

Article

Parity-Time Symmetry Enabled Band-Pass Filter Featuring High Bandwidth-Tunable Contrast Ratio

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Abstract: Integrated optical filters based on microring resonators play a critical role in many applications, ranging from wavelength division multiplexing and switching to channel routing. Bandwidth tunable filters are capable of meeting the on-demand flexible operations in complex situations, due to their advantages of scalability, multi-functionality, and being energy-saving. Recent studies have investigated how parity-time (PT) symmetry coupled-resonant systems can be applied to the bandwidth-tunable filters. However, due to the trade-off between the bandwidth-tunable contrast ratio and insertion loss of the system, the bandwidth-tunable contrast ratio of this method is severely limited. Here, the bandwidth-tunable contrast ratio is defined as the maximum bandwidth divided by the minimum bandwidth. In this work, we show that a high bandwidth-tunable contrast ratio and low insertion loss of the system can be achieved simultaneously by increasing the coupling strength between the input port and the resonant. Theoretical analysis under different coupling states reveals that the low insertion loss can be obtained when the system initially operates at the over-coupling condition. A high bandwidth-tunable contrast ratio PT-symmetry band-pass filter with moderate insertion loss is shown on the Silicon platform. Our scheme provides an effective method to reduce the insertion loss of on-chip tunable filters, which is also applicable to the high-order cascaded microring systems.

Keywords: bandpass tunable filter; parity-time symmetry; microresonators



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1. Introduction

Highly integrated photonic technology requires on-demand scalable hardware. A key device for wavelength division multiplexing systems is the functional optical filter with immediate applications for optical communication and switching networks [1,2]. Among the integrated optical devices, microresonator-based instruments have been extensively explored in a broad range of applications, including all-optical signal processing [3–5], lasing [6,7], quantum optics [8], wavelength filtering [9–11], etc. Dynamic tuning of optical microresonators [12–18] has been shown to provide new functionalities for on-chip optical communications and information processing. By simply fabricating microheaters on the microresonators, the resonance of the microresonators can be adjusted through the thermal drift effect [19–21]. The inherent amplitude-frequency responses of microresonators are usually applied to add, drop, or block selected wavelength channels, which is naturally suitable for designing on-chip bandwidth tunable filters. To date, various tunable filter implementations based on microresonators have been studied, promising to meet the great needs of big data and intelligent optical communications—for instance, employing the Mach–Zehnder interferometers (MZI) combined microresonators for simply tuning the input and output coupling strength to control the resonant linewidth [12]; using a multiple microrings

cascading structure to reach a large tunable bandwidth [22]; applying tandem microrings for broadening transmittance for band-pass filters [23,24]; and utilizing some other structures to control the tunable-bandwidth [25–30]. All these schemes have analyzed the spectral response of the filter, and the proof-of-principle devices showed the results matched well with the theoretical prediction. However, these designs are relatively complicated.

Parity-time (PT) symmetry [31–33] has shown promise in numerous applications since its first observation in optics [32], such as ring laser [6], sensing [34], optoelectronic oscillator [35], coherent perfect absorption in coupled rings [36], etc. Recent explorations on PT-symmetry in optical coupled-resonant systems open the doors for bandwidth manipulation [36,37]. By introducing appropriate loss on the aimed resonance, i.e., setting the suitable parameters of the coupling strengths between the rings and between the ring and the bus-waveguide, the mode splitting can be prevented near the exceptional point (EP) where the phase transition occurs [6,33,34,36]. Our recent work has realized a simple and practical bandwidth-tunable band-pass filter by introducing the PT-symmetry coupled-resonant system into the band-pass filters [37]. Benefiting from the flexible adjustability of the system, a PT-symmetry enabled band-pass filter has a wide range of applications. However, due to the trade-off between the bandwidth-tunable contrast ratio and insertion loss of the system, the PT-symmetry band-pass filter faces a considerable challenge in the high bandwidth-tunable contrast ratio applications. The tunable microring bandwidth systems [13] have been widely utilized in many applications, not only in communications but in Q-switched lasers [38], tunable modulators [39], tunable time delay lines [40], stopping light devices [41], etc. In this work, we reveal the physics behind this trade-off by applying the temporal coupled mode theory (TCMT) [42] and the generalized critical coupling (GCC) condition proposed in [43,44]. The trade-off between the bandwidth-tunable contrast ratio and insertion loss of the system is mitigated by increasing the coupling strength between the input port and the resonant. We further theoretically study the feasibility of this scheme by showing a high bandwidth-tunable contrast ratio PT-symmetry band-pass filter with moderate insertion loss.

2. Principle of Operation

The PT-symmetry coupled resonant structure with microheaters has been demonstrated to realize the bandwidth tunable band-pass filter [37], the schematic diagram of which is shown in Figure 1a. The system consists of the main cavity and the auxiliary cavity with an intercavity coupling rate of g , and two bus waveguides coupled to the main cavity and auxiliary cavity, respectively. The signal launches into the waveguide from the bottom left port, and the filtered light leaves from the top right side, as indicated in Figure 1a. The intrinsic decay rates of the main ring and the auxiliary ring are γ_{i1} and γ_{i2} , and the coupling decay rates of the main cavity and the auxiliary cavity between the bus-waveguides are γ_{c1} and γ_{c2} , respectively. For the coupled resonant device, the sizes of the two rings are set to be the same, which results in the same free spectral range (FSR), and the same intrinsic decay rate $\gamma_{i1} = \gamma_{i2}$. According to the temporal coupled mode theory (TCMT) [42], the coupled mode equation of the system can be expressed as $idA/dt = HA - i\sqrt{\gamma_{c1}} \begin{pmatrix} a_{in} & 0 \end{pmatrix}^T$, where $A = \begin{pmatrix} a_1 & a_2 \end{pmatrix}^T$ is the modal field vector, a_1 and a_2 are the intracavity fields of the main ring and the auxiliary ring, a_{in} is the input modal field, and ω_1 and ω_2 are the resonant frequencies of the main ring and the auxiliary ring, respectively. Furthermore, the non-Hermitian Hamiltonian H reads

$$H = \begin{bmatrix} \omega_1 - i(\gamma_{i1} + \gamma_{c1})/2 & g \\ g & \omega_2 - i(\gamma_{i2} + \gamma_{c2})/2 \end{bmatrix} \quad (1)$$

