Supplementary Information to "Sensitivity of the Penman-Monteith Reference Evapotranspiration Equation to Meteorological Variables for Puerto Rico"

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Appendix S1. Aerodynamic and energetic components of the ASCE standardized reference evapotranspiration (ET$_o$) equation

The American Society of Civil Engineers (ASCE) standardized grass-reference Penman–Monteith reference evapotranspiration equation ([1]; Equation (1) in the main manuscript associated with these supplementary materials) can be written as a sum of an energetic and an aerodynamic (mass transfer) component as follows:

$$ET_o = ET_{o,energetic} + ET_{o,aerodynamic} \tag{S1.1}$$

$$ET_{o,energetic} = \frac{0.408\,\Delta\,(R_n-G)}{\Delta + \gamma(1+0.34u_2)} \tag{S1.2}$$

$$ET_{o,aerodynamic} = \frac{\gamma\frac{900}{T+273}u_2(e_s-e_a)}{\Delta + \gamma(1+0.34u_2)} \tag{S1.3}$$

where ET_o is the daily reference evapotranspiration (mm/day), R_n is the net radiation at the crop surface (MJ/m^2/day), G is the soil heat flux density at the soil surface (MJ/m^2/day), which is relatively small with respect to R_n and, hence, neglected for daily timesteps, T is the mean daily air temperature at 2-meter height (°C), u_2 is the mean daily wind speed at 2-meter height (m/s), e_s is the saturation vapor pressure at 2-meter height (kPa), e_a is the mean actual vapor pressure at 2-meter height (kPa), Δ is the slope of the saturation vapor pressure–temperature curve (kPa/°C), and γ is the psychrometric constant (kPa/°C).

The fraction of the total ET_o due to the energetic and aerodynamic components can be written, respectively, as follows:

$$ET_{o,energetic\ fraction} = ET_{o,energetic}/ET_o \qquad (S1.4)$$

$$ET_{o,aerodynamic\ fraction} = ET_{o,aerodynamic}/ET_o \qquad (S1.5)$$

The different terms of the ASCE standardized ET_o equation can be written in terms of the basic five meteorological variables, as in GOES-PRWEB [2] and specified below. Wind speeds measured at a height different from the required 2 meters above the surface can be transformed to 2 m-height wind speeds based on:

$$u_2 = u_{z_w} * \frac{4.87}{\log(67.8 * z_w - 5.42)} \qquad (S1.6)$$

where u_2 is the mean daily wind speed at 2-meter height (m/s) and u_{z_w} is the mean daily wind speed at height z_w above the surface (meters).

The albedo (α) of the reference grass is set as:

$$\alpha = 0.23 \qquad (S1.7)$$

The daily average air temperature at 2-meter height (T, °C) is computed as the average of the daily maximum (T_{max}, °C) and minimum (T_{min}, °C) air temperature at 2-meter height:

$$T = 0.5(T_{max} + T_{min}) \qquad (S1.8)$$

The daily average saturation vapor pressure (e_s, kPa) is computed as the average of the saturation vapor pressure at the daily maximum temperature ($e_s(T_{max})$) and the saturation vapor pressure at the daily minimum temperature ($e_s(T_{min})$), which are based on the following approximation to the Clausius–Clapeyron relationship [3]:

$$e_s(T_{max}) = 0.6108 * \exp\left(17.27 \frac{T_{max}}{T_{max} + 237.3}\right) \qquad (S1.9)$$

$$e_s(T_{min}) = 0.6108 * \exp\left(17.27 \frac{T_{min}}{T_{min} + 237.3}\right) \qquad (S1.10)$$

$$e_s = 0.5\left(e_s(T_{max}) + e_s(T_{min})\right) \qquad (S1.11)$$

The daily average actual vapor pressure (e_a, kPa) is computed as the product of the daily mean relative humidity (RH_{mean}) as a fraction and the daily average saturation vapor pressure:

$$e_a = \frac{RH_{mean}}{100} e_s \qquad (S1.12)$$

Although Equation (S1.12) is used to estimate e_a in GOES-PRWEB, it is not one of the preferred methods and is given a preference ranking of 4 by the ASCE (Table 3 of [1]). The use of daily maximum and minimum relative humidity (RH_{max} and RH_{min}) for estimating e_a is recommended by the ASCE over the use of RH_{mean} and is given a ranking of 2. Although not presented here, we investigated the effect of using RH_{max} and RH_{min} instead of Equation (S1.12) for estimating e_a in Puerto Rico based on GOES-PRWEB data. Island wide, we found that annual ET_o based on e_a computed from Equation (S1.12) (RH_{mean}) was up to 12% lower than with e_a computed from RH_{max} and RH_{min}. The difference between the two methods for estimating e_a was generally smaller, at around 5–7% for our main focus region at the location of the irrigated farms in 2015, which are predominantly on the south coast.

The atmospheric pressure (P, kPa) is estimated from the site elevation above mean sea level (z, meters) based on the following simplified formulation of the Universal Gas Law:

$$P = 101.3 \left(\frac{293 - 0.0065\ z}{293} \right)^{5.26} \qquad (S1.13)$$

The slope of the saturation vapor pressure vs. temperature curve (Δ, kPa/$^\circ$C) is computed as:

$$\Delta = 4098\ \frac{0.6108\ exp\left(\frac{17.27\ T}{T + 237.3} \right)}{(T + 237.3)^2} \qquad (S1.14)$$

The psychrometric constant (γ, kPa/$^\circ$C) is computed from the atmospheric pressure (P) and the latent heat of vaporization ($\lambda = 2.45$, MJ/kg) based on:

$$\gamma = 0.00163 \frac{P}{\lambda} \tag{S1.15}$$

The unitless inverse relative distance factor (squared) for the earth-sun (d_r) is computed based on the Julian day (J) as:

$$d_r = 1 + 0.033 \cos\left(2 \frac{\pi}{365} J\right) \tag{S1.16}$$

The solar declination angle (δ, radians) is given by:

$$\delta = 0.409 \sin\left(2 \frac{\pi}{365} J - 1.39\right) \tag{S1.17}$$

The sunset hour angle (ω_s, radians) is computed from the latitude (φ, radians) and the solar declination angle (δ, radians) as:

$$\omega_s = \mathrm{acos}(-\tan(\varphi)\tan(\delta)) \tag{S1.18}$$

The incoming extraterrestrial solar radiation (R_a, MJ/m^2/d):

$$R_a = \left(\frac{24}{\pi}\right) d_r \ G_{sc} \left(\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)\right) \tag{S1.19}$$

where G_{sc} is the solar constant (4.92 MJ/m^2/h).

The clear-sky incoming solar radiation (R_{so}, MJ/m^2/d) is estimated as a function of station elevation above mean sea level (z) and the extraterrestrial solar radiation from:

$$R_{so} = (0.75 + (2 * 10^{-5})z)R_a \tag{S1.20}$$

The ratio of the actual incoming solar radiation (R_s, MJ/m^2/d) and the clear-sky incoming solar radiation (R_{so}, MJ/m^2/d) is used to estimate the dimensionless cloudiness function (f_{cd}) for historical atmospheric constituent conditions:

$$f_{cd} = \max\left\{0.05, \left[\min\left(1.0, \left(1.35 \frac{R_s}{R_{so}} - 0.35\right)\right)\right]\right\} \tag{S1.21}$$

Net long-wave radiation (R_{nl}, MJ/m^2/d) is estimated as:

$$R_{nl} = \sigma\, f_{cd}\, * \frac{(0.34 - 0.14\,\sqrt{e_a})[(T_{max} + 273.2)^4 + (T_{min} + 273.2)^4]}{2} \tag{S1.22}$$

where σ is the Stefan–Boltzman constant (4.901 x 10^{-9} MJ/K^4/m^2/d).

Net short-wave or solar radiation (R_{ns}, MJ/m^2/d) is computed from the albedo (α) and the incoming solar radiation:

$$R_{ns} = (1 - \alpha) * R_s \tag{S1.23}$$

Net radiation (R_n, MJ/m^2/d) is computed as:

$$R_n = R_{ns} - R_{nl} \tag{S1.24}$$

Appendix S2. Computation of daily meteorological variables from hourly WRF output

Hourly saturation vapor pressure (e_s) is calculated from Equation (S1.9) based on hourly air temperature at 2 m height and converted to units of mb (hPa). Hourly relative humidity (*RH*, %) at 2 m is computed from mixing ratio at 2 m (*r*, kg/kg), surface atmospheric pressure (*P*, mb), and saturation vapor pressure as follows:

$$RH = 100 * \left(\frac{rP/(r+\varepsilon)}{e_s}\right) \tag{S2.1}$$

where ε is the ratio of the molar mass of water vapor to that of dry air (0.622). The numerator in Equation (S2.1) is equivalent to the hourly actual vapor pressure as described in [4].

