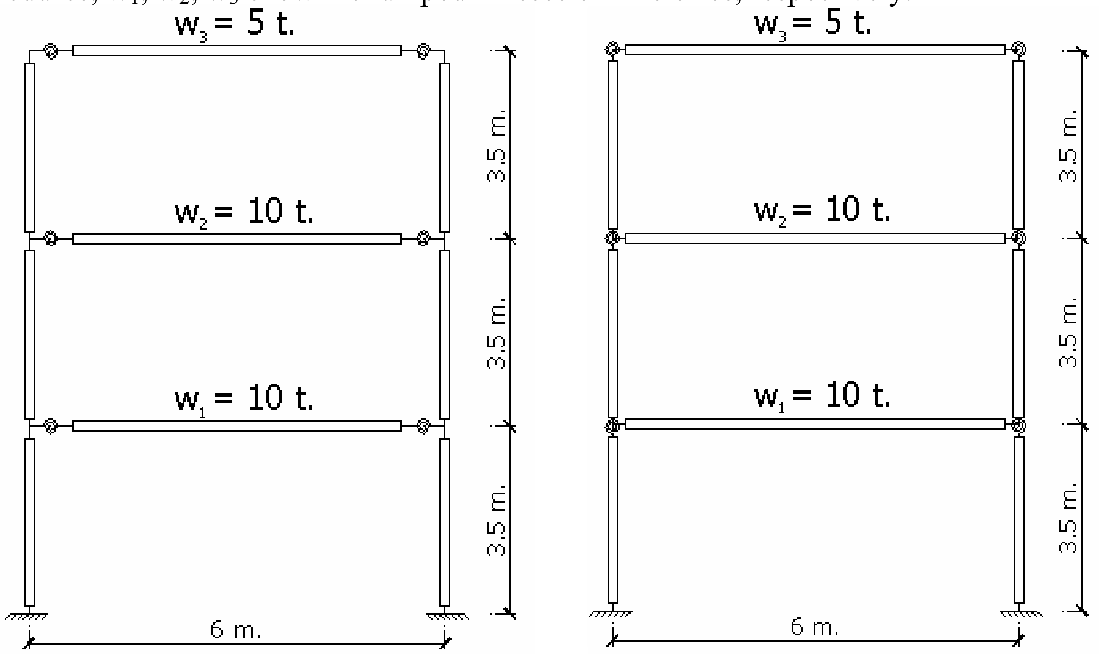

Figure 5. Semi-rigid models for the present analysis

Table 1. Reducing coefficients

Connection model	Reducing coefficient (α_r)		Lateral rigidity (t / m)	
	First Model	Second Model	First Model	Second Model
Semi-rigid (2000 tm/rd)	0.0231	0.2688	850.14	9874.67
Semi-rigid (5000 tm/rd)	0.0542	0.2932	1989.48	10769.66
Semi-rigid (20000 tm/rd)	0.1657	0.3725	6087.48	13648.36
Semi-rigid (10^{20} tm/rd)	0.5663	0.5663	20801.14	20801.14
Rigid	0.5663	0.5663	20801.14	20801.14

The results of the conducted analysis are given for each mod of vibration below.

Table 2. Dynamic results of 1st mod

Connection model	Vibration Frequencies (rd/sec)		Vibration Periods (sec)	
	First Model	Second Model	First Model	Second Model
Semi-rigid (2000 tm/rd)	14.9442	50.3316	0.4204	0.1234
Semi-rigid (5000 tm/rd)	22.8610	53.1897	0.2748	0.1181
Semi-rigid (20000 tm/rd)	39.9894	59.8779	0.1571	0.1049
Semi-rigid (10^{20} tm/rd)	73.9213	73.9213	0.0850	0.0850
Rigid	73.9213	73.9213	0.0850	0.0850

Table 3. Dynamic results of 2nd mod

Connection model	Vibration Frequencies (rd/sec)		Vibration Periods (sec)	
	First Model	Second Model	First Model	Second Model
Semi-rigid (2000 tm/rd)	40.8282	139.1478	0.1539	0.0452
Semi-rigid (5000 tm/rd)	62.4575	145.3168	0.1006	0.0432
Semi-rigid (20000 tm/rd)	109.2531	163.5894	0.0575	0.0384
Semi-rigid (10^{20} tm/rd)	201.9568	201.9568	0.0311	0.0311
Rigid	201.9568	201.9568	0.0311	0.0311