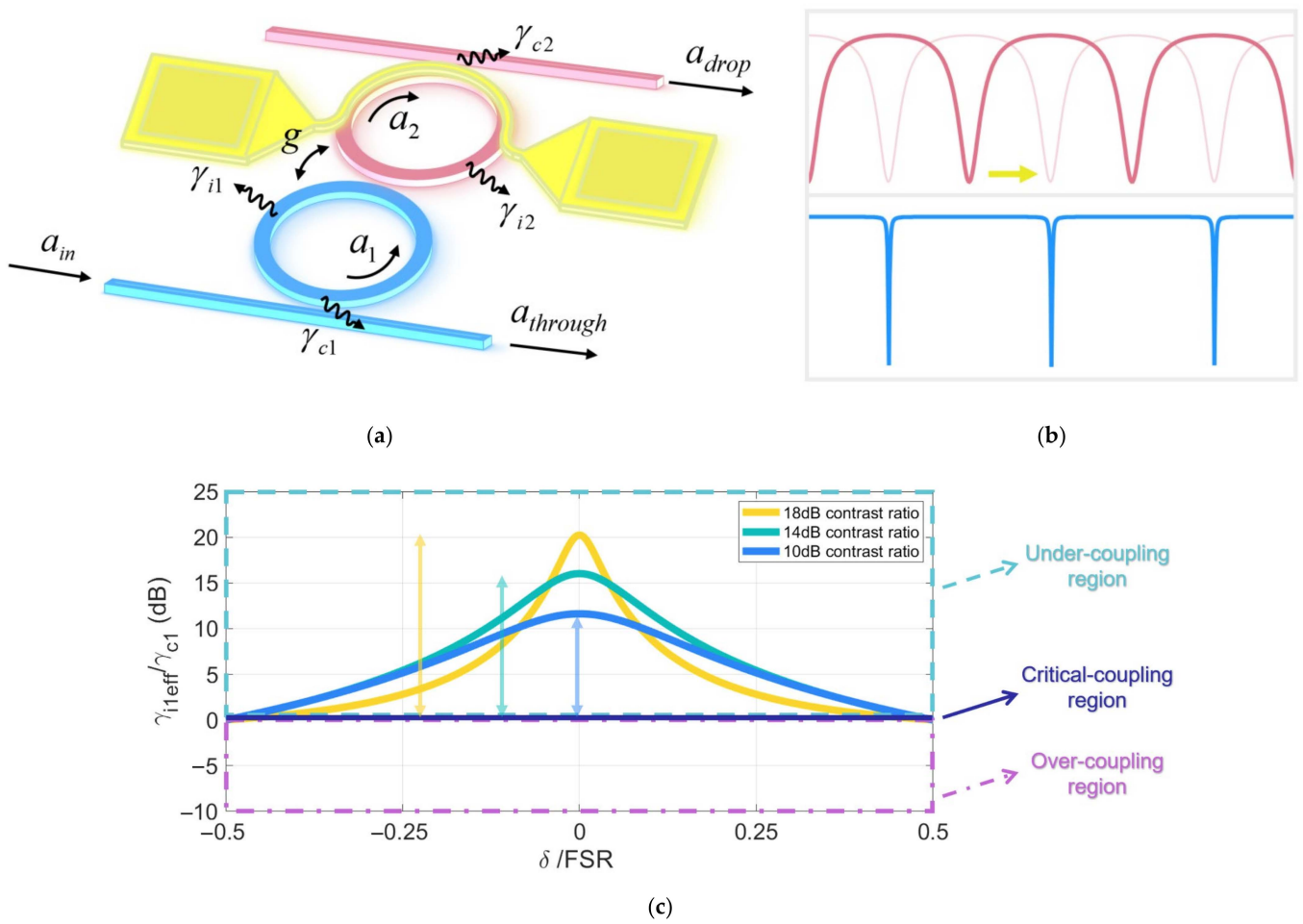


Figure 1. Parity-time (PT) symmetry enabled bandwidth-tunable band-pass filter with the trade-off between the bandwidth tunable contrast ratio and insertion loss. (a) Schematic diagram of the proposed coupled resonant device structure. (b) Schematic of adjusting detuning between two rings. (c) Evolution of the ratio of the effective propagation loss to the coupling loss of the main cavity ($\gamma_{1eff}/\gamma_{c1}$) as a function of the detuning δ/FSR .

We define the δ as the frequency detuning of two rings, satisfying $\delta = (\omega_2 - \omega_1)/2\pi$. The microheater is fabricated upon the auxiliary ring to adjust the δ by tuning the resonant frequency [19], as shown in the schematic diagram of Figure 1b. Blue and red lines in Figure 1b represent the resonance positions of the main cavity and the auxiliary cavity, respectively. In this work, it is only needed to adjust a resonance across one FSR of the microresonator; thus, thermal tuning is completely sufficient. It should be noted that the initial detuning ($|\delta/FSR| = 0.5$) and the alignment ($\delta/FSR = 0$) of the two rings, corresponding to the positions of minimum and maximum system bandwidth, are represented by the dark and light red lines in Figure 1b, respectively. Following our previous work [37], the system is operated close to the EP ($4g \approx \gamma_{c2} - \gamma_{c1}$) at the alignment, where the maximum and minimum bandwidths of the system are $\Delta\omega_{max} = (\gamma_{i1} + \gamma_{c1} + \gamma_{i2} + \gamma_{c2})/2$ and $\Delta\omega_{min} = \gamma_{i1} + \gamma_{c1}$. To evaluate the system performance, the bandwidth-tunable contrast ratio is defined as the system maximum bandwidth divided by minimum bandwidth:

$$\Delta\omega_{ratio} = \frac{\Delta\omega_{max}}{\Delta\omega_{min}} = \frac{1}{2} + \frac{\gamma_{i2} + \gamma_{c2}}{2(\gamma_{i1} + \gamma_{c1})} \quad (2)$$