The 10 m-height wind speed (u_{10}, m/s) is computed from its zonal (u_{z10}, m/s) and meridional (u_{m10}, m/s) components based on:

$$u_{10} = \sqrt{u_{z10}^2 + u_{m10}^2} \tag{S2.1}$$

The times for the hourly temperature at 2 meters (*T*, K), hourly relative humidity (*RH*, %), hourly downward short-wave radiation at the surface (R_s, MJ/m^2/d), and 10 m wind speed (u_{10}, m/s) are then converted from Greenwich Meridian Time to local time in Puerto Rico (Atlantic

Standard Time). Finally, daily maximum and minimum temperatures are computed from 24 hourly values and converted to °C (T_{max} and T_{min}, respectively), and daily means of RH (RH_{mean}), R_s, and u_{10} are computed.

Appendix S3. Analytical sensitivity coefficient equations

Defining the numerator of the American Society of Civil Engineers (ASCE) standardized grass-reference Penman–Monteith evapotranspiration equation (Equation (1) in the main manuscript associated with these supplementary materials) as f and the denominator as g gives:

$$ET_o = \frac{f}{g} \tag{S3.1}$$

where

$$f = 0.408\,\Delta\,(R_n - G) + \gamma\,\frac{900}{T+273}u_2(e_s - e_a) \tag{S3.2}$$

$$g = \Delta + \gamma(1 + 0.34u_2) \tag{S3.3}$$

Then, based on the quotient rule, Equation (2) in the main manuscript associated with these supplementary materials, can be re-written as:

$$S_{V_i} = \left(\frac{f\prime}{f} - \frac{g\prime}{g}\right) * V_i \tag{S3.4}$$

where

f' is the partial derivative of f with respect to V_i;

g' is the partial derivative of g with respect to V_i.

These functions and derivatives were computed for each of the following variables (V_i): daily maximum (T_{max}) and minimum (T_{min}) air temperature at 2 meter height, daily mean relative humidity (RH_{mean}), daily mean incoming solar radiation (R_s), and 2 m-height wind speed (u_2) as follows.

Derivatives for daily maximum air temperature (T_{max}):

$$\frac{\partial \Delta}{\partial T_{max}} = \frac{-8*4098*0.6108\,exp\left(\frac{17.27\,T}{T+237.3}\right)(2T+237.3(2-17.27))}{(2T+2*237.3)^4} \tag{S3.5}$$

$$\frac{\partial R_n}{\partial T_{max}} = -2\sigma f_{cd}(T_{max} + 273.2)^3 \left(0.34 - 0.14\sqrt{\frac{e_s * RH_{mean}}{100}}\right) +$$

$$\left(0.14\sigma f_{cd}\sqrt{\frac{RH_{mean}}{200}}\right)\frac{[(T_{max} + 273.2)^4 + (T_{min} + 273.2)^4]}{2}\left[\frac{17.27}{T_{max}+237.3} - \right.$$

$$\left.\frac{17.27 T_{max}}{(T_{max}+237.3)^2}\right]\left[\frac{e_s(T_{max})}{2\sqrt{e_s(T_{max})+ e_s(T_{min})}}\right] \tag{S3.6}$$

$$f_1' = 0.408\left[\frac{\partial \Delta}{\partial T_{max}}(R_n - G) + \Delta\frac{\partial R_n}{\partial T_{max}}\right] \tag{S3.7}$$

$$f_2' = \frac{0.5*900\,\gamma u_2(1-RH_{mean}/100)e_s(T_{max})\left[\frac{17.27}{T_{max}+237.3} - \frac{17.27 T_{max}}{(T_{max}+237.3)^2}\right]}{T+273.2} -$$

$$\frac{900\,\gamma u_2(1-RH_{mean}/100)e_s}{2(T+273.2)^2} \tag{S3.8}$$

$$f' = f_1' + f_2' \tag{S3.9}$$

$$g' = \frac{\partial \Delta}{\partial T_{max}} \tag{S3.10}$$

Derivatives for daily minimum air temperature (T_{min}):

$$\frac{\partial \Delta}{\partial T_{min}} = \frac{-8*4098*0.6108\,exp\left(\frac{17.27\,T}{T+237.3}\right)(2T+237.3(2-17.27))}{(2T+2*237.3)^4} \tag{S3.11}$$

$$\frac{\partial R_n}{\partial T_{min}} = -2\sigma f_{cd}(T_{min} + 273.2)^3 \left(0.34 - 0.14\sqrt{\frac{e_s * RH_{mean}}{100}}\right) +$$

$$\left(0.14\sigma f_{cd}\sqrt{\frac{RH_{mean}}{200}}\right)\frac{[(T_{max} + 273.2)^4 + (T_{min} + 273.2)^4]}{2}\left[\frac{17.27}{T_{min}+237.3} - \right.$$

$$\left.\frac{17.27 T_{min}}{(T_{min}+237.3)^2}\right]\left[\frac{e_s(T_{min})}{2\sqrt{e_s(T_{max})+ e_s(T_{min})}}\right] \tag{S3.12}$$

$$f_1' = 0.408\left[\frac{\partial \Delta}{\partial T_{min}}(R_n - G) + \Delta\frac{\partial R_n}{\partial T_{min}}\right] \tag{S3.13}$$

$$f_2' = \frac{0.5*900\,\gamma u_2(1-RH_{mean}/100)e_s(T_{min})\left[\frac{17.27}{T_{min}+237.3}-\frac{17.27T_{min}}{(T_{min}+237.3)^2}\right]}{T+273.2} -$$

$$\frac{900\,\gamma u_2(1-RH_{mean}/100)e_s}{2(T+273.2)^2} \tag{S3.14}$$

$$g' = \frac{\partial\Delta}{\partial T_{min}} \tag{S3.15}$$

$$f' = f_1' + f_2'{}' \tag{S3.16}$$

Derivatives for daily mean relative humidity (RH_{mean}):

$$f' = 0.408\,\Delta\left(0.14\sigma f_{cd}\sqrt{e_s}\right)\frac{[(T_{max}+273.2)^4+(T_{min}+273.2)^4]}{2*20\sqrt{RH_{mean}}} - \frac{9\gamma u_2 e_s}{T+273.2} \tag{S3.17}$$

$$g' = 0 \tag{S3.18}$$

Derivatives for daily mean incoming solar radiation (R_s):

$$f' = 0.408\,\Delta\left\{(1-\alpha) - \frac{1.35\sigma}{R_{so}}\left(0.34 - \right.\right.$$

$$\left.\left.0.14\sqrt{e_s}\,\right)\frac{[(T_{max}+273.2)^4+(T_{min}+273.2)^4]}{2}\right\} \tag{S3.19}$$

$$g' = 0 \tag{S3.20}$$

Derivatives for daily mean 2 m-height wind (u_2):

$$f' = \frac{900\,\gamma(e_s-e_a)}{T+273.2} \tag{S3.21}$$

$$g' = 0.34\,\gamma \tag{S3.22}$$

Appendix S4. Maps of long-term (2009–2017) monthly average meteorological variables and ET_o components