Table 4. Dynamic results of 3rd mod

Connection model	Vibration Frequencies (rd/sec)		Vibration Periods (sec)	
	First Model	Second Model	First Model	Second Model
Semi-rigid (2000 tm/rd)	55.7723	190.0794	0.1127	0.0331
Semi-rigid (5000 tm/rd)	85.3186	198.5065	0.0736	0.0317
Semi-rigid (20000 tm/rd)	149.2425	223.4673	0.0421	0.0285
Semi-rigid (10^{20} tm/rd)	275.8781	275.8781	0.0228	0.0228
Rigid	275.8781	275.8781	0.0228	0.0228

4. CONCLUSION

The subject has been studied extensively for last 10 years. These studies were mostly about statical analysis and also, these connections models were not investigated together. Dynamic analysis of these connection models have been studied recently.

However, none of them is about the reducing coefficients and lateral rigidity values for different models.

In this study, two semi-rigid frame models were used. The connection flexibility was modeled by linear elastic rotational springs. A computer program was written to obtain the reducing coefficients for each model and dynamic analysis was performed for different types of each semi-rigid frame model.

In the first model, flexible connections were located at the intersection of beam and column. Four different spring coefficients were used for the connection flexibility. In the second model, flexible connections were located at the ends of the beam. Indeed, four different coefficients were used for representing the connection flexibility. The reducing coefficients and the dynamic characteristics were determined for each connection type. Dynamic properties were investigated with reference to modal attributes.

This study compares these two connection models and sheds lights on the design of steel structures. For the structural design; this study gives designers information about the differences which can be occurred by modeling. The importance of modeling is one of the main aims of the study like determining reducing coefficients, lateral rigidity values, dynamic characteristics such as vibration periods and frequencies.

The study indicates that connection models have influences on the dynamic characteristics of frames. The location of the linear elastic connection springs affects the behavior and the lateral rigidity of semi-rigid frames. Linear elastic springs of the first model decrease the lateral rigidity values more than linear elastic springs of the second model. The dynamic results were represented in terms of vibration frequencies (rd/sec) and periods (sec). For each spring coefficient, vibration frequencies of the first model are lower than vibration frequencies of the second model in all modes. As a result, vibration periods of the first model are higher than vibration periods of the second model.

In practice, since beam and columns are connected at the ends of beam element, the second model represents the structural behavior better than the first model. For this reason, the second model is more suitable in steel frame design.

As the linear elastic rotational spring coefficients increase, the behavior of frames becomes more rigid. For the ultimate values of linear elastic rotational spring coefficients, there were no differences with the dynamic results of the rigid frame.

4. REFERENCES

1. E.M. Lui and A. Lopes, Dynamic analysis and response of semi rigid frames, Journal of Engineering Structures, 19, 644-654, 1997.
2. R. Bjorhovde, A. Colson and J. Brozetti, A classification system for beam to column connections, Journal of Structural Engineering, 116, 3059-3076, 1990.

3. H.H. Çatal, Yapı ve deprem mühendisliğinde matris yöntemler, Izmir, 2002.
4. R. Salatic and M. Sekılovic, Nonlinear analysis of frames with flexible connections, Computers and Structures, 79, 1097-1107, 2001.
5. W.F. Chen, Y.Goto and R.Liew, Stability design of semi rigid frames, New York, Wiley, 1993.
6. W.F. Chen and S.E. Kim, LRFD Steel Design Using Advance Analysis, Florida, CRC Press, 1997.
7. R.K. Livesley, Matrix Methods of Structural Analysis, Great Britain: Pergamon Press, 1975.
8. J.C. Awkar and E.M. Lui, Seismic analysis and response of multistory semi-rigid frames, Journal of Engineering Structures,30, 425-441, 1997.
9. A.K. Chopra, Dynamics of structures: Theory and applications to Earthquake engineering, Englewood Cliffs, NJ, Prentice Hall, 1995.
10. R.W. Clough and J. Penzien, Dynamics of structures 2nd ed., New York, McGraw-Hill, 1993.