Equation (2) shows that the system can realize a high bandwidth-tunable contrast ratio when $\gamma_{c2} \gg \gamma_{c1}$. Note that when the intrinsic losses of the rings are small (for high

Q factor rings cases), i.e., $\gamma_{c2} \gg \gamma_{i1}$ and $\gamma_{c2} \gg \gamma_{i2}$, the filter can also benefit from a high tunable contrast ratio. However, [37] only achieves a bandwidth-tunable contrast ratio of about 5.78 (28.8~166.6 GHz) with the low system insertion loss, which is limited by the trade-off between bandwidth-tunable contrast ratio and insertion loss of the system.

To ensure the injected signal can be launched into the rings completely, the through port of the system is operated in the critical-coupling condition ($\gamma_{i1} \approx \gamma_{c1}$) at the initial detuning [37]. Obviously, when the through port is extinct at the critical coupling, the input signal is entirely transferred to the drop port, resulting in the system insertion loss achieving the minimum. Here, the bandwidth-tunable contrast ratio is found to be

$$\Delta\omega_{ratio} \approx \frac{1}{2} + \frac{\gamma_{c2}}{4\gamma_{c1}} \quad (3)$$

However, due to the influence of the auxiliary ring, the coupling state of the through port changes significantly during detuning. The system insertion loss increases gradually once the coupling state of the through port deviates from the critical-coupling condition. According to the GCC condition [43,44], the effective loss rate of the main cavity induced by the auxiliary ring and the output waveguide of the auxiliary ring is calculated as $\gamma_{1eff} = 4g^2/(\gamma_{i2} + \gamma_{c2})$, when $\delta = 0$. Note that the effective propagation loss of the main ring is defined as $\gamma_{i1eff} = \gamma_{i1} + \gamma_{1eff}$. Therefore, the coupled mode equation of the dual-ring system can be inferred as

$$\frac{da_1}{dt} = -i\omega_1 a_1 - \frac{\gamma_{i1eff} + \gamma_{c1}}{2} a_1 - \sqrt{\gamma_{c1}} a_{in} \quad (4)$$

Moreover, considering the influence of the detuning δ of the two rings, the effective propagation loss of the main cavity can be calculated as

$$\gamma_{i1eff} = \gamma_{i1} + \frac{4g^2}{\gamma_{i2} + \gamma_{c2} + \frac{4\delta^2}{\gamma_{i2} + \gamma_{c2}}} \quad (5)$$

According to the GCC condition, the main cavity is critically coupled to the through-port waveguide when $\gamma_{c1} = \gamma_{i1eff}$. Defining $\Delta\gamma = \gamma_{i1eff}/\gamma_{c1}$ as the degree of deviation between the system coupling state and the critical coupling condition. It is foreseeable that as $\Delta\gamma$ deviates from 0dB, the system coupling state will gradually shift from the critical coupling condition, and the system insertion loss will increase. In other words, the insertion loss of the system is positively related to $\Delta\gamma$, and $\Delta\gamma$ reaches the maximum $\Delta\gamma_{max} \approx (\gamma_{c2} - \gamma_{c1})^2/4\gamma_{c2}\gamma_{c1} + 1$ when the two rings are aligned. When the through port is operated in the critical-coupling condition at the initial detuning and the system has a high bandwidth-tunable contrast ratio, the relationship between $\Delta\omega_{ratio}$ and $\Delta\gamma_{max}$ can be obtained according to Equation (3):

$$\Delta\omega_{ratio} \approx \Delta\gamma_{max} - \frac{1}{2} \quad (6)$$

Equation (6) indicates that the system bandwidth-tunable contrast ratio $\Delta\omega_{ratio}$ is proportional to $\Delta\gamma_{max}$, confirming that the insertion loss of the system increases with the improving bandwidth-tunable contrast ratios. Furthermore, the evolution of $\Delta\gamma$ as a function of δ/FSR under different system bandwidth-tunable contrast ratios are plotted with three colored lines in Figure 1c. The yellow, green, and blue lines correspond to the systems with bandwidth-tunable contrast ratios of 18 dB, 14 dB, and 10 dB, respectively. The areas circled in the green and purple lines represent the system operating in the under-coupling and over-coupling conditions, respectively. The dark blue line means that the main cavity is critically coupled to the through port. The system working parameters here are $\gamma_{i1} = \gamma_{i2} = 2.37$ GHz. The parameters used for the yellow, green, and blue lines are: $\gamma_{c1} = 3.42$ GHz, $\gamma_{c2} = 369.37$ GHz, $g = 182.39$ GHz, $\gamma_{c1} = 24.51$ GHz,

$\gamma_{c2} = 1043.7$ GHz, $g = 505.52$ GHz, and $\gamma_{c1} = 114.23$ GHz, $\gamma_{c2} = 1847.6$ GHz, $g = 874.79$ GHz, respectively. It can be shown that when $|\delta/FSR| = 0.5$, the $\Delta\gamma$ of three lines are all 0dB, which means that the through ports of systems are operated in the critical-coupling condition at the initial detuning. According to the conclusion of the GCC condition, the effective propagation loss γ_{i1eff} of the main cavity increases as the detuning δ decreases. The coupling states of the through port will depart from critical-coupling to the under-coupling condition with the reduction of δ , corresponding to the lines in Figure 1c, since the effective propagation loss of the main cavity gradually increases while the coupling loss of that remains the same. All these changes lead to the insertion loss growing significantly as the coupling state of the through port gradually deviates from the critical-coupling condition. The colored lines drawn in Figure 1c show that the system having a higher bandwidth-tunable contrast ratio corresponds to a larger insertion loss, which matches well with the theoretical expectations (Equation (6)).