GOES-PRWEB data can be found at: https://pragwater.com (accessed on 25 February 2021 and 31 August 2021).
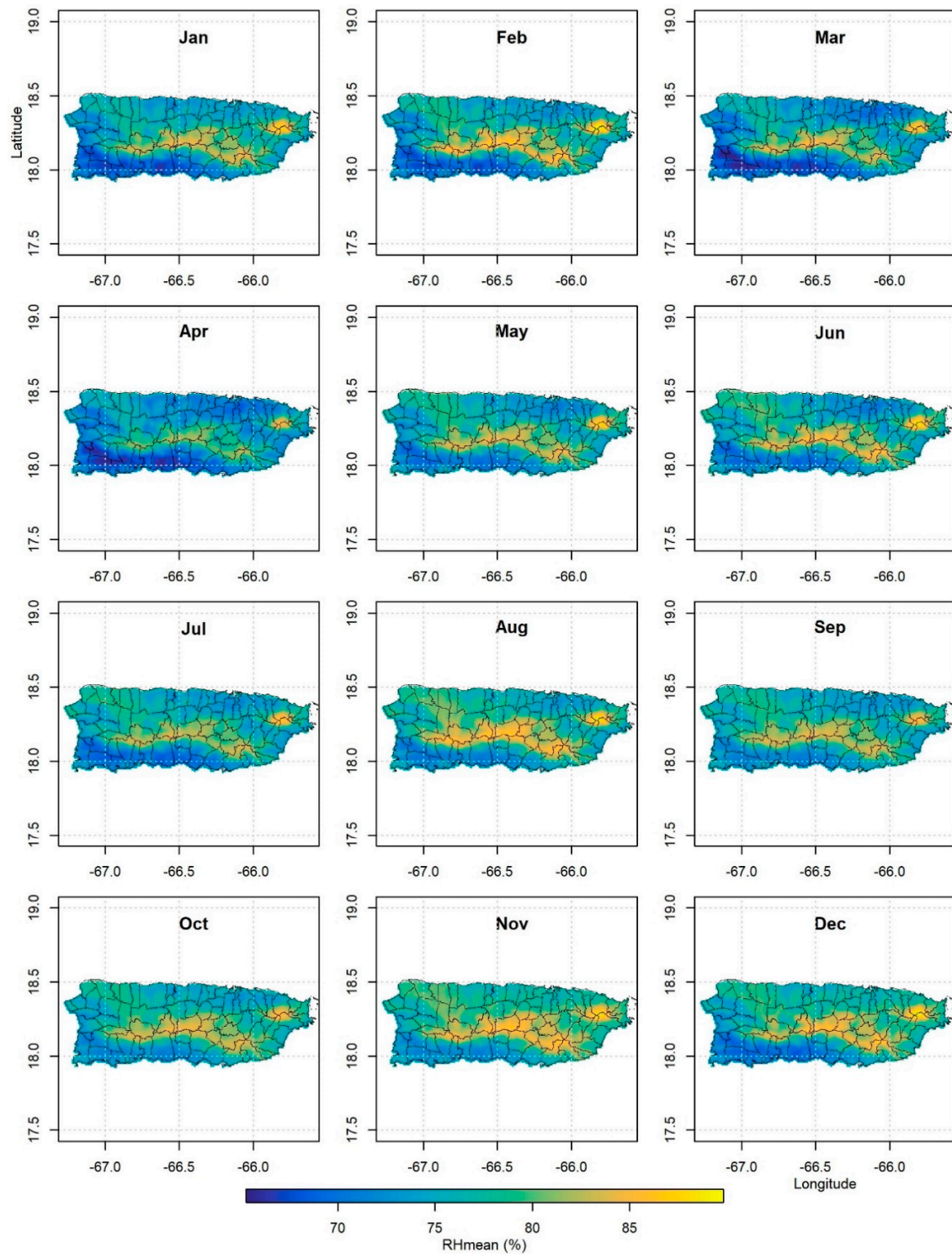
Figure S1. Long-term (2009–2017) monthly average mean relative humidity (RH_{mean}). The black lines show the location of the municipalities in Puerto Rico.
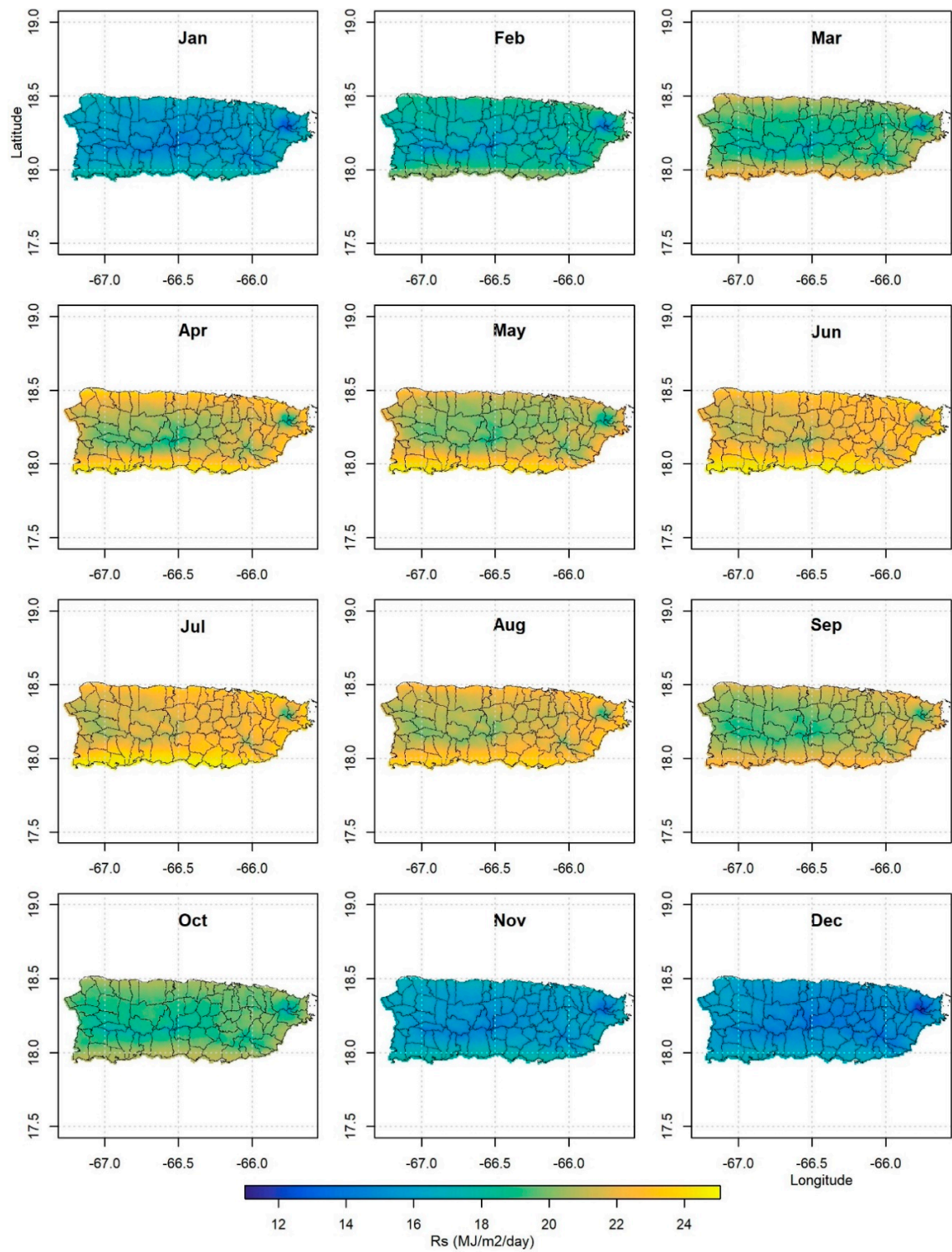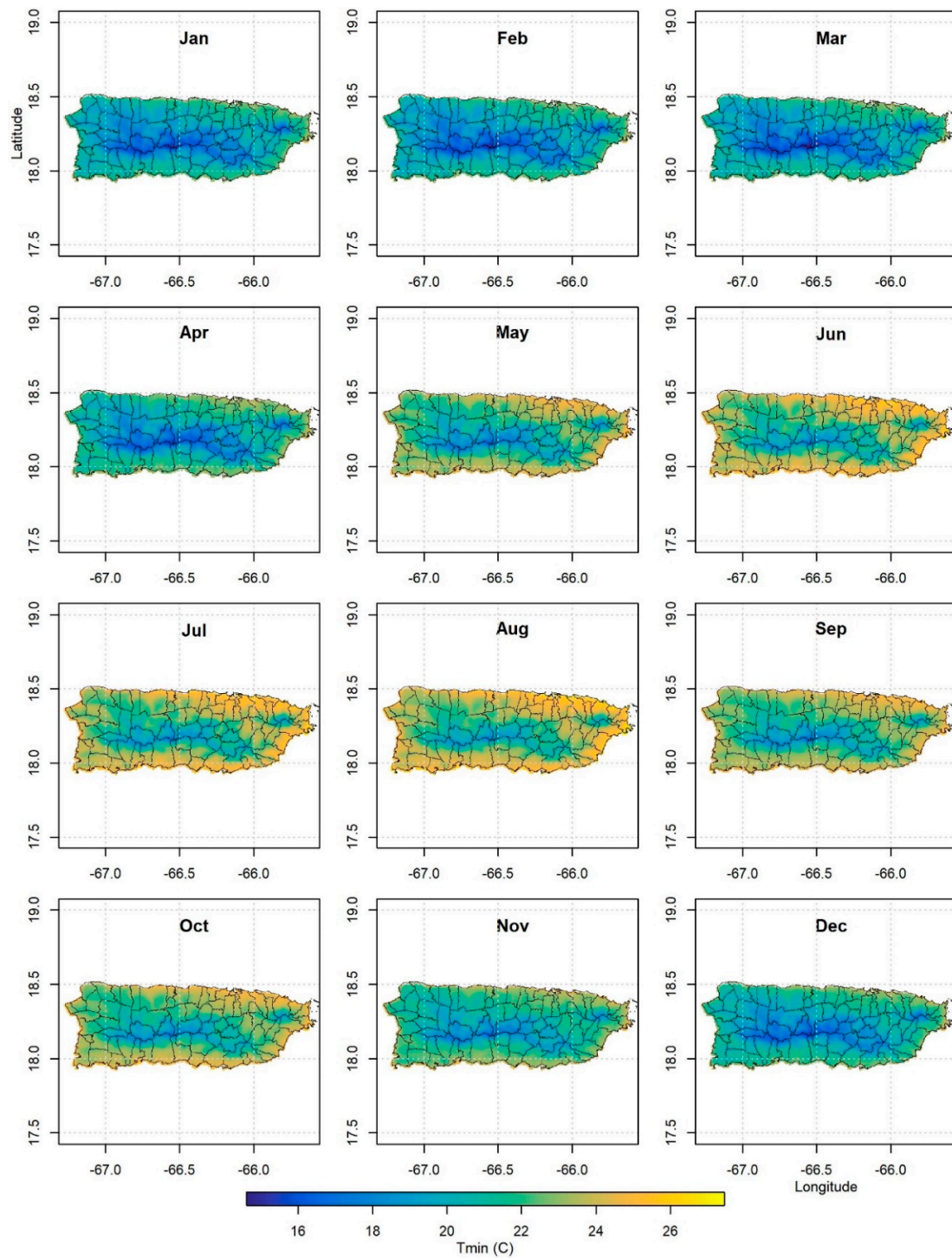
Figure S2. Long-term (2009–2017) monthly average incoming solar radiation (R_s). The black lines show the location of the municipalities in Puerto Rico.

Figure S3. Long-term (2009–2017) monthly average maximum air temperature (T_{max}). The black lines show the location of the municipalities in Puerto Rico.

Figure S4. Long-term (2009–2017) monthly average minimum air temperature (T_{min}). The black lines show the location of the municipalities in Puerto Rico.
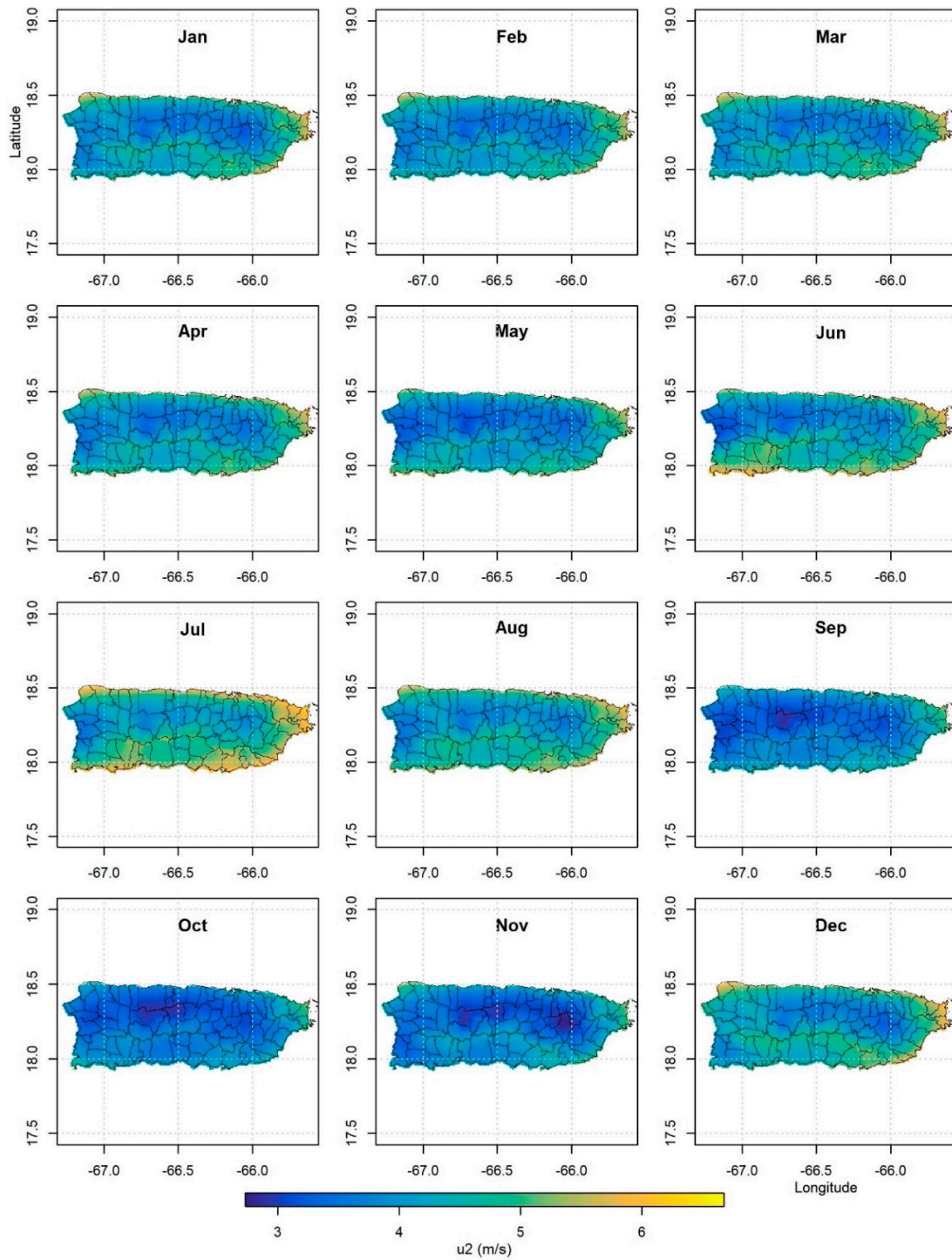
Figure S5. Long-term (2009–2017) monthly average wind speed (u_2). The black lines show the location of the municipalities in Puerto Rico.
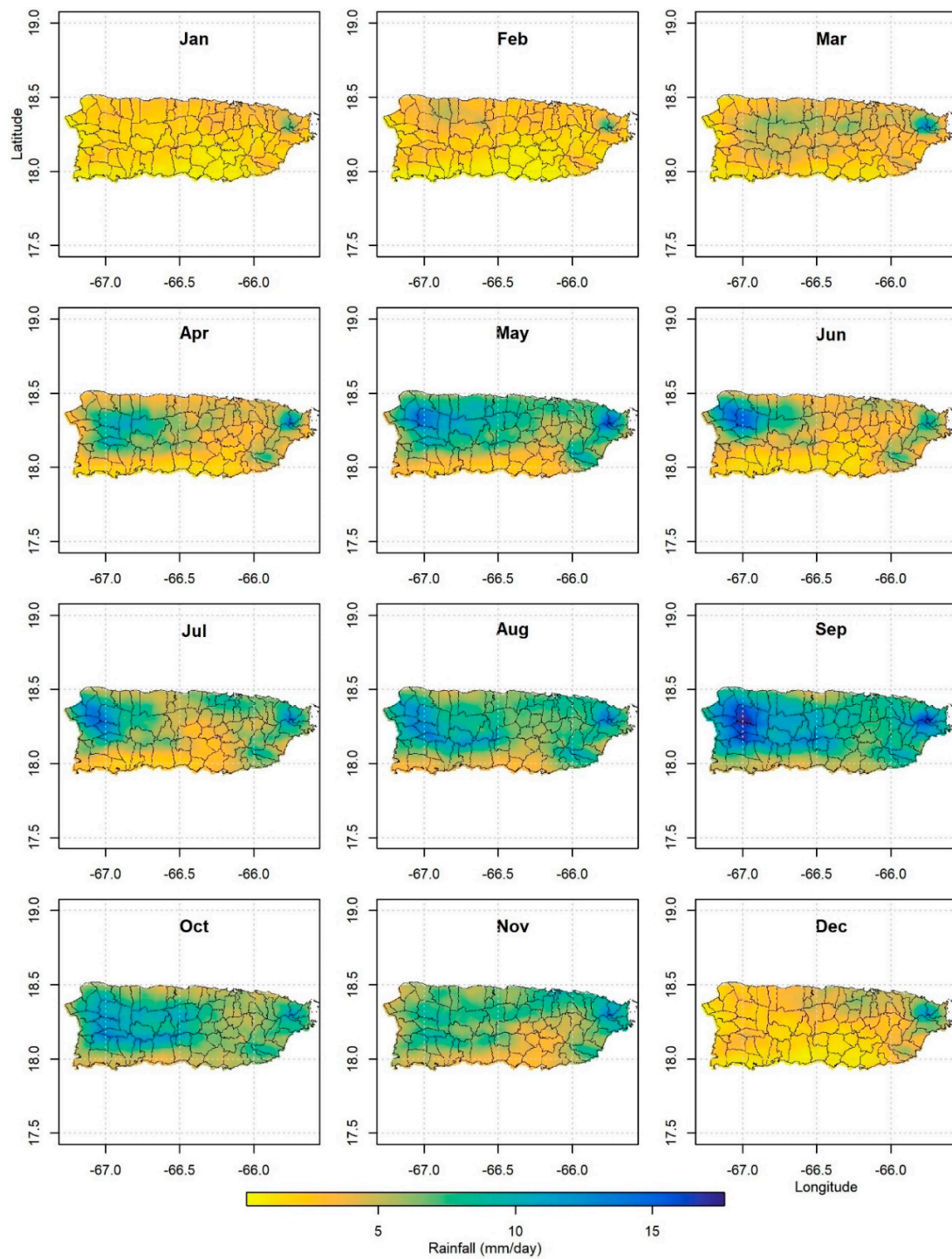
Figure S6. Long-term (2009–2017) monthly average daily rainfall rate. The black lines show the location of the municipalities in Puerto Rico.