To demonstrate the PT-symmetry coupled-resonant band-pass filter with a high bandwidth-tunable contrast ratio, we propose a scheme to mitigate the trade-off between the bandwidth-tunable contrast ratio and insertion loss of the system. Since the deviation of the system coupling state from the critical-coupling condition will lead to high insertion loss, the system with low insertion loss can be realized once the coupling state of the system can be maintained near the critical-coupling condition throughout the detuning. By increasing the coupling strength between the through port and the main cavity, we modify the evolution of system coupling states with decreasing detuning as slightly over-coupling changes to critical-coupling, finally reaching under-coupling, which keeps the entire insertion loss variation within a small threshold. The schematic diagrams of the systems operating in critical coupling, under-coupling, and over-coupling conditions at the initial detuning are given in Figure 2a–c, respectively. Note that the different length curves of γ_{c1} are utilized to represent the coupling strength between the through port and main cavity in Figure 2a–c. Figure 2d indicates the evolution of $\Delta\gamma$ with δ/FSR under different initial coupling states, where the dark blue, green, and purple lines correspond to the systems with initial coupling states as critical coupling, under-coupling, and over-coupling conditions, respectively. The system working parameters here are $\gamma_{i1} = \gamma_{i2} = 2.37$ GHz, $\gamma_{c2} = 1043.7$ GHz, $g = 505.52$ GHz. Parameters used for the dark blue, green, and purple lines are: $\gamma_{c1} = 24.51$ GHz, $\gamma_{c1} = 3.95$ GHz, and $\gamma_{c1} = 151.97$ GHz, respectively. The dark blue line shows that the insertion loss of the system keeps increasing as δ decreases. As the detuning decreases, the coupling state of the system, corresponding to the green line in Figure 2d, is further away from the critical-coupling condition. Here, the main cavity is strongly under-coupled at the alignment, where the input field is hardly coupled into the system and fewer field outputs from the drop port, resulting in extremely high system insertion loss. Note that with the decrease of δ , the coupling state of the system corresponding to the purple line first approaches critical coupling and then moves away from the critical-coupling condition, which keeps the entire evolution of the system coupling state near the critical-coupling condition, ensuring that the system insertion loss is maintained in a small threshold. It is foreseeable that the system operating on the purple line can enable the high bandwidth-tunable contrast ratio PT-symmetry coupled-resonant band-pass filter with low insertion loss.

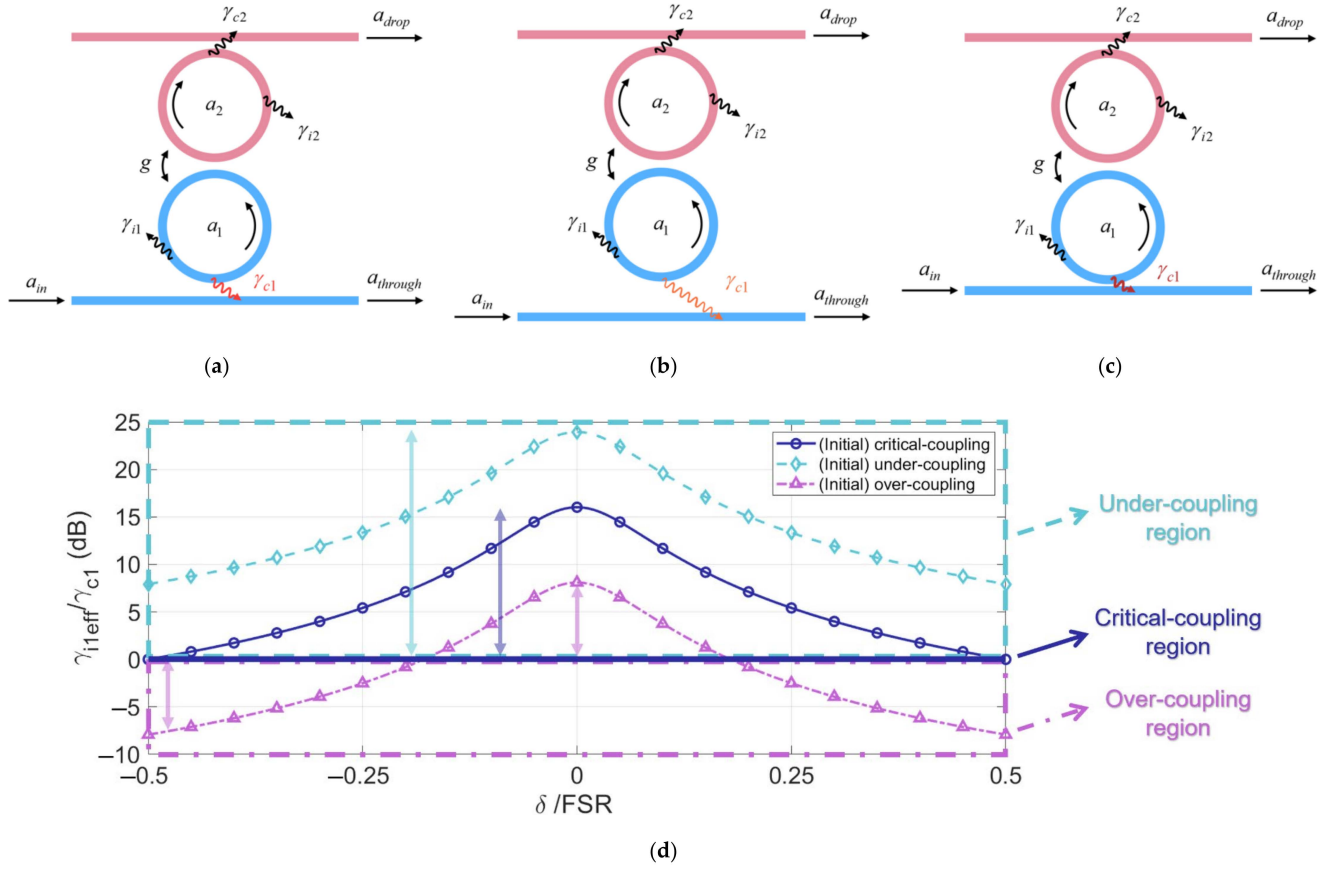


Figure 2. Evolution of system coupling state with detuning under different initial coupling states. (a–c) show the schematic diagrams of the system operating at the critical-coupling, under-coupling, and over-coupling conditions at initial detuning, respectively. (d) The evolution of the ratio of the effective propagation loss to the coupling loss of the main cavity ($\gamma_{i1eff}/\gamma_{c1}$) as a function of the detuning δ/FSR .

3. Results and Discussion

Although TCMT can characterize the physics behind the system well, it cannot accurately simulate the entire parameter space. More importantly, it is only suitable for low-loss and weakly coupled systems. In order to evaluate and optimize the system performance under various coupling states, a rigorous analysis of our approach is captured by the transfer matrix method (TMM) [42]. Figure 3a shows the notations used in the TMM of the system, where $\sigma_{j=1,2}$ is the roundtrip field attenuation factor of the resonators, k_2 is the field coupling coefficient between the resonators, k_1, k_3 are the field coupling coefficients of the main cavity and the auxiliary cavity with the waveguides, respectively, and $r_{j=1,2,3}$ is the transmission coefficient, where $r_j^2 + k_j^2 = 1$ in the approximation of lossless coupling. E_{in} is the input field, and $E_{through}$ is the output field at the through port; E_{drop} is the output field at the drop port. The transmission spectra of the through port and drop port can be calculated as

$$T_{through_TMM} = \left| \frac{E_{through}}{E_{in}} \right|^2 = \left| \frac{r_1 - \sigma_1 \tau \exp(i\varphi_1)}{1 - r_1 \sigma_1 \tau \exp(i\varphi_1)} \right|^2 \quad (7)$$

$$T_{drop_TMM} = \left| \frac{E_{drop}}{E_{in}} \right|^2 = \left| \frac{ik_1 k_2 k_3 \sqrt{\sigma_1} \exp(i\varphi_1/2) \sqrt{\sigma_2} \exp(i\varphi_2/2)}{(1 - r_1 \sigma_1 \tau \exp(i\varphi_1))(1 - r_2 \sigma_2 r_3 \exp(i\varphi_2))} \right|^2 \quad (8)$$

where $\tau = (r_2 - \sigma_2 r_3 \exp(i\varphi_2)) / (1 - r_2 \sigma_2 r_3 \exp(i\varphi_2))$ and $\varphi_{j=1,2}$ is the phase shift of input light traveling per roundtrip in the resonators. The system insertion loss is defined as the maximum loss at the drop port when the resonances are tuned, i.e., the maximal $|T_{drop_max}|$.

It is worth noting that in this work, all the simulation parameters are based on the Silicon platform. Moreover, the lines in Figure 3b indicate that the insertion loss of the drop port varies with δ/FSR under the same coupling coefficients k_2, k_3 in different initial coupling conditions of critical-coupling, under-coupling, and over-coupling, respectively. The dark blue, green, and purple lines correspond to the systems operating in the critical-coupling, under-coupling, and over-coupling conditions at initial detuning, with the insertion loss range of 1.23~10.91 dB, 2.94~15.56 dB, and 0.46~6.5 dB, respectively. Note that the initial system coupling states of under-coupling and over-coupling conditions are realized by setting the transmittance of the through port to -6dB at the initial detuning. The solid dark blue line in Figure 3b corresponds to the through port that operates at the critical-coupling condition at the initial, showing that the initial insertion loss of the drop port is about 1.7 dB, where the insertion loss is mainly caused by the intrinsic loss of the system. The insertion loss of the drop port is slightly reduced with the tuning of δ/FSR , caused by the co-effect of the two supermodes. Note that the effective propagation loss of the main cavity is considerably increased at the alignment, where the system represented by the dark blue line enters the strongly under-coupled regime and has a high insertion loss. The green and purple lines feature a larger insertion loss at the initial detuning since, at this time, the through port is not entirely extinct and some energy escapes. Since the system corresponding to the green line is under the stronger under-coupling condition throughout the tuning process, the overall insertion loss of the green line is larger than that of the dark blue line. Moreover, the purple line indicates that the insertion loss of the drop port first decreases to near 0 dB and then increases, corresponding to the evolution of the through port coupling states from over-coupling to critical coupling to under-coupling conditions, which matches well with theoretical expectations. The parameters used for the dark blue, green, and purple lines are: $k_1 = 0.082, k_2 = 0.206, k_3 = 0.59, k_1 = 0.048, k_2 = 0.206, k_3 = 0.59$ and $k_1 = 0.142, k_2 = 0.206, k_3 = 0.59$, respectively. The parameters of the micro-ring based on the Si platform: the length of the ring is 62.83 μm , and the intrinsic quality (Q) factor of the rings is 5.1×10^5 .