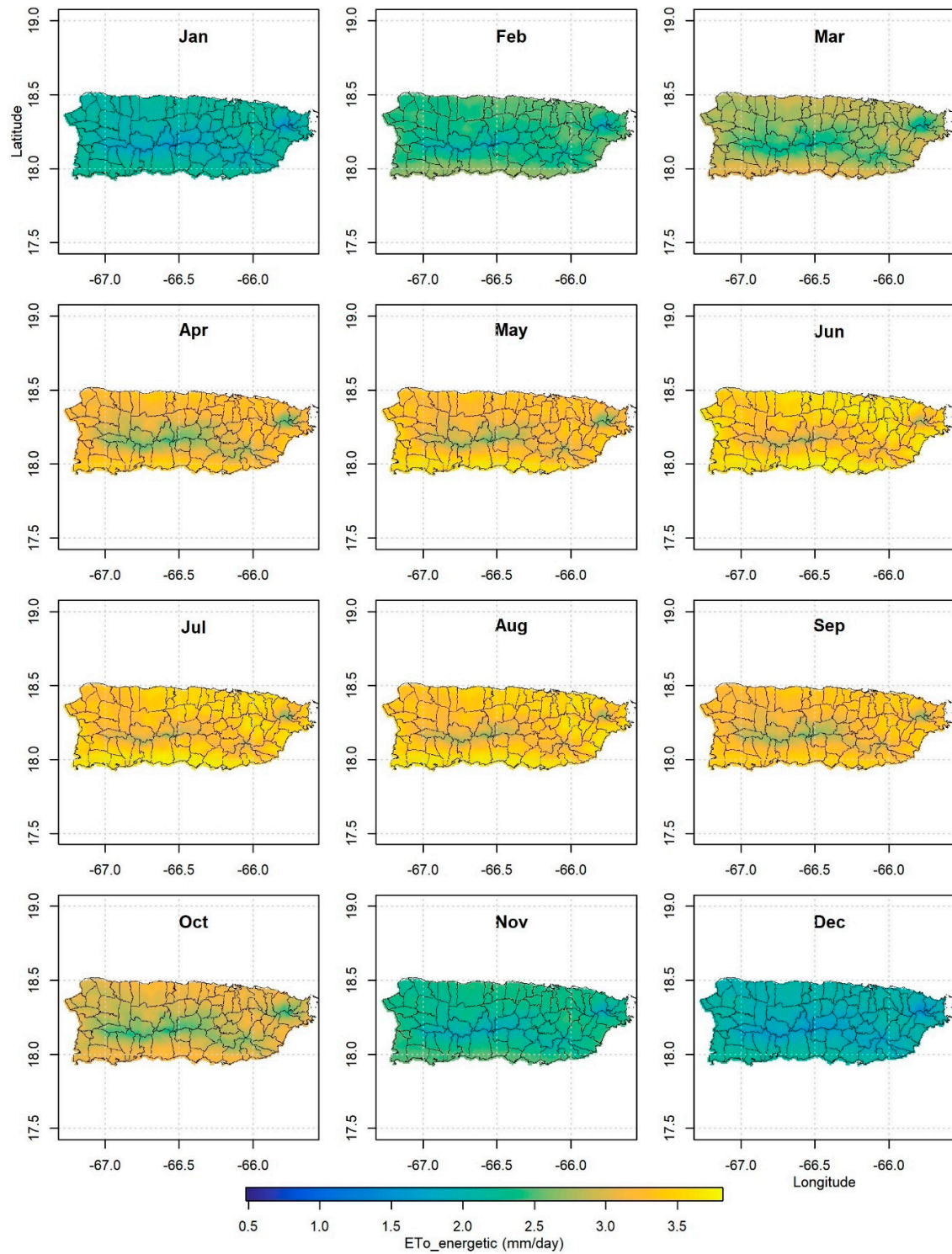
Figure S7. Long-term (2009–2017) monthly average energetic component or reference evapotranspiration ($ET_{o,energetic}$). The black lines show the location of the municipalities in Puerto Rico.
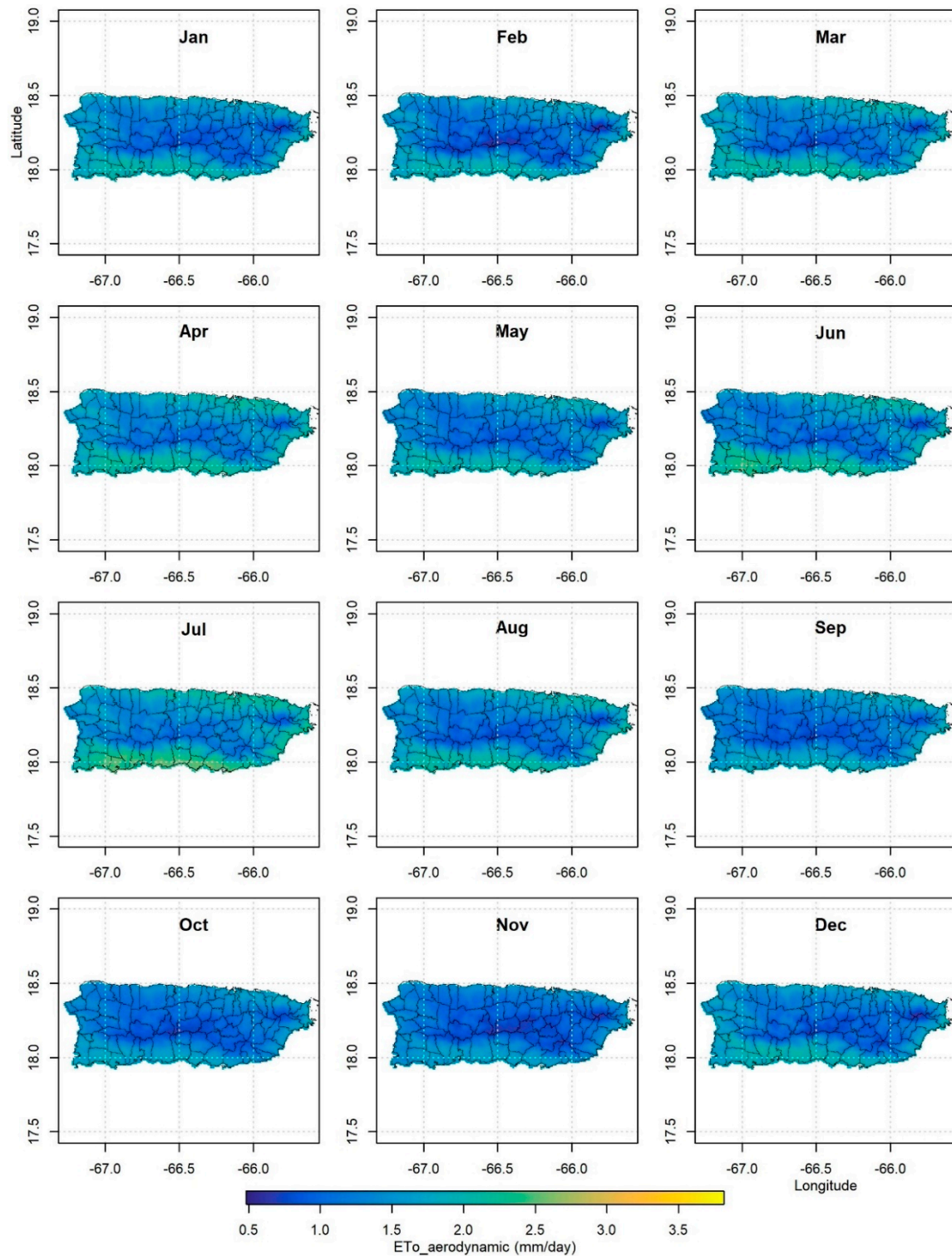
Figure S8. Long-term (2009–2017) monthly average aerodynamic component of reference evapotranspiration ($ET_{o,aerodynamic}$). The black lines show the location of the municipalities in Puerto Rico.
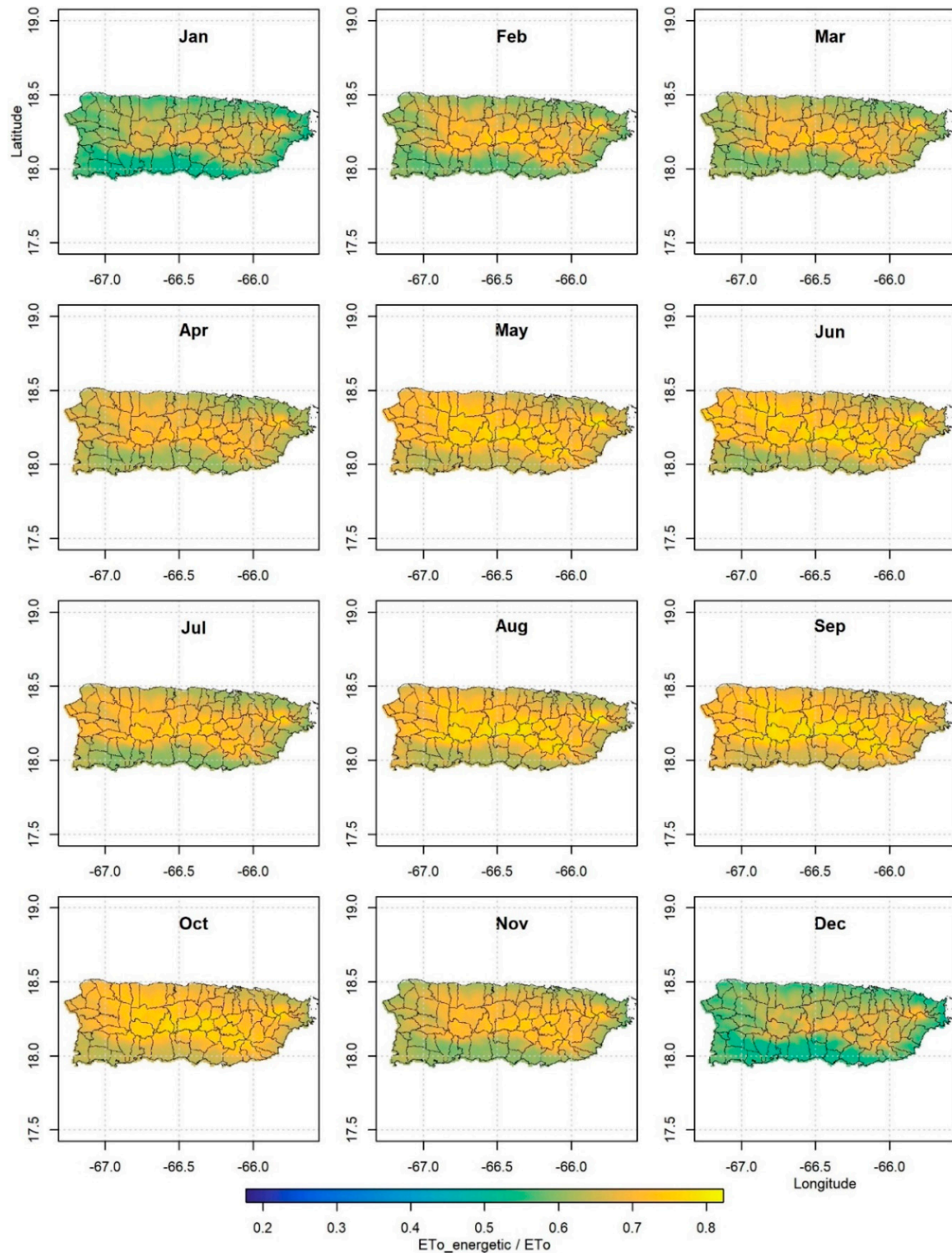