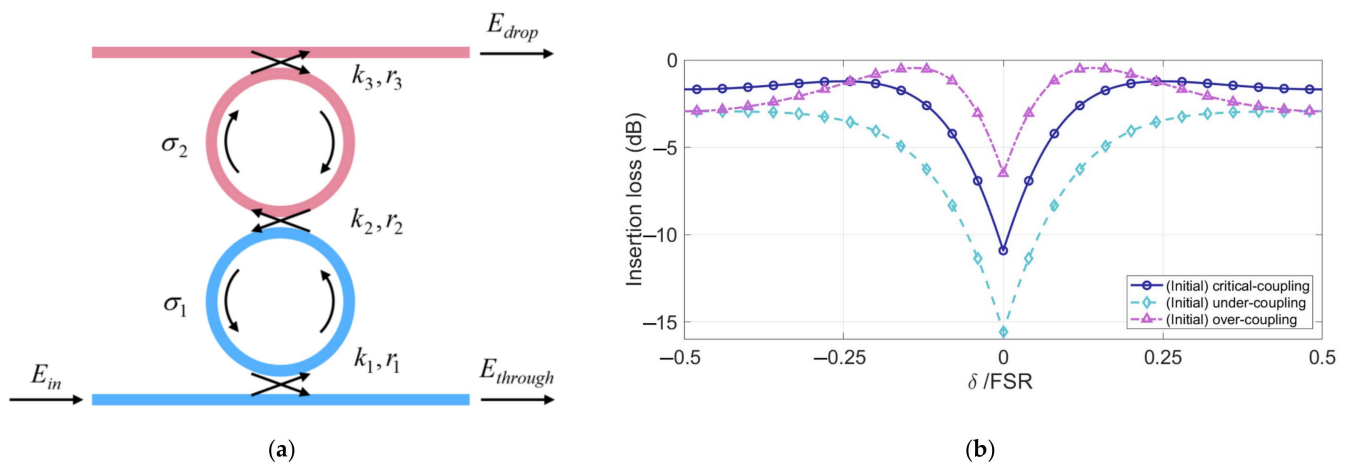


Figure 3. Impact of different coupling states on the system insertion loss. (a) Proposed device structure under the transfer matrix method (TMM). (b) The insertion loss of the drop port as a function of δ/FSR under different initial coupling conditions.

Figure 4a–c gives the intensity distribution of the PT system under critical/under/over-coupling conditions at the initial detuning simulated by COMSOL Multiphysics, respectively. Note that they correspond to the dark blue, green, and purple lines under the condition of $|\delta/FSR| = 0.5$ in Figure 3. It is clear that when critical coupling is reached, i.e., in Figure 4a, the through port is entirely extinct. At this point, the insertion loss is about 1dB, consistent with the dark blue line prediction in Figure 3b. The same results can be concluded under the under coupling or over coupling. The simulation results of

COMSOL Multiphysics show excellent consistency with our TMM theoretical model, reflecting the effectiveness of the TMM. The radius of the microring used in Figure 4 is 10 μm . The gaps corresponding to the coupling coefficients k_1, k_2, k_3 used in the simulation are: $\text{gap}_1 = 167 \text{ nm}$, $\text{gap}_2 = 70 \text{ nm}$, $\text{gap}_3 = 43 \text{ nm}$, $\text{gap}_1 = 225 \text{ nm}$, $\text{gap}_2 = 70 \text{ nm}$, $\text{gap}_3 = 43 \text{ nm}$ and $\text{gap}_1 = 122 \text{ nm}$, $\text{gap}_2 = 70 \text{ nm}$, $\text{gap}_3 = 43 \text{ nm}$, for the critical/under/over coupling states, respectively.

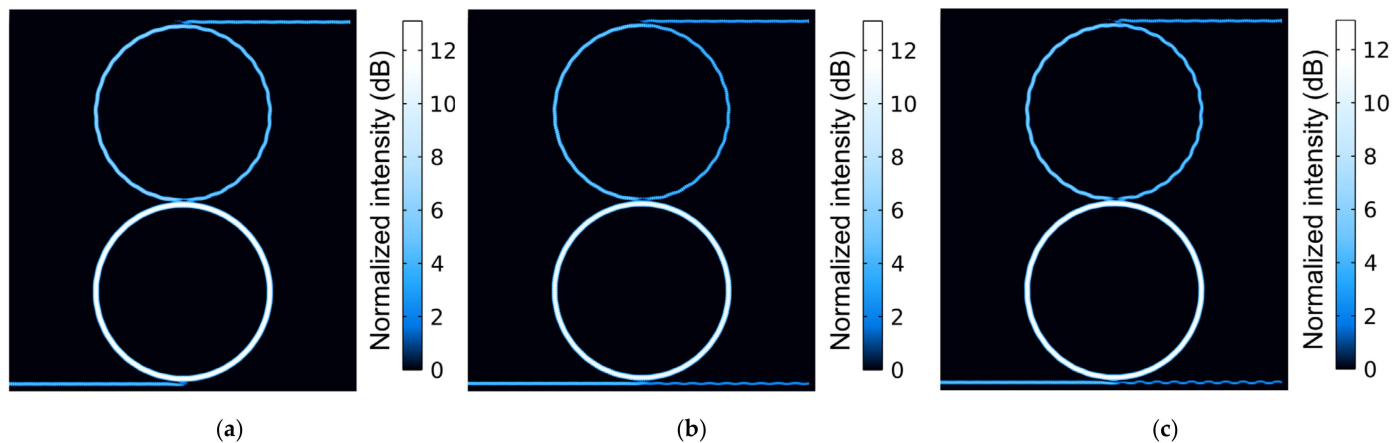


Figure 4. The intensity distribution of system under different initial coupling states obtained by COMSOL Multiphysics. (a–c) show the intensity distribution of the system operating at the critical-coupling, under-coupling, and over-coupling conditions at the initial detuning, respectively.

Figure 5a,b show the transmission spectrum of the drop port in various coupling conditions at the initial detuning and alignment, respectively. Here, the dark blue, green, and purple lines correspond to the systems operating in the critical-coupling, under-coupling, and over-coupling conditions at initial detuning, respectively. The parameters used here are the same as those in Figure 3. It is evident that the system corresponding to the under-coupling condition of the through port has the smallest minimum bandwidth, which induces a high bandwidth-tunable contrast ratio. In comparison, the minimum bandwidth of the system in the over-coupling condition is moderately larger, but its overall insertion loss is significantly reduced. By slightly sacrificing the minimum bandwidth of the system, we exploit the simultaneous realization of a high bandwidth-tunable contrast ratio and a low insertion loss, which is of great significance to the PT-symmetry coupled-resonant filter.

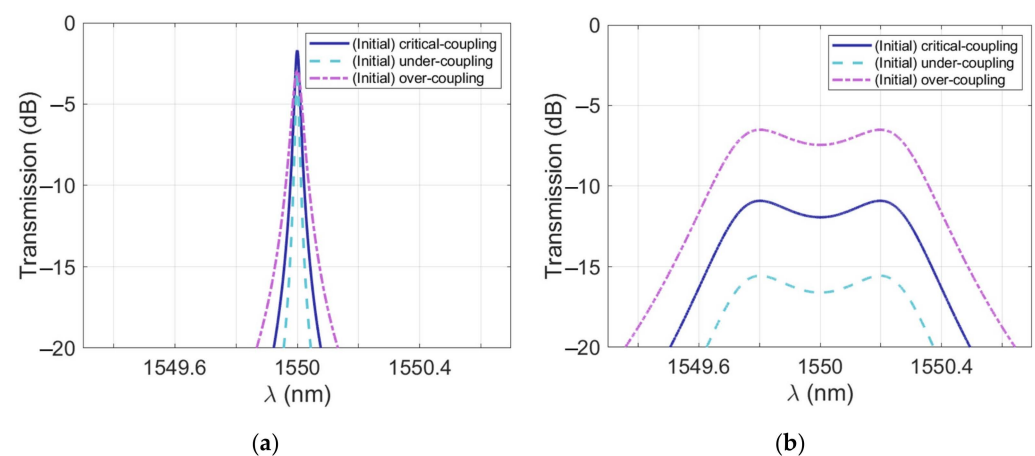


Figure 5. Bandwidth-tunable range of the system under different initial coupling states. (a,b) The transmission spectra of the drop port at the minimum and maximum bandwidth of the system, where the system bandwidth of different colored lines are 2.35 GHz, 1.53 GHz, 4.65 GHz and 85.02 GHz, 84.64 GHz, 86.17 GHz, respectively.

Furthermore, the system performance under different coupling states is obtained by scanning the coupling coefficients in a substantial parameter space. The diagrams in Figure 6 show the system bandwidth-tunable contrast ratio as functions of the coupling coefficients k_2 , k_3 , where the contour lines represent the distribution of system insertion loss. Here, Figure 6a–c correspond to the bandwidth-tunable contrast ratio evolution of the system operating in the under-coupling, critical-coupling, and over-coupling conditions at the initial detuning, respectively. The dark blue, green, and purple points represent the systems, corresponding to the same colored lines in Figures 3 and 5, with the insertion loss of 10.91 dB, 15.56 dB, and 6.5 dB, respectively. The parameters used here are the same as those in Figure 3. Note that the extinction ratio of the system is above 20 dB in a substantial parameter space, which meets the application expectations. The white areas are specifically unsuitable for signal processing, corresponding to the PT-symmetric region, where the excessive splitting in the transmission profile (i.e., with more than 3 dB in-band ripple compared to the transmission maxima) at the alignment results in signal distortion. It should be noted that each point in the diagrams is obtained by tuning δ/FSR in the range of $-0.5 \sim 0.5$. In Figure 6a, the main cavity is set to the critical coupling condition [43,45] at the initial detuning, where the extinction ratio of the through port is 0. Obviously, the area of the high bandwidth-tunable contrast ratio is scattered in the upper right of Figure 6a, which is due to the fact that the coupling loss of the auxiliary cavity is relatively larger than that of the main cavity ($k_3 > k_1$), matching well with the predicted result of TCMT. However, the system insertion loss is too high (>11 dB) in the high bandwidth-tunable contrast ratio region, significantly reducing system performance. Figure 6b,c show the diagram of the system bandwidth-tunable contrast ratio as a function of the coupling coefficient at the two initial coupling states of under-coupling and over-coupling, respectively. It can be shown that under the same coupling coefficient k_2, k_3 , the insertion loss in Figure 6c is smaller than that in Figure 6a and is much smaller than that in Figure 6b, which shows the low insertion loss feature of the system operating under the over-coupling condition at the initial. It is worth noting that the coupling coefficients k_2, k_3 are the same for the three colored points in Figure 6, corresponding to the three relevant colored lines in Figure 3b. The purple dot in Figure 6c, operating in the over-coupling condition at the initial detuning, achieves the bandwidth-tunable contrast ratio of ~ 12.8 dB and the system insertion loss of 6.5 dB, which indicates that we have realized the high bandwidth-tunable contrast ratio PT-symmetry enabled band-pass filter with moderate insertion loss. It is foreseeable that the PT-symmetry coupled-resonant band-pass filter could be applied under various platforms, and the system bandwidth-tunable contrast ratio can be improved by reducing the intrinsic loss of materials and increasing the fabrication accuracy. Note that the PT-symmetry enabled band-pass filter can achieve the high bandwidth-tunable contrast ratio with ultra-low insertion loss if the coupling state of the through port can be kept at the critical-coupling condition during the detuning, which is expected to be realized by adjusting the coupling loss of the main cavity in real-time.

Moreover, Figure 7 shows the system performance under different degrees of over-coupling states. The coupling coefficients k_2, k_3 used here are the same as the red dots in Figure 6c. The dark blue dashed line in Figure 7 corresponds to the k_1 when the system is operating at the critical-coupling state. The regime on the right of the dashed line represents the system operating in the over-coupling region. As coupling coefficient k_1 increases, the over-coupling degree of the system becomes stronger. The purple dot in Figure 7 corresponds to the system operating at the purple dot in Figure 6c, the initial through port transmittance of which is -6 dB. It can be seen that the bandwidth-tunable contrast ratio of the system decreases with the increase of k_1 since the minimum bandwidth of the system increases with the stronger over-coupling. The evolution of the system insertion loss in Figure 7b indicates that the system insertion loss first decreases and then increases as k_1 increases. This means that a moderate over-coupling can reduce the insertion loss of the system. When the degree of over-coupling is too strong, too much light is coupled from the main ring to the through port, resulting in a significant reduction of the light transfers

to the drop port and the increased system insertion loss. It is evident that the system can achieve an insertion loss below 6.5 dB and the bandwidth-tunable contrast ratio between ~8.5 dB and ~12.8 dB, when k_1 is between 0.142 and 0.257, which shows the robustness of the system to different over-coupling degrees.