Figure S9. Long-term (2009–2017) monthly average energetic fraction of reference evapotranspiration ($ET_{o,energetic\,fraction}$). The black lines show the location of the municipalities in Puerto Rico.
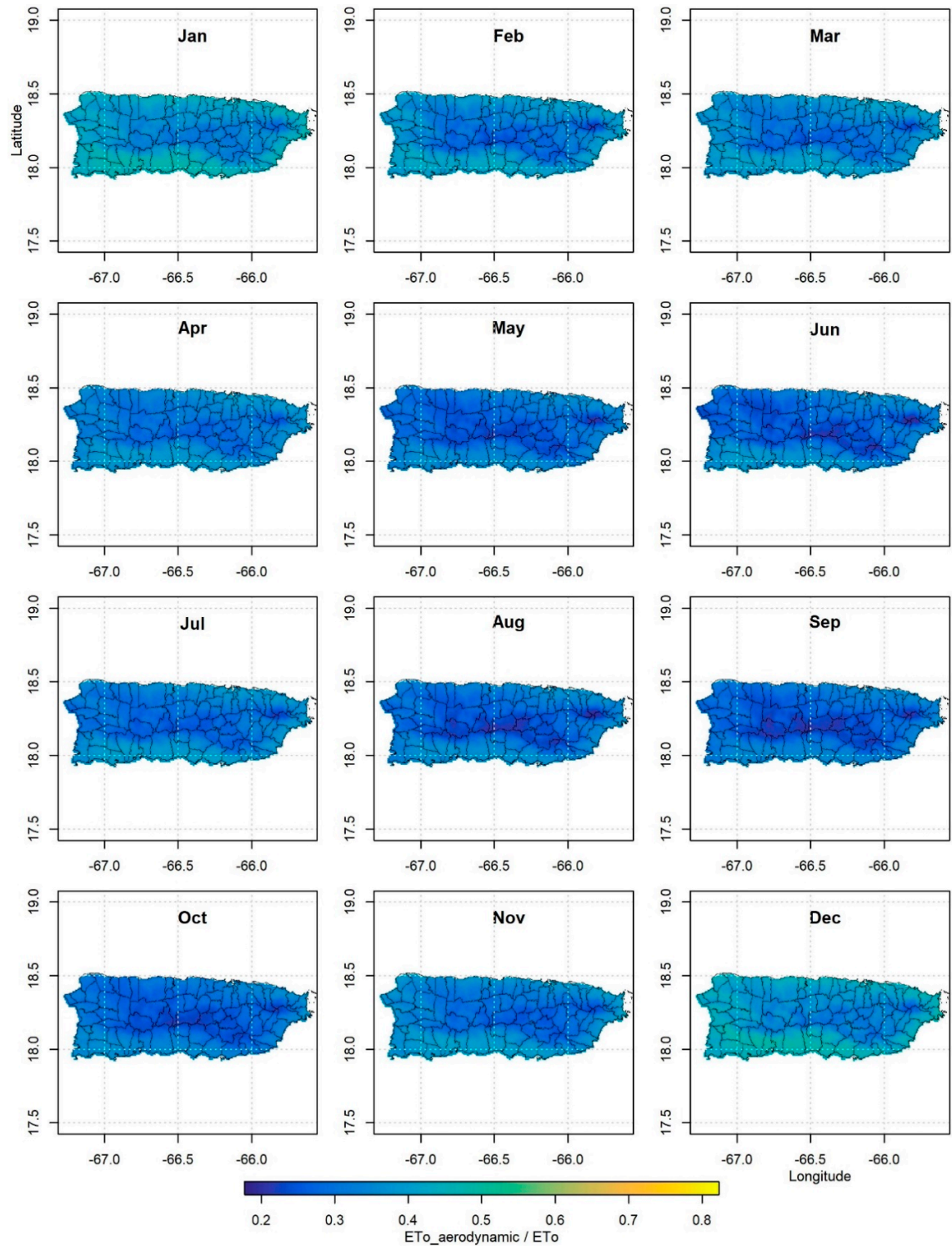
Figure S10. Long-term (2009–2017) monthly average aerodynamic fraction of reference evapotranspiration (*ET$_{o,aerodynamic fraction}$*). The black lines show the location of the municipalities in Puerto Rico.
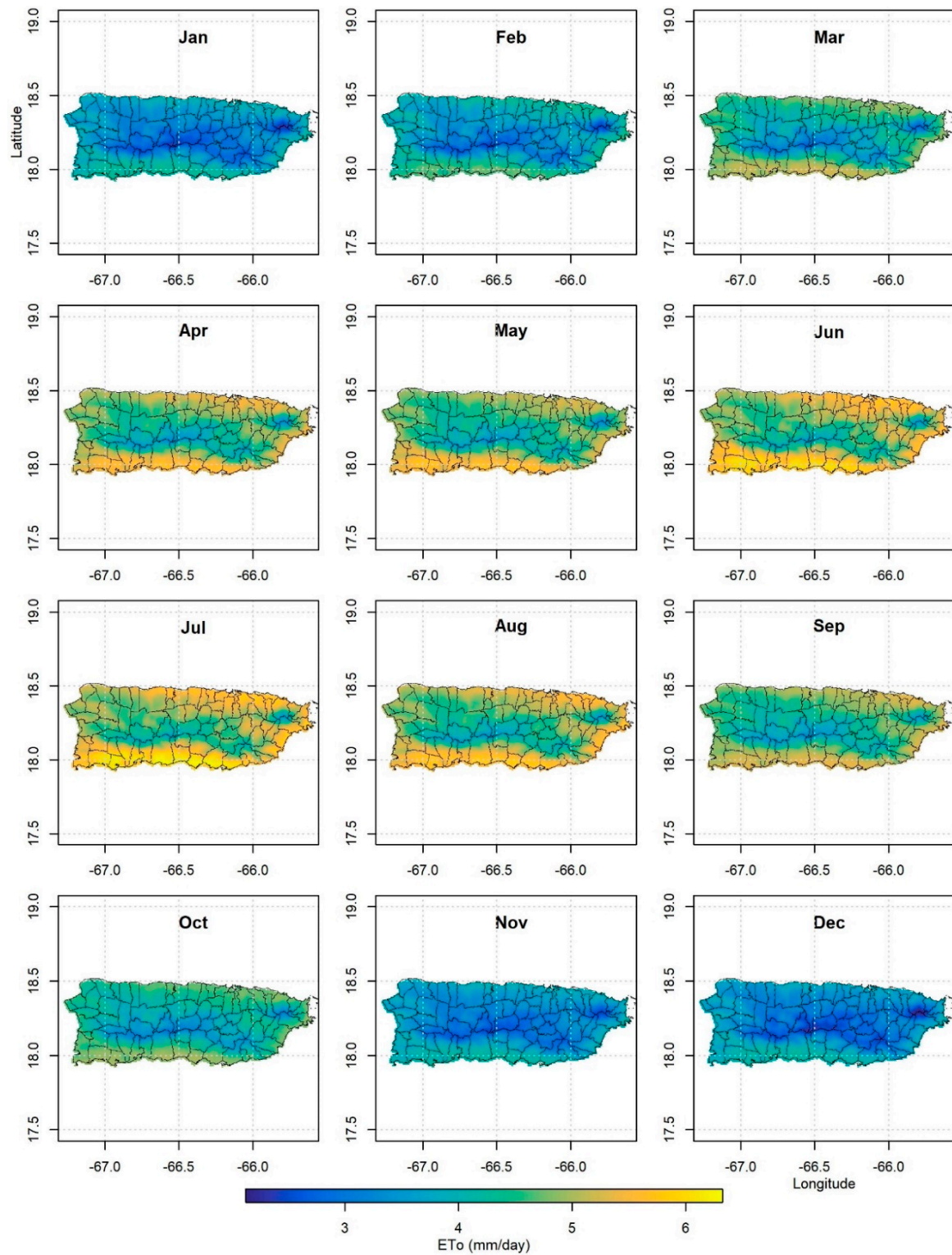
Figure S11. Long-term (2009–2017) monthly average daily evapotranspiration (ET_o). The black lines show the location of the municipalities in Puerto Rico.