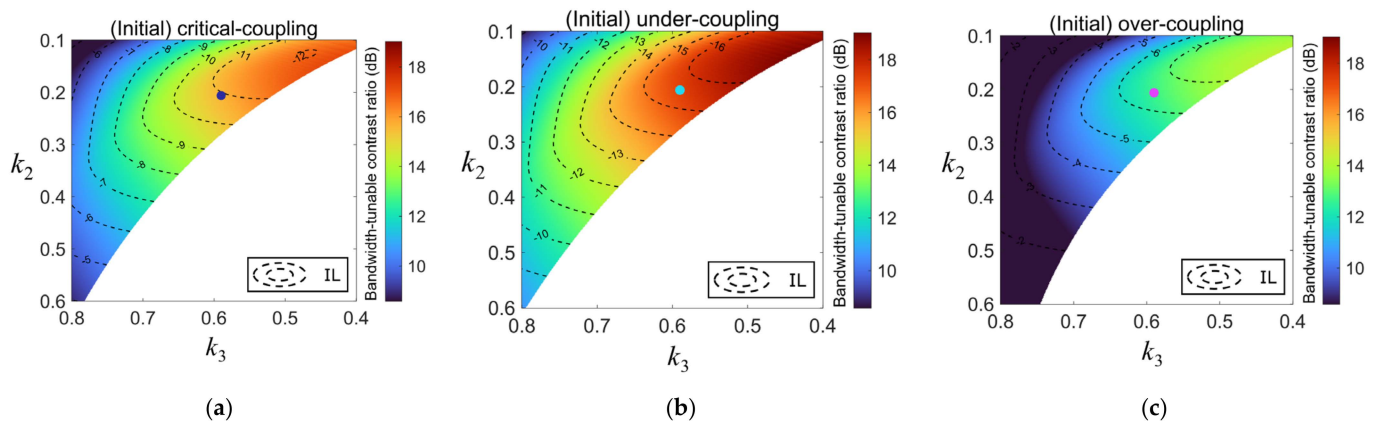


Figure 6. Bandwidth-tunable contrast ratio of the system under different initial coupling states. (a–c) indicate the evolution of the system bandwidth-tunable contrast ratios as a function of the coupling coefficient k_2, k_3 , corresponding to the systems operating in the critical-coupling, under-coupling, and over-coupling conditions at initial detuning, respectively. The contour lines represent the distribution of system insertion loss (IL).

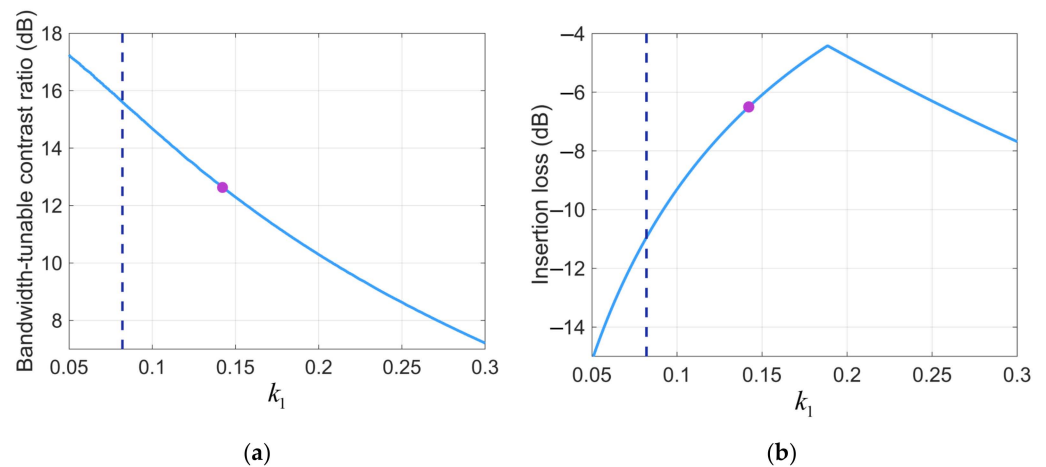


Figure 7. System performance under different initial coupling states. (a,b) The evolution of the bandwidth-tunable contrast ratio and insertion loss of the system with the coupling coefficient k_1 , respectively.

4. Conclusions

In conclusion, we theoretically demonstrate a scheme that mitigates the trade-off between the bandwidth-tunable contrast ratio and insertion loss of the PT-symmetry enabled band-pass filter. A high bandwidth-tunable contrast ratio system with moderate insertion loss is obtained simultaneously. We reveal the trade-off in PT-symmetry coupled-resonant band-pass filter systems through TCM and GCC conditions. Moreover, the effect of the through port coupling states at initial detuning on the system performance is discussed. The trade-off between the bandwidth-tunable contrast ratio and insertion loss of the system is solved by increasing the coupling strength between the through port and the main cavity, which is useful for realizing the band-pass filter with a high bandwidth-tunable contrast ratio. It is worth noting that we realize a reduction (−4.4 dB) in the insertion loss by slightly sacrificing the minimum bandwidth (2.3 GHz) of the system, which is proven by TMM. The low insertion loss features of the system initially operating

in the over-coupled condition are demonstrated by scanning the coupling coefficients in substantial parameter space. Note that the system insertion loss can be further reduced if the coupling loss of the main cavity can be adjusted in real-time. The implementation of our scheme is simple and elegant, and only the thermal tuning is sufficient. The system applies to various material platforms for experimental implementations, showing potential in a wide range of applications. Finally, the scheme we proposed solves the core problem of the PT-symmetry enabled band-pass filter, making it applicable to more scenarios.

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