Appendix S5. Performance of GOES-PRWEB wind speed at 2 meters and sensitivity to errors in wind speed

Table S1. Stations with homogeneous wind speed data in Puerto Rico (shown in Figure S12); % diff. and R^2 are the percent difference and coefficient of determination, respectively, between GOES-PRWEB and station wind speeds converted to 2 m height according to [1] when sensor elevation data are available. Source datasets include MESONET (CariCOOS, [5]), NOAA's Center for Operational Oceanographic Products and Services (NOAA CO-OPS; [6]) tidal stations, NOAA's Global Summary of the Day (NOAA GSOD; [7]), NOAA's National Weather Service (NOAA NWS, [6]), and National Interagency Fire Center Remote Automatic Weather Stations (RAWS; [8]).

Station name	Dataset	Latitude	Longitude	Wind sensor elevation (m)	% diff.	R^2
Gurabo	MESONET	18.26	-65.99	10.0	128	0.34
San Juan Naval	MESONET	18.46	-66.13	10.0	2	0.53
Las Mareas	MESONET	17.93	-66.16	10.0	3	0.59
Del Rey Marina	MESONET	18.29	-65.63	10.0	14	0.62
Yabucoa-El Negro	MESONET	18.05	-65.83	10.0	9	0.57
Fajardo	NOAA CO-OPS	18.34	-65.63	6.4	33	0.39
Yabucoa Harbor	NOAA CO-OPS	18.06	-65.83	6.4	106	0.28
San Juan	NOAA CO-OPS	18.46	-66.12	8.1	8	0.49
Magueyes Island	NOAA CO-OPS	17.97	-67.05	6.6	132	0.08
PENUELAS, RQ	NOAA GSOD	17.97	-66.77	N/A	-12	0.45
FERNANDO LUIS RIBAS DOMINICCI AIRPORT	NOAA GSOD	18.46	-66.10	N/A	-11	0.20
FAJARDO, RQ	NOAA GSOD	18.33	-65.63	N/A	4	0.45
SAN JUAN, RQ	NOAA GSOD	18.47	-66.12	N/A	-10	0.35
MEDIA LUNA LA PARGUERA, RQ	NOAA GSOD	17.93	-67.05	N/A	-23	0.46
YABUCOA HARBOR	NOAA GSOD	18.05	-65.83	N/A	59	0.25
Aguadilla AP	NOAA NWS	18.50	-67.13	9.1	18	0.64
ROOSEVELT ROADS	NOAA NWS	18.25	-65.63	9.1	34	0.49
San Juan LMM AP	NOAA NWS	18.45	-66.00	9.1	33	0.49
Cabo Rojo	RAWS	17.97	-67.16	6.1	14	0.52
Camp Santiago	RAWS	18.00	-66.28	6.1	27	0.39
Guanica	RAWS	17.95	-66.88	6.1	127	0.50
Maricao	RAWS	18.16	-67.03	6.1	225	0.34

(a) Stations with wind data

(b) Percent difference between GOES-PRWEB and station wind speed

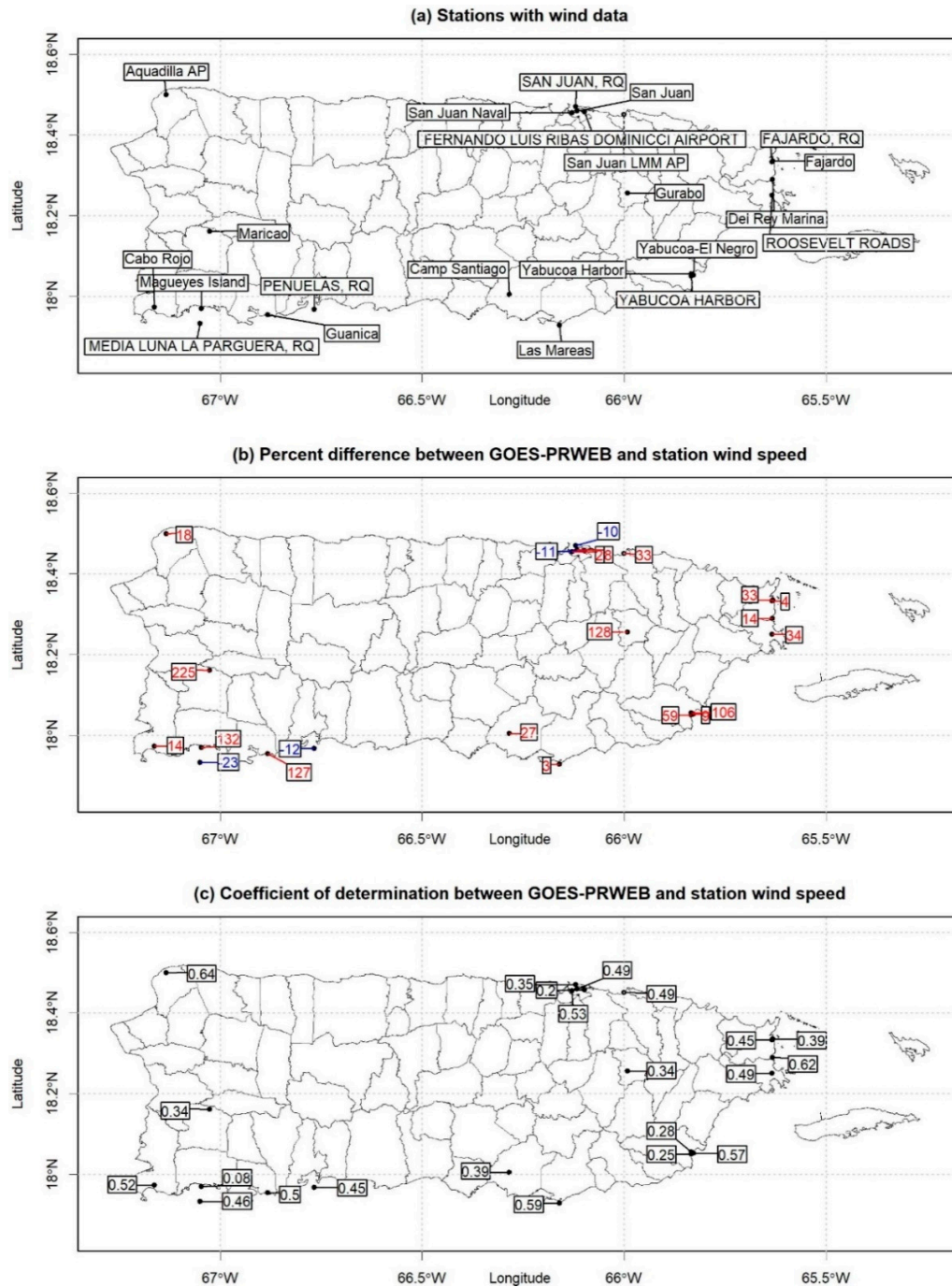## (c) Coefficient of determination between GOES-PRWEB and station wind speed

Figure S12. (a) Location of stations with homogeneous wind speed data in Puerto Rico (station names are case-sensitive and refer to those in Table S1); **(b)** percent difference and **(c)** coefficient of determination (R^2) between GOES-PRWEB and station wind speeds converted to 2 m height according to [1] when sensor elevation data are available. The black lines show the location of the municipalities in Puerto Rico. Base from 1:20,000 USGS Digital Line Graphs municipality boundaries, shown in geographic coordinate system and World Geodetic System 1984 datum.
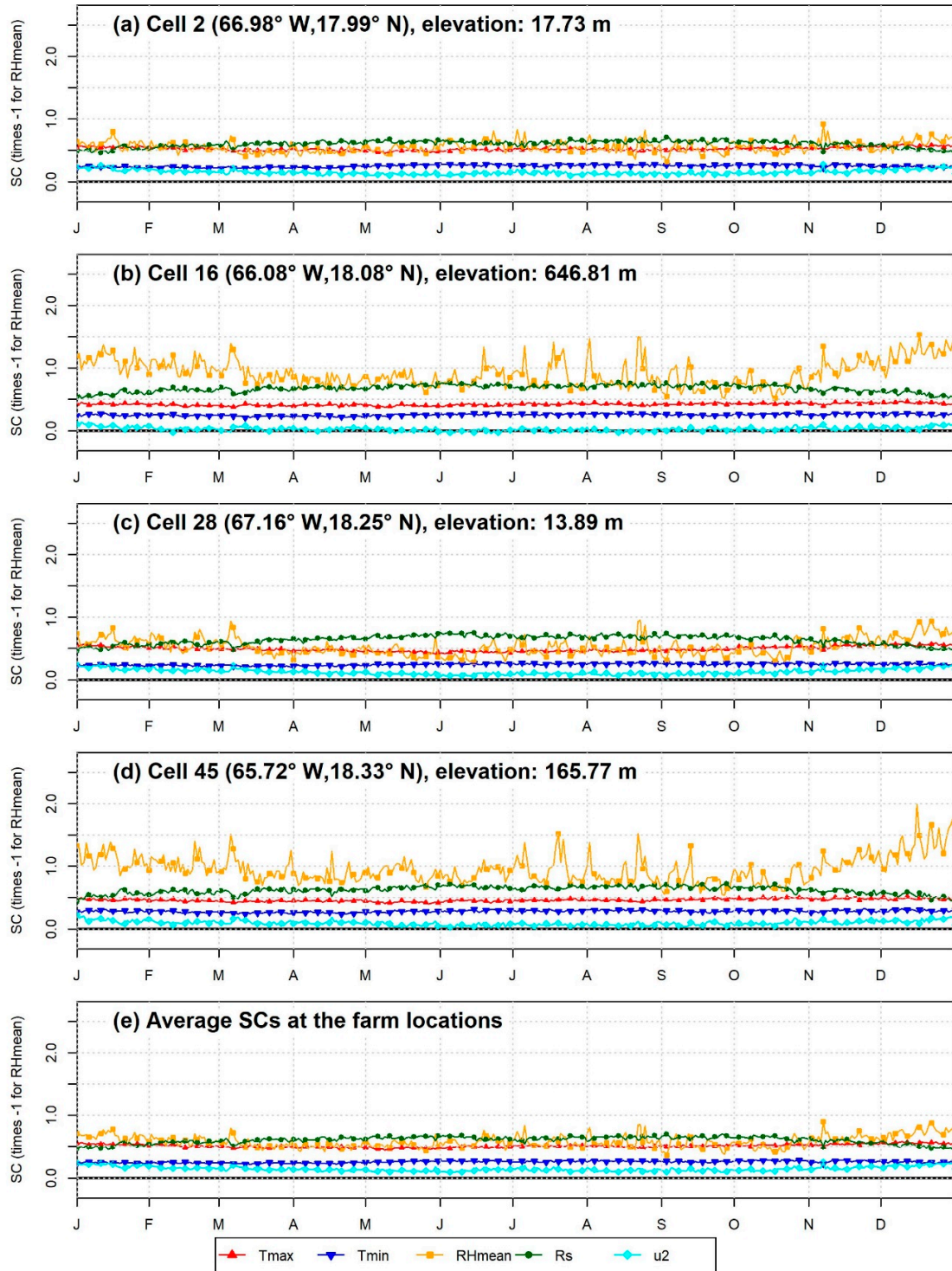
Figure S13. Daily timeseries of sensitivity coefficients (SCs) with wind speed multiplied by 0.67 at: (a)-(d) GOES-PRWEB grid cell locations (Figure 1) and (e) average at 2015 irrigated farm locations. Markers shown every 5 days. Cell 2 is on the southwest coast, cell 16 is located near the Sierra de Cayey, cell 28 is on the west coast, cell 45 is located near the Luquillo Mountains.

Figure S14. Daily timeseries of sensitivity coefficients (SCs) with wind speed multiplied by 0.5 at: (a)-(d) GOES-PRWEB grid cell locations (Figure 1), and (e) average at 2015 irrigated farm locations. Markers shown every 5 days. Cell 2 is on the southwest coast, cell 16 is located near the Sierra de Cayey, cell 28 is on the west coast, cell 45 is located near the Luquillo Mountains.
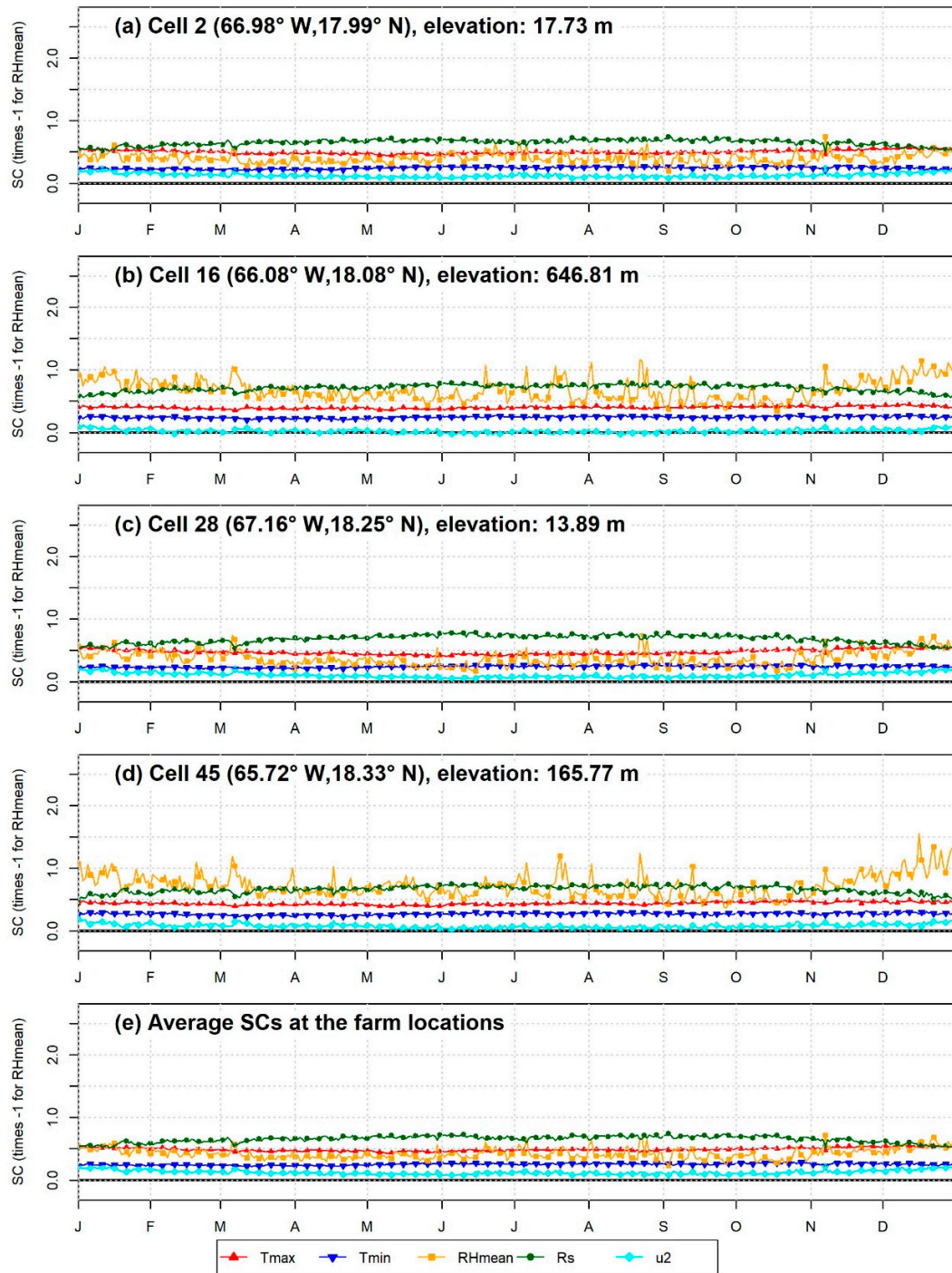
Table S2. Island-wide average absolute relative SCs for case with original GOES-PRWEB 2 m wind speeds and wind speeds multiplied by factors equal to 0.5 and 0.67.

Island-wide average absolute relative SC for	Based on original u_2	Based on $0.67u_2$	Based on $0.5u_2$
RH_{mean}	0.98	0.69	0.50
R_s	0.57	0.64	0.68
T_{max}	0.50	0.47	0.45
T_{min}	0.27	0.25	0.25
u_2	0.12	0.10	0.09

References